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Multiagent Pursuit-Evasion Problem with the Pursuers Moving at Uncertain Speeds

Fuhan Yan[1,2] · **Jiuchuan Jiang**[3] · **Kai Di**[1,2] · **Yichuan Jiang**[1,2] · **Zhifeng Hao**[1]

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Abstract

The multiagent pursuit-evasion problems have been widely investigated in related areas. Previous studies usually assumed that the pursuers move at certain speeds. However, in many circumstances the above assumption does not match the peculiarities of real pursuit-evasion cases in which the pursuers' speeds may be uncertain. Therefore, this paper investigates the multiagent pursuit-evasion problem under the situation in which the pursuers move at uncertain speeds. The new problems of multiagent pursuit-evasion caused by the uncertainty of the pursuers' speeds include: 1) many previous strategies plan pursuers' paths based on their speeds, but the uncertainty of speeds will make the pursuers move to worthless target points; 2) previous strategies usually let each pursuer move to a scheduled location, but the uncertainty of speeds may make some pursuers fail to reach the scheduled locations punctually. Aiming at addressing these problems, we present the strategy which lets each pursuer flexibly help the slow neighboring pursuer. As the pursuers' speeds are uncertain, the optimal decision of pursuers cannot be calculated directly. Thus, we analyze the alternative decision space of pursuers, which contains the decisions that may be optimal and does not contain the obviously bad decisions (such as moving away from the evader). Then, we compare the decisions in the alternative decision space based on simulated annealing resulting that the optimal decision may be selected after repeatedly comparing different decisions. The experimental results show that our strategy can generally outperform previous strategies when the pursuers' speeds are uncertain.

Keywords Multiagent pursuit-evasion problem · Uncertain speeds · Pursuing strategy

1 Introduction

The multiagent pursuit-evasion problems have been widely investigated in related areas [1, 4, 7, 11, 12, 19, 21, 22]. The studies of multiagent pursuit-evasion are involved with many real-world applications, such as missile guidance and defense [21, 22], robots confrontation [7], and unmanned aerial vehicles (UAVs) control [1, 26]. Especially, capturing a faster evader based on the cooperation of multiple slower pursuers is quite important [9, 20, 25]. For instance, the terrorists may use the drones to launch strikes; thus, Tokyo police try to use drones with nets to catch other drones [26]. In this case, always assuming that police drones are faster than the suspected drones is infeasible. Instead, assuming that the pursuers are slower and designing suitable cooperation method for the pursuers is able to contend against different enemy evader. Previous studies usually focus on the cooperation among the pursuers moving at certain speeds [1–4, 6, 7, 9, 11, 12, 15, 16, 19–22, 25, 28]. In detail, the pursuers have the same fixed speeds [3, 11, 12, 19, 20], or the speeds of different pursuers are different, but the speed of each pursuer is fixed [25]. However, in some real cases, an agent cannot always move at a certain speed, and the actual speed may fluctuate uncontrollably when this agent moves [18]. For instance, a quadruped robot's speed is influenced by touchdown angle so that the speed may fluctuate because of the undulations of the ground [14, 18].

Therefore, this paper explores the multiagent pursuit-evasion problem in which the pursuers cannot move at

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✉ Yichuan Jiang
yjiang@seu.edu.cn

[1] School of Mathematics and Big Data, Foshan University, Foshan 528000, China

[2] School of Computer Science and Engineering, Southeast University, Nanjing 211189, China

[3] School of Computer Science and Engineering, Nanyang Technological University, Singapore 639798, Singapore

certain speeds. In our study, the evader's speed is constant and higher than the pursuers' maximum speed; the pursuers' actual speeds will fluctuate between the maximum speed and a lower bound. Because the pursuers are slower than the evader, the evader may be captured only if multiple pursuers cooperate effectively. However, the uncertainty of the pursuers' speeds makes against the team collaboration. In detail, 1) the uncertainty of speeds will mislead the pursuers which plan their paths based on their speeds; 2) because of the uncertainty of the speeds, some pursuers may fail to punctually reach the locations where they plan to reach. Therefore, the capture success ratios based on previous certain-speed pursuers-oriented strategies will decrease because of the uncertainty of the pursuers' speeds.

In order to increase the capture success ratios when the pursuers' speeds are unstable, we present the strategy that can address the problems caused by the uncertainty of the pursuers' speeds. As mentioned above, uncertain speeds may mislead the strategies resulting that planning path for the pursuers based on their current speeds is infeasible. Besides, some pursuers may fail to reach the scheduled locations punctually. Hence, we present a rule that lets each pursuer flexibly help the slow neighboring pursuer control the encircled formation consisting of pursuers. However, as the pursuers' speeds are uncertain, it is hard to directly obtain the moving direction that is the most helpful in controlling the encircled formation consisting of pursuers. In this case, we first analyze the alternative decision space of pursuers. This decision space should contain the moving directions that may be the optimal decision and does not contain the obviously bad moving direction. For instance, moving away from the evader is a bad choice and not helpful in decreasing the gaps among pursuers. Then, we combine the rule mentioned above with simulated annealing to compare the moving directions in the alternative decision space. Therefore, the optimal decision may be selected after repeatedly comparing different decisions based on simulated annealing.

Extensive experiments are presented to show the performance of our strategies in various cases. The experimental results show that, our strategy based on simulated annealing is better than previous strategies when the pursuers' speeds are uncertain and similar to previous classical strategies when the speeds are certain. In addition, based on the experimental data, we verify the analyses on the problems caused by the uncertainty of the pursuers' speeds and deeply discuss the advantages and disadvantages of different pursuing strategies. The main conclusions in experiments include: 1) when other parameters are fixed, the variety of the pursuers' maximum speed and the variety of the number of the pursuers will not influence the relative merits of different pursuing strategies; 2) the time interval between the fluctuations of the pursuers' speeds can influence the relative merits of different pursuing strategies; and 3) when the pursuers' speeds are uncertain, the heterogeneity of the pursuers' maximum speed and minimum speed will not obviously influence the performance of the pursuing strategies.

The rest of this paper is organized as follows. In Section 2, we introduce related works. Section 3 shows problem formulation. Then, we present the analyses and pursuing strategy in Section 4. The experiments are provided in Section 5. Finally, we conclude this paper in Section 6.

2 Related Works

Pursuit-evasion problem has been widely discussed in previous studies [1–4, 6, 7, 9, 11, 12, 15, 16, 19–22, 25, 28]. These studies of pursuit-evasion problem can be divided into: 1) pursuit-evasion in discrete world [3, 11, 12, 16] or 2) pursuit-evasion in continuous world [1, 2, 4, 6, 9, 19–22, 25, 28]. However, to the best of our knowledge, previous studies have not considered the case that the pursuers' speeds are uncertain, no matter in discrete world or continuous world.

The studies of pursuit-evasion in discrete world usually assume that the world consists of grids [3, 11, 12, 16]. The agent in the grid world can just move between grids, and the direction of agent is limited. These studies [3, 11, 12] usually do not care the influence of the pursuers' speeds. In discrete world, the evader and the pursuers can move to near grid every one time step. Therefore, there is no uncertainty of the pursuers' speeds in discrete world. For instance, Huang et al. [11] study pursuit-evasion games which are discrete games, and the players are able to move to an adjacent node in each step. In [12], the players are also able to move to an adjacent node in each step. Besides, Barrett et al. [3] adopt the problem formulation that the world is a toroidal grid. They use a single prey and four predators, with only left, right, up, down, and no-op movements. Agents start in random positions and select their actions simultaneously at each time step [3]. As these studies assume the pursuers move from one grid to adjacent grid without the uncertainty of speeds, the strategies in these studies cannot solve the problem presented in this paper.

On the other hand, in continuous world, agents can move in any directions, and the influence of their speeds is usually discussed [1, 2, 4, 6, 9, 13, 19–22, 25, 28]. However, these studies always assume that pursuers' speeds are certain. For instance, Raboin et al. [19] assume that each agent is a holonomic point robot with a fixed maximum velocity. In [4], the values of the pursuers' speeds are known accurately. There is no uncertainty of the pursuers' speeds [4]. Besides, Ramana and Kothari [20] study the pursuit-evasion game, where all pursuers have equal fixed speed, and the evader's speed is higher. As these studies are based on the pursuers moving at certain speeds, the strategies in these studies may lead to low capture success ratios when pursuers' speeds are uncertain. In detail,

- Many previous pursuing strategies [9, 13, 20] let the pursuers plan their paths based on their speeds. If the pursuers' speeds are unstable, their paths depending on their speeds may consist of some worthless target points.

- Previous strategies [9, 13, 20] for stable-speed pursuers usually let each pursuer complete its own sub-task independently from others, such as moving to a scheduled location to adjust the encircled formation consisting of pursuers without caring whether others can reach the scheduled locations punctually. The instability of pursuers' speeds may make some pursuers fail to reach the scheduled locations punctually, thereby a big gap may be generated among the pursuers, through which the evader can pass.

Thus, designing a feasible pursuing strategy aiming at addressing the problems caused by the uncertainty of the pursuers' speeds is necessary.

What is more, Alexopoulos et al. [1] have discussed the case that the pursuers do not move at fixed speeds. They study the problem of pursuit-evasion games between two pursuing and one evading unmanned aerial vehicle (UAV) in a 3-D environment which is unbounded and without obstacles. However, all UAVs have implemented a velocity controller and can control their velocities. Therefore, in [1], the UAVs' velocities are certain because the UAVs can control their velocities and know the accurate values of their velocities in next time step. Moreover, the evader is assumed 20% slower than the pursuers in [1], while we discuss the cooperation among the pursuers that are slower than the evader. The strategy in [1] also cannot deal with the multiagent pursuit-evasion problem where the pursuers' speeds are uncertain and smaller than the evader's speed.

In addition, we have briefly discussed the uncertainty of the pursuers' speeds in [27] which is an extended abstract presented in AAMAS 2017. In [27], we have presented a pursuing strategy based on reinforcement learning. The analyses of decision space which supports the reinforcement learning algorithm are similar to that in this paper. However, because of the space limitation of extended abstract, these detailed analyses have not been shown in [27]. Besides, the results of [27] shown that the pursuing strategy based on reinforcement learning is worse than traditional classical strategies when the pursuers' speeds are the same constant. The reason is that when the speeds are constant, there is no uncertainty so that learning process becomes insignificant. Besides, the random decision in learning process decreases the performance. Although reinforcement learning may finally converge to the optimal solution, the limitation of the rate of convergence make the reinforcement learning algorithm [27] fail to perfectly solve the pursuit-evasion problem with the pursuers moving at uncertain speeds. Therefore, in this paper, we analyze this problem more deeply and present another algorithm based on simulated annealing, which is better than previous strategies when the pursuers' speeds are uncertain and is not worse than previous strategies when the pursuers' speeds are certain.

Chung et al. [7] present a survey of search and pursuit-evasion in mobile robotics. Chung et al. point out that the previous studies of pursuit-evasion games usually assume that the pursuers move at the same constant speed, and these studies focus on the other factors (such as sensing factor) in pursuit-evasion game. Chung et al. [7] also do not show that previous studies have discussed the uncertainty of the pursuers' speeds.

Overall, previous studies [1–4, 6, 7, 9, 11, 12, 15, 16, 19–22, 25, 28] usually assume that pursuers can move at certain speeds. Therefore, the strategies in these studies cannot solve the problems caused by the uncertainty of the pursuers' speeds:

3 Problem Formulation

3.1 Multiagent Pursuit-Evasion Problem Based on the Pursuers Moving at Uncertain Speeds

In this paper, we study the pursuit-evasion problem in a continuous open world without frontier [6, 20]. Let there be n pursuers and one single evader. The evader and pursuers are set as circles, and the diameters of them are all set as one unit distance. It is assumed that both the evader and pursuers have enough visual ranges. In other words, the environment is assumed full observable. In this paper, as we focus on the uncertainty of the pursuers' speeds, the setting of sensing range is simplified. Some previous studies have discussed the influence of sensing limitation [5, 7]. If our technology needs to be used in the cases that the pursuers have limited sensing range, our technology can be combined with the results of these previous studies [5, 7].

The evader's speed is higher than the pursuers' maximum speed. Let v_e denote the evader's speed. Besides, let v_p denote the pursuers' maximum speed. We have $v_e > v_p$. It is worth noting that the pursuers cannot always move at the maximum speed v_p. The pursuers' actual speeds are initially v_p but may fluctuate between the maximum speed v_p and a lower bound $v_p{}'$. Let $v_{p_i}(t)$ denote the actual speed of pursuer P_i at time t. $v_{p_i}(0) = v_p$, and $v_{p_i}(t) \in [v_p{}', v_p]$. In real world, v_p is determined by the property of the pursuers, and $v_p{}'$ may be determined by the environment (such as the undulations of the ground). The time interval between the fluctuations of v_{p_i} is assumed as f_i. Let $f_i = 1 + u_i$ where u_i obeys poisson distribution ($p(u_i = k) =$

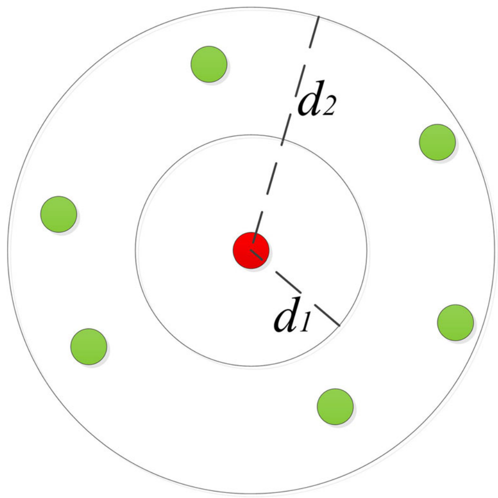

Fig. 1 The initial locations of the evader and pursuers

$\frac{\lambda^k}{k!}e^{-\lambda}$). Moreover, $u_0, u_1 ... u_n$ are independent identically distributed. Poisson distribution is usually used in previous studies to model the random phenomenon in real world [10, 17]. Therefore, this distribution is also selected in this paper. In addition, f_i is a variable and reset when v_{p_i} changes. If $\lambda = 0$, let $f_i \equiv 1$, which represents the minimum time interval.

At the beginning of the pursuit-evasion game, the pursuers are randomly located within the vicinity of the evader. Because of the randomization, the pursuers may initially form an incompact encircled formation. Then, the pursuers need to shrink the encircled formation. Conversely, if the pursuers do not form an encircled formation at the beginning of the pursuit-evasion game, the capture can be considered failing initially [6, 20]. As shown in Fig. 1, the pursuers are located randomly in an annulus (the red circle represents the evader while the green circles represent the pursuers). The distance between the evader and each pursuer is randomly set in $[d_1, d_2]$.

Then, we present some important definitions:

Definition 1 Capture condition If the distance between the evader and a pursuer is small enough, the evader is captured [6, 20]. In this paper, the pursuers must touch the evader to capture it.

Definition 2 Failure condition As the pursuit-evasion game is in an open world without frontier, if the evader is out of the encircled formation consisting of pursuers, it can be considered escaping successfully [6, 20]. The condition of a failing capture is that all the pursuers are on the same side of the evader, such as shown in Fig. 2. Because $v_p < v_e$, the pursuers will never capture the evader in Fig. 2.

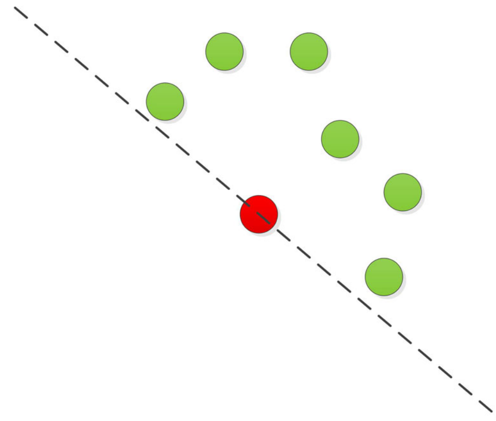

Fig. 2 A case of failing capture

Definition 3 Capture success ratio As the pursuers are randomly located, the pursuers may capture the evader in some cases but fail in other cases. Capture success ratio is the ratio of successful capture in repeated games with various initial states [9]. Let R denote the capture success ratio.

Finally, we can define the problem in this paper as the following.

Multiagent Pursuit-Evasion Problem Given n pursuers which initially surround an evader. The evader's speed is known as v_e. The pursuers' speeds randomly fluctuate in $[v_p{}', v_p]$. The problem is to find a suitable strategy for the pursuers to maximize the capture success ratio, i.e.,

Maximize R

Subject to:

$$v_{p_{i\in[1,n]}}(t) \in [v_p{}', v_p]; \tag{1}$$

$$v_p < v_e; \tag{2}$$

$$v_{p_{i\in[1,n]}}(t \in [0, f_i)) = v_p; \tag{3}$$

$$v_{p_{i\in[1,n]}}(t) \neq v_{p_{i\in[1,n]}}(t + f_i); \tag{4}$$

$$f_i = 1 + u_i; \tag{5}$$

$$p(u_i = k) = \frac{\lambda^k}{k!}e^{-\lambda}. \tag{6}$$

3.2 The Evader's Escape Strategy

The pursuit-evasion game is an adversarial game. The strategy of the evader will influence the result of pursuit-evasion game. In order to effectively test the performance of the pursuing strategy, we need to select a suitable escape strategy.

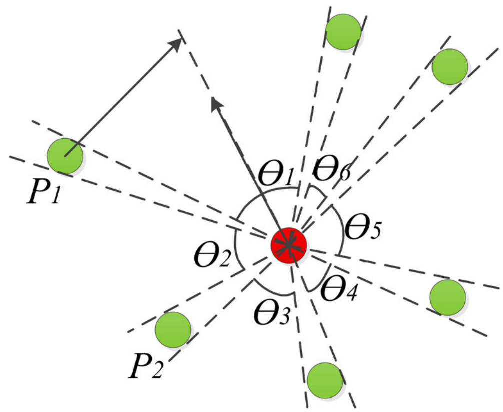

Fig. 3 A case of escape strategy

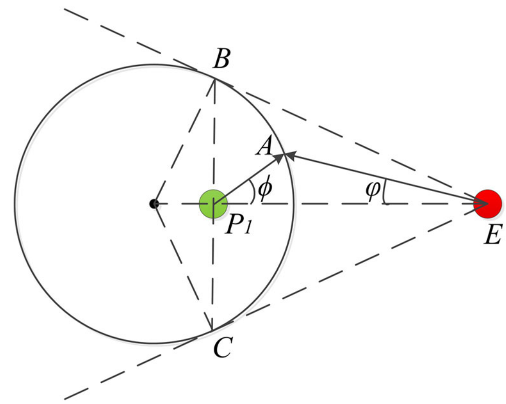

Fig. 4 Apollonius circle

In previous studies [9, 20, 23, 28], two escape strategies were usually selected: 1) the evader moves away from the closest pursuer; 2) the evader selects the largest gap among the pursuers to pass through. As mentioned in Section 3.1, the pursuers are initially located surrounding the evader. In this case, the first escape strategy is helpful in increasing the capture time instead of decreasing the capture success ratio. Therefore, the first strategy is infeasible in this problem. Considering the problem statements in Section 3.1, the second escape strategy is a feasible strategy.

Then, the details of the escape strategy which is used in this paper are shown as following:

1. As shown in Fig. 3, the evader will initially compare the angles $\theta_1, \theta_2 ... \theta_n$ and select the maximum one among these angles. Then, the evader moves along the angular bisector of the maximum angle.
2. If the evader ensures that it can escape successfully without changing its moving direction, it will keep the current moving direction. In detail, if the angle towards which the evader moves is larger than $2arcsin\frac{v_p}{v_e}$, the evader will keep the current moving direction.
3. If the condition in step 2 is not satisfied and another angle becomes the largest, the evader will change its moving direction and select the new largest angle.

4 Analyses and Strategy Design

In this section, we first discuss the problems caused by the uncertainty of the pursuers' speeds and then design the pursuing strategy that can address these problems.

4.1 The Problems Caused by the Uncertainty of the Pursuers' Speeds

Previous strategies are usually based on Apollonius Circle [9, 13, 20], such as shown in Fig. 4. In Fig. 4, $\frac{|P_1A|}{v_{p_1}} = \frac{|EA|}{v_e}$. In other words, Apollonius Circle consists of the points at which pursuer P_1 may meet the evader. Therefore, Apollonius Circle depends on the speeds and the current locations of pursuer P_1 and the evader. In detail, let (x_e, y_e) denote the coordinate of the evader and (x_{p_1}, y_{p_1}) denote the coordinate of pursuer P_1. It can be known based on previous studies that, the center of the Apollonius Circle is $(\frac{x_{p_1}-(\frac{v_{p_1}}{v_e})^2 x_e}{1-(\frac{v_{p_1}}{v_e})^2}, \frac{y_{p_1}-(\frac{v_{p_1}}{v_e})^2 y_e}{1-(\frac{v_{p_1}}{v_e})^2})$, and the radius of the Apollonius Circle is $\frac{\frac{v_{p_1}}{v_e}\sqrt{(x_e-x_{p_1})^2+(y_e-y_{p_1})^2}}{1-(\frac{v_{p_1}}{v_e})^2}$ [9, 13, 20]. It means that v_{p_1} can significantly influence the Apollonius Circle. Therefore, when v_{p_1} is uncertain, the Apollonius Circle cannot be calculated accurately. As mentioned in Section 3.1, $v_p{}'$ may be determined by the environment. It means that $v_p{}'$ is not necessarily known. If $v_p{}'$ is known, the Apollonius Circle can be calculated based on $\frac{v_p+v_p{}'}{2}$ so that the fluctuation of v_{p_i} may not lead to obviously bad result. However, if $v_p{}'$ is unknown, the pursuers planning path based on Apollonius Circle may move to worthless target locations when their speeds fluctuate. Then, the capture success ratios will decrease.

Lemma 1 *The pursuers' speeds are assumed uncertain. If the evader does not change its initial moving direction and the pursuers plan their paths based on Apollonius Circle, calculating Apollonius Circle based on* $\frac{v_p+v_p{}'}{2}$ *will lead to higher capture success ratio than that based on* v_p *or* v_{p_i}.

Proof The capture success ratio represents the average performance of the pursuing strategies. Therefore, it needs to be proved that $\frac{v_p+v_p{}'}{2}$ is averagely better than v_p or v_{p_i}. Even if calculating Apollonius Circle based on $\frac{v_p+v_p{}'}{2}$ may be worse than that based on v_p or v_{p_i} in some particular cases, the proof is not influenced.

As the pursuers' speeds fluctuate in $[v_p{}', v_p]$, a pursuer moves averagely $\frac{\Delta t(v_p+v_p{}')}{2}$ unit distance in Δt unit time.

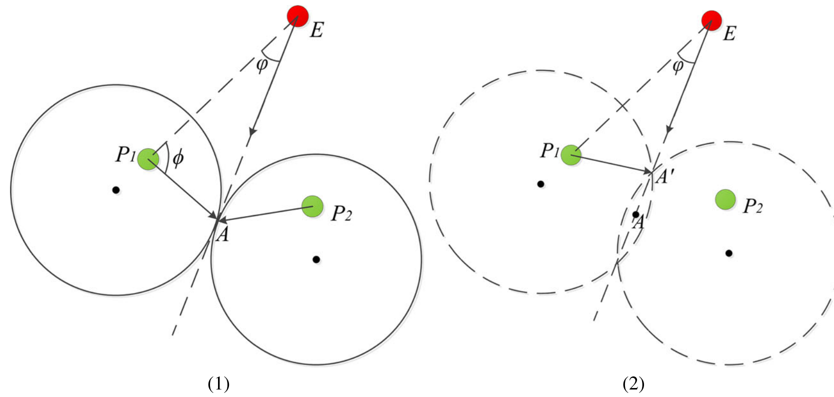

Fig. 5 The strategy based on apollonius circle

As shown in Fig. 5 (1), it is assumed that the Apollonius Circles of pursuer P_1 and pursuer P_2 are calculated based on $\frac{v_p+v_p{}'}{2}$, and there is only one intersection point of the two Apollonius Circles. In this case, P_1A is perpendicular to P_1E, and $\frac{2|P_1A|}{v_p+v_p{}'} = \frac{|EA|}{v_e}$ [20]. Therefore, pursuer P_1 and pursuer P_2 can capture the evader only if they let $\phi = \frac{\pi}{2}$.

If the pursuers calculate Apollonius Circles based on v_p, the radius of Apollonius Circles will be larger than that based on $\frac{v_p+v_p{}'}{2}$. Then, pursuer P_1 and pursuer P_2 will not select $\phi = \frac{\pi}{2}$ if the Apollonius Circles are based on v_p, resulting that they cannot capture the evader.

Moreover, if pursuer P_1 calculates Apollonius Circles based on v_{p_1}, the fluctuation of v_{p_1} will make the Apollonius Circle changes repeatedly. Then, the Apollonius Circles influence the moving direction of pursuer P_1 resulting that P_1 will move to some worthless locations. As shown in Fig. 5 (2), if $v_{p_1} > \frac{v_p+v_p{}'}{2}$ at a time step, P_1 will move towards point A'. In Fig. 5 (2), point A corresponds to that in Fig. 5 (1). On the average, P_1 can just meet the evader at point A, and it cannot meet the evader at point A'. Even if v_{p_1} changes to $\frac{v_p+v_p{}'}{2}$ in the subsequent time steps, P_1 has wasted some time resulting that P_1 cannot intercept the evader.

Overall, when the pursuers can averagely capture the evader based on the Apollonius Circles depending on $\frac{v_p+v_p{}'}{2}$, they cannot do based on the Apollonius Circles depending on v_p or v_{p_i}. Therefore, calculating Apollonius Circle based on $\frac{v_p+v_p{}'}{2}$ will lead to higher capture success ratio than that based on v_p or v_{p_i}. □

In addition, even if $v_p{}'$ is known, there is another problem caused by the uncertainty of the pursuers' speeds. Previous strategies usually let each pursuer complete its own sub-task, such as reaching the scheduled location to control the encircled formation consisting of pursuers [9, 13, 20]. The uncertainty of pursuers will make some pursuers fail to reach the scheduled locations punctually so that the evader has more chances to escape. For instance, in Fig. 6, it is assumed that $\angle P_1EP_2$ is the largest angle initially. The evader will initially escape by moving along the angular bisector of $\angle P_1EP_2$. In Fig. 6, the green hollow circles represent the locations of pursuers at next moment. Based on the Apollonius Circles, pursuer P_1 and P_2 move to point A to intercept the evader. In this case, $\angle P_1EP_3$ may increase with the move of pursuer P_1. If the pursuers move at certain speeds, pursuer P_3 can move to a scheduled location to control the angle $\angle P_1EP_3$ so that $\angle P_1EP_3$ will not become very large [9, 13, 20]. However, if the pursuers move at uncertain speeds, pursuer P_3 may move very slowly and fail to reach the scheduled location punctually. Then, $\angle P_1EP_3$ will become large fast. If $\angle P_1EP_3$ becomes the largest, the evader will change its direction and easily escape by moving along the angular bisector of $\angle P_1EP_3$.

To sum up, two problems are caused by the uncertainty of the pursuers' speeds: 1) the uncertainty of speeds lead to inaccurate Apollonius Circle (especially when $v_p{}'$ is unknown) so that the pursuers adopting the strategies [9, 13, 20] based on Apollonius Circle often move to worthless

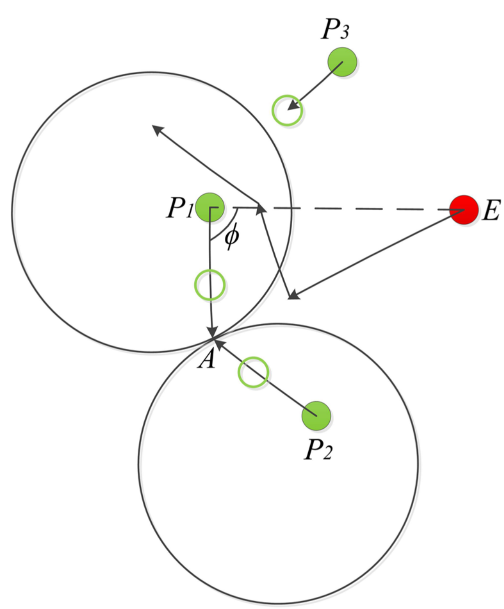

Fig. 6 A case of the problem caused by the uncertainty of the pursuers' speeds

target locations; 2) previous strategies [9, 13, 20] let pursuers move to scheduled locations, but the uncertainty of speeds will make some pursuers fail to reach the scheduled locations punctually. Therefore, the capture success ratios based on previous strategies will decrease because of the uncertainty of the pursuers' speeds.

4.2 The Idea of Our Pursuing Strategy

In order to increase the capture success ratios when pursuing a faster evader based on the pursuers moving at uncertain speeds, we present the strategy that is more feasible. Besides, traditional classical pursuing strategies may be optimal when the pursuers' speeds are certain. Therefore, in this paper, the goal is to present a pursuing strategy that is better than previous strategies when the pursuers' speeds are uncertain and is not worse than previous strategies when the pursuers' speeds are certain. As analyzed in Section 4.1, two problems should be considered when designing our pursuing strategy. At first, our strategy does not depend on the accurate values of the pursuers' speeds. Secondly, as some pursuers may fail to reach scheduled locations punctually, our strategy let each pursuer flexibly select direction to help the slower neighboring pursuers control the encircled formation consisting of pursuers. However, as the pursuers' speeds are uncertain, it is hard to directly obtain the moving direction that is the most helpful in achieving our purpose. Therefore, we analyze the alternative decision space of pursuers and compare the moving directions in this decision space based on the change of state. Our strategy can be summarized as following:

- At first, we analyze the alternative decision space of the pursuers. The decision space of a pursuer can be represented by the value range of ϕ (ϕ is shown in Figs. 4 and 5). We exclude some values of ϕ which will obviously lead to bad results and then obtain a suitable value range of ϕ.
- Then, we discuss how to select the value of ϕ in the suitable value range. We combine our analyses with simulated annealing to compare the values of ϕ in the value range.

4.3 The Suitable Value Range of ϕ

In this subsection, we investigate how to exclude some directions which are obviously bad choices for the pursuers, and finally obtain the suitable decision space which is represented by the value range of ϕ.

When $\varphi \geq \frac{\pi}{2}$ or $\varphi < \frac{\pi}{2}$ (φ is shown in Figs. 4 and 5), the pursuers adopt different strategies. The reason is that, when $\varphi \geq \frac{\pi}{2}$, the pursuer can never capture the faster evader if the evader does not change its direction. Conversely, when $\varphi < \frac{\pi}{2}$, the pursuer may be able to intercept the evader.

We first analyze the suitable value range of ϕ if $\varphi < \frac{\pi}{2}$. In this problem, the evader and pursuers can move in a plane with unlimited directions. It means $\phi \in [0, 2\pi]$. However, $\phi \in [\pi, 2\pi]$ are obviously bad choices when $\varphi < \frac{\pi}{2}$. For instance, although P_1 in Fig. 6 needs to consider the change of $\angle P_1EP_3$, its current goal is to decrease $\angle P_1EP_2$. If ϕ is in $[\pi, 2\pi]$, $\angle P_1EP_2$ will increase fast so that the evader can escape without changing its direction. Therefore, ϕ should be selected in $[0, \pi]$.

Then, Fig. 7 shows the influence of $\phi \in [0, \pi]$. The points in Fig. 7 (1), (2) and (3) correspond to that in each other figures. In Fig. 7 (3), it is assumed that $\angle A_0P_1E = \frac{\pi}{2}$ and $\beta_1 = \beta_2$. When pursuer P_1 tries to intercept the evader that will not change the moving direction, moving towards point A_0 is better than any other points on the direction $\overrightarrow{EE'}$. The reason is that $\frac{|P_1A_0|}{|A_0E|} = \frac{sin\varphi}{sin\frac{\pi}{2}}$ is smaller than $\frac{|P_1A_j|}{|A_jE|}$ (A_j is any other point on the direction $\overrightarrow{EE'}$). In

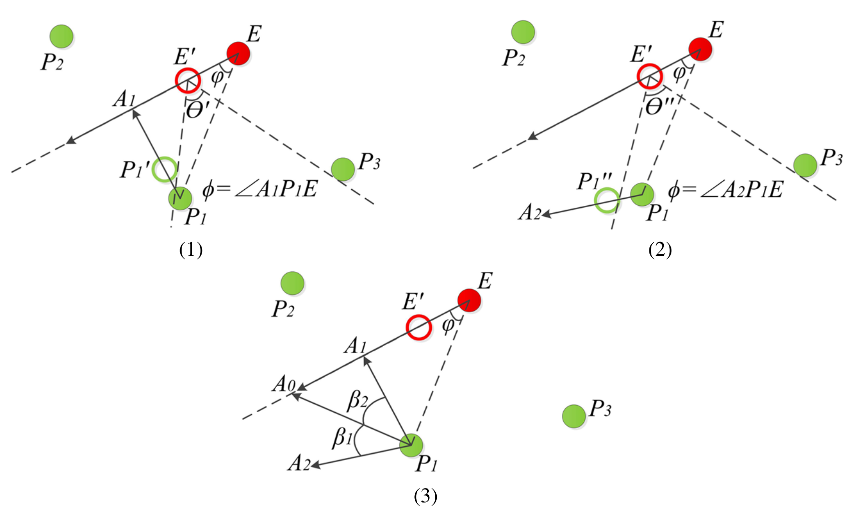

Fig. 7 The influence of ϕ when $\varphi < \frac{\pi}{2}$

other words, $\phi = \angle A_0 P_1 E = \frac{\pi}{2}$ is the most helpful in decreasing $\angle P_1 E P_2$. However, as the evader may change its direction, the suitable value of ϕ should also be helpful in inhibiting the increase of $\angle P_1 E P_3$. As shown in Fig. 7 (1) and (2), the green hollow circles P_1' and P_1'' represent the possible locations of pursuer P_1 at next moment, and the red hollow circle E' means the possible location of the evader at next moment. It can be observed in Fig. 7 (1) and (2) that $\theta' < \theta''$. It means that $\phi = \angle A_1 P_1 E$ is more helpful in inhibiting the increase of $\angle P_1 E P_3$ than $\phi = \angle A_2 P_1 E$. It is worth noting that $|\angle A_1 P_1 E - \frac{\pi}{2}| = |\angle A_2 P_1 E - \frac{\pi}{2}| > 0$. Therefore, compared with $\phi = \angle A_0 P_1 E = \frac{\pi}{2}$, letting $\phi = \angle A_1 P_1 E$ or $\phi = \angle A_2 P_1 E$ both inhibit the increase of $\angle P_1 E P_3$ by moderately ignoring the increase of $\angle P_1 E P_2$. As $\phi \in [0, \frac{\pi}{2}]$ and $\phi \in [\frac{\pi}{2}, \pi]$ lead to similar effects while $\phi \in [0, \frac{\pi}{2}]$ is better, ϕ should be selected in $[0, \frac{\pi}{2}]$.

In addition, ϕ should not be too small. At least, $|P_1 A|$ and $|EA|$ in Fig. 4 should both increase or decrease along with the change of ϕ. If $|EA|$ decreases and $|P_1 A|$ increases along with the change of ϕ, the evader may escape easily. In Fig. 4, based on the law of sines [8],

$$\frac{|P_1 A|}{sin\varphi} = \frac{|EA|}{sin\phi} = \frac{|P_1 E|}{sin(\pi - \varphi - \phi)}. \tag{7}$$

Besides,

$$sin(\pi - \varphi - \phi) = sin(\varphi + \phi). \tag{8}$$

Therefore,

$$|EA| = \frac{sin\phi}{sin(\varphi + \phi)} |P_1 E| \tag{9}$$

and

$$|P_1 A| = \frac{sin\varphi}{sin(\varphi + \phi)} |P_1 E|. \tag{10}$$

If $|EA|$ and $|P_1 A|$ are considered as the functions of ϕ, the derivatives are

$$\begin{aligned} |EA|' &= \frac{cos\phi sin(\varphi + \phi) - sin\phi cos(\varphi + \phi)}{sin^2(\varphi + \phi)} |P_1 E| \\ &= \frac{sin\varphi cos^2\phi + sin\varphi sin^2\phi}{sin^2(\varphi + \phi)} |P_1 E| \geq 0 \end{aligned} \tag{11}$$

and

$$|P_1 A|' = \frac{-sin\varphi cos(\varphi + \phi)}{sin^2(\varphi + \phi)} |P_1 E|. \tag{12}$$

As $\phi \in [0, \frac{\pi}{2}]$, $|P_1 A|' \geq 0$ when $\phi \in [\frac{\pi}{2} - \varphi, \frac{\pi}{2}]$, and $|P_1 A|' \leq 0$ when $\phi \in [0, \frac{\pi}{2} - \varphi]$. It means that $|EA|'|P_1 A|' \geq 0$ when $\phi \in [\frac{\pi}{2} - \varphi, \frac{\pi}{2}]$. Therefore, ϕ should be in $[\frac{\pi}{2} - \varphi, \frac{\pi}{2}]$ if $\varphi < \frac{\pi}{2}$.

Then, we discuss the suitable value range of ϕ if $\varphi \geq \frac{\pi}{2}$. The strategy of the pursuers is less important when $\varphi \geq \frac{\pi}{2}$ than that when $\varphi < \frac{\pi}{2}$. The reason is as mentioned above, when $\varphi \geq \frac{\pi}{2}$, the pursuers cannot intercept the evader and just need to control the encircled formation. In previous studies, the pursuers usually adopt two simple strategies, moving towards the current location of the evader ($\phi = 0$) [6] or moving in the same direction with the evader ($\phi = \pi - \varphi$) [2].

As the evader will move, moving towards the current location of the evader can be considered as moving towards a point between two locations of the evader at two moments. The points in Fig. 8 (1) and (2) correspond to that in each other figure. The green hollow circles P_2' and P_2'' represent the possible locations of pursuer P_2 at next moment, and the red hollow circle E' means the possible location of the evader at next moment. It can be observed in Fig. 8 (1) and (2) that $\theta_1' > \theta_1''$ and $\theta_2' < \theta_2''$. It means that P_2 moving towards the current location of the evader is more helpful in decreasing $\angle P_2 E P_3$ while P_2 moving in the same direction with the evader is helpful in decreasing $\angle P_1 E P_2$. Therefore, the pursuers can select the suitable value of ϕ in $[0, \pi - \varphi]$ depending on circumstances in pursuit-evasion process when $\varphi \geq \frac{\pi}{2}$. Then, P_2 may flexibly help P_1 or P_3 depending on their actual speeds.

Overall, ϕ should be selected in $[\frac{\pi}{2} - \varphi, \frac{\pi}{2}]$ if $\varphi < \frac{\pi}{2}$ and be selected in $[0, \pi - \varphi]$ if $\varphi \geq \frac{\pi}{2}$.

4.4 Selecting the Value of ϕ in Suitable Value Range

After excluding the obviously bad directions, we discuss how to select the value of ϕ in the suitable value range. When making a decision, each pursuer should consider both two angles which are between it and its two neighboring pursuers. In detail, let $\theta_1 = \angle P_1 E P_2$ and $\theta_2 = \angle P_1 E P_3$ in

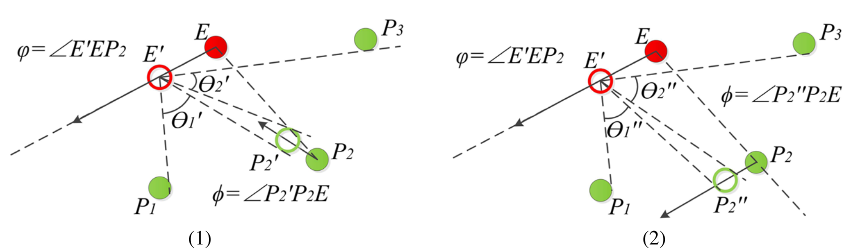

Fig. 8 The influence of ϕ when $\varphi \geq \frac{\pi}{2}$

Fig. 6. If θ_1 and θ_2 are considered as the function of ϕ, $\theta_1{}'(\phi)$ and $\theta_2{}'(\phi)$ are the derivatives. Then, P_1 needs to decrease both $\theta_1{}'(\phi)$ and $\theta_2{}'(\phi)$. Besides, as $\theta_1 > \theta_2$ at this moment, decreasing $\theta_1{}'(\phi)$ is more important than decreasing $\theta_2{}'(\phi)$. Therefore, in Fig. 6, pursuer P_1 needs to minimize

$$\theta'(\phi) = w_1\theta_1{}'(\phi) + w_2\theta_2{}'(\phi). \tag{13}$$

Here, w_1 and w_2 are the weight parameters. w_1 and w_2 should be determined based on the relative size of θ_1 and θ_2. Therefore, let $w_1 = \frac{\theta_1}{\theta_1+\theta_2}$ and $w_2 = \frac{\theta_2}{\theta_1+\theta_2}$ so that larger angle will become more important. It is worth noting that, as the pursuers' speeds are uncertain, $\theta_1{}'(\phi)$ and $\theta_2{}'(\phi)$ are also uncertain and can hardly be estimated accurately. Therefore, it is hard for pursuers to make decisions based on Eq. 13. In this case, we can refer to simulated annealing to compare the discrete values of ϕ and then select the optimal decision. Simulated annealing is a classical metaheuristic and may lead to optimal decision in this problem [24].

Based on the analyses in Section 4.3, the value of ϕ has been limited in $[\frac{\pi}{2} - \varphi, \frac{\pi}{2}]$ if $\varphi < \frac{\pi}{2}$ or in $[0, \pi - \varphi]$ if $\varphi \geq \frac{\pi}{2}$. Then, we discretize the range so that the values of ϕ can be compared in simulated annealing algorithm based on evaluation function. If $\varphi < \frac{\pi}{2}$, $\phi = \frac{\pi}{2} - \varphi + \frac{a\varphi}{k}$, $(a = 0, ..., k)$. Correspondingly, if $\varphi \geq \frac{\pi}{2}$, $\phi = \frac{a(\pi-\varphi)}{k}$, $(a = 0, ..., k)$. Moreover, the evaluation function can be based on the Eq. 13. It is assumed that $\theta_1(t)$ and $\theta_2(t)$ are the two angles between a pursuer and its two neighboring pursuers at time t. Then, let a_t denote the action selected by the pursuer at time t. Therefore,

$$C(a_t) = w_1[\theta_1(t) - \theta_1(t+\Delta t)] + w_2[\theta_2(t) - \theta_2(t+\Delta t)]. \tag{14}$$

Here, Δt represents the minimum interval between the time points at which the agents make decisions, and $w_1 = \frac{\theta_1(t)}{\theta_1(t)+\theta_2(t)}$, $w_2 = \frac{\theta_2(t)}{\theta_1(t)+\theta_2(t)}$. The pursuers initially select the upper bound of the range of ϕ and compare a with $a - 1$ or $a + 1$. Overall, the pursuing strategy based on simulated annealing is shown as Algorithm 1. There is no centralized control in Algorithm 1. The time complexity of Algorithm 1 is $O(1)$.

In Algorithm 1, T is a parameter which belongs to simulated annealing algorithm [24]. If $T \to 0$, simulated annealing algorithm will become greedy algorithm. As greedy algorithm may lead to local optimal results, parameter T is used to add a chance for the algorithm to converge to the global optimal solution. In this paper, parameter T can be set referring to previous studies which discussed simulated annealing algorithm [24]. Besides, lines 14 and 16 are the key steps in the algorithm. Therefore, we present an instance to explain lines 14 and 16:

Algorithm 1 The pursuing strategy based on simulated annealing

1: Input the angle φ, time t, speed v_p and v_e.
2: **if** $t = 0$ **then**
3: Initialize T
4: $a_t \leftarrow k$
5: **else**
6: **if** $t = 2\Delta t$ **then**
7: $T \leftarrow |C(a_{\Delta t}) - C(a_0)|$
8: **if** $a_{t-\Delta t} = k$ **then**
9: $a_t \leftarrow k - 1$
10: **else if** $a_{t-\Delta t} = 0$ **then**
11: $a_t \leftarrow 1$
12: **else**
13: **if** $C(a_{t-\Delta t}) \geq C(a_{t-2\Delta t})$ or with probability $e^{\frac{[C(a_{t-\Delta t})-C(a_{t-2\Delta t})]}{T}}$ **then**
14: $a_t \leftarrow 2a_{t-\Delta t} - a_{t-2\Delta t}$
15: **else**
16: $a_t \leftarrow 2a_{t-2\Delta t} - a_{t-\Delta t}$
17: $T \leftarrow \beta T$
18: **if** $\varphi \geq \frac{\pi}{2}$ **then**
19: $\phi \leftarrow \frac{a_t(\pi-\varphi)}{k}$
20: **else**
21: $\phi \leftarrow \frac{\pi}{2} - \varphi + \frac{a_t\varphi}{k}$

– It is assumed that $a \in \{0, 1, ...10\}$, $a_{t-2\Delta t} = 5$ and $a_{t-\Delta t} = 6$. In this case, $2a_{t-\Delta t} - a_{t-2\Delta t} = 7$ and $2a_{t-2\Delta t} - a_{t-\Delta t} = 4$. Then, if $C(a_{t-\Delta t}) \geq C(a_{t-2\Delta t})$, $C(6)$ is better than $C(5)$. Therefore, $a = 6$ is accepted, and $a = 7$ will be tested. Besides, a worse action may also be accepted with a probability resulting that $a = 7$ may also be tested at time step t even if $C(a_{t-\Delta t}) < C(a_{t-2\Delta t})$. It means line 14 will be performed. Conversely, if $a = 6$ is not accepted, the action which is close to $a = 5$ but is not $a = 6$ will be tested at time step t. Then, line 16 will be performed.

After Algorithm 1 is presented, we can show the analyses of Algorithm 1.

Lemma 2 *If the evader will not change its initial moving direction, Algorithm 1 will lead to the same capture success ratio with the pursuing strategy which calculates Apollonius Circle based on* $\frac{v_p+v_p{}'}{2}$.

Proof In Fig. 9, if pursuer P_1 and pursuer P_2 which plan their paths based on Apollonius Circle (depending on $\frac{v_p+v_p{}'}{2}$) are able to capture the evader, a condition should be satisfied: there is at least one intersection point of the Apollonius Circles of pursuer P_1 and pursuer P_2.

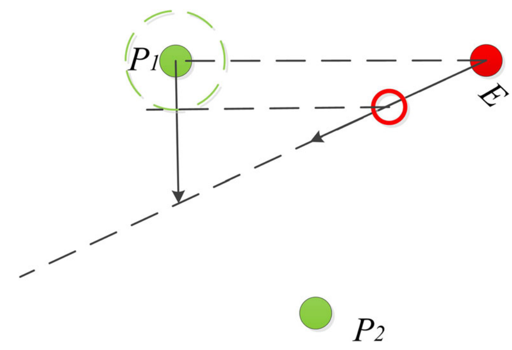

Fig. 9 The selection of moving direction

If there is only one intersection point of the Apollonius Circles of pursuer P_1 and pursuer P_2, let A denote this point (such as in Fig. 5). In this case, P_1A is perpendicular to P_1E [20]. Besides, let $\overline{v_p} = \frac{v_p+v_p{'}}{2}$ so that $\frac{|P_1A|}{\overline{v_p}} = \frac{|EA|}{v_e}$. After Δt unit time, the evader moves $v_e\Delta t$ unit distance. Then, $\frac{|EA|-v_e\Delta t}{v_e} = \frac{|EA|}{v_e} - \Delta t = \frac{|P_1A|}{\overline{v_p}} - \Delta t = \frac{|P_1A|-\Delta t\overline{v_p}}{\overline{v_p}}$. This case is as shown in Fig. 9. In Fig. 9, the red hollow circle denotes the location of the evader after Δt unit time. The green imaginary line denotes the possible location of pursuer P_1 after Δt unit time. It can be known that $\phi = \frac{\pi}{2}$ is the most helpful in decreasing $\angle P_1EP_2$. As $\angle P_1EP_2$ is the largest gap among pursuers, decreasing $\angle P_1EP_2$ is the most important. Therefore, Algorithm 1 will let $\phi = \frac{\pi}{2}$ so that Algorithm 1 will lead to the same result with the pursuing strategy which calculates Apollonius Circle based on $\frac{v_p+v_p{'}}{2}$.

If there is two intersection points of the Apollonius Circles of pursuer P_1 and pursuer P_2, $\angle P_1EP_2$ will increase when pursuer P_1 does not select $\phi = \frac{\pi}{2}$. There will be only one intersection point of the Apollonius Circles of pursuer P_1 and pursuer P_2 after some time. Then, the conclusion is consistent with above analyses.

If there is no intersection point of the Apollonius Circles of pursuer P_1 and pursuer P_2, neither Algorithm 1 nor the pursuing strategy based on Apollonius Circle can lead to a successful capture.

Overall, if the evader will not change its initial moving direction, Algorithm 1 will lead to the same capture success ratio with the pursuing strategy which calculates Apollonius Circle based on $\frac{v_p+v_p{'}}{2}$. □

Then, based on Lemma 1 and Lemma 2, Theorem 1 is presented.

Theorem 1 *The performance of Algorithm 1 will not be worse than that of the pursuing strategy which just depends on Apollonius Circle.*

Proof When the pursuers' speeds are unstable, pursuer P_i can calculate Apollonius Circle based on v_p, $\frac{v_p+v_p{'}}{2}$ or v_{p_i}. Lemma 1 shows that, if the evader will not change its moving direction, calculating Apollonius Circle based on $\frac{v_p+v_p{'}}{2}$ will lead to higher capture success ratio than that based on v_p or v_{p_i}. Then, Lemma 2 shows that, if the evader will not change its initial moving direction, Algorithm 1 will lead to the same result with the pursuing strategy which calculates Apollonius Circle based on $\frac{v_p+v_p{'}}{2}$. Therefore, if the evader will not change its initial moving direction, Algorithm 1 will not be worse than the pursuing strategy which just depends on Apollonius Circle. It means that the two pursuers which generate the largest gap among the pursuers can select the optimal actions when performing Algorithm 1.

If the evader changes its moving direction, the change of the gaps except the largest gap will also influence the results. Because of the uncertainty of the pursuers' speeds, the gaps change differently in repeated games. Accordingly, Algorithm 1 will let the pursuers help the neighboring pursuers flexibly. In other words, Algorithm 1 seeks the balance among the gaps, such as shown in Fig. 10 (the green hollow circles represent the possible locations of pursuers at next moment, and the red hollow circle means the possible location of the evader at next moment.). Even if some pursuers move slowly, the neighboring pursuers can

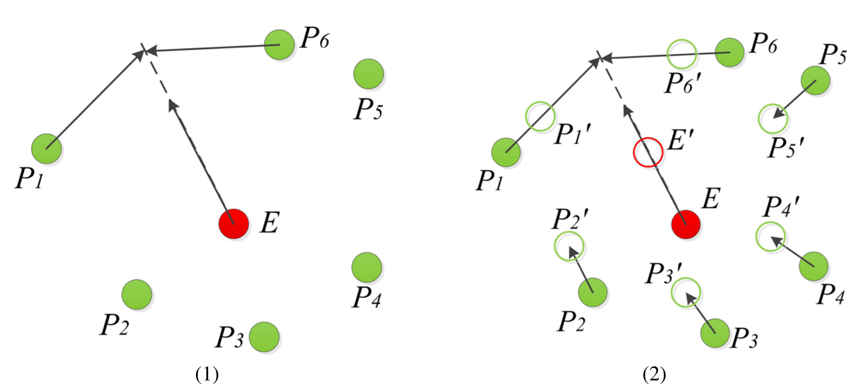

Fig. 10 The performance of Algorithm 1 when the evader changes its moving direction

help them seeks the balance among the gaps. Therefore, when the evader changes its moving direction, it needs to pass through a gap which is close to the average size of the gaps. On the other hand, the pursuing strategies which just depend on Apollonius Circle cannot balance the gaps because these strategies do not consider the uncertainty of the pursuers' speeds. For instance, if pursuer P_5 does not consider the change of $\angle P_4EP_5$ and P_4's speed is smaller than its expected speed, $\angle P_4EP_5$ will increase fast. In this case, when the evader changes its moving direction, it just needs to pass through a gap which is obviously larger than other gaps. Therefore, Algorithm 1 is more helpful in preventing the escape of the evader.

Overall, the performance of Algorithm 1 will not be worse than that of the pursuing strategy which just depends on Apollonius Circle. □

5 Simulation Results

In this section, we test our strategy based on various simulation experiments. Simulating continuous time, the evader and the pursuers can make decisions and change their directions per 0.1 unit time. In other words, $\Delta t = 0.1$ in Algorithm 1. Each experiment is performed with 5000 replications, and the ratio of the replications in which the pursuers capture the evader successfully is shown as capture success ratio. In each replication, the locations of pursuers are reset randomly.

We select the contrast strategies in the studies which adopted the similar assumptions with this paper, i.e., the game is in an open world and the evader escapes successfully when it is out of the encircled formation consisting of pursuers. In detail, the strategies in [6, 13, 27] are compared with our strategy:

- Strategy 1: The pursuers adopt the strategy that is shown in Algorithm 1 with $\beta = 0.9$ and $k = 10$. The parameter setting is inspired by [24].
- Strategy 2: The pursuers adopt the strategy that is presented in [27]. This strategy is based on reinforcement learning.
- Strategy 3: The pursuers adopt the strategy presented in [13]. This strategy is based on the Apollonius Circle. Apollonius Circle in Strategy 3 depends on the pursuer's current speed v_{p_i}.
- Strategy 4: The pursuers also adopt the strategy presented in [13]. Apollonius Circle in Strategy 4 depends on the average speed $\frac{v_p+v_p{}'}{2}$.
- Strategy 5: The pursuers adopt the strategy presented in [6]. This strategy is not based on the Apollonius Circle but also depends on the pursuers' speeds. Besides, this strategy depends on the assumption that the pursuers have the same speed. Therefore, Strategy 5 is based on $\frac{v_p+v_p{}'}{2}$ instead of v_{p_i}.

Strategy 3 and Strategy 4 are the same strategy presented in [13] and calculate Apollonius Circles based on different speeds. This strategy let pursuers predict the intersection point of the Apollonius Circles of pursuers.

Strategy 5 consists of three processes: besiege, shrink and capture [6]. The pursuers maintain and adjust the encirclement in the first process. When some conditions are satisfied, the pursuers shrink the encirclement and finally capture the evader. Each process is based on the accurate values of the pursuers' speeds.

5.1 The Influence of the Pursuers' Maximum Speed and Minimum Speed

The uncertainty of the pursuers' speeds is the main issue in this paper, and the uncertainty is influenced by the parameters v_p, $v_p{}'$ and the parameter λ in poisson distribution. Therefore, in this subsection, we present extensive experimental data to discuss the influence of the parameters v_p, $v_p{}'$ and λ. Four cases are discussed:

1. v_p changes while other parameters are fixed;
2. $v_p{}'$ changes while other parameters are fixed;
3. λ changes while other parameters are fixed;
4. it is assumed that the pursuers have heterogeneous v_p and $v_p{}'$.

In addition, the increase of v_e is similar to the decrease of v_p and $v_p{}'$. Therefore, we do not extra test these strategies with different values of v_e.

5.1.1 The Capture Success Ratios with Different v_p

In Fig. 11, $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_e = 2$, $\lambda = 0$ and $v_p{}' = 0$. This figure shows the performance of five strategies when v_p changes from 1 to 2. As shown in Fig. 11, Strategy 1 is always the best. Strategy 2 is the second best strategy. The advantage of our strategies is quite obvious. It means when other parameters are fixed, the variety of the pursuers' maximum speed will not influence the advantage of our strategies.

Strategy 3 and Strategy 4 are the same strategy that is based on Apollonius Circle, and the Apollonius Circles in Strategy 3 and Strategy 4 depend on different values of the pursuers' speeds. The performance of Strategy 3 is worse than Strategy 4. Strategy 5 is better than Strategy 3 but a little worse than Strategy 4. Generally, the strategies which do not consider the uncertainty of the pursuers' speeds lead to similar results in this paper and are worse than our algorithm.

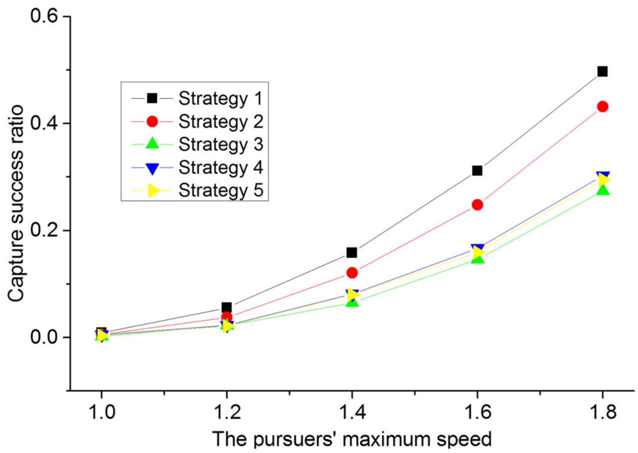

Fig. 11 The capture success ratios with different v_p

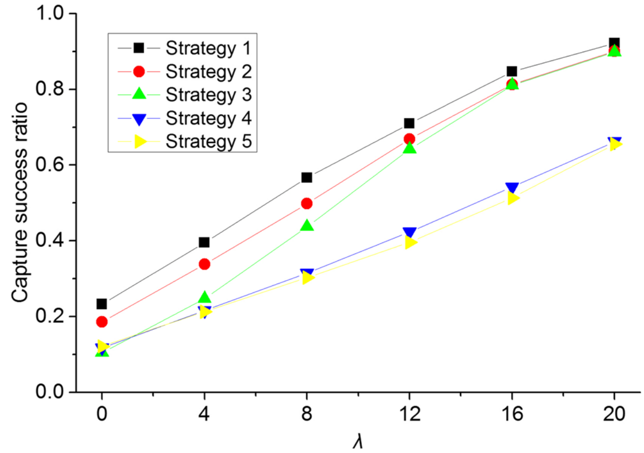

Fig. 13 The capture success ratios with different λ

5.1.2 The Capture Success Ratios with Different $v_p{}'$

The capture success ratios with different $v_p{}'$ is also shown. In Fig. 12, $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_p = 1.5$, $\lambda = 0$ and $v_e = 2$. As shown in Fig. 12, the capture success ratios based on Strategy 1 is the highest when $v_p{}' \leq 1.1$. When $v_p{}' \geq 1.3$, the capture success ratios based on Strategy 1 are very close to that based on Strategy 3 and 4. Generally, Strategy 1 is the best strategy in Fig. 12. Besides, Strategy 2 is better than Strategy 3 and 4 when $v_p{}' \leq 0.9$. When $v_p{}'$ is large enough, Strategy 3 and 4 are better than Strategy 2. Strategy 5 is still better than Strategy 3 and worse than Strategy 4.

5.1.3 The Capture Success Ratios with Different λ in Poisson Distribution

In Fig. 13, $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_e = 2$, $v_p = 1.5$ and $v_p' = 0$. The parameter λ changes from 0 to 20. If λ is larger, the pursuers' speeds are more certain, and certain speeds benefit the pursuers. Therefore, the capture success ratios based on all strategies increase along with the increase of λ. Besides, the most outstanding phenomenon in Fig. 13 is that the capture success ratio based on Strategy 3 increases very obviously. When the pursuers' speeds fluctuate frequently, Strategy 3 is worse than Strategy 4 and Strategy 5 because $\frac{v_p + v_p{}'}{2}$ is closer to the average speed. Conversely, when λ becomes large, v_{p_i} leads to more accurate Apollonius Circle so that Strategy 3 becomes better than Strategy 4 and leads to similar results with Strategy 2 which has considered the uncertainty of the pursuers' speeds. However, Strategy 3 is still worse than Strategy 1. The reason is that the time interval of the fluctuation of the pursuers' speeds is random variable (obeys poisson distribution). Although λ is large, the time interval may also be a small value because of the randomness. In this case, Strategy 1 which can effectively address the problems caused by the uncertainty of the pursuers' speeds still leads to higher capture success ratio.

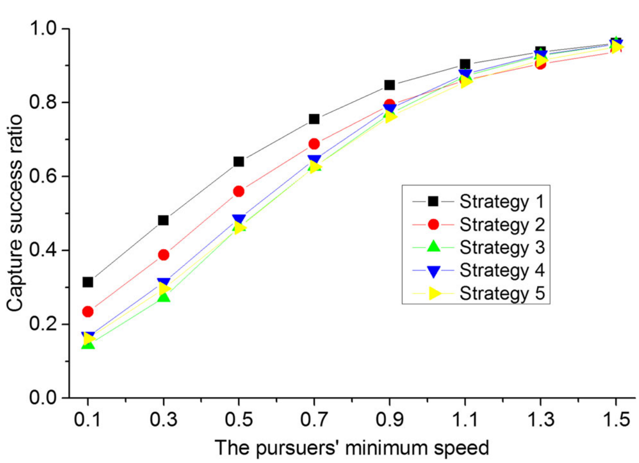

Fig. 12 The capture success ratios with different $v_p{}'$

5.1.4 The Capture Success Ratios with heterogeneous v_p' and v_p

In the other figures, we assume that each pursuer has the same v_p' and v_p. In Fig. 14, we also show the cases that the pursuers have heterogeneous v_p' and v_p.

In Fig. 14 (1), it is assumed that the maximum speed of each pursuer obeys uniform distribution in $[1.5 - v', 1.5 + v']$. Certainly, the maximum speed of each pursuer does not change in the pursuit-evasion process. The other parameters are fixed: $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_e = 2$, $\lambda = 0$ and $v_p' = 0$. In Fig. 14 (2), it is assumed that the minimum speed of each pursuer obeys uniform distribution in $[0.5 - v', 0.5 + v']$. The minimum speed of each pursuer also does not change in the pursuit-evasion process. The other parameters are fixed: $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_e = 2$,

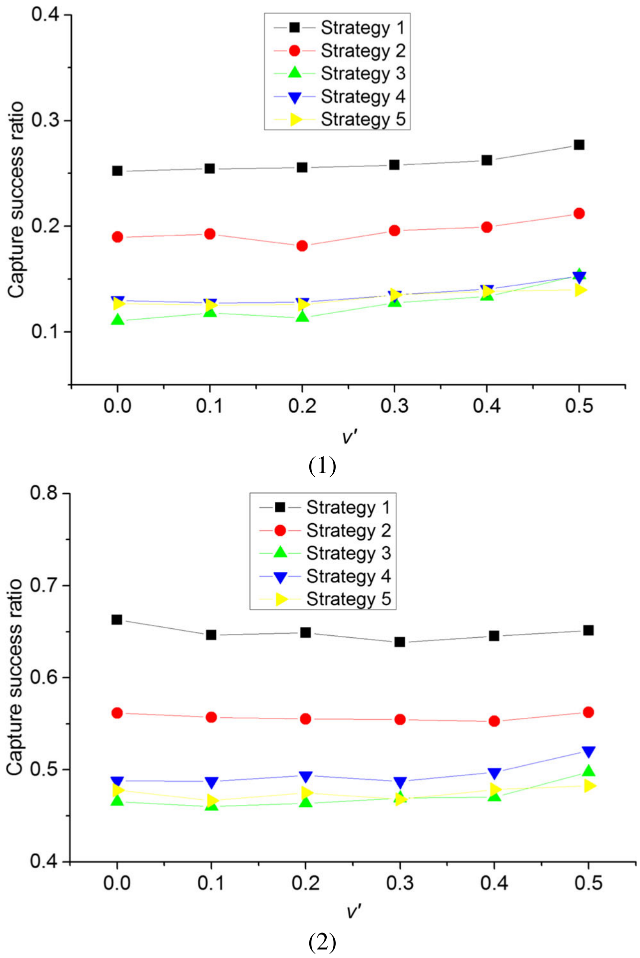

Fig. 14 The capture success ratios with heterogeneous v'_p and v_p

$\lambda = 0$ and $v_p = 1.5$. Besides, v_p and v'_p which are used in the pursuing strategies are replaced by the average values.

Based on Fig. 14, it is known that the heterogeneity of v'_p and v_p will not obviously influence the performance of the pursuing strategies. The reason is that the pursuers' actual speeds are uncertain so that v_{p_i} itself is heterogeneous even if v'_p and v_p are homogeneous. Therefore, when the pursuers' speeds are uncertain, there is not any outstanding difference between the cases where v'_p and v_p are homogeneous or heterogeneous.

5.1.5 Brief Summary

Based on Figs. 11, 12, 13 and 14, we have discussed the influence of the uncertainty of the pursuers' speeds. The five strategies have been tested in different cases.

Overall, our pursuing strategy based on simulated annealing is always the best strategy. If other parameters are fixed, the variety of the pursuers' maximum speed will not influence the relative merits of the strategies. Besides, when the pursuers' speeds fluctuate frequently, calculating

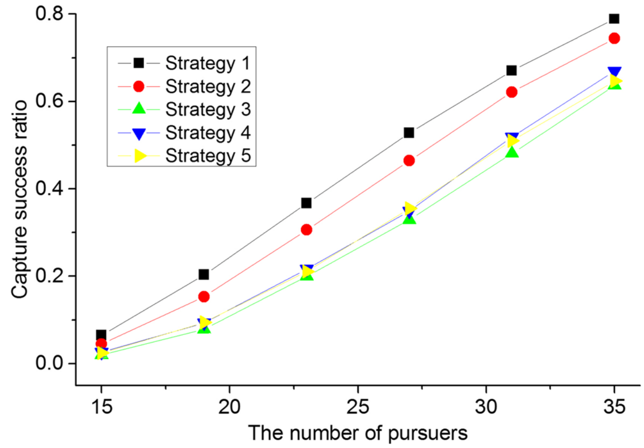

Fig. 15 The capture success ratios with different n

Apollonius Circle based on the pursuers' average speed is better than the pursuers' actual speeds. However, when the pursuers' speeds fluctuate not frequently, calculating Apollonius Circle based on the pursuers' actual speeds is better. In addition, the heterogeneity of the pursuers' maximum speed and minimum speed will not obviously influence the performance of the pursuing strategies.

5.2 The Influence of the Number and the Initial Locations of Pursuers

Then, we discuss the influence of the number and the initial locations of pursuers. The parameters n, d_1 and d_2 do not directly influence the uncertainty of the pursuers' speeds. However, when the pursuers move at uncertain speeds, the influence of these parameters may be different from that in previous studies. Therefore, we also show the capture success ratios with different n, d_1 and d_2.

5.2.1 The Capture Success Ratios with Different *n*

In Fig. 15, we test the five strategies with different number of pursuers. The other parameters are fixed. In detail, $d_1 = 20$, $d_2 = 40$, $v_p = 1.5$, $v_e = 2$, $\lambda = 0$ and $v_p{}' = 0$.

As shown in Fig. 15, the capture success ratios increase along with the increase of n no matter which strategy is adopted by pursuers. Strategy 1 always leads to higher capture success ratio than other strategies. Strategy 2 is always the second best. The advantage of our strategy is quite obvious no matter how much n is. It means when other parameters are fixed, the variety of the number of pursuers will not influence the advantage of our strategy.

Strategy 4 is also better than Strategy 5 while Strategy 5 is better than Strategy 3. It means the variety of n does not influence the disadvantage of calculating Apollonius Circles based on corresponding pursuers' current speeds. To sum

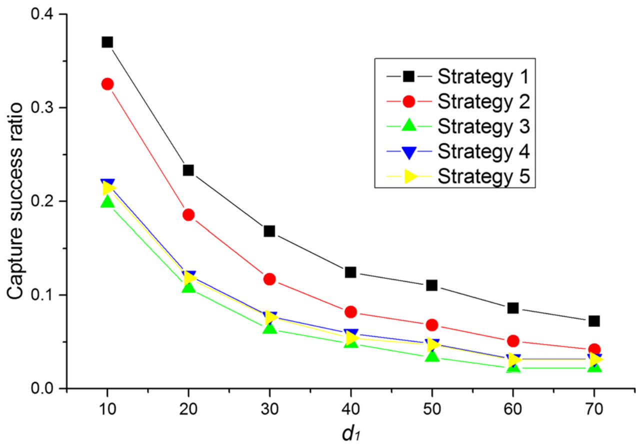

Fig. 16 The capture success ratios with different $\frac{d_2+d_1}{2}$

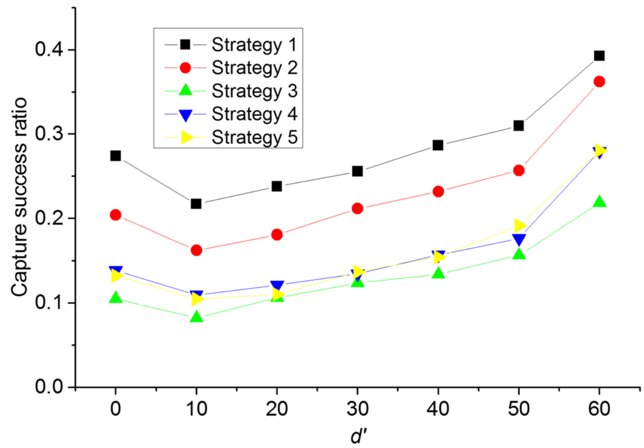

Fig. 17 The capture success ratios with different $d_2 - d_1$

up, there is not any surprising phenomenon in Fig. 15. The increase of n will lead to higher capture success ratios but does not influence the relative merits of different pursuing strategies.

5.2.2 The Capture Success Ratios with Different d_1 and d_2

Then, we discuss the influence of d_1 and d_2. In Fig. 16, $n = 20$, $d_2 = d_1 + 20$, $v_p = 1.5$, $v_e = 2$, $\lambda = 0$ and $v_p{}' = 0$. In other words, the mathematical variance of the initial distance between each pursuer and the evader is fixed. When d_1 and d_2 are larger, the pursuers will be located farther from the evader. Then, the capture success ratios will also become lower. Certainly, Strategy 1 is always the best even if d_1 and d_2 are quite large.

In Fig. 17, $n = 20$, $v_p = 1.5$, $v_e = 2$, $\lambda = 0$ and $v_p{}' = 0$. $\frac{d_2+d_1}{2}$ is always 30. Let $d' = d_2 - d_1$. The phenomena in Fig. 17 are quite interesting. The capture success ratios based on the five strategies all decrease at first and then increase with the increase of d'. When d' is small, the initial distances among pursuers and the evader are similar. In this case, the pursuers can coordinate with each other efficiently. For instance, the Apollonius Circle of one pursuer is more likely to intersect with the Apollonius Circle of another pursuer. When d' increases, the distances among pursuers may increase so that the evader has more chances to escape. On the other hand, $d' = 60$ and $\frac{d_2+d_1}{2} = 30$ means $d_1 = 0$. The initial distance between a pursuer and the evader may be very small. Then, the evader is more likely to be captured by the pursuers which are initially very close to it. Therefore, the capture success ratios increase when d_1 approaches zero.

5.2.3 Brief Summary

Based on Figs. 15, 16 and 17, we have discussed the influence of the number and the initial locations of pursuers. Overall, Strategy 1 is still the best strategy in the three figures. There is not any surprising experimental phenomenon when the number of pursuers changes. The increase of the number of pursuers will lead to higher capture success ratios but does not influence the relative merits of different pursuing strategies.

It is worth noting that, when the initial locations of the pursuers change, there is an interesting experimental phenomenon. When average distance between each pursuer and the evader is fixed, the capture success ratios based on the five strategies all decrease at first and then increase with the increase of the mathematical variance of the initial distance between each pursuer and the evader.

5.3 Verifying the Analyses Presented in Section 4

In order to understand the disadvantages of previous pursuing strategies that are based on Apollonius Circle more deeply, we verify the analyses presented in section 4.1 based on the experimental data in Fig. 18. In Sections 5.1 and 5.2, we have discussed the influence of various parameters on the pursuit-evasion game and show that our strategy based on simulated annealing is better than other pursuing strategies in different cases. However, the experimental data in Sections 5.1 and 5.2 are not enough helpful in deeply understanding the disadvantages of previous pursuing strategies. In this subsection, we assume that the evader will not change the moving direction in pursuit-evasion process. In other words, only the first step of the escape strategy presented in section 3.2 is adopted. Then, Fig. 18 (1) shows the capture success ratios with $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_p = 1.5$, $\lambda = 0$ and $v_e = 2$. In Fig. 18 (2), $d_1 = 20$, $d_2 = 40$, $n = 20$, $v_p = 1.5$, $v_p' = 0$ and $v_e = 2$.

Compared with Fig. 12, the capture success ratios in Fig. 18 (1) are all higher. It means that the evader keeping

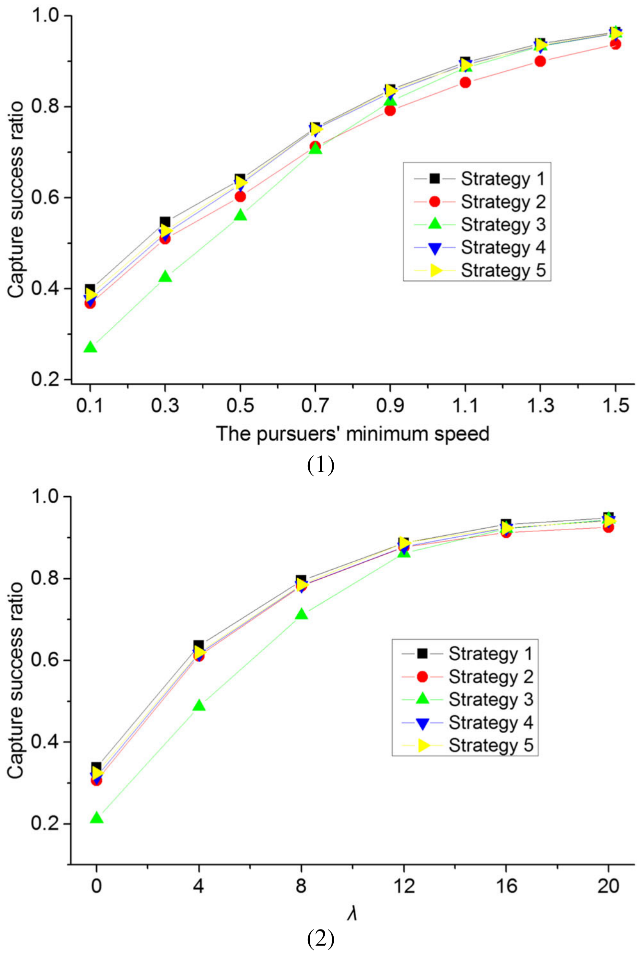

Fig. 18 The capture success ratios when the evader does not change its initial moving direction

the initial moving direction is good for the pursuers. Besides, as shown in Fig. 18 (1), the capture success ratios based on Strategy 1 are always very close to that based on Strategy 4. This phenomenon accords with Lemma 2. In Fig. 12, the capture success ratios based on Strategy 4 are obviously lower.

Based on the analyses presented in section 4.1, it has been known that:

- The uncertainty of speeds lead to inaccurate Apollonius Circle (especially when v_p' is unknown) so that the pursuers adopting the strategies [9, 13, 20] based on Apollonius Circle often move to worthless target locations;
- Previous strategies [9, 13, 20] let pursuers move to scheduled locations, but the uncertainty of speeds will make some pursuers fail to reach the scheduled locations punctually. The evader can escape successfully by changing moving direction and finding the pursuers which fail to reach the scheduled locations punctually.

It means that Strategy 4 is mainly influenced by the second problem caused by the uncertainty of the pursuers' speeds. In Fig. 18 (1), the second problem does not exist. Therefore, Strategy 4 can lead to similar capture success ratios with Strategy 1. In addition, although the second problem does not exist in Fig. 18 (1), the capture success ratios based on Strategy 3 are also lower than that based on Strategy 1 and 4. The reason is that Strategy 3 is influenced by both the first and second problems caused by the uncertainty of the pursuers' speeds. The pursuers adopting Strategy 3 will usually move towards worthless target points.

Figure 18 (2) can be compared with Fig. 13. In Fig. 13, Strategy 3 becomes better than Strategy 4 and Strategy 5 when λ is large. However, in Fig. 18 (2), Strategy 4 and Strategy 5 lead to similar results with Strategy 1 and Strategy 2. Strategy 3 becomes the worst in Fig. 18 (2). The reason is just as shown in Lemma 1. If the evader does not change its moving direction, Strategy 4 is averagely better than Strategy 3 whatever λ is.

Overall, the analyses in section 4.1 can be verified based on the experimental data in Figs. 12, 13 and 18.

6 Conclusions

In this paper, we have studied the multiagent pursuit-evasion problem in the situation in which the pursuers move at uncertain speeds. The pursuers' maximum speed is a known constant. However, the pursuers cannot always move at the maximum speed, and their actual speeds fluctuate between the maximum speed and a lower bound. The evader's speed is a known constant and larger than the pursuers' maximum speed. Previous studies usually assumed that the pursuers can move at certain speeds; thus, the capture success ratios based on previous pursuing strategies decrease when the pursuers move at uncertain speeds. In order to increase the capture success ratio, we presented the feasible pursuing strategy considering the problems caused by the uncertainty of the pursuers' speeds. We excluded some bad moving directions to reduce the decision space of pursuer. Then, each pursuer selects moving direction based on the rule that controls the gaps between it and its neighboring pursuers, and the rule is combined with simulated annealing. Therefore, the faster pursuer can help the slower neighboring pursuer to inhibit the expansion of the gap between them.

Extensive experiments have been presented to show the performance of our strategies in various cases. The experimental data shown that our strategies can lead to higher capture success ratios than previous strategies in the situations where the pursuers' speeds are uncertain. When the pursuers' speeds are certain, our strategy based on simulated annealing leads to similar capture success ratios with previous strategies. When other parameters are fixed, the variety of the pursuers' maximum speed and the variety of the number of the pursuers will not influence the relative merits of different pursuing strategies. Besides, the time interval between the fluctuations of the pursuers' speeds can influence the relative merits of different pursuing strategies. When the pursuers' speeds are uncertain, the heterogeneity of the pursuers' maximum speed and minimum speed will not obviously influence the performance of the pursuing strategies.

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Fuhan Yan received the B.S. degree and M.S. degree at the School of Computer Science and Engineering, Southeast University, Nanjing, China, in 2013 and 2015, respectively. He is currently pursuing the Ph.D. degree at the Distributed Intelligence and Social Computing Laboratory, School of Computer Science and Engineering, Southeast University, Nanjing, China. His current research interest is multiagent systems. He has published several scientific articles in refereed journals and conference proceedings, such as the European Conference on Artificial Intelligence (ECAI), the International Conference on Autonomous Agents and Multiagent Systems (AAMAS), and Physica A: Statistical Mechanics and its Applications.

Jiuchuan Jiang is now with the School of Computer Science and Engineering, Nanyang Technological University, Singapore. His main research interests include multiagent systems and artificial intelligence. He has published several scientific articles in refereed journals and conference proceedings, such as the IEEE Transactions on Parallel and Distributed Systems, the IEEE Transactions on Systems, Man, and Cybernetics: Systems, the Information Processing & Management, the Information Processing Letters, and the International Conference on Autonomous Agents and Multiagent Systems (AAMAS).

Kai Di received the B.S. degree at the School of Information Science and Technology, Southwest Jiaotong University, Chengdu, China, in 2016. He is currently pursuing the Ph.D degree at the Distributed Intelligence and Social Computing Laboratory, School of Computer Science and Engineering, Southesat University, Nanjing, China. His current research interest is multiagent systems.

Yichuan Jiang received his PhD degree in computer science from Fudan University, Shanghai, China, in 2005. He is currently a full professor with the School of Computer Science and Engineering, Southeast University, Nanjing, China. His main research interests include multiagent systems, social computing, and social networks. He has published more than 80 scientific articles in refereed journals and conference proceedings, such as the IEEE Transactions on Parallel and Distributed Systems, the IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, the IEEE Transactions on Systems, Man, and Cybernetics-Part C: Applications and Reviews, the IEEE Transactions on Systems, Man, and Cybernetics: Systems, the IEEE Transactions on Cybernetics, the ACM Transactions on Autonomous and Adaptive Systems, the Journal of Autonomous Agents and Multi-Agent Systems, the Journal of Parallel and Distributed Computing, the International Joint Conference on Artificial Intelligence (IJCAI), the International Conference on Autonomous Agents and Multiagent Systems (AAMAS), and the European Conference on Artificial Intelligence (ECAI). He won the best paper award from PRIMA06 and best student paper awards twice from ICTAI13 and ICTAI14. He is a senior member of IEEE.

Zhifeng Hao received the Ph.D degree in applied mathematics from Nanjing University, Nanjing, China, in 1995. He is now a full professor with the School of Mathematics and Big Data, Foshan University, China. His research interests include artificial intelligence and big data. He has published several scientific articles in refereed journals and conference proceedings, such as the IEEE Transactions on Neural Networks and IEEE Transactions on Knowledge and Data Engineering.