Journal of Intelligent & Robotic Systems (2019) 95:229–238
https://doi.org/10.1007/s10846-018-0820-x

Aerial-Underwater Systems, a New Paradigm in Unmanned Vehicles

D. Mercado[1] · M. Maia[1] · F. J. Diez[1]

Received: 30 September 2017 / Accepted: 20 March 2018 / Published online: 3 April 2018

Abstract

Unmanned Aerial-Underwater Vehicles (UAUVs) arise as a new kind of unmanned system capable of performing equally well in multiple mediums and seamlessly transitioning between them. This work focuses in the modeling and trajectory tracking control of a special class of air-underwater vehicle with full torque actuation and a single thrust force directed along the vehicle's vertical axis. In particular, a singularity-free representation is required in order to orient the vehicle in any direction, which becomes critical underwater in order to direct the thrust force in the direction of motion and effectively overcome the increased drag and buoyancy forces. A quaternion based representation is used for this purpose. A hierarchical controller is proposed, where trajectory tracking is accomplished by a Proportional-Integral-Derivative (PID) controller with compensation of the restoring forces. The outer trajectory tracking control loop provides the thrust force and desired orientation. The latter is fed to the inner attitude control loop, where a nonlinear quaternion feedback is employed. A gain scheduling strategy is used to deal with the drastic change in medium density during transitions. The proposed scheme is studied through numerical simulations, while real time experiments validate the good performance of the system.

Keywords Multi-medium systems · UAVs · UUVs · Singularity-free · Trajectory tracking

1 Introduction

Unmanned Aerial Vehicles (UAVs) have received increased attention from the industry and academic communities in recent years, leading to fast development and great success in their application in a wide range of fields [1]. Similarly, Unmanned Underwater Vehicles (UUVs) have also been improved and found to be increasingly utilized [2]. Such a success in the research of unmanned vehicles allows us to think about new boundaries to break, and pursue goals that seemed unrealistic or unfeasible in years past. That is the case in the conception of unmanned vehicles that can equally perform in multiple mediums, and seamlessly transition between them. The concept of such a vehicle has been present since 1930 [3, 4] when the Denmark navy and the Soviet Union conceived, respectively, a submarine plane and a flying underwater boat. Also, in recent years, control of an aerial underwater vehicle with four air rotors and four underwater rotors, using an Euler angles representation, was proposed in [5, 6]. However, all of these works rested only in the conception and design phase, and were never implemented. This idea is no longer a far away illusion, but a reality, and we can find some early examples in literature [7], where a suitable combination of the studies in UAVs and UUVs opens the door for a whole new set of opportunities in what we call Unmanned Aerial-Underwater Vehicles (UAUV) or more generally, multi-medium vehicles. A video of one of the first full working UAUV prototypes can be seen at https://www.youtube.com/watch?v=-JbcP98rw74.

Indeed, UAUVs combine the potential of aerial and underwater vehicles, but also their limitations. For example, GPS (Global Positioning System) signals have been used for positioning in a wide range of applications with UAVs, but they are normally non-existent underwater. On the other hand, equipment specially designed for underwater applications do not satisfy the light-weight requirements imposed by the limited payload capacity of UAVs. Another interesting particularity is that aerial vehicles require a thrust force pointing mostly upwards to overcome the gravity. In fact, most applications for aerial vehicles only require to stabilize the orientation of the vehicle around the origin. Meanwhile, underwater, the buoyancy force can be used to compensate for gravity, but the thrust force must

✉ F. J. Diez
diez@rutgers.edu

[1] Department of Mechanical and Aerospace Engineering
Rutgers, The State University of New Jersey, 98 Brett Road,
Piscataway, NJ 08854-8058, USA

be pointed in the direction of movement. This is also true to be able to effectively overcome the higher drag force underwater.

There exist a wide variety of different configurations for UAVs and UUVs, but not all of them are well suited for both scenarios. In this work we are particularly interested in multirotor vehicles with full torque actuation and a single force direction. This kind of underactuated vehicles includes the most used configurations for multirotor UAVs. As stated elsewhere [7, 17], the octo-quadcopter configuration is well suited for transitioning between mediums and consequently is selected for the present study.

We can find in the literature different representations for the pose of the vehicle [8], where the Euler angles [9, 10], quaternions [11–13] and the special orthogonal group SO(3) [14, 15] are the most common ones. However, Euler angle representations suffer from singularities. This is not usually a big problem in most UAVs applications, since the vehicle only needs to be operated close the equilibrium and far away from the singularity. Nevertheless, singularity-free representations, such as quaternions or the SO(3) group have been studied and are preferred when aggressive maneuvers and acrobatics are required. From these two, quaternions offer the minimal representation (only four parameters) and is the most efficient computationally, but present ambiguities (two different quaternions represent the same orientation). On the other hand, the SO(3) group requires nine parameters and in consequence larger computations.

Furthermore, in UUVs with a thrust force vector pointing only in one direction, the use of a singularity-free representation becomes critical for a near neutrally buoyant or positively buoyant vehicle. For instance, in order to dive, it must be able to face upside-down and direct the thrust force downwards.

This paper is a continuation of the work presented in ICUAS 2017 [16], where a singularity -free representation employing quaternions was proposed along with a control strategy for the altitude and orientation. Also, in [17] the experimental platform "Naviator" (see Fig. 1) was presented together with some preliminary experimental results, using an Euler angles representation. In the present work the singularity-free attitude controller in [16] is extended to accomplish trajectory tracking in position. This is acknowledged in a hierarchical scheme where the desired orientation is used as a virtual control input for the trajectory tracking control. Furthermore, real-time experimental results are provided to validate the performance of the attitude controller.

The remainder of the paper is organized as follows: Section 2 presents the mathematical background in quaternion algebra. Then, Section 3 introduces the dynamic model of an air-underwater UAUV in a singularity-free representation. The employed control strategy is provided in Section 4 followed by a study of its performance through numerical simulations in Section 5, where two cases are considered: aggressive maneuvers in air and downward navigation underwater. A preliminary experiment of a vertical loop underwater is then discussed in Section 6 and finally, Section 7 presents some conclusions and perspectives.

2 Preliminaries

While singularity-free representations are required in aerial vehicles only for special applications involving aggressive maneuvers, in underwater underactuated vehicles with only a thrust force direction, it is an essential requirement in order to effectively overcome the large drag forces and specially to go downwards when the vehicle is positive buoyant or close to neutrally buoyant (weight lower or almost equal to the buoyancy force, respectively). To solve this constraint, we choose a quaternion based representation. Quaternions are an extension of complex numbers and can be represented as

$$\mathbf{q} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} = \begin{bmatrix} q_0 \\ \bar{q} \end{bmatrix} = [q_0\ q_1\ q_2\ q_3]^T \tag{1}$$

for a quaternion $\mathbf{q}$, with scalar and vector components q_0 and $\bar{q}$, respectively, and the axes $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$ satisfying

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1$$
$$-\mathbf{ij} = \mathbf{ji} = \mathbf{k}, \quad -\mathbf{jk} = \mathbf{kj} = \mathbf{i}, \quad -\mathbf{ki} = \mathbf{ik} = \mathbf{j} \tag{2}$$

Quaternions with unitary norm, i.e.

$$|\mathbf{q}| = \sqrt{\mathbf{q}^T\mathbf{q}} = 1 \tag{3}$$

are called unit quaternions, and can be used to represent three dimensional rotations. In fact, they are the most

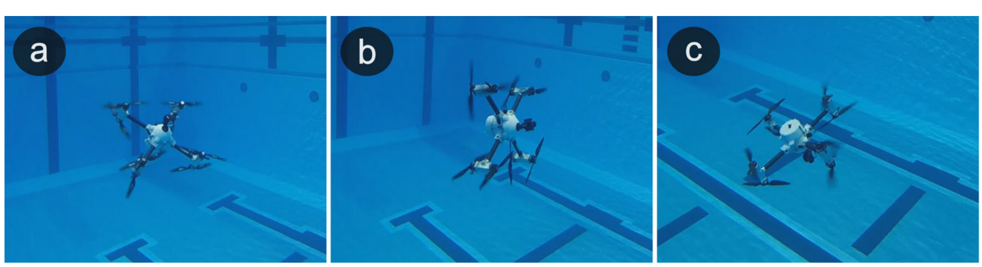

Fig. 1 Experimental platform "Naviator" performing a flip maneuver underwater, shown approaching a forward pitch of 45° in (**a**), 90° in (**b**) and 180° in (**c**)

compact form to represent rotations without singularities. Given an unit axis of rotation $\hat{\mathbf{k}}$ and an angle α, the vector part of the quaternion of rotation $\bar{q}$ and the scalar part q_0 satisfy

$$q_0 = \cos(\alpha/2), \bar{q} = \hat{\mathbf{k}} \sin(\alpha/2) \tag{4}$$

Rotation of a vector by a quaternion $\mathbf{q}$ can be represented by means of a rotation matrix $\mathbf{R}(\mathbf{q})$ of the form

$$\mathbf{R}(\mathbf{q}) = \mathbf{I}_3 + 2q_0\bar{q}_\times + 2\bar{q}_\times^2 \tag{5}$$

where $\mathbf{I}_3$ stands for the three by three identity matrix and $\bar{q}_\times$ is the skew-symmetric matrix of the vector $\bar{q}$. Unit quaternions present ambiguities since they double cover the SO(3) group ($\mathbf{R}(\mathbf{q}) = \mathbf{R}(-\mathbf{q})$). For further details about quaternion algebra refer to [18].

3 Dynamic Model

UAUVs can be modeled as a rigid body evolving in a three-dimensional space, where the medium properties may change abruptly, i.e. density, viscosity. Let us consider an earth-fixed inertial coordinate frame $\mathbf{I}$ and a body fixed coordinate frame $\mathbf{B}$ (see Fig. 2). Define the position vector $\boldsymbol{\xi} = [x,\ y,\ z]^T$, along with the quaternion of rotation $\mathbf{q}$ describing the vehicle's orientation. Then, using the Newton-Euler formalism, the equations of motion of an air-water UAUV (refer to [19, 20] for more details about modeling of air, respectively underwater vehicles) can be found as

$$m\ddot{\boldsymbol{\xi}} = T\mathbf{R}(\mathbf{q})\mathbf{e_3} - g(m - \rho V)\mathbf{e_3} - \rho\mathbf{D}_{\boldsymbol{\xi}}(\dot{\boldsymbol{\xi}}) \tag{6}$$

$$\mathbf{J}\dot{\boldsymbol{\Omega}} = \boldsymbol{\tau} - \boldsymbol{\Omega} \times \mathbf{J}\boldsymbol{\Omega} - \rho\mathbf{D}_{\Omega}(\boldsymbol{\Omega}) \tag{7}$$

with a mass m, gravity constant g, volume of the vehicle V and inertia matrix $\mathbf{J}$. For simplicity, we consider a perfect alignment of the centers of mass and buoyancy with the body-fixed frame $\mathbf{B}$. $e_3 = [0,\ 0,\ 1]^T$ is an unitary vector,

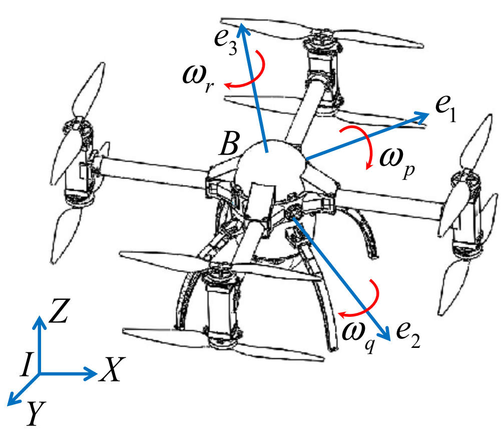

Fig. 2 Unmanned Aerial-Underwater Vehicle coordinate frames

and $\mathbf{R}(\mathbf{q}) \in SO(3)$ is the rotation matrix from body to inertial-fixed frame as defined in equation (5). The drag forces $\rho\mathbf{D_i}()$ with $i \in \{\xi, \Omega\}$ will be discussed later. $T \in \mathbb{R}^+$ represents the total thrust produced by the rotors and along with the torques $\boldsymbol{\tau} = [\tau_{e1},\ \tau_{e2},\ \tau_{e_3}]^T$ constitute the only control inputs to the system. The angular rates in the body-fixed frame are denoted by $\boldsymbol{\Omega} = [\omega_p,\ \omega_q,\ \omega_r]^T$, and are related to the quaternion time derivative $\dot{\mathbf{q}}$ as follows

$$\dot{\mathbf{q}} = \frac{1}{2}\begin{bmatrix} -\bar{q}^T \\ q_0\mathbf{I}_3 + \bar{q}_\times \end{bmatrix}\boldsymbol{\Omega} \tag{8}$$

From the equations of motion (6), (7) we can observe two main differences with respect to the classic models used to represent only aerial vehicles. First, the explicit notation of the medium density ρ. The dramatic change of density between mediums (around 1.275 kg/m^3 for air and 1×10^3 kg/m^3 for water) represents the main challenge while dealing with UAUVs. The second difference comes as a direct consequence of the density change, it is the inclusion of the drag force $\mathbf{F_D}$ and the buoyancy of the vehicle F_B. These two phenomena are often negligible in aerial vehicles, but become crucial for larger density values, as is the case underwater. The buoyancy force acts in opposite direction to the gravity and is equal in magnitude to the weight of the fluid displaced by the vehicle, i.e.

$$F_B = \rho V g \tag{9}$$

On the other hand, the drag $\mathbf{F_D}$ is a resistive force exerted in opposition to the motion of the body with respect to the surrounding fluid, and can be described by

$$\mathbf{F_D} = \rho\mathbf{D_i}(\mathbf{v}) = \frac{1}{2}\mathbf{C_D}(\mathbf{R_e})\mathbf{A}\rho\mathbf{v} \circ |\mathbf{v}| \tag{10}$$

where we separate the medium density ρ from the function $\mathbf{D_i}(\mathbf{v})$ with $i \in \{\xi, \Omega\}$, in order to highlight the influence of the density in the drag force. The drag force depends on a characteristic area A and the relative velocity $\mathbf{v}$ between the object and the fluid in the corresponding axis (for a still fluid $\mathbf{v} = \{\dot{x},\ \dot{y},\ \dot{z}\}$ for translational motions and $\mathbf{v} = \{\omega_p,\ \omega_q,\ \omega_r\}$ for rotational ones). The drag coefficient C_D is a function of the Reynolds number R_e, which for a kinematic viscosity ν and a characteristic diameter D, is given by

$$R_e = \frac{vD}{\nu} \tag{11}$$

We select a multirotor in an octo-quadcopter configuration, as depicted in Fig. 2. It is composed of four coaxial rotor pairs, four rotors spinning clock-wise and four spinning counter clock-wise in order to cancel out the reactive torques. This kind of configuration is ideal for UAUVs to transition between mediums, since the top rotors can spin at different speeds than the bottom ones, according to the

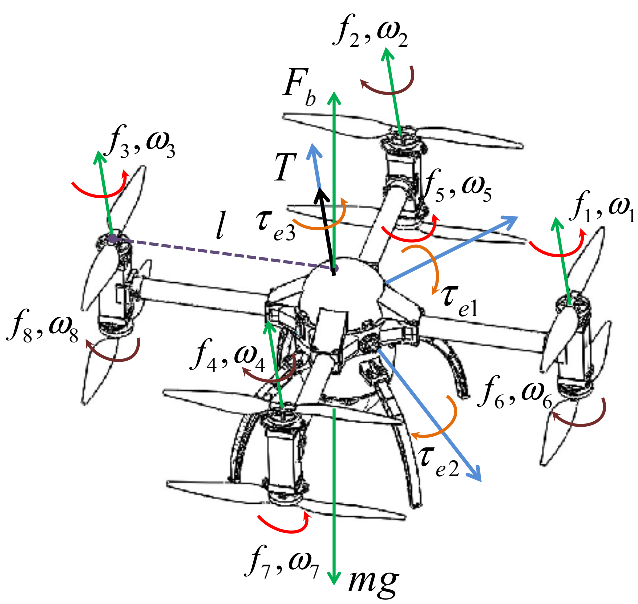

Fig. 3 Forces and moments on the UAUV in an octo-quadcopter configuration

medium they are immersed in [7]. The forces and moments exerted in the vehicle (see Fig. 3) are

$$\begin{bmatrix} T \\ \boldsymbol{\tau} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -l & l & l & -l & l & -l & -l & l \\ l & l & -l & -l & l & l & -l & -l \\ -b & b & -b & b & b & -b & b & -b \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_8 \end{bmatrix} \tag{12}$$

and

$$f_i = {\omega_i}^2 \rho K_m \tag{13}$$

where f_i, ω_i are respectively the force and the angular speed produced by rotor i. The constants l, b and K_m stand for the distance from the rotor to the center of mass of the vehicle, an angular speed to torque constant and the motor's constant, respectively. As defined above, $T \in \mathbb{R}^+$ and $\boldsymbol{\tau} \in \mathbb{R}^3$ are respectively the trust force and torque inputs produced by the rotors in this particular configuration. Once more we note the influence of the medium density in the generation of forces and torques by the motors.

4 Control Strategy

Trajectory tracking control of UAUVs is addressed in this section. Provided that all the states are available for the controller, the control objective consists of following a time varying trajectory in position ξ_d. Note than in practice, localization of UAUVs is a complicated and challenging task since GPS signals are non-existent underwater and acoustic sensors are generally too heavy to be carried in air. Taking into account that the vehicle under consideration is underactuated, with a single thrust force direction, the trajectory tracking is accomplished thanks to the fact that the rotational dynamics are much faster than the translational ones. Also, due to the increased drag and buoyancy forces underwater, the thrust force must be directed in the direction of motion for the vehicle to be able to overcome these forces. Hence, the desired orientation of the vehicle can be used as a new virtual control input $\boldsymbol{\mu} = T\mathbf{R_d}(\mathbf{q})\mathbf{e_3}$ to steer the vehicle's position to its desired value, using a PID controller with compensation of the restoring forces. Then, a nonlinear quaternion feedback is used to control the vehicle's attitude. Finally, a gain scheduling strategy is employed to deal with the abrupt change in medium density. A block diagram with the overall control strategy is depicted in Fig. 4.

4.1 Trajectory Tracking Position Control

Defining the trajectory tracking position error $\bar{\xi} = \xi - \xi_d$, we propose, for a constant density, a PID controller with compensation of the restoring forces (weight and buoyancy) as follows

$$\boldsymbol{\mu} = g(m - \rho V)\mathbf{e_3} + m\ddot{\boldsymbol{\xi}}_d - \mathbf{K}_{\xi p}\bar{\xi} - \mathbf{K}_{\xi d}\dot{\bar{\xi}} - \mathbf{K}_{\xi i}\int \bar{\xi}\, dt \tag{14}$$

where the diagonal gain matrices $\mathbf{K}_{\xi p}$, $\mathbf{K}_{\xi d}$ and $\mathbf{K}_{\xi i}$ are positive definite. The desired thrust force is computed as the magnitude of the new virtual control input $\boldsymbol{\mu}$, i.e.

$$T = \|\boldsymbol{\mu}\| \tag{15}$$

the desired quaternion can be extracted from the virtual input orientation as in [21]. For a specific yaw orientation in the inertial frame ψ_p, the desired rotation matrix takes the form

$$\boldsymbol{R}_d = [\boldsymbol{b}_1, \boldsymbol{b}_2, \boldsymbol{b}_3] \tag{16}$$

where the basis are given by

$$\boldsymbol{b}_3 = \frac{\boldsymbol{\mu}}{T}; \boldsymbol{b}_2 = \boldsymbol{b}_3 \times \boldsymbol{b}_\psi; \boldsymbol{b}_1 = \boldsymbol{b}_2 \times \boldsymbol{b}_3 \tag{17}$$

assuming that the unitary vector $\boldsymbol{b}_\psi = [\cos(\psi_p)\ \ \sin(\psi_p)\ \ 0]^T$ is not parallel with $\boldsymbol{b}_3$. Then, the desired quaternion $\mathbf{q_d}$ can be extracted from the rotation matrix $\boldsymbol{R}_d = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{12} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$ such as

$$\mathbf{q_d} = \begin{bmatrix} \frac{1}{2}\sqrt{1 + t_r} \\ (r_{23} - r_{32})/(4q_{d0}) \\ (r_{31} - r_{13})/(4q_{d0}) \\ (r_{12} - r_{21})/(4q_{d0}) \end{bmatrix} \tag{18}$$

with t_r as the trace of the matrix $t_r = r_{11} + r_{22} + r_{33}$, and q_{d0} as the scalar part of the desired quaternion $\mathbf{q_d}$.

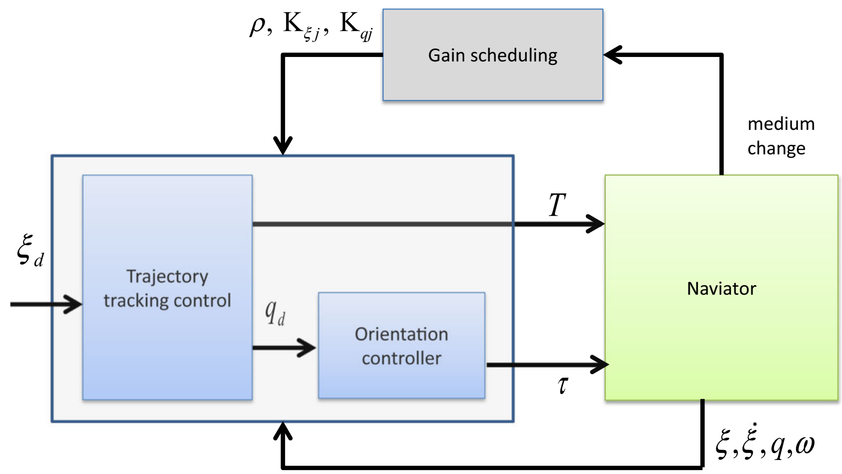

Fig. 4 Control strategy block diagram

4.2 Attitude Control

Once the desired quaternion $\mathbf{q_d}$ is extracted from the trajectory tracking control, a nonlinear quaternion feedback is proposed inspired by the work on fully actuated underwater vehicles [13]. Defining the quaternion error as $\tilde{\mathbf{q}} = \mathbf{q_d}^{-1}\mathbf{q}$, the following control law renders the attitude subsystem stable for a constant density

$$\boldsymbol{\tau} = -\mathbf{K}_{qd}\Omega - \mathbf{K}_{qp} sgn(\tilde{q}_0)\tilde{\bar{\mathbf{q}}} \tag{19}$$

where $sgn()$ is the sign function s.t. $sgn(q_0) = \frac{q_0}{\|q_0\|}$ for $q_0 \neq 0$. $\mathbf{K}_{qd}$ and $\mathbf{K}_{qp}$ are diagonal positive definite gain matrices.

To deal with the drastic change in density, we propose a gain scheduling controller [22]. A gain scheduler is an adaptive control scheme to deal with a change in the process conditions when such a change is well known and can be easily detected. It results in a non linear feedback whose parameters change as a function of the operating conditions, in this case the change of medium. This strategy is easy to implement and well suited for UAUVs, where one or more low-cost water sensors can be easily added to the vehicle. When the medium changes, the control parameters are updated to their adequate values for the respective medium, along with the corrected density value. Then, defining P as a set of parameters and control gains which depend on the medium, such as $P = (\rho, \nu, \mathbf{K_{\xi p}}, \mathbf{K_{\xi i}}, \mathbf{K_{\xi d}}, \mathbf{K_{qp}}, \mathbf{K_{qd}})$, the set P is updated as

$$P = \begin{cases} P_{air} & z > 0 \\ P_{water} & z < 0 \end{cases} \tag{20}$$

where P_{air}, P_{water} are the set of suitable parameters for each medium, previously selected.

5 Numerical Simulations

In order to study the behavior of UAUVs in different mediums and test the performance of the proposed control algorithms, several simulations were carried out with the help of MATLAB-Simulink®. In this section, we present some results for tracking of a spiral trajectory in both mediums, air and water. Table 1 presents the parameters employed during the simulations.

5.1 Aerial Operation

As confirmed through Figs. 5, 6, 7 and 8, aerial operation of multirotors requires the vehicle to operate in an orientation close to zero, directing the thrust force vector mostly upwards to compensate for gravity. Also, small angle variations will produce relatively large accelerations and fast response. The trajectory consists of a spiral with 4 m radius and 20 s period, as depicted in three-dimensions in Fig. 5. The vehicle starts −2 m away from the desired trajectory along the x axis, 1 m in y and 2 m in z, as can be appreciated in Fig. 6, where the position (top) and the position errors (bottom) are depicted. We can observe that the errors quickly converge to zero and always remain small. In particular, the vehicle starts 2 m over the desired

Table 1 Simulation parameters

$m[k_g]$	3.85	$g[k_g m/s^2]$	9.81
$V[m^3]$	4×10^{-3}	$\mathbf{J} \times 10^{-3}$	$\mathbf{I}_3[69, 69, 11.9]^T$
$K_m \times 10^{-5}$	8.49	b	0.1
$l[m]$	0.27	$\mathbf{C_{Dv}A_v}$	0.45
$\mathbf{C_{D\Omega}A_{D\Omega}}$	1	$\mathbf{K}_{qp}$	$\mathbf{I}_3(200)$
$\mathbf{K}_{qd}$	$\mathbf{I}_3(5)$	(air) $\mathbf{K_{\xi p}}$	99
(air) $\mathbf{K_{\xi d}}$	50	(air) $\mathbf{K_{\xi i}}$	0.01
(water) $\mathbf{K_{\xi p}}$	50	(water) $\mathbf{K_{\xi d}}$	0.5
(water) $\mathbf{K_{\xi i}}$	0.01		

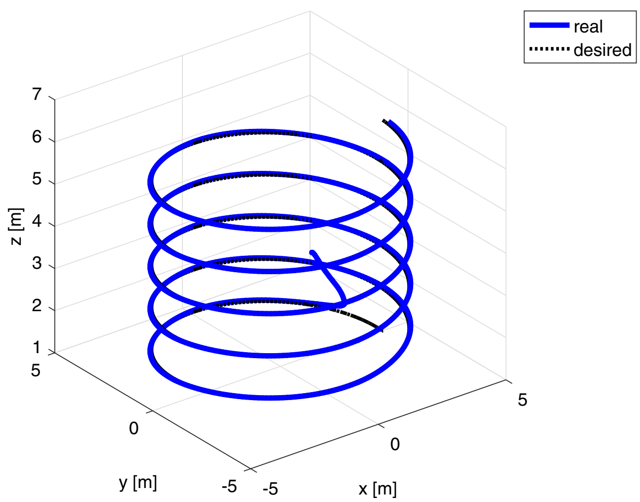

Fig. 5 3D visualization of the trajectory tracking in air. The desired trajectory consists of a spiral with 4 m amplitude and 20 s period. The vehicle is able to follow the spiral trajectory with good performance

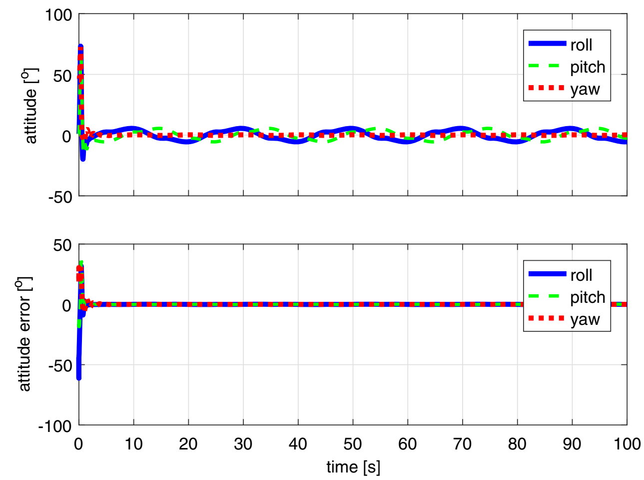

Fig. 7 Trajectory tracking attitude in air. The desired orientation is provided by the position controller. The attitude (top) remains mostly close to the origin to compensate gravity. The attitude errors (bottom) quickly converge to the origin and stay close to zero

position in altitude, and has to turn upside down in order to reach the target reference. This can be seen from Fig. 7, where the vehicle's attitude (top) and the attitude errors (bottom) are depicted. Euler angles are used only to show the results in a more intuitive way. Once the drone is close to the desired trajectory, it turns again to direct the thrust vector opposite to the gravity, and the vehicle operates within small angles while in air. The orientation controller shows good performance as the errors quickly converge to the equilibrium point.

Figure 8 presents the control inputs, with thrust on top and torques at the bottom. From this Figure, we can highlight that the thrust force converges to the magnitude of the weight of the vehicle and operates around this point, since its main objective in air is to compensate for gravity.

5.2 Underwater Operation

In contrast with aerial operation, underwater, the vehicle must deal with larger drag and buoyancy forces produced by the significant increase in the medium density of about three orders of magnitude. In this sense, the vehicle's behavior must vary as well. The buoyancy force can be used by design to compensate gravity, leaving the thrust only to translate the vehicle. We consider a slightly buoyant vehicle ($g < \rho V$), since this property allows for an easy recovery

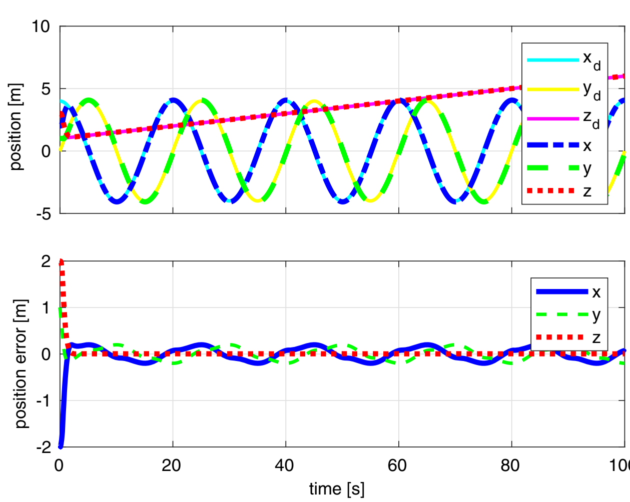

Fig. 6 Trajectory tracking position in air. Real (dashed lines) vs desired position (continuous lines) on top. The position errors (bottom) quickly converge to zero and remain well bounded

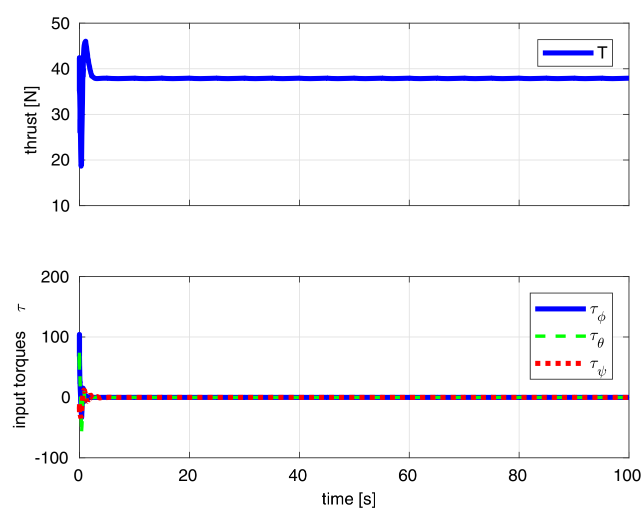

Fig. 8 Trajectory tracking control inputs in air. The thrust force (top) is used mainly to compensate for gravity. Small changes in the torque inputs (bottom) produce significant motions of the vehicle in air

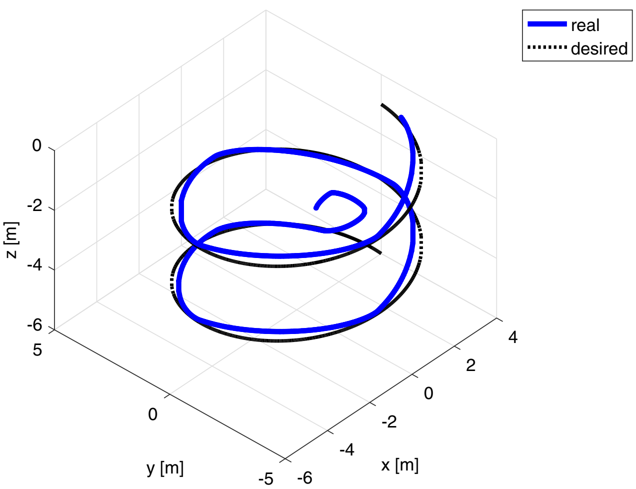

Fig. 9 3D visualization of the trajectory tracking underwater. The desired trajectory consists of a spiral with 4 m amplitude and 50 s period. The larger drag force underwater acts as a natural damper to the system, hindering the motion of the vehicle. Henceforth, only slower trajectories are accomplished underwater

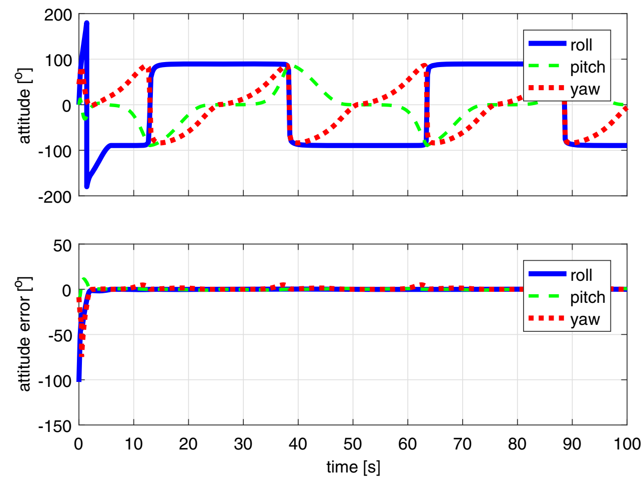

Fig. 11 Trajectory tracking attitude underwater. The desired orientation is provided by the position controller. Due to the increased drag and buoyancy forces underwater, the thrust force must be pointed in the direction of motion, hence, more aggressive orientations are commanded (top). The attitude errors (bottom) quickly converge to the origin and remain well bounded

in case of system failure in a real scenario. Therefore, the only way for the drone to dive is to turn upside down directing the thrust force downwards. Also, the increased drag force acts against the motion of the vehicle, such that it is required to direct the thrust force vector in the direction of motion. Hence the importance of a singularity-free representation is highlighted in this case. All of this can be observed in Figs. 9, 10, 11 and 12, where the operation of the drone during a trajectory tracking mission underwater is studied. The desired trajectory, as seen in Fig. 9 consists of a spiral with radius of 4 m and a period of 50 s. Note that the spiral chosen underwater is much slower than the one in air, because the motion of the vehicle is much slower underwater due to the larger drag force, as is verified in Fig. 10 where the position (top) and position errors (bottom) are presented. Although the trajectory tracking is accomplished satisfactorily, we can observe a little decay in the tracking performance, with slower response resulting in larger errors which is expected.

As stated earlier, to best overcome the large drag force, the thrust force vector is directed toward the direction of motion, resulting in larger rotations. This can be analyzed from Fig. 11, where the orientation of the drone (top) is shown along with the orientation errors. We can observe

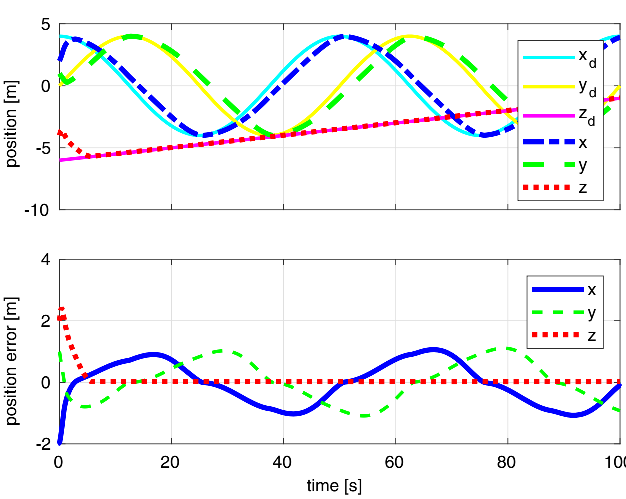

Fig. 10 Trajectory tracking position underwater. Real (dashed lines) vs desired position (continuous lines) on top. The position errors (bottom) remain bounded, but the increased drag force underwater slows down the system response, resulting in larger errors

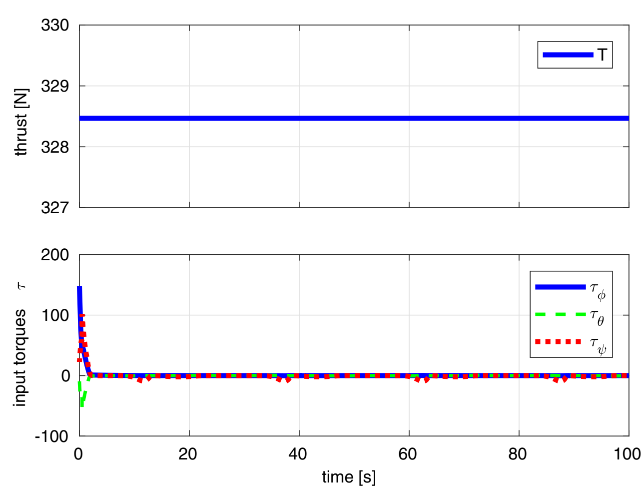

Fig. 12 Trajectory tracking control inputs underwater. In order to overcome the larger drag force and keep track of the desired position, the thrust force (top) is saturated to its maximum and oriented in the direction of motion. Input torques (bottom)

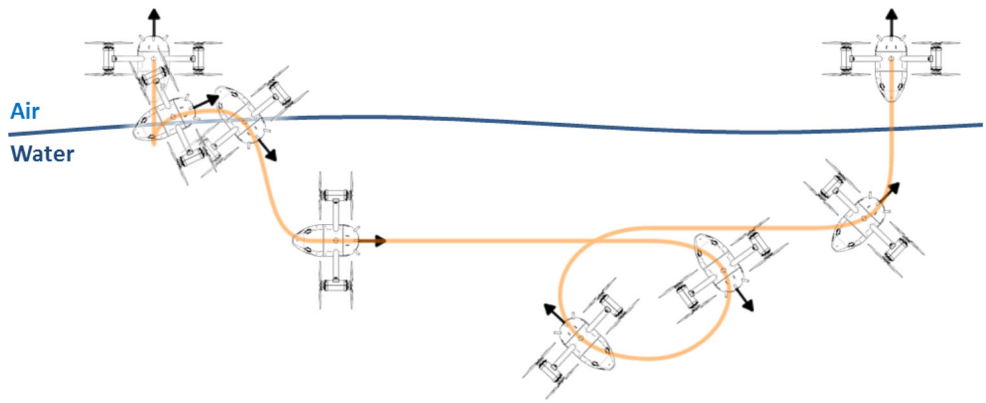

Fig. 13 An illustration of the mission conducted experimentally showing the Naviator performing a vertical loop underwater

aggressive angle changes to drive the vehicle to the desired position, specially in roll. It is interesting to notice that the thrust force was saturated to the maximum during the entire mission, as can be seen in Fig. 12 in which the control input signals are presented, with thrust on top and torques at the bottom.

From the simulations we can conclude that the proposed control strategies are suitable for trajectory tracking of underactuated UAUVs with full torque actuation and a single thrust force, in multiple mediums. We can also appreciate the main effects of the density change for different mediums, and the resulting difference in the behavior of the vehicle.

6 Experiments

To verify the results of the numerical simulation, experiments were carried out on the fifth prototype iteration of the experimental platform, the "Naviator-NV5" (see Fig. 1). The rotor configuration and corresponding multirotor dynamics of the actual vehicle align closely with the dynamic model presented in Section 3. Briefly, some of the most important characteristics of the Naviator-NV5 are in its customized rotor driver firmware, autopilot firmware and rugged watertight construction, which allow the vehicle to operate at low rotor speeds when deep underwater and high rotor speeds when in the air. Additional water and altitude/depth sensors together with the inertial measurement units make it possible to directly measure four states of the vehicle $\mathbf{y}_m = [\,\Omega^{\mathrm{T}}\ z\,]^{\mathrm{T}}$, which are utilized in the proposed quaternion control strategies. Prior to using quaternions, the vehicle was limited to conducting missions with pitch angles limited to $|\theta| < 90°$ due to Euler angle singularities, which meant that it was impossible for the thrust vector to have a negative vertical component to drive the vehicle downward in the case of neutral or positive buoyancy. Therefore, the vehicle was designed to be slightly negatively buoyant and essentially hover in water, but with very little thrust required. In that scheme, to descend deeper into the water, the rotors spin extremely slow or not at all, only turning on to correct attitude during the descent. More details on the platform and the angle control, using Euler angles, of the Naviator-NV5 with angles approaching (but not hitting) the singularity while maintaining depth is found in [17]. With quaternions, the singularity problem is no longer a concern and to demonstrate the ability of the vehicle to go further than 90° on the pitch, a vertical loop maneuver underwater was showcased experimentally as shown in Fig. 13. Although transition in and out of water is depicted, it has already been shown in prior works that the vehicle is capable of seamless transition and since the topic is on singularity-free control of a multi-medium vehicle, the focus will be on the underwater vertical loop. Note that to transition from air

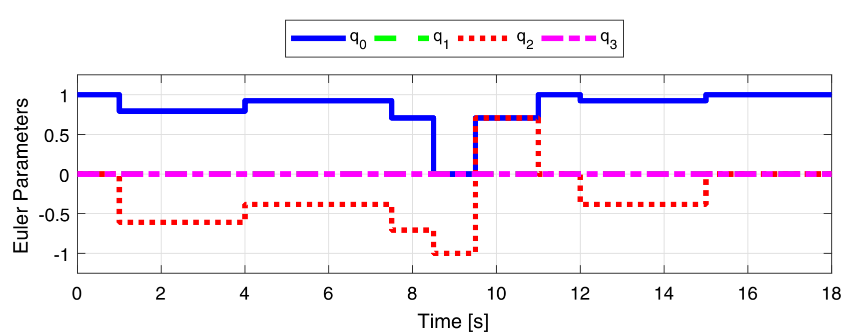

Fig. 14 The input quaternion references throughout the underwater vertical loop mission showing the value of each Euler parameter over time

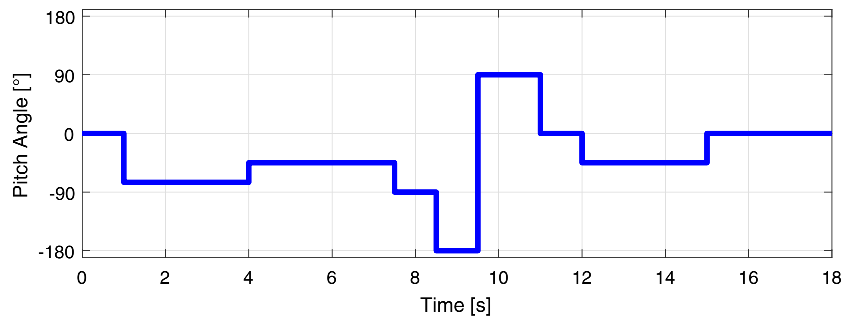

Fig. 15 The value of the pitch angle throughout the underwater vertical loop mission obtained from converting the Euler parameters in Fig. 14 to Euler angle where throughout the mission, the roll and yaw references are zero

to water, for the case under consideration of positive buoyancy, a half flip maneuver is required at the surface or close to it, in order to direct the thrust force downwards. To drive the vehicle through the vertical loop, quaternion step inputs were uploaded to the Naviator-NV5 and the vehicle executed these inputs autonomously. Throughout the mission, the depth input reference was fixed and during moments when the vehicle's thrust vector vertical component made it impossible to control depth, a default fixed throttle value was used. The quaternion inputs are shown in Fig. 14 and their converted values in a more intuitive pitch angle is shown in Fig. 15.

A video of the experiment showing several runs of the same mission can be found in https://youtu.be/QwOMgMPBoBQ and confirms that a vertical loop was successfully executed and matches very closely qualitatively with the desired mission shown in Fig. 13. Furthermore, despite a step reference being used as the input instead of a gradual trajectory, the vehicle was able to converge to the desired state satisfactorily.

7 Conclusions and Perspectives

In this work we have studied a relatively new class of unmanned vehicles that is capable of effectively operating in different mediums and seamlessly transitioning between them, the Unmanned Aerial-Underwater Vehicles. More in particular, we considered underactuated multirotor vehicles with full torque actuation and a single thrust force pointing along the vehicle's vertical direction.

A quaternion based representation was employed to avoid singularities, enabling aggressive flight maneuvers in air, and more importantly, downward navigation underwater when the vehicle is positively or neutrally buoyant.

A PID with compensation of the restoring forces was proposed for trajectory tracking in position, were the desired orientation is used as a virtual control input. Then, a nonlinear quaternion feedback was used for attitude control. Numerical simulation studies verified the validity of the proposed algorithms for multiple mediums, after a suitable adaptation of the parameters and gains by means of a gain scheduling strategy.

The results of the simulations were further supplemented by real-time experiments on a multi-medium multirotor platform "Naviator-NV5" which showed that the proposed quaternion control strategy allows the vehicle to conduct vertical loops underwater, even under aggressive step input references.

UAUVs present a wide range of applications, opportunities and challenges which combine those of aerial and underwater vehicles. Localization of aerial underwater vehicles appears as a major challenge without a straightforward solution, since GPS signals are blocked in denser mediums such as water, and acoustic sensors normally used underwater are too heavy of a payload for small aerial vehicles. Future works are encouraged in this regard.

Acknowledgments This work was supported by Office of Naval Research (ONR), Grant No. N00014-15-2235 with Dr. Thomas McKenna serving as Program Manager.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

References

1. Kumar, V., Michael N.: Opportunities and challenges with autonomous micro aerial vehicles. In: International Symposium on Robotics Research, Flagstaff (2011)
2. Kinsey, J., Eustice, R., Whitcomb, L.: A survey of underwater vehicle navigation: recent advances and new challenges. In: Proceedings of the IFAC Conference of Maneuvering and Control of Marine Craft, p. 88. Lisbon (2006)
3. Unknown: Denmark amazing submarine plane. In: Modern Mechanics and Inventions, vol. 25, pp. 74–75 (1930)
4. Petrov, G.: Flying submarine. www.Airforce.ru - russian air force. http://www.airforce.ru/aircraft/miscellaneous/flying_submarine/index.htm (1995). Accessed 15 July 2017

5. Drews, P. .Jr., Neto, A., Campos, M.: Hybrid unmanned aerial underwater vehicle: modeling and simulation. In: International Conference on Intelligent Robots and Systems (IROS), Chicago (2014)
6. Neto, A., Mozelli, L., Drews, P. .Jr., Campos, M.: Attitude control for an hybrid unmanned aerial underwater vehicle: a robust switched strategy with global stability. In: International Conference on Robotics and Automation (ICRA). Sweden, Stockholm (2016)
7. Maia, M., Soni, P., Diez-Garias, F.: Demonstration of an aerial and submersible vehicle capable of flight and underwater navigation with seamless air-water transition. arXiv:1507.01932 (2015)
8. Wen, J., Kreutz-Delgado, K.: The attitude control problem. IEEE Trans. Autom. Control. **36**(10), 1148–1162 (1991)
9. Castillo, P., Lozano, R., Dzul, A.: Modelling and Control of Mini-flying Machines. Springer, Londres (2005)
10. Hamel, T., Mahony, R., Lozano, R., Ostrowsky, J.: Dynamic modeling and configuration stabilization for a X4-Flyer. In: 15th Triennial IFAC World Congress. Barcelona (2002)
11. Cassau, P., Sanfelice, R., Cunha, R., Cabecinhas, D., Silvestre, C.: Robust global trajectory tracking for a class of underactuated vehicles. Automatica **58**, 90–98 (2015)
12. Joshi, S., Kelkar, A., Wen, J.: Robust attitude stabilization of spacecraft using nonlinear quaternion feedback. IEEE Trans. Autom. Control. **40**(10), 1800–1803 (1995)
13. Fjellstad, O., Fossen, T.: Quaternion feedback regulation of underwater vehicles. In: Proceedings of the Third IEEE Conference on Control Applications (1994)
14. Lee, T.: Global exponential attitude tracking controls on SO(3). IEEE. Trans. Autom. Control. **60**(10), 2837–2842 (2015)
15. Frazzoli, E., Dahleh, M., Feron, E.: Trajectory tracking control design for autonomous helicopters using a backstepping algorithm. In: Proceedings of the American Control Conference, Chicago (2000)
16. Mercado, D., Maia, M., Diez, F.: Aerial-underwater systems, a new paradigm in unmanned vehicles. In: International Conference on Unmanned Aircraft Systems (2017), Miami (2017)
17. Maia, M., Mercado, D., Diez, F.: Design and implementation of multirotor aerial-underwater vehicles with experimental results. In: International Conference on Intelligent Robots and Systems (IROS), Vancouver (2017)
18. Trawny, N., Roumeliotis, S.: Indirect Kalman filter for 3d attitude estimation. Technical report, University of Minnesota, Department of Computing Science and Engineering (2005)
19. Roskam, J.: Airplane flight dynamics and automatic flight controls. Roskam Aviation and Engineering Corporations, Lawrence (1982)
20. Fossen, T.: Guidance and Control of Ocean Vehicles. Wiley, England (1994)
21. Zhao, S., Dong, A., Farrell, J.: Quaternion-based trajectory tracking control of VTOL-UAVs using command filtered backstepping. In: American Control Conference (ACC), Washington (2013)
22. Astrom, K., Wittenmark, B.: Adaptive Control, 2nd edn, ch. 9, pp. 390–417. Addison-Wesley, Reading (1995)

D. Mercado was born in Mexico City, Mexico, on July 24, 1987. He received his B.S. degree in Mechatronics Engineering from the Universidad Panamericana in Mexico in 2010, the M.Sc. degree in Electrical Engineering option Mechatronics from CINVESTAV-IPN, Mexico in 2012, and the Ph.D. in Automation, Embedded Systems and Robotics from the University of Technology of Compiègne, France in 2015. His research topics include modeling and control of unmanned aerial and/or underwater vehicles, autonomous navigation, real-time embedded applications, data fusion and computer vision.

M. Maia received his Ph.D. in Mechanical and Aerospace Engineering from the School of Engineering, Rutgers University, in 2018. His interests are in robotics, propulsion and controls, and he worked primarily as an applied systems and control research engineer during his academic career at the Applied Fluids Laboratory in the Department of Mechanical and Aerospace Engineering at Rutgers University. Under the direction of his advisor, Prof. F. Javier Diez, Dr. Maia invented the Naviator, a vehicle capable of operating in both air and water, which secured funding for further development from the Office of Naval Research (ONR). Dr. Maia is also the CTO and member of SubUAS LLC, that commercializes the Naviator for both military and commercial applications.

F. J. Diez is the head of the Applied Fluids Laboratory in the Department of Mechanical Engineering at Rutgers University. He is an author of over 100 journal articles, conference papers, and technical publications, and has given over 20 invited lectures, in the areas of flow control, experimental fluids, and unmanned vehicles. He has also served as a reviewer of multiple journals and government evaluation panels. Dr. Diez's primary research interests lie in the areas of aerodynamics and propulsion. These include UAV/UUV platform development, fluidic sensors and actuators, active flow control, turbulent flows, laser flow diagnostics and multiphase flows, all of which have important and diverse applications. His research has been funded by industry and government agencies such as NSF, ONR, NASA, FAA, DARPA, and AFOSR among others. Dr. Diez is also the CEO/founder of SubUAS LLC, that commercializes the Naviator, (a hybrid air/underwater drone) for both military and commercial applications.