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Adaptive Control? But is so Simple!

A Tribute to the Efficiency, Simplicity and Beauty of Adaptive Control

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Abstract In a recent paper, a few pioneers of adaptive control review the classical model reference adaptive control (MRAC) concept, where the designer is basically supposed to conceive a model of the same order as the (possibly very large) plant, and then build an adaptive controller such that the plant is stable and ultimately follows the behavior of the model. Basically, adaptive control methods based on model following assume full-state feedback or full-order observers or identifiers. These assumptions, along with supplementary prior knowledge, allowed the first rigorous proofs of stability with adaptive controllers, which at the time was a very important first result. However, in order to obtain this important mathematical result, the developers of classical MRAC took the useful *scalar* Optimal Control feedback signal and made it into an adaptive gain-vector of basically of the same order as the plant, which again had to multiply the plant state-vector in order to finally end with another scalar adaptive control feedback signal. It is quite known today, however, what happens when this requirement is not satisfied, and when "unmodeled dynamics" distorts the controller based on these ideal assumptions. Even though much effort has been invested to maintain stability in spite of so-called "unmodeled dynamics," in some applications, such as large flexible structures and other real-world applications, even if one can assume that the order of the plant is known, one just cannot implement a controller of the same order as the plant (or even a "nominal" or a "dominant" part of the plant), before even mentioning the complexity of such an adaptive controller. Without entering the argument around their special reserve in relation to claimed efficiency of the particular L1-Adaptive Control methodology, this paper first shows that, after the first successful proof of stability and even under the same basic full-state availability assumption, the adaptive control itself can be reduced to just *one* adaptive gain (which multiplies *one* error signal) in single-input-single-output (SISO) systems and, as a straightforward extension, an m*m gain matrix in an m-input-m-output (MIMO) plant. Not only is stability not affected, but actually the simplified scheme also gets rid of most seemingly "inherent" problems of the adaptive control represented by classical MRAC. Moreover, proofs of stability have all been based on the so-called Barbalat's lemma which seems to require very strict uniform continuity of signals. The apparent implications are that any discontinuity, such as square-wave input commands or just some occasionally discontinuous disturbance, may put stability of adaptive control in danger, without even mentioning such things as

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impulse response. Instead, based on old yet amazingly unknown extensions of LaSalle's Invariance Principle to *nonautonomous* nonlinear systems, recent developments in stability analysis of nonlinear systems have mitigated or even eliminated most apparently necessary prior conditions, thus adding confidence in the robustness of adaptive scheme in real world situations.

Keywords Control systems · Adaptive control · Stability · Nonlinear systems · Autonomous and nonautonomous systems

1 Introduction

In a recent paper [80], a few pioneers of adaptive control review the classical model reference adaptive control (MRAC) concept, where the designer is basically supposed to conceive a model of the same order as the (possibly very large) plant, and then build an adaptive controller such that the plant is stable and ultimately follows the behavior of the model. Adaptive control methods based on model following assume full-state feedback or full-order observers or identifiers. These assumptions, along with supplementary prior knowledge, allowed the first rigorous proofs of stability with adaptive controllers.

It is quite known today, however, what happens when this requirement is not satisfied, and when "unmodeled dynamics" distorts the controller based on these ideal assumptions. Besides, the adaptive controller is basically of same order as the plant and again multiplies a state-vector. Even though much effort has been invested to maintain stability in spite of so-called "unmodeled dynamics," in some applications, such as large flexible structures and other real-world applications, even if one can assume that the order of the plant is known, one just cannot implement a controller of the same order as the plant (or even a "nominal" or a "dominant" part of the plant), before even mentioning the complexity of such an adaptive controller. Moreover, proofs of stability based on the so-called Barbalat's lemma seem to require very strict uniform continuity of signals. The implications seem to be that any discontinuity, such as square-wave input commands or just some occasionally discontinuous disturbance, put stability of adaptive control in danger, without even mentioning such things as impulse response.

Without entering the argument around their special reserve towards claimed efficiency of the particular L1-Adaptive Control methodology, which then seems to strongly dominate [80] (for a response see [67, 94]), recent papers [27, 29] remind us that, while the first basic adaptive Model Following concepts required the controlled plant *itself* to be strictly positive real (SPR), the move towards the classical MRAC methodology [80] managed to mitigate this condition and might have been the first adaptive control scheme which, assuming full-state availability, showed how to fulfill an SPR condition and ended with the first *rigorous* proofs of stability, without requiring the plant itself to be SPR.

Therefore, our intention is first of all to emphasize the importance of the MRAC original idea of satisfying a passivity conditions that allowed the guarantee of stability of adaptive control. More than 35 years ago, classical MRAC developers managed to introduce Lyapunov-style stability analysis and thus to end with the first rigorous proofs of stability of adaptive control systems. However, 35 years later, it will also be shown that this idea can be simplified and adjusted to fit the real-world applications and the basic knowledge that usually is available out there. In this context, even though this is not explicitly mentioned in [80], references [27, 29] show that, along with other assumptions, this success was first of all based on the use of the useful yet *scalar* Optimal Control combination signal $\mathbf{b}^T\mathbf{P}\mathbf{x}(t)$. Nevertheless, even though most certainly at the time this was a *first and necessary* step for seeking the proof of stability, a second look shows that, by using this *scalar* signal again in order to build a *full-order* adaptive gain-vector, classical MRAC might have spread its useful stabilizing properties around. Actually, because the Optimal Control would be $u(t) = r^{-1}\mathbf{b}^T\mathbf{P}\mathbf{x}(t)$, it shows that the signal $\mathbf{b}^T\mathbf{P}\mathbf{x}(t)$ already *is the* scalar state-feedback signal (with only the addition of a proper scaling parameter). In an unknown system, the proper weight (here, r^{-1}) is not known and so, invites adaptive thinking and yet, is this the reason for making this scalar state-feedback signal back into an adaptive gain-vector of the order of the plant?

Therefore, the Simple Adaptive Control idea is that, following along the same lines of Optimal Control and even under *the same basic full-state availability* *assumption* as classical MRAC, the *adaptive* control itself can be reduced to just *one* adaptive gain (which multiplies only *one* error signal) in SISO systems and, as a straightforward extension, an *m*m* gain matrix in an *m*-input-*m*-output MIMO plant. Not only is stability *not* affected, but the simplified scheme actually seems to get rid of most seemingly "inherent" problems of classical MRAC.

Moreover, the long experience of classical control design teaches us that any plant "model" is only an approximate representation of the actual plant, approximation which can only reproduce some of the actual plant properties in such a way that some control design is possible. Some of these properties are obtained by frequency response, wind-tunnel experiment, etc. Even if for a moment one considers the plant to be perfectly linear time invariant (LTI), its "representation" could be just a step-response line in the time-domain or a frequency response line in the frequency domain. This line may show a plant of very large order n=30, 150, 2000, or even infinite. However, using large order models in design may introduce numerical problems that may make any design impossible. Therefore, while a very reduced model, such as n=3 or 5, could be too crude, one may decide to use a model of order, say, n=10 or 15, just good enough to reproduce the experimental data or, in other words, the actual plant *properties*, with sufficient approximation. In this context, one cannot help but admire the outstanding advances of classical control design based on good use of these approximate representations of real-world plants.

However, one is also warned *that not too much weight* can be awarded to the model and to its order. Moreover, if in the LTI case, some unmodeled dynamics may not affect the design too much if it does not affect the reproduction of the experimental data too much, things are not necessarily so in the nonstationary case, when even unmodeled dynamics that may not affect stability in the LTI world, can lead to total destruction in the nonstationary world [29, 91, 147].

On the other hand, there is almost no case when the lack of precise knowledge can actually keep the classical control designer from performing *some* basic design. Planes and missiles fly, robots perform fast and precise tasks, etc. Nevertheless, because some basic design using the parameters of some *nominal* plant has to perform with actual parameters of the actual plant, which could be not only different from the nominal, but also nonlinear *and* time-varying during the operation, the safety of operation of real plant is heavily checked for the predictable operational conditions using Monte-Carlo and similar tests. However, because one controller must guarantee safe operation under various conditions, performance *has* to be limited. Here, the adaptive control idea of fitting the right control parameter to the right situation is very attractive, assuming however that, first of all, stability of the *nonstationary* adaptive control system is guaranteed. This takes us all the way back to the so-called MIT-rule in adaptive control [136, 177], which we consider a very clever and even ingenious *Engineering* idea. However, as it ended in an unfortunate disaster, it also proved that, in the nonstationary world, even ingenuity may not be enough and that theoretical *guarantee* of stability is needed before one can think of improving performance.

The Simple Adaptive Control methodology of this paper attempts to explain why a classical control designer would want to even consider Adaptive Control. In this context, right from its beginning, the Simple Adaptive Control methodology proved to be good control, with very good results. This approach is based on the idea that Adaptive Control is not called to do everything, but rather to improving the limited performance of classical design. Nevertheless, as the mere use of nonstationary gains (even within the assumable "admissible domain") may lead to total instability [147], the Simple Adaptive Control methodology first tries to establish what available prior knowledge about the actual plant *properties* that was available for classical design could also *guarantee* stability with nonstationary adaptive control. This is only a first, yet vital, conditions that then allows also reaching better performance. Like any nonstationary and nonlinear system methodology, progress here has been slow and required patience, as intermediate theoretical results might have looked very poor.

We recall that, even though now it is the basis for any modern stability analysis, the Lyapunov stability approach itself needed some 60 years before people started really paying attention to it. Moreover, after its first happy and successful applications, it soon became clear that finding an adequate Lyapunov function which has a negative *definite* derivative is not that common in other than class-room examples. However, after many small steps that followed Lyapunov, and in particular, based on *largely unknown* extensions of LaSalle's Invariance Principle to *nonautonomous* nonlinear systems, recent developments in stability analysis have mitigated or eliminated most apparently necessary prior conditions, thus adding confidence in the robustness of adaptive and nonlinear control schemes in realistic situations [27–30].

Before the following presentation, it is worth recalling the following disclaimer that begins reference [91]: "At no stage is it intended to claim that the control problem is simple, or that adaptive control methods are meant to obviate the need for diligent learning and accumulation of knowledge or for the patient modeling of plants, or to replace established control principles. This being said, readers are encouraged to test the described adaptive techniques, either with the numerous and various examples in this book or with their own examples. Only this way can one get some feeling and understanding of the authors' own surprise and enthusiasm with the performance that can be obtained using minimal prior knowledge about the controlled plant."

In spite of the disclaimer above, as a result of the advances in both the theory of simple adaptive control and in the general nonlinear systems stability analysis, this paper can claim that *the same conditions which are sufficient to guarantee stability with classical control design are also sufficient to guarantee stability with the specific methodology called Simple Adaptive Control (SAC)*. This basic guarantee is only the *first and necessary* step, which then allows improving performance to levels that would not have seemed to be obtainable otherwise.

Section 2 gives a brief review of Optimal Control that the Simple Adaptive Control approach will later carry into the world of uncertainty. Section 3 first reviews the Model Reference Adaptive Control methodologies, their important contributions to first rigorous proofs of stability and shows how further development can simplify the MRAC algorithm and at the same time mitigate some of its apparently inherent problems. Section 4 revisits the passivity conditions that can guarantee stability of adaptive control schemes. Section 5 shows how the Simple Adaptive Control (SAC) approach simplifies the classical MRAC and moves from basically state-feedback approach towards output-feedback control in system that satisfy the "almost passivity" conditions. Section 6 then shows that basic stability and stabilizability properties of classical control systems are sufficient prior knowledge that can *guarantee* stability of the SAC scheme through parallel feedforward. Section 7 presents a brief review of old and new results in stability analysis, meant to eliminate the confusion that seems to exist in present literature. Its ultimate result, the new Theorem of Stability, is a direct extension of the original Lyapunov Stability theorem to the case of *semidefinite* Lyapunov derivative. In turn, it now guarantees stability of adaptive control systems, without requiring the "customary" uniform continuity conditions and other apparently necessary conditions.

2 Optimal Control or Control *before* Adaptive Control

The approach of this paper to Simple Adaptive Control (SAC) assumes that adaptation is not meant to replace, but only to help classical control design. In this context, it intends to show that, even if one assumes that full knowledge on the plant to be controlled is available, fitting the right control parameters to the right operational situation can improve performance. Moreover, adaptive techniques are also meant to maintain satisfactory performance in spite of uncertainty or time-variation of plant parameter.

Note: Contrary to what some classical MRAC Colleagues may claim, this section is not meant "to teach the well known Optimal Control." On the contrary, it is mentioned to emphasize the useful properties of Optimal Control solution that classical MRAC actually spoiled in order to end with a first successful mathematical result. Therefore, 35 year later, we show that classical MRAC not only can be simplified, but also with much better properties.

Towards the following presentation, it is useful to first recall the basic Optimal Control Theory for LTI systems of the form

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t). \tag{2}$$

The Optimal Controller that minimizes the criterion

$$J = \int_0^t \left(\mathbf{x}^T(\tau)\mathbf{Q}\mathbf{x}(\tau) + \mathbf{u}^T(\tau)\mathbf{R}\mathbf{u}(\tau) \right) d\tau \tag{3}$$

where both $\mathbf{Q}$ and $\mathbf{R}$ are Positive Definite symmetric, is given by the *state-feedback* control

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t) = -\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}\mathbf{x}(t) \tag{4}$$

Here, $\mathbf{P}$ is the solution of the algebraic Riccati equation (ARE)

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T\mathbf{P} = -\mathbf{Q} \tag{5}$$

The Optimal Control solution requires using the solution (4). However, we rewrite (5) as

$$\mathbf{P}\left(\mathbf{A} - \frac{1}{2}\mathbf{B}\mathbf{K}\right) + \left(\mathbf{A} - \frac{1}{2}\mathbf{B}\mathbf{K}\right)^T \mathbf{P} = -\mathbf{Q} \tag{6}$$

Equation (6) is the Lyapunov equation for the closed-loop system and shows that using half the value of the Optimal gain already guarantees asymptotic stability of the closed-loop system. The optimal gain $\mathbf{K}$ in Eq. (4), which is double the stabilizing gain in Eq. (6), implies that the Optimal Control solution guarantees (at least) 6 dB stability gain margin from below.

If, for convenience (for now), we denote $\mathbf{P}\mathbf{B} = \mathbf{C}^T$ and $\frac{1}{2}\mathbf{R}^{-1} = \mathbf{K}_e$, and define a fictitious 'output' signal $\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t)$, we get from Eq. (6)

$$\mathbf{P}\left(\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C}\right) + \left(\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C}\right)^T \mathbf{P} = -\mathbf{Q} \tag{7}$$

Therefore, the closed loop system

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C})\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{8}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{9}$$

satisfies the Strict Positive Realness relations

$$\mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C}) + (\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C})^T\mathbf{P} = -\mathbf{Q} \tag{10}$$

$$\mathbf{P}\mathbf{B} = \mathbf{C}^T \tag{11}$$

Therefore, the answer to the question raised by Kalman's seminal paper [90] "When is Linear System Optimal?" is "When it satisfies a Strict Positive Realness (SPR) condition."

What is important for the following developments is that if one can use the *particular* state combination $\mathbf{y}(t) = \mathbf{B}^T\mathbf{P}\mathbf{x}(t) = \mathbf{C}\mathbf{x}(t)$, the entire *remaining* Optimal Controller is just a $m * m$ square gain $\mathbf{K}_e$ in the multivariable case, or just a simple *scalar* gain k_e in the SISO case. Moreover, not only does this gain selection shown above stabilize the closed loop system, but also *any* positive addition to this gain maintains stability. This property is in particular important for Nonlinear and Adaptive Control because, as shown in [27, 29], one can also use any *nonlinear and nonstationary* gain $\mathbf{K}(\mathbf{x}, t) = \mathbf{K}_e + \Delta(\mathbf{x}, t)$ where all that must be known about the nonlinear addition is that $\Delta(\mathbf{x}, t) \geq 0$ (or uniformly Positive Semi-Definite for matrices). With such a controller, the Lyapunov algebraic equation for the new closed-loop system is

$$\begin{aligned}\mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K}(\mathbf{x}, t)\mathbf{C}) \;+\; (\mathbf{A} - \mathbf{B}\mathbf{K}(\mathbf{x}, t)\mathbf{C})^T\mathbf{P} &= \mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C}) \\ &\quad + (\mathbf{A} - \mathbf{B}\mathbf{K}_e\mathbf{C})^T\mathbf{P} \\ &\quad -2\mathbf{C}^T\Delta(\mathbf{x}, t)\mathbf{C} \leq -\mathbf{Q} < 0\end{aligned} \tag{12}$$

and it shows that the closed-loop system remains stable for any $\Delta(x, t) \geq 0$, even arbitrarily large (see also [91], pp. 48–49).

In order to prevent eventual comments that SAC may *require* high gains, we mention that the above property only *guarantees* that stability is *not* threatened if, during adaptation stage the adaptive gains must increase. The adaptive algorithm then makes sure that the loop-gain increases if stability is in danger and yet, no more than needed. Moreover, it is worth emphasizing that this *guarantee* of stability is only a *first* though vital step, which ultimately allows obtaining superior performance *without* the need for high loop-gains.

3 Model Reference Adaptive Control

3.1 First Model Reference Adaptive Control Configurations

First attempts at using adaptive control techniques, such as the MIT-rule [136, 177], were developed during the sixties and were based on intuitive and possibly ingenious ideas, yet they ended in failure, mainly because at the time there was not very much knowledge of stability analysis with nonstationary parameters. Modern methods of stability analysis that had been developed by Lyapunov at the start of the 19th century [110] were not broadly known, much less used, in the West [63, 148]. Nevertheless, we remind the MIT approach because, once stability conditions *can* be analyzed and properly guaranteed, fitting the right control gain to the right situation ultimately leads to clear advantages upon using fixed controllers.

After the initial problems with adaptive control techniques of the sixties, stability analysis has become a center point in new developments related to adaptive control. Participation of some of the leading researchers in the control community at the time, such as Monopoly, vanAmerongen, Narendra, Landau, Åström, Grimble, Kokotovič, Goodwin, Morse [3, 11, 12, 55, 61, 62, 81, 102, 103, 117, 118, 120, 128–130] and many others [5, 47, 68, 82, 97, 98, 114, 124, 127, 135, 143, 151, 154, 174], added a remarkable contribution to the better modeling and to the understanding of systems adaptive control methodologies.

New tools and techniques have been used or specifically developed for rigorous stability analysis and they finally led to successful proofs of stability, mainly based on the Lyapunov approach. The standard adaptive control methodology was the Model Reference Adaptive Control approach which, as its name states, basically requires the "bad" - or just not so good - plant to follow the behavior of a "good" Model Reference.

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m\mathbf{x}_m(t) + \mathbf{B}_m\mathbf{u}_m(t) \tag{13}$$

$$\mathbf{y}_m(t) = \mathbf{C}_m\mathbf{x}_m(t) \tag{14}$$

Here, before even mentioning adaptation, we recall a very basic idea of Model Following, where the control signal that feeds the plant is a linear combination of the Model state variables

$$\mathbf{u}(t) = \sum k_i x_{mi}(t) = \mathbf{K}\mathbf{x}_m(t) \tag{15}$$

In the deterministic case [54], if the plant closed-loop system were just stable and the plant parameters were fully known, one could compute the gains of the corresponding feedforward signals from the model in such a way that would force the not-so-good plant to asymptotically behave exactly like the good Model, or $\mathbf{x}(t) \to \mathbf{x}_m(t)$ and correspondingly $\mathbf{y}(t) \to \mathbf{y}_m(t)$. This way, external signals supplied by the Model allowed good tracking with very low errors and even asymptotically perfect tracking, *without* requiring high gain and bandwidth from the problematic closed-loop system.

When the plant parameters are not (entirely) known, one is naturally led to use adaptive control gains. The basic idea is that the plant is fed a control signal that is a linear combination of the model state through some gains. If not all gains are correct, the plant does not exactly behave like the Model, and its measured output differs from the output of the Model Reference. The resulting "tracking error"

$$\mathbf{e}_y(t) = \mathbf{y}_m(t) - \mathbf{y}(t) \tag{16}$$

can be monitored and used to generate adaptive gains. The basic idea of adaptation is like that: assume that one component of the control signal that is fed to the plant comes from the variable x_{mi} through the gain k_{xi}. If the gain is not perfectly correct, this component contributes to the tracking error and therefore the tracking error and the component x_{mi} are correlated. This correlation is used to generate the adaptive gain $\dot{k}_{x_i}(t) = \gamma_i e_y(t) x_{m_i}(t)$, where γ_i is just a parameter that affects the rate of adaptation. The adaptation continues until, under appropriate conditions to be further discussed, the correlation diminishes and ultimately vanishes and therefore the gain derivative tends to zero and the gain itself is (hopefully) supposed to go to a constant value. In vectorial form,

$$\dot{\mathbf{K}}_x(t) = \Sigma\gamma_i e_y(t) x_{mi}(t) = \mathbf{e}_y(t)\mathbf{x}_m^T(t)\Gamma_x \tag{17}$$

$$\mathbf{u}(t) = \Sigma k_{xi} x_{mi}(t) = \mathbf{K}_x(t)\mathbf{x}_m(t) \tag{18}$$

As also observed, there are various other components that can be added to improve the performance of the MRAC system such as $\dot{\mathbf{K}}_u(t) = \mathbf{e}_y(t)\mathbf{u}_m^T(t)\Gamma_u$, so the total control signal is

$$\mathbf{u}(t) = \mathbf{K}_x(t)\mathbf{x}_m(t) + \mathbf{K}_u(t)\mathbf{u}_m(t) \tag{19}$$

The basic approach was able to generate the first rigorous proofs of stability that showed that not only the tracking error but actually the entire state error

$$\mathbf{e}_x(t) = \mathbf{x}_m(t) - \mathbf{x}(t) \tag{20}$$

asymptotically vanishes. This result implied that the plant behavior would asymptotically reproduce the stable model behavior. In particular, the Lyapunov stability technique revealed the prior conditions that had to be satisfied in order to guarantee stability and allowed getting rigorous proofs of stability of the adaptive control system. Because along with the dynamics of the state or the state error, adaptive control systems have also introduced the adaptive gains dynamics, the positive definite quadratic Lyapunov function had to contain both the errors and the adaptive gains and usually had the form

$$\begin{aligned} V(t) = {} & \mathbf{e}_x^T(t)\mathbf{P}\mathbf{e}_x(t) \\ & + tr\left[\left(\mathbf{K}(t) - \widetilde{\mathbf{K}}\right)\Gamma^{-1}\left(\mathbf{K}(t) - \widetilde{\mathbf{K}}\right)^T\right] \end{aligned} \tag{21}$$

Here, $\widetilde{\mathbf{K}}$ is a set of the ideal gains that could perform perfect model following if the parameters were known, and that the adaptive control gains were supposed to asymptotically reach.

This approach ended in the first rigorous proof of stability and yet, in spite of successful proofs of stability, there was no use of these adaptive control techniques in practice, because of some of the problems that are inherent to the basic MRAC approach. The weakest point of basic MRAC was that stability of the adaptive control could be guaranteed *only* if the original plant were Strictly Passive (SP), which in LTI systems implies that its input-output transfer function is Strictly Positive Real (SPR). (We note that, while the plant should be rigorously called SP and only its transfer function should be called SPR, these names are very often interchanged in LTI systems.)

Nevertheless, the basic Model Following is another idea that is worth keeping here. In other words, once stability *can be guaranteed*, good or even perfect following can be achieved with external signals that come from the model, without having to require high gains and bandwidth from the internal closed-loop system.

3.2 Classical Model Reference Adaptive Control

It is interesting that at same time, because they were dealing with large systems, other researchers attempted a reduced order version of MRAC, that they called such modest names as "adaptive command generator tracking," or "adaptive *output* model following" and finally "simple adaptive control (SAC)" [13, 15, 35, 37, 40, 159–163, 175]. What was also interesting, the researchers observed that, although passivity (or SPR for LTI systems) was very important, the Plant to be controlled was not required to be "perfectly" SPR. As a first relaxation, it was sufficient if the Plant was separated by some unknown output feedback from being SPR and so, because only a constant feedback would separate them from being SPR, they were ultimately called "Almost SPR (ASPR)[15, 37]." Although, as we show below, these works were a direct and natural continuation of classical MRAC, the new direction was not well received by most main representatives of adaptive control community, which had difficulties accepting the new ***Almost*** SPR concept instead of the customary SPR concept. Moreover, no matter what applications were presented, a pretty common response was "another example cannot make this bad control method good."

However, because, along with the first proofs of stability, it became clear that basic MRAC required conditions that were not inherently satisfied by realistic applications, the mainstream adaptive control community had to also abandon the basic MRAC configuration and to replace it with the so-called "classical" MRAC configuration. Stability conditions were then fulfilled by first assuming full Plant state availability, then other elements such as full-order adaptive observers, etc., and can be found in the references cited above. As described in the recent [80], because "classical" MRAC was developed for SISO plants, the Plant has the basic representation

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}(u(t) + \theta^{*T}\mathbf{x}(t)) \tag{22}$$

The plant is required to follow the Model

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m\mathbf{x}_m(t) + \mathbf{b}_m u_m(t) \tag{23}$$

Under the basic assumptions that $\mathbf{A} = \mathbf{A}_m$ is a known Hurwitz matrix, θ^* an unknown constant vector, and vector $\mathbf{b}$ is known, the "classical" MRAC control solution that must force the Plant state $\mathbf{x}$ to ultimately follow the Model vector $\mathbf{x}_m$, is

$$\dot{\theta}(t) = \gamma\mathbf{e}^T(t)\mathbf{P}\mathbf{b}\mathbf{x}(t) \tag{24}$$

$$u(t) = -\theta^T\mathbf{x}(t) + \mathbf{k}_0^* u_m(t) \tag{25}$$

Here, $\mathbf{e}(t) = \mathbf{x}(t) - \mathbf{x}_m(t)$ is the "state error," $\mathbf{k}_0^*$ is a matching coefficient (to be defined below) and $\mathbf{P}$ is the solution of the Lyapunov stability equation

$$\mathbf{P}\mathbf{A} + \mathbf{A}^T\mathbf{P} = -\mathbf{I} \tag{26}$$

Substituting the control (25) in Eq. (22) gives

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{b}_m u_m(t) - \mathbf{b}\left(\theta^T(t) - \theta^{*T}\right)\mathbf{x}(t) \\ &\quad + \mathbf{b}k_0^* u_m(t)\end{aligned} \tag{27}$$

Subtract the Model Eq. (23) to get the error equation

$$\begin{aligned}\dot{\mathbf{e}}(t) &= \mathbf{A}\mathbf{e}(t) - \mathbf{b}\left(\theta^T(t) - \theta^{*T}\right)\mathbf{x}(t) \\ &\quad + (\mathbf{b}k_0^* - \mathbf{b}_m)u_m(t)\end{aligned} \tag{28}$$

Here, in order to eliminate the 'disturbing' last term, one actually assumes that supplementary prior knowledge is available such that the ”matching condition” $\mathbf{b}k_0^* = \mathbf{b}_m$ is satisfied and one gets

$$\dot{\mathbf{e}}(t) = \mathbf{A}\mathbf{e}(t) - \mathbf{b}\left(\theta^T(t) - \theta^{*T}\right)\mathbf{x}(t) \tag{29}$$

For convenience of the following development, we now define

$$\mathbf{P}\mathbf{b} = \mathbf{c}^T \tag{30}$$

We notice that simultaneous satisfaction of the two relations (26) and Eq. (30) is the so-called Strict Passivity (SP) property of systems (with Strictly Positive Realness (SPR) transfer functions in LTI systems). Therefore, by assuming that the Plant (or a “dominant” part of the Plant) is stable and also assuming the availability of (a desired combination of) the *full state vector* x, classical MRAC manages to satisfy an SPR-like condition and to end with a successful proof of stability of classical MRAC *under ideal conditions*. First of all, because only the ideal closed-loop configuration was required to satisfy the SPR condition, the original plant, now $\{\mathbf{A} + \mathbf{b}\theta^{*T}, \mathbf{b}, \mathbf{c}\}$, was at the distance of a constant feedback from being SPR. Therefore, by using state-feedback, classical MRAC developers actually ’reinvented’ the ASPR property. Besides, quite a bit of perfect knowledge has been used in order to end with the simple Eq. (29).

Still, developers of classical MRAC point to some problems related to its use. If the actual state order is larger than the nominal order, the so-called “unmodeled dynamics” may affect stability of the adaptive control system. Although in its most ideal form (29) one can prove asymptotic stability of the Adaptive Control system, transient response of MRAC is limited by the need of using only *slow* adaptation rate. Furthermore, it is commonly accepted that even small disturbances or deviations in Eq. (29) may lead to “bursting” or other unwanted phenomena [5].

The following development will show how similar assumptions can lead to simplified adaptive controllers of much lesser order, fewer computing demands and fewer problems, and how further simplification and demands have ultimately lead to the Simple Adaptive Control methodology.

3.3 Towards Using Output Error

Before moving on from classical MRAC, we revisit one of its very basic assumptions. Equation (22) is based on the assumption that the unknown Plant system matrix, which we will call $\mathbf{A}_p$, can be decomposed into $\mathbf{A}_p = \mathbf{A}_m + \mathbf{b}\Theta^*$. This decomposition inherently assumes controllability of $\{\mathbf{A}_p, \mathbf{b}\}$ and $\{\mathbf{A}_m, \mathbf{b}\}$, which allows pole-placing with state feedback. In other words, the state feedback that uses the combination $\mathbf{e}^T\mathbf{P}\mathbf{b}$ (or $\mathbf{b}^T\mathbf{P}\mathbf{e}$) or $\mathbf{b}^T\mathbf{P}\mathbf{x}$ can stabilize the plant. However, while in Optimal Control the useful signal $\mathbf{b}^T\mathbf{P}\mathbf{x}$ needs just *one* more appropriate scalar coefficient in order to be *the state feedback* signal that results in the *scalar* optimal command, classical MRAC controller seems to need both the *state* error $\mathbf{e}$ *and* the Plant *state* $\mathbf{x}$ in the computation of the adaptive gain *vector* $\dot{\theta}(t)$. Moreover, the large resulting *gain-vector* is again multiplied by the entire *state-vector* in order to finally obtain the *scalar* control signal.

Therefore, towards the simplified techniques of Simple Adaptive Control, we will only refer to the original Plant system matrix $\mathbf{A}_p$. The plant is

$$\dot{\mathbf{x}}(t) = \mathbf{A}_p\mathbf{x}(t) + \mathbf{b}u(t) \tag{31}$$

with the “output” signal

$$y(t) = \mathbf{b}^T\mathbf{P}\mathbf{x}(t) = \mathbf{c}\mathbf{x}(t) \tag{32}$$

The plant is required to track the Model

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m\mathbf{x}(t) + \mathbf{b}_m u_m(t) \tag{33}$$

$$y_m(t) = \mathbf{c}_m\mathbf{x}_m(t) \tag{34}$$

Continuing the Optimal Control idea, we assume that, with proper control, the plant could be forced to move along some bounded “ideal trajectories” that would perform perfect tracking. The ideal control $u^*(t)$ and the ideal trajectories $\mathbf{x}^*(t)$ that allow perfect following satisfy the plant equations

$$\dot{\mathbf{x}}^*(t) = \mathbf{A}_p\mathbf{x}^*(t) + \mathbf{b}u^*(t) \tag{35}$$

$$y^*(t) = \mathbf{c}\mathbf{x}^*(t) \tag{36}$$

Because the ideal trajectories perform perfect tracking, we can write

$$y^*(t) = \mathbf{c}\mathbf{x}^*(t) = \mathbf{c}_m\mathbf{x}_m(t) = y_m(t) \tag{37}$$

We define the “state error” as the difference between the ideal trajectory and actual trajectory

$$\mathbf{e}(t) = \mathbf{x}^*(t) - \mathbf{x}(t) \tag{38}$$

Subtracting Eq. (31) from Eq. (35) gives the state error equation

$$\dot{\mathbf{e}}(t) = \mathbf{A}_p\mathbf{e}(t) + \mathbf{b}(u^*(t) - u(t)) \tag{39}$$

and the output tracking error is then

$$\begin{aligned} e_y(t) &= y_m(t) - y(t) = y^*(t) - y(t) \\ &= \mathbf{cx}^*(t) - \mathbf{cx}(t) = \mathbf{ce}(t) \end{aligned} \tag{40}$$

As shown above, even though classical MRAC also makes use of the special combination that makes the *scalar* signal $e_y(t)$, it then uses it in order to compute a *full state-vector* adaptive control gain $\theta(t)$ by again multiplying $e_y(t)$ by the *entire* Plant state-vector $\mathbf{x}(t)$, which results in n multiplications and n integrations for a system of order n. Furthermore, the resulting adaptive gain $\theta(t)$ again multiplies the *entire* state vector $\mathbf{x}(t)$, thus performing n more multiplications in order to obtain the *scalar* control $u(t)$.

Instead, in order to avoid both the complexity and various problems that seem to appear in classical MRAC even with the use of *full-state* vector, one could follow along the lines of Optimal Control for the *error* system (39)-(40). Therefore, under *same* conditions shown above for classical MRAC, we modify the adaptive controller to just be

$$u(t) = k(t)\mathbf{b}^T\mathbf{Pe}(t) = k(t)\mathbf{ce}(t) \tag{41}$$

and assume that there exist some constant gain k^* such that the "ideal control" is

$$u^*(t) = k^*\mathbf{b}^T\mathbf{Pe}(t) = k^*\mathbf{ce}(t) \tag{42}$$

Here, we note that when the Plant reaches and moves along an ideal trajectory, the ultimate ideal system equation is

$$\dot{\mathbf{x}}^*(t) = \mathbf{A}_p\mathbf{x}^*(t) \tag{43}$$

$$y^*(t) = \mathbf{cx}^*(t) \tag{44}$$

because, in the ideal case, the plant is perfectly tracking and $e_y(t) = 0$. The error equation is then

$$\dot{\mathbf{e}}(t) = \mathbf{A}_p\mathbf{e}(t) - \mathbf{b}k(t)\mathbf{ce}(t) \tag{45}$$

Subtracting and adding $\mathbf{b}k^*\mathbf{ce}(t)$ gives

$$\dot{\mathbf{e}}(t) = \left(\mathbf{A}_p - \mathbf{b}k^*\mathbf{c}\right)\mathbf{e}(t) - \mathbf{b}\left(k(t) - k^*\right)\mathbf{ce}(t) \tag{46}$$

or

$$\dot{\mathbf{e}}(t) = \mathbf{A}_{pc}\mathbf{e}(t) - \mathbf{b}(k(t) - k^*)e_y(t) \tag{47}$$

where $\mathbf{A}_{pc} = \mathbf{A}_p - \mathbf{b}k^*\mathbf{c}$.

This way, the *entire* Adaptive Control has been reduced to just a *scalar* adaptive gain (in SISO case) that only multiplies the *scalar* output error. Even before we define a specific rule for the nonstationary gain $k(t)$, we select the Lyapunov function

$$V(t) = \mathbf{e}^T(t)\mathbf{Pe}(t) \tag{48}$$

Differentiating Eq. (48) along the trajectories of Eq. (47) gives

$$\begin{aligned} \dot{V}(t) &= \mathbf{e}^T(t)(\mathbf{PA}_{pc} + \mathbf{A}_{pc}^T\mathbf{P})\mathbf{e}(t) \\ &\quad -2e_y^T(t)\left(k(t) - k^*\right)e_y(t) \end{aligned} \tag{49}$$

At this stage and *only for starters*, we stay with classical MRAC assumption that state-feedback combination *can* stabilize the Plant. In other words, even though the matrix P has been computed in Eq. (26) using the *Model* system matrix, the resulting *Plant* state-feedback can indeed stabilize the *Plant*. In other words, we expect the *fictitious* closed-loop system to satisfy a stability equation of the form

$$\mathbf{PA}_{pc} + \mathbf{A}_{pc}^T\mathbf{P} = -\mathbf{Q} \tag{50}$$

for some (unknown) Positive Definite Symmetric matrix $\mathbf{Q}$. This gives

$$\dot{V}(t) = -\mathbf{e}^T(t)\mathbf{Qe}(t) - 2e_y^T(t)\left(k(t) - k^*\right)e_y(t) \tag{51}$$

Therefore, as Eq. (51) shows, under this simple output feedback configuration, the error system can end being asymptotically stable if the adaptive algorithm only guarantees that the adaptive gain will reach values that are beyond some minimal value, independently of the specific selection of the adaptive gain rule, as long as it keeps $k(t) - k^*$ nonnegative.

Eventual presence of a bounded external disturbance will naturally affect the behavior, yet would only appear as just another input. For any bounded disturbance that could affect the negativity of the Lyapunov derivative and lead to increase in the tracking error, the derivative would again becomes negative for some bounded error. Therefore, the system would just be a decent bounded-input-bounded-output system and *no peculiar phenomenon* (bursting, etc.) can occur. Besides, even though we want to keep the adaptive gains as small as possible, no negative effect on stability of the adaptive control system occurs if, due to noise or disturbances, the adaptive gains happen to increase beyond the minimal desired value.

The error system represented by Eq. (39) and Eq. (40) has been obtained by replacing the *full-state* adaptive feedback control with just *one* output adaptive feedback control gain and, besides its simplicity of implementation, this basic configuration that does not mix the error with the Plant state vector also manages to avoid some problems that may appear with classical MRAC in less than ideal situations. Moreover, while classical MRAC has become a problem of identification of plant parameters, with impressive works showing the ”equivalence of indirect and direct Adaptive Control,” the simplified approach keeps the Control developer and applicant focused on the Control problem, namely, fitting the right control gains to the right situation.

Still, we only presented this configuration as a basic idea that we do not further pursue here, because in next sections we also gradually reduce the basic assumption from full-state availability towards just input and output availability. Furthermore, although the SPR conditions are *not* inherently fulfilled in the real-world, we will show that the use of the *same prior knowledge* that exists in *any (non-adaptive) classical design* allows modifying the plant such that it satisfies the desired conditions that can provide the guarantee of stable behavior with the Adaptive Control.

4 On Passivity Conditions

Although Positive Realness or, more precise, Passivity of systems has been first introduced in networks [52], it has also demonstrated its usefulness in dynamic systems within the context of “absolute stability,” when Popov [142] showed that stability of a system for non-stationary gains that can arbitrarily vary within some range is *guaranteed* if the system is Positive Real. As already mentioned, it has also been applied to dynamic systems by Kalman [90] in the context of optimality (and stabilizability). As also mentioned, Positive Realness has also been shown to be a useful property that allows the proof of stability with adaptive controllers. At present, it is useful to present the *state-space* representation of the SPR conditions which seems to be the most useful for successful proofs of stability using Lyapunov stability theory.

Definition 1 A linear time-invariant system with a state-space realization $\{\mathbf{A}_K, \mathbf{B}, \mathbf{C}\}$, where $\mathbf{A}_K \in \mathbf{R}^{n*n}$, $\mathbf{B} \in \mathbf{R}^{n*m}$, $\mathbf{C} \in \mathbf{R}^{m*n}$, with the m*m transfer function $\mathbf{T}(s) = \mathbf{C}(sI - \mathbf{A}_K)^{-1}\mathbf{B}$, is called “strictly passive (SP)” and its transfer function “strictly positive real (SPR)” if there exist two positive definite symmetric (PDS) matrices, $\mathbf{P}$ and $\mathbf{Q}$, such that the following two relations are simultaneously satisfied [6]:

$$\mathbf{P}\,\mathbf{A}_K + \mathbf{A}_K^T\mathbf{P} = -\mathbf{Q} \tag{52}$$

$$\mathbf{PB} = \mathbf{C}^T \tag{53}$$

The relation between the state-space conditions (52)-(53) and the strict positive realness of the corresponding transfer function has been treated elsewhere [83, 176]. Relation (52) is the common algebraic Lyapunov equation and shows that an SPR system is asymptotically stable. One can also show that conditions (52)-(53) also imply that the system is strictly minimum-phase, yet simultaneous satisfaction of both conditions (52)-(53) is far from being guaranteed even in stable and minimum-phase systems, and therefore the SPR condition seemed much too demanding. (Some colleagues in the general control community would also ask: if the system is already minimum-phase and asymptotically stable why would one need adaptive controllers?)

For a long time, the so-called passivity condition had been considered very restrictive (and rather obscure) and for quite some time the adaptive control community has been trying to drop this condition and do without it. The passivity condition has been somewhat mitigated when it was shown that stability with adaptive controllers could be guaranteed even for the non-SPR system (1)-(2) if there exists a constant output feedback gain (unknown and not needed for implementation), such that the *fictitious* closed-loop system with the system matrix

$$\mathbf{A}_K = \mathbf{A} - \mathbf{B}\widetilde{\mathbf{K}}_e\mathbf{C} \tag{54}$$

is SPR, namely, it satisfies the passivity conditions (52)-(53). Because in this case the original system (1)-(2) was only separated from strict passivity by a simple constant output feedback, it was called “Almost Strictly Passive(ASP)” and its transfer function “Almost Strictly Positive Real (ASPR)” [18, 37].

At the time, this “mitigation” of the passivity conditions did not make a great impression, because it was not clear what systems would actually satisfy the new conditions. (Some even claimed that because SPR seemed to be just another name for the void class of systems, the “new” class of ASPR systems was only adding the ’boundary.’) Nonetheless, some ideas were available. Because a constant output gain feedback was supposed to stabilize the system, it seemed apparent that the original plant was not required to be stable. Also, because it was known that SPR systems were minimum-phase and because it was easy to see that (53) implies that the product **CB** is Positive Definite Symmetric (PDS), it was intuitive to assume that minimum-phase systems with Positive Definite Symmetric **CB** were natural ASPR candidates [37]. Indeed, simple Root-locus techniques were sufficient to prove this result in SISO systems, and many examples of minimum-phase MIMO systems with **CB** product PDS were shown to be ASPR [18, 37]. However, it was not clear how many of such MIMO systems actually were ASPR. Because the ASPR property can be stated as a simple condition and because it is the main condition needed to guarantee stability with adaptive controllers, it is useful to present here the ASPR theorem for the general multi-input-multi-output systems [21]:

Theorem 1 *Any linear time-invariant system with the state-space realization* $\{\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}\}$, *where* $\boldsymbol{A} \in \boldsymbol{R}^{n*n}$, $\boldsymbol{B} \in \boldsymbol{R}^{n*m}$, $\boldsymbol{C} \in \boldsymbol{R}^{m*n}$, *with the m*m transfer function* $\boldsymbol{T}(s) = \boldsymbol{C}(sI - \boldsymbol{A})^{-1}\boldsymbol{B}$, *that is minimum-phase and where the matrical product* ***CB*** *is PDS is "almost strictly passive (ASP)" and its transfer function "almost strictly positive real (ASPR)."*

Although the original plant is not SPR, a (fictitious) closed-loop system satisfies the SPR conditions, or in other words, there exist two positive definite symmetric (PDS) matrices, ***P*** *and* ***Q****, and a positive definite gain* $\widetilde{\boldsymbol{K}}_e$ *such that the following two relations are simultaneously satisfied:*

$$\boldsymbol{P}(\boldsymbol{A} - \boldsymbol{B}\widetilde{\boldsymbol{K}}_e\boldsymbol{C}) + (\boldsymbol{A} - \boldsymbol{B}\widetilde{\boldsymbol{K}}_e\boldsymbol{C})^T\boldsymbol{P} = -\boldsymbol{Q} \tag{55}$$

$$\boldsymbol{P}\boldsymbol{B} = \boldsymbol{C}^T \tag{56}$$

It is useful noting that the ASPR property includes the desired $\boldsymbol{PB} = \boldsymbol{C}^T$ *combination and so, it directly provides the desired control signal that is the basis for Optimal and Adaptive Control as output signal, without requiring full-state availability.*

As a matter of fact, an early proof of Theorem 1 has been available in the Russian literature since 1976 [56, 59]*, yet it was not known in the West. Here, many other works have independently rediscovered, reformulated, and further developed the idea (see* [20, 173] *and in particular* [21] *and references therein for a brief history and for a simple and direct, algebraic, proof of this important statement).*

Theorem 1 has managed to explain the rather obscure passivity conditions with the help of new conditions that could be understood by control practitioners. It is useful to notice an important property that was hinted in the section on Optimal Control and that makes an ASPR system to be a good candidate for stable adaptive control: if a plant is minimum-phase and its input-output matrical product ***CB*** *is Positive Definite Symmetric (PDS), it is stabilizable via some static Positive Definite (PD) output feedback. Furthermore, if the output feedback gain is increased beyond some minimal value, the system remains stable even if the gain increase is nonstationary. At this stage, Theorem 1 tells us that ASPR systems do provide us with the appropriate desired signal* $y(t) = \boldsymbol{B}^T\boldsymbol{P}x(t)$, *without requiring full-state availability. Later on, we show how to use basic available knowledge in order to get this combination in real-world application to systems that, inherently, are not necessarily ASPR.*

Notice that the required positivity of the product ***CB*** *could be expected, as it seemed to be a generalization of the sign of the transfer function that allows using negative feedback in LTI systems. However, although at the time it seemed to be absolutely necessary for the ASPR conditions, the required* ***CB*** *symmetry proved to be rather difficult to fulfill in practice, and in particular in adaptive control systems where the plant parameters are not known.*

After many attempts that have ended in failure, a recent publication [41] *finally managed to eliminate the need for a symmetric* ***CB****. First, it was easy to observe that the Lyapunov function remains positive definite if the gain term is rewritten as follows:*

$$V(t) = \boldsymbol{e}_x^T(t)\boldsymbol{P}\boldsymbol{e}_x(t) + tr\left[\boldsymbol{W}\left(\boldsymbol{K}(t) - \widetilde{\boldsymbol{K}}\right)\Gamma^{-1}\left(\boldsymbol{K}(t) - \widetilde{\boldsymbol{K}}\right)^T\right] \tag{57}$$

Here, ***W*** *is a Positive Definite matrix. This new formulation allowed the extension of useful passivity-like properties to a new class of systems that was called WASP, through the following definition:*

Definition 2 Any linear time-invariant system with state-space realization $\{\mathbf{A}, \mathbf{B}, \mathbf{C}\}$, where $\mathbf{A} \in \mathbf{R}^{n*n}$, $\mathbf{B} \in \mathbf{R}^{n*m}$, $\mathbf{C} \in \mathbf{R}^{m*n}$, with the m*m transfer function $\mathbf{T}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$, is called "W-almost strictly passive (WASP)" and its transfer function "W-almost strictly positive real (WASPR)," if there exist two positive definite *symmetric* (PDS) matrices, $\mathbf{P}$ and $\mathbf{Q}$, a positive definite matrix $\mathbf{W}$, and a positive definite gain $\widetilde{\mathbf{K}}_e$ such that the following two conditions are simultaneously satisfied:

$$\mathbf{P}(\mathbf{A}-\mathbf{B}\widetilde{\mathbf{K}}_e\mathbf{C})+(\mathbf{A}-\mathbf{B}\widetilde{\mathbf{K}}_e\mathbf{C})^T\mathbf{P}=-\mathbf{Q} \tag{58}$$

$$\mathbf{PB}=\mathbf{C}^T\mathbf{W} \tag{59}$$

This new definition can be used with the following theorem [25, 26, 41]:

Theorem 2 *Any linear time-invariant system with state-space realization $\{\boldsymbol{A},\boldsymbol{B},\boldsymbol{C}\}$, where $\boldsymbol{A}\in\boldsymbol{R}^{n*n}$, $\boldsymbol{B}\in\boldsymbol{R}^{n*m}$, $\boldsymbol{C}\in\boldsymbol{R}^{m*n}$, with the m*m transfer function $\boldsymbol{T}(s)=\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}$, is "W-almost strictly passive (WASP)" in accord with Definition 2 if the eigenvalues of the not necessarily symmetric matrix product $\boldsymbol{CB}$ are located in the right half-plane. If, in addition, $\boldsymbol{CB}$ is also diagonalizable and its eigenvalues are real and positive, matrix $\boldsymbol{W}$ is also PDS.*

Note: We notice that the eigenvalues of a nonsymmetric matrix $\boldsymbol{M}$ can be different from the eigenvalues of its symmetric part $\boldsymbol{M}_s=(\boldsymbol{M}+\boldsymbol{M}')/2$. Therefore, while a symmetric matrix with real and positive eigenvalues is Positive Definite, this is not necessarily so when the matrix is not symmetric. Conversely, it is interesting noting that a nonsymmetric matrix $\boldsymbol{M}$ can be Positive Definite although its eigenvalues are not real and positive, if its symmetric part $\boldsymbol{M}_s$ is Positive Definite Symmetric. Therefore, while only the elimination of $\boldsymbol{CB}$ symmetry condition was sought, the final result implies that $\boldsymbol{CB}$ does not have to be either symmetric or positive definite, as long as its eigenvalues are properly located in the right half-plane.

While [41] only extended the ASP condition for the symmetric $\boldsymbol{W}$ and only for the output stabilization case, this was a first mitigation of a condition that has been around for more than 40 years. Nevertheless, it was very tempting to try to eliminate (almost) any restriction on $\boldsymbol{CB}$. Because any $\boldsymbol{CB}$ product with eigenvalues anywhere in the right half-plane would allow existence of non-symmetric $\boldsymbol{W}$, this can result in a considerable mitigation of passivity conditions. Although a non-definite term containing $(\boldsymbol{W}-\boldsymbol{W}')\left(\boldsymbol{K}(t)-\widetilde{\boldsymbol{K}}\right)$ (that does not necessarily vanish unless $\boldsymbol{W}$ is symmetric) appears in the Lyapunov derivative, recent research [26] shows that the nondefinite term is dominated by the negative definite terms and cannot affect stability with respect to boundedness. Besides, it was shown elsewhere [23] that the feedforward adaptive gains perform a steepest descent minimization of the tracking errors, forcing the adaptive gains $\boldsymbol{K}(t)$ to approach the ideal gains $\widetilde{\boldsymbol{K}}$. Therefore, even though not a direct result of Lyapunov stability approach (yet), tests do show asymptotically perfect following [26] and so, symmetry of $\boldsymbol{W}$ is not actually needed and stability of Adaptive Control can be maintained under the most mitigated conditions.

5 Simple Adaptive Control (SAC), or the simplified Approach to Model Reference Adaptive Control

Next sections will show that those ingenious adaptive control ideas and the systematic stability analysis they introduced had finally led to adaptive control systems that, using the prior knowledge *usually available* for design, can guarantee stability robustness and superior performance when compared with alternative, non-adaptive, methodologies. This section will first assume that at least one of the *almost* passivity conditions presented above holds and will deal with a particular methodology that first of all seems to eliminate the need for plant order and therefore can mitigate the problems related to "unmodeled dynamics" and "persistent excitation." Subsequent sections will then extend the feasibility of the methodology to those real-world systems that do not inherently satisfy any passivity conditions.

The beginning of the alternative adaptive control approach can be found in the intense activities at Rensselaer (RPI), where such researchers as Kaufman, Sobel [159, 160], Barkana, Balas [15, 35, 40], Wen[13, 175], Ozcelik[137, 138, 140] and others were trying to use customary adaptive control techniques with large order MIMO systems, such as planes, large flexible structures, etc. It did not take long to realize that it was impossible to consider implementing controllers of the same order as the plant, or even of the order of a "nominal" plant. Besides, those were inherently MIMO systems, while customary MRAC techniques at the time were only dealing with SISO systems.

Towards using reduced order adaptive controllers, the Optimal Control idea was adopted and, following this idea, a direct adaptive output feedback component was added to the adaptive algorithm (that otherwise is very similar to the very basic MRAC algorithms), namely,

$$\mathbf{u}(t) = \mathbf{K}_e(t)\mathbf{e}_y(t) + \mathbf{K}_x(t)\mathbf{x}_m(t) + \mathbf{K}_u(t)\mathbf{u}_m(t) = \mathbf{K}(t)\mathbf{r}(t) \tag{60}$$

where we denote the reference vector

$$\mathbf{r}(t) = \left[\, \mathbf{e}_y^T(t)\;\; \mathbf{x}_m^T(t)\;\; \mathbf{u}_m^T(t) \,\right]^T \tag{61}$$

and the adaptive gain

$$\mathbf{K}(t) = \left[\, \mathbf{K}_e(t)\;\; \mathbf{K}_x(t)\;\; \mathbf{K}_u(t) \,\right]. \tag{62}$$

Because this approach actually uses the model as a command generator, it was called Adaptive Command Generator Tracker. However, because it also uses low-order models and, in particular, low-order adaptive *controllers*, it was ultimately called Simple Adaptive Control (SAC) [18].

Here, it is important to also review one issue that was emphasised by the authors of [80]. They criticize the filter that L1-Adaptive Control approach uses in the internal loop, because it reduces the loop-bandwidth. On the other hand, one may understand that it could be dangerous to let an adaptive control system attempt to follow *any* command and most probably, this argument stands behind the supplementary filter. In the SAC approach, "filtering" the command is exactly the role of the Model. Instead of supplying the adaptive control plant such a command as step-input and then let it try to track it, the plant is supplied and asked to track the time response of the model, which represents the behavior that the plant can be expected and required to provide. Therefore, even if the parameters of an helicopter are not known, an appropriate model reference should only require the plant to behave as a good helicopter and not as a dog-fighter. The difference is that the model is outside the loop and does not affect the plant closed-loop. Moreover, as discussed in [29] and below, supplementary signals from the model alow high level of performance *without* stressing the internal loop.

Before we discuss the differences between the SAC approach and classical MRAC, it is useful to first dwell over the special role of the direct output feedback term. If the plant parameters were known, one could choose an appropriate gain $\widetilde{\mathbf{K}}_e$ and stabilize the plant via constant output feedback control $\mathbf{u}(t) = -\widetilde{\mathbf{K}}_e\mathbf{y}(t)$.

As we already mentioned above, it was known that an ASPR system (or, as we now know, a minimum-phase plant with appropriate **CB** product [21]) could be stabilized by a positive definite output feedback gain. Furthermore, it was known that ASPR systems are high-gain stable, so stability of the plant is maintained if the gain value happens to go arbitrarily high beyond some minimal value. Whenever one may have sufficient prior knowledge to assume that the plant is ASPR, yet may not have sufficient knowledge to choose a good control gain, one can use the tracking error to generate the adaptive gain

$$\dot{\mathbf{K}}_e(t) = \mathbf{e}_y(t)\mathbf{e}_y^T(t)\Gamma_e \tag{63}$$

and the control

$$\mathbf{u}(t) = \mathbf{K}_e(t)\mathbf{e}_y(t) \tag{64}$$

As hinted in the section on Optimal Control, it was shown that this adaptive gain addition is able to avoid some of the most difficult inherent problems related to the standard MRAC and to add robustness to its stability. Although it was developed as a natural compensation for the low-order models and was just one more element of the Simple (Model Reference) Adaptive Control methodology, it is worth mentioning that, similarly to the first proof of the ASPR property, the origins of this specific adaptive gain can again be found in an earlier work of Fradkov [56] in the Russian literature. Moreover, later on, this gain received a second birth and became very popular after 1983 in the context of adaptive control "when the sign of high-frequency gain is unknown." First in this context [65, 121, 132] and then after a very rigorous mathematical treatment [51], it also received a new name and it is sometimes called the Byrnes-Willems gain. Its useful properties have been thoroughly researched and some may even call this one adaptive gain Simple Adaptive Control as they were apparently able to show that it can do "almost" everything [78, 115]. Indeed, because an ASPR system is high-gain stable, it seems very attractive to let the adaptive gain increase to high values in order to achieve good performance that is represented by small tracking errors. However, although at first thought one may find that high gains are very attractive, a second thought and some more engineering experience with the real world applications make it clear that high gains may lead to saturations and may excite vibrations and other disturbances. These disturbances may not have even appeared in the nominal plant model that was used for design and may not be felt in the real-world plant *unless* one uses those very high gains. Furthermore, as the motor or the plant dynamics would always require an input signal in order to keep moving and tracking the desired trajectory, it is quite clear that the tracking error cannot be zero or very small unless one uses very high gains indeed.

On the other hand, designers of tracking systems know that appropriate feedforward signals that come from the desired trajectory can help achieving low-error or even perfect tracking *without* requiring the use of dangerously high gains (and, correspondingly, exceedingly high bandwidth) in the closed-loop. In the non-adaptive world, the use of feedforward signals could be problematic because, unlike the feedback loop, any errors in the feedforward parameters are directly and entirely transmitted to the output tracking error. Here, the adaptive control methodology can demonstrate an important advantage on the non-adaptive techniques, because the feedforward parameters are finely tuned by the very tracking error they intend to minimize. The issues discussed here and the need for feedforward again seem to show the intrinsic importance of the basic Model Following idea, and again point to the need for a model. However, the difference between the Model Reference used by the basic MRAC and the model used by SAC is that this time the so-called "Model" does not necessarily have to reproduce the plant, besides incorporating the desired input-output behavior of the plant. At the extreme, it could be just a first-order pole that performs a reasonable step-response, or otherwise a low-order system of order just sufficiently high to generate the desired trajectory. As it generates the command, this "model" can also be called [49] "Command Generator" and the corresponding technique "Command Generator Tracker (CGT)."

In summary, the adaptive control system monitors all available data, namely, the tracking error, the model states and the model input command (see Fig. 1) and uses them to generate the adaptive control signals

$$\dot{\mathbf{K}}_e(t) = \mathbf{e}_y(t)\mathbf{e}_y^T(t)\Gamma_e \tag{65}$$

$$\dot{\mathbf{K}}_x(t) = \mathbf{e}_y(t)\mathbf{x}_m^T(t)\Gamma_x \tag{66}$$

$$\dot{\mathbf{K}}_u(t) = \mathbf{e}_y(t)\mathbf{u}_m^T(t)\Gamma_u \tag{67}$$

that using the concise notations (30)-(31) give

$$\dot{\mathbf{K}}(t) = \mathbf{e}_y(t)\mathbf{r}^T(t)\Gamma \tag{68}$$

and the control

$$\begin{aligned}\mathbf{u}(t) &= \mathbf{K}_e(t)\mathbf{e}_y(t) + \mathbf{K}_x(t)\mathbf{x}_m(t) \\ &\quad + \mathbf{K}_u(t)\mathbf{u}_m(t) = \mathbf{K}(t)r(t)\end{aligned} \tag{69}$$

It is worth noting that, initially, SAC was meant to be a modest alternative to MRAC with apparently very modest aims and that also seemed to be restricted by the new conditions. Nevertheless, at the time it probably was the only adaptive technique that could have been used in MIMO systems and with large systems, and therefore was quite immediately adopted by many researchers and practitioners in such diverse applications as flexible structures [14, 42, 75–77, 107, 125, 156], flight control including reconfiguration [22, 43, 60, 122, 123, 152, 180], flexible spacecraft [74, 111] improving performance of existing autopilot[150], power systems [33, 144–146, 181], robotics [34, 168, 169], motor control [19, 155, 165], systems with time-delay [157], drug infusion and systems with saturation constraints [139, 140], DC/DC boost converter [88, 89], structural performance improvement [4, 44–46], vibration suppression of piezoelectric smart structures [131], satellite mission life extension [71–73, 101], water hydraulic servo systems [84, 85, 112, 113, 141, 166, 178], pneumatic motion [133], active magnetic bearing systems [64], control of voltage in proton exchange membrane fuel cell [126], air-fuel ratio control [92] electrical stimulation for upper limb motion [99], fractional-order systems[100, 164] modified Delta operator and form for intelligent systems [7], magnetic levitation [179], quadrotor helicopter [53], and others [39, 48, 79, 95, 109, 116, 119, 167].

Nonetheless, although it was quite simple to implement, the theory around SAC was not simple at all and many tools that, slowly and certainly, revealed themselves over the years, were initially lacking to support its qualities. It subsequently not only required developing new analysis tools but also, probably more important, better understanding of their implications, before they could be properly used so that they ultimately managed to highlight the very useful properties of SAC. Finally, after developments that had spanned over more than 30 years, SAC has in fact proved to

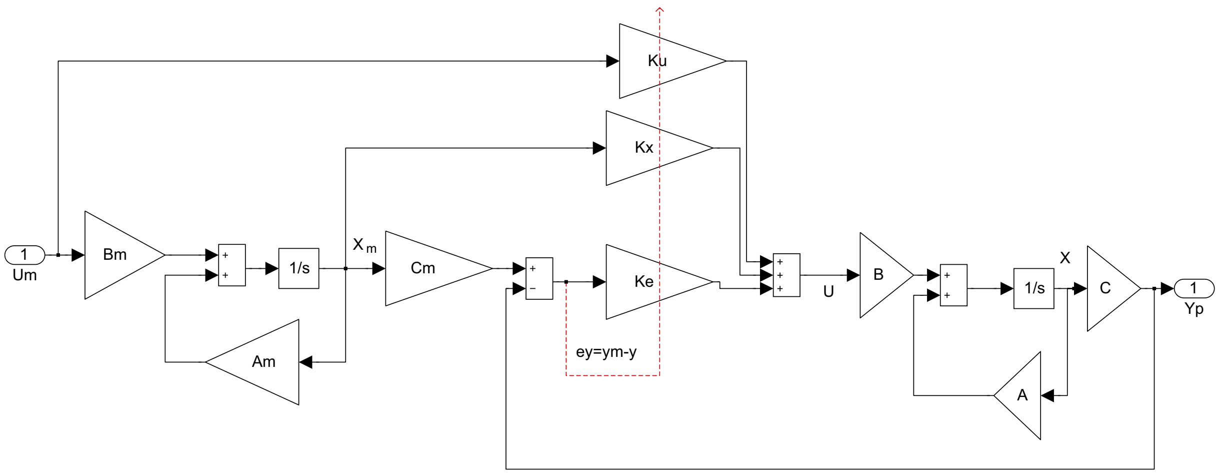

Fig. 1 SAC

be the stable MRAC, because right from the beginning it avoids some difficulties that are inherent in the standard MRAC. In particular, it is useful to notice that SAC first assumes and then attempts to fulfill some desirable Plant *properties*, and does not necessarily deal with Plant *parameters* and in particular plant *order*, so there is no "unmodeled dynamics." Also, because basically the stability of the system rests on the direct output feedback adaptive gain, the model is immaterial in this context and of course there is no need to even mention "sufficient excitation."

Besides, as we will later show, and as it was observed by almost all practitioners that have tried to use it, SAC proved to be good control. In this context, while the standard MRAC may have to explain why it does not work when it is supposed to work, SAC may have to explain why it does work even in cases when the (*sufficient*) conditions are not fully satisfied. Although, similarly to any nonstationary control, in Adaptive Control it is very difficult to find the very minimal conditions that would keep the system stable, the need to explain why SAC seemed to demonstrate some robustness even when the basic sufficient conditions are not satisfied, ultimately provided the motivation for continuously extending the domain of feasibility of SAC by further mitigating these *sufficient* conditions that guarantee stability. We must also note that, in those cases when basic conditions *are* fulfilled, they are always sufficient to guarantee the stability of the adaptive control system, with no exceptions and no counterexamples. In this respect, along with the proof of stability we will also again mention that the so-called "counterexamples" to MRAC [69] become just trivial, stable, and well behaving examples for SAC [26].

It is also important to note the role of the various components of the SAC control signal (69). Even though the term "Simple Adaptive Control (SAC)" was initially intended for the combination (69), other approaches may try to further "simplify" it and may only use the simple controller

$$\mathbf{u}(t) = \mathbf{K}_e(t)\mathbf{e}_y(t) \tag{70}$$

(i.e., $\mathbf{K}_x(t) = 0$ and $\mathbf{K}_x(t) = 0$). This saves a few real-time computations and may result in reasonable similar performance. However, this apparent saving may come at some heavy cost. Although we emphasised the importance of the main gain $\mathbf{K}_e(t)$ for the guarantee of *stability* with SAC, Eq. (70) shows that, having a control signal that can keep the plant moving at low tracking errors may require *very high* internal gain $\mathbf{K}_e(t)$ values. On the other hand, the control signal (69) shows that, in principle, as far as tracking at low or even zero errors is concerned, $\mathbf{K}_e(t)$ is allowed to be zero, as the control signal comes from the model through the appropriate feedforward gains $\mathbf{K}_x(t)$ and $\mathbf{K}_x(t)$. Therefore, if, for example, the initial value of the main gain is $\mathbf{K}_e(0) = 1$, one may see that, during the entire adaptation, $\mathbf{K}_e(t)$ may hardly move and only when called and may get values between 1 and 2. On the other hand, even though the tracking errors may look similar, if one uses $\mathbf{K}_x(t) = 0$ and $\mathbf{K}_x(t) = 0$, the remaining main gain $\mathbf{K}_e(t)$ may reach such high values as 40 or even 70, with all potentially negative effects of high gains and high loop-bandwidth on the actual system, with its unmodeled dynamics, hidden nonlinearities, oscillatory modes, etc.

It is worth mentioning that, although not common among ”classical” MRAC developers, some seem to have become aware [70] that, instead, ”classical” MRAC has problems moving from SISO to MIMO and from state-feedback to output feedback as, in any case, it uses lots of adaptive gains and looks for various appropriate ”decompositions” that would allow satisfaction of various ”matching” conditions. They even recommend considering SAC for large systems [70]. The question is if even for a SISO system of order, say, 6 it is worth using 6 adaptive gains when one single gain does the job.

6 On Parallel Feedforward

We noted that, in real-world, systems do not necessarily satisfy the ASPR condition, and then the so-called “parallel feedforward configuration (PFC)” can be added to allow the augmented system to satisfy the ASPR conditions [17, 18, 37]. We must also mention that, in spite of its successful application in many practical and difficult adaptive control applications, some much respected readers still feel uneasiness at the PFC idea, which is “adding something in parallel with the actual plant and therefore does not control the actual plant any more” (Fig. 2) and this comment deserves our attention.

In spite of encouraging results with large flexible structures (which, nevertheless, required collocation of sensors and actuators), not many other examples of ASP systems seemed to be readily available in practice. Therefore, the idea of maybe adjusting the plant to be controlled in such a way that may satisfy an ASP relation started being investigated.

The first idea was raised by an apparent problem with adaptive control of discrete-time systems [15, 36], where SP (and ASP) relations require the system to be of the form $\{A, B, C, D\}$ with non-singular D, while most systems are of the form $\{A, B, C\}$. As a result, passivity relations were considered unattainable in discrete-time systems. However, it was observed that in some systems, if in parallel to the discrete transfer function $G(z)$ one adds an appropriate constant matrix D, the augmented plant $G_a(z) = G(z)+D$ does satisfy an ASP relation. In other words, there exists some constant K_e such that the fictitious closed-loop system $T(z) = G_a(z)/(1 + K_eG_a(z))$ *is* SP [15, 36].

An interesting result was that a small addition could allow implementing ASP configurations even of unstable *and* non-minimum phase systems that, otherwise, were usually considered taboo for adaptive control (unless some very precise prior knowledge on the exact location of the problematic zeros of the assumable unknown plant could be assumed to be available). The first explanation came with the observation that,

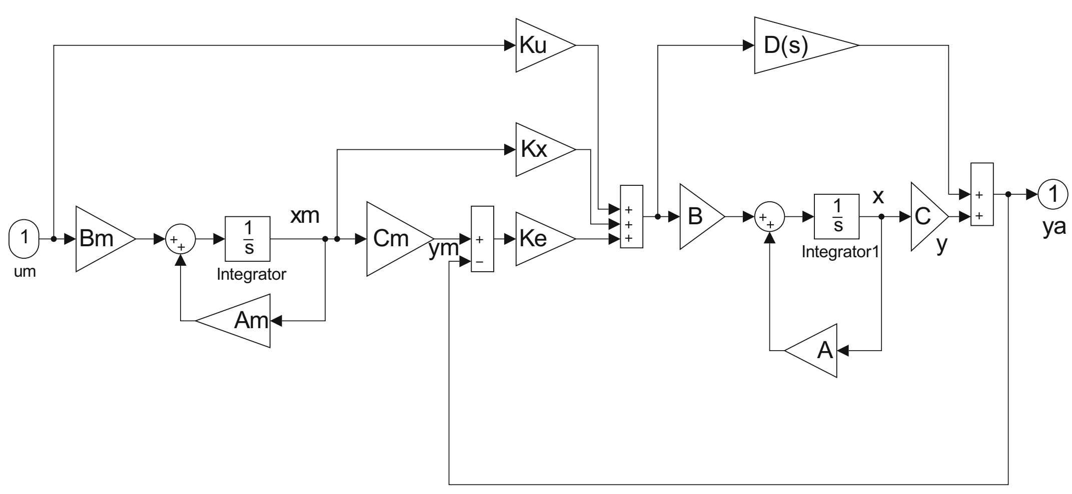

Fig. 2 Augmented system with PFC

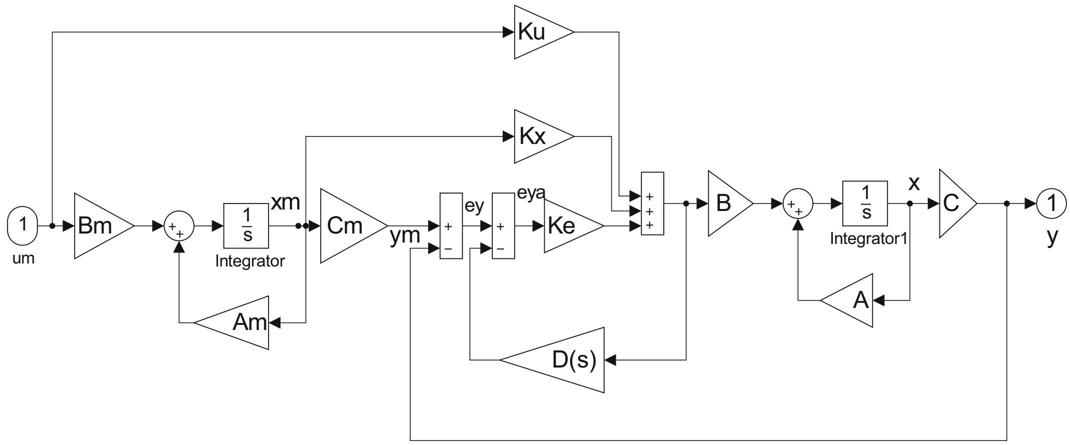

Fig. 3 Actual implementation with PFC

if the (continuous *or* discrete) system $G(\bullet)$ is stabilizable by some constant feedback K, then the inverse $D = 1/K$ is the "parallel feedforward" that makes the augmented system ASP [37, 38]. In an attempt to avoid direct input-output gains in continuous-time systems, the idea was first extended to simple dynamic feedforward of the form $D(s) = 1/[K(1+s/w)]$, and then further extended to general stabilizing controllers $C(s)$.

In general, if the (linear *or* nonlinear, continuous *or* discrete) system $G(\bullet)$ is stabilizable through some configuration $C(\bullet)$, then the augmented system $G_a(\bullet) = G(\bullet) + D(\bullet)$, where $D(\bullet) = 1/C(\bullet)$, is minimum-pase (or it has stable zero dynamics, or has a stable inverse). The augmented system is then ASP if the relative degree of $G_a(\bullet)$ is zero or one [16–18, 24, 91]. We mention that the references deal with MIMO systems and this brief description here deals with SISO plants only because of the convenience of notation and presentation.

At this stage, maybe it is worth mentioning that the terms "parallel feedforward" or "shunt" [15, 18, 36, 37, 57, 58, 86, 87, 114] (represented by D(s) in Fig. 2) has been introduced because, for the guarantee of stability of the adaptive control, the controlled plant must indeed satisfy the required ASP conditions. However, only the adaptive controller must "see" an *augmented* ASP plant, because, in fact, nothing is added "in parallel" to the real plant (same way as no plane or motor axis has to be bent in order to use position feedback). Thus, the PFC actually is only a part of the controller and is only a supplementary feedback around the main adaptive gain $\mathbf{K}_e(t)$ (Fig. 3).

Moreover, as new developments show, assume that a "best" linear design (or at the end of some adaptive control design) is $G(s)$ and so, its tracking error is

$$e(s) = \frac{1}{1 + G(s)} u(s). \tag{71}$$

The designer then realizes that the performance could be less than satisfactory and yet, for improving this performance, one would have to add another ideal, possibly improper, controller $C(s)$ and also with some high gain. Instead, within the SAC methodology, one is advised to use the inverse of $C(s)$, $D(s) = 1/C(s)$, as PFC "in parallel" with the plant. As recently shown, the ultimate tracking error of the actual plant with SAC controller and with PFC is [172]

$$e(s) = \frac{1}{1 + C(s)G(s)} u(s). \tag{72}$$

In other words, the adaptive control system *with* the assumed PFC "ballast" achieves the performance of the most ideal linear system *without* PFC. For a simple example, the desired addition could be a supplementary PD controller $C(s) = K(1 + s/s_0)$, requiring a supplementary, precise and expensive velocity sensor or perfect differentiation, along with some high gain

K, while, instead, the PFC configuration is only some simple, inoffensive, first-order pole with low gain

$$D(s) = \frac{1}{K(1+s/s_0)}. \quad (73)$$

While on paper the use of C(s) or of its inverse D(s) may look similar, in reality is the difference between using some advanced hardware components, and just changing a few lines in the computer software [31].

While one may object that this good behavior occurs only *after* adaptation, this is only for those who got used with the "slow adaptation" rule that classical MRAC so strongly demands. Instead, the *guarantee* of stability that PFC adds to the adaptive controller frees SAC from the customary fear of fast adaptation. Therefore, the adaptation time with SAC is practically almost negligible when compared with any expected time constants and so, as test results show, the supplementary error introduced by the use of PFC "in parallel" with $G(s)$ only separates the actual plant from the most *ideal* perfect following behavior. As a result, this supplementary error is usually much lower than otherwise and does not reduce, but rather *improves* SAC performance in comparison with the "best" classical design $G(s)$ [32, 150].

For another attempt at easy introduction of "parallel" feedforward idea, let us assume that the plant $G(s)$ is known to be stabilizable and that the designer would be allowed to chose any *constant* gain from the minimal value K_{min} (say, $K_{min} = 1$) to maximal value K_{max} (say, $K_{max} = 100$). However, the steady-state tracking error with the low value $K_{min} = 1$ is $e(s) = G(s)/(1 + G(s))u(s)$ and does not fit good response to maneuvers. On the other hand, the highest admissible value $K_{max} = 100$ results in the small tracking error $e(s) = G(s)/(1 + 100G(s))u(s)$, yet would only amplify noise and maybe excite oscillations and nonlinearities when the plant is supposed to be quiet and performance is not required. Therefore, the designer would like to define rules that would use nonstationary gains $K(t)$, such that they fit the right situation. However, time varying gains do not guarantee stability any longer, even if they remain within the so-called "admissible domain" $K_{min} < K(t) < K_{max}$ [29, 91]. Instead, we suggest using the small added value $D = 1/K_{max} = 1/100$ in "parallel" with the plant. The word "parallel" was written within quotation marks, because it only represents the idea that an augmented plant is made ASP with parallel feedforward. In fact, D is part of the controller and is connected in feedback around the main adaptive loop gain $K_e(t)$ (Fig. 3). With this simple constant gain feedback (and for a moment ignoring the feedforward gains), the "effective adaptive gain" that the plant sees is $K_{eff}(t) = K_e(t)/(1 + DK_e(t))$. While, within the SAC approach, the "parallel" feedforward concept allows using the almost passivity properties for the proof of stability, the $K_{eff}(t)$ above also reminds various "projections" that were shown to improve stability of various adaptive control scheme. In particular, even if the adaptive gain $K_e(t)$ would tend to diverge, the plant would receive at most the maximal admissible value K_{max}. *The "small" difference is that, while direct use of the upper 'admissible' bound K_{max} does not guarantee stability with nonstationary adaptive gains, the "parallel" feedforward configuration concept does.*

One can see how even members of the "classical" MRAC community may use Simple Adaptive Control techniques *and* PFC if they have to deal with a really large *and* MIMO system [1, 2].

Nevertheless, for a long time the existence of suitable PFC that can make real-world systems into ASPR had been questioned. Therefore, in recent papers the following results have been established:

1. PFC that renders plant ASPR always exist for any proper plant (stable or unstable, minimum or non-minimum phase plant, SISO and MIMO) (see [150] , Appendix);
2. The whole set of PFCs has been parameterized;
3. The duality of PFC and negative feedback had been established [149];
4. The set of all PFCs is larger than the set of all stabilizing controllers.

For the question of robustness of passification with unmodeled dynamics see [171]. Moreover, even though one could expect the adaptive controller to ultimately require same high loop-gain values as the ideal classical controller, the numerous applications show that the PFC is only needed to provide the *guarantee* of stability. In turn, this guarantee then facilitates computation of the appropriate values for the desired feedforward adaptive gains, $\mathbf{K}_x(t)$ and $\mathbf{K}_u(t)$ (Fig. 3), which ultimately allow reaching the desired performance *without* requiring high $\mathbf{K}_e(t)$ gains and/or high bandwidth from the internal loop. As tests show, even though the actual tracking error is $e(s) = G(s)/(1 + 100G(s))u(s)$, the main adaptive loop-gain $K_e(t)$ barely moves above the initial value $K_e(0) = 1$.

7 New Results in Nonlinear Systems Stability Analysis

We recall that the dynamics of the Adaptive Control systems contains both the errors $\mathbf{e}(t)$ and the adaptive gains $\mathbf{K}(t)$ (or $\theta(t)$), while the Lyapunov derivative is only a function of $\mathbf{e}(t)$. Thus, although the Lyapunov derivative is Negative Definite in terms of $\mathbf{e}(t)$, it is only Negative *Semi*definite in terms of the entire space $\{\mathbf{e}(t), \mathbf{K}(t)\}$ and this is less than the Lyapunov Theorem requires. Therefore, new developments were needed in order to allow stability analysis of nonlinear systems when the derivative of the Lyapunov function is not exactly Negative Definite.

Unless mentioned otherwise, this section deals with stability of *non-autonomous* nonlinear systems (where the time explicitly appears) of the form

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, t). \tag{74}$$

Because of the extreme importance of proper stability analysis of the Adaptive Control systems, we present here a few old and new results[27, 29, 30].

Lyapunov Direct Method The so-called Lyapunov direct method is the methodology that allowed analysis of nonlinear systems stability without requiring to actually solve the nonlinear differential equations for all initial conditions. Lyapunov proposed to associate with the system an appropriate function, say positive definite and radially unbounded. Such functions have later been called "Lyapunov functions" $V(\mathbf{x})$, where $\mathbf{x}$ is the entire state vector. The Lyapunov theorem states that if the derivative of $V(\mathbf{x})$ *along the trajectories of the system* is negative definite, then the system is globally asymptotically stable.

Although Lyapunov works have been published at the start of 20th century [110], it took more than another half a century before they became the basic tool upon which modern stability analysis is based (for good presentations of Lyapunov direct method, see, for example the excellent books [63] and [148]). Nevertheless, in spite of the initial enthusiasm, pretty soon developers were forced to realize that in real world applications it is not easy to find an appropriate Lyapunov function with a negative definite derivative. The main difficulty that limited the applicability of the direct Lyapunov approach was the fact that, in most applications, the derivative of the Lyapunov function usually was at most negative semidefinite. Here, things started looking pretty complex.

Krasovskii-LaSalle Invariance Principle First extensions to Lyapunov-style approach for the case when the derivative of the Lyapunov function is only negative semidefinite were first attributed in the West to LaSalle [106], yet now are also attributed to Krasovskii and Barbashin [96] . Their result has become known as the Krasovskii-LaSalle Invariance Principle, that was only covering strictly autonomous functions of the form $\dot{\mathbf{x}}(t) = f(\mathbf{x})$ (where the time does not explicitly show). In this case, they show that bounded trajectories end within the domain that is defined by $\dot{V}(t) = 0$. We notice that, unlike the strictly negative definite case, this result does not necessarily imply asymptotic stability as its implications depend on the meaning of $\dot{V}(t) = 0$.

Moreover, most non-trivial control systems are *nonautonomous* of the form (74) and, as one may usually read in the professional literature "unfortunately, this system is nonautonomous and so, the invariance principle cannot be applied."

Therefore, because control systems are nonautonomous, new extensions to the basic Lyapunov stability theory have been sought.

Barbalat's Lemma One of these extensions was provided by Barbalat's lemma that we state here as it appears in [158]:

Lemma 1 *(Barbalat's Lemma): If the differentiable function $V(t)$ has a finite limit as $t \to \infty$ and if $\dot{V}(t)$ is uniformly continuous (or $\ddot{V}(t)$ is bounded), then $\dot{V}(t) \to 0$ as $t \to \infty$.*

Barbalat's Lemma is very simple and, therefore, very attractive. Furthermore, under some conditions, it allowed to finally show that the function $\dot{V}(t)$ ultimately vanishes and in many cases even it allowed reaching the desirable asymptotic stability or asymptotically perfect tracking conclusion. Nevertheless, it also leaves the burdensome impression that any input command, distortion or disturbance that may affect the uniform continuity of Lyapunov derivative may affect the proof and, therefore, the very guarantee of *stability of nonlinear systems. However, as we show below, it is only because Barbalat's lemma deals with the functions and not with the systems that it imposes those strict conditions on continuity of functions and even of their derivative. These conditions may happen to hold in some systems, yet if they are not satisfied under less than ideal conditions, it is not necessarily a result of some lack of stability.*

These conditions are needed because, as most Colleagues and some of the best publications seem to think, "Unfortunately, LaSalle's Invariance Principle, and better call it 'Krasovskii-LaSalle Invariance Principle,' only covers 'time-invariant' (or, more exactly, autonomous) systems." Is it indeed so? Yes, yet only if one still sticks to the 60s.

(The Real) LaSalle's Invariance Principle As a matter of fact, extensions of LaSalle's Invariance Principle to nonautonomous systems have been available at least since 1976 [8–10, 104, 105], and have been immediately adopted and used since 1980 for such special problems as adaptive control of large space structures and other applications (see for example [91] for a proof and a brief presentation of the theory along with some early examples). Nonetheless, as classical books in nonlinear systems [93, 153, 158, 170] and even recent publications [66, 108] seem to show, either the Principle is largely unknown or, at least, has remained misunderstood.

Therefore, it is important to first emphasize LaSalle's simple and ingenious idea. Instead of dealing with the properties of some general *function*, LaSalle's Invariance Principle goes back to establishing some milder conditions on the *system*. Using the notation $|f| = \sqrt{f_1^2 + f_2^2 + ... f_n^2}$, satisfaction of one of the following two assumptions *along trajectories* of a system is *checked*:

1) $|f(\mathbf{x}(t), t)|$ is uniformly bounded for any *bounded* $\mathbf{x}$.

   or

2) $\int_\alpha^\beta |f(\mathbf{x}, \tau)| d\tau = \mu(\beta - \alpha)$.

We note that in LaSalle's formulation, the function $\mu(\tau)$ is a "modulus of continuity," to be discussed in continuation. Furthermore, LaSalle's Invariance Principle deals with the more general case when not all trajectories are necessarily bounded and so, it only attempts to locate the limit points of those trajectories that are bounded. Therefore, its Lyapunov function is only required to be bounded from below and is not necessarily required to be Positive Definite.

Under these assumptions, we can present LaSalle's Invariance Principle for nonautonomous systems:

Theorem 3 *(LaSalle's Invariance Principle): Consider the nonlinear non-autonomous system (74). Assume that there exists a Lyapunov function $V(\boldsymbol{x})$, which is bounded from below, and that its derivative $\dot{V}(x, t)$ along the trajectories of (74) is Negative Semi-Definite and satisfies a relation of the form $\dot{V}(\boldsymbol{x}, t) \leq W(\boldsymbol{x}) \leq 0$. Now, define the domain $\Omega = \{\boldsymbol{x} | W(\boldsymbol{x}) = 0\}$. Then, if one of the assumptions 1 or 2 holds, all bounded trajectories ultimately reach the domain Ω* [105], [91].

Of course, whenever the case is and the appropriate Lyapunov function is found, one may show that all trajectories are bounded. Besides, LaSalle's formulation is that bounded trajectories end within "an invariant set within the domain Ω." The actual meaning of this formulation may look pretty obscure and it will be explain later below.

The new Invariance Principle As recently observed [28], even the milder conditions that LaSalle's original formulation *seems* to impose are both difficult to satisfy in realistic applications and also are not necessarily needed. Therefore, a new Invariance Principle for nonautonomous systems was presented [28] in a form that is appropriate for the problems we discuss and that further relaxes even the milder conditions of LaSalle. Here, satisfaction of one of the following two assumptions is *checked* along trajectories of a system:

A) $|f(\mathbf{x}(t), t)|$ is uniformly bounded for any *bounded* $\mathbf{x}$.

   or

B) $\int_\alpha^\beta |f(x(\tau), \tau)| d\tau$ is bounded along any *bounded* trajectory $\mathbf{x}(t)$ and for any *finite* time interval $p = \beta - \alpha$.

Note that, while Assumption A is identical to Assumption 1 in LaSalle's formulation, our presentation of Assumption B above is different from LaSalle's original presentations. In LaSalle's formulation, the function $\mu(\tau)$ in Assumption 2 is a "modulus of continuity," from which one can imply that the trajectory is supposed to be a continuous function of time. Here, we replaced the modulus of continuity by a simple bound, because although continuity is desirable, it cannot be guaranteed in practical environments and it is not necessarily needed for stability.

For a simple illustration, one may compare the "decent" uniformly continuous converging function $x_1(t) = e^{-t}$ with the equally converging, though not continuous, ladder function described by $x_2(t) = e^{-(k+1)}$ for $k < t \leq k+1$, k=0,1,2,... Nor is continuity actually needed for stability under the Invariance Principle approach, because, as the proof of stability shows, the only condition that is needed is that the trajectory, continuous or not continuous, *cannot* pass an infinite distance in finite time.

On another point, LaSalle's works could only deal with those Lyapunov derivatives that satisfy a relation of the form $\dot{V}(x,t) \leq W(x) \leq 0$. However, although in many cases this relation could be sufficient, at a second look it may *unnecessarily* restrict the applicability of stability theory. For example, while for $\dot{V}_1(x,t) = -x^2(2 + sint)$ one can define $W(x) = -x^2$ and then write $\dot{V}_1(x,t) \leq W(x) \leq 0$, this is not possible for $\dot{V}_2(x,t) = -x^2(1 + sint)$. Nevertheless, one *can* define $W(x) = -x^2$ and $g(t) = 1 + sint$ and get $\dot{V}_2(x,t) \leq W(x)g(t) \leq 0$. It is clear that $\dot{V}_2(x,t) \leq 0$ or, in other words, that $\dot{V}_2(x,t)$ is *uniformly* negative semi-definite. In a more general case, such as $\dot{V}_3(x,t) = -x_1^2(1 + sint) - x_2^2(1 + cost)$ it is still clear that $\dot{V}_3(x,t)$ is *uniformly* negative semi-definite although it cannot be written in any one of the (more convenient) previous forms. Therefore, whenever needed, we will show that one can directly deal with *uniformly* positive and negative definite *explicit* functions of time.

Theorem 4 *(The new Invariance Principle): Consider the nonlinear non-autonomous system (74). Assume that there exists a Lyapunov function $V(\boldsymbol{x})$ which is bounded from below and that its derivative $\dot{V}(\boldsymbol{x},t)$ along the trajectories of (74) is Negative Semi-Definite, i.e., satisfies $\dot{V}(\boldsymbol{x},t) \leq 0$. Now, define the domain Ω where the Lyapunov derivative equals zero, $\Omega = \{\boldsymbol{x}|\dot{V}(\boldsymbol{x},t) = 0)\}$, and the restricted domain Ω_i where the Lyapunov derivative is identically zero (i.e., not just equal zero), $\Omega_i = \{\boldsymbol{x}|\dot{V}(\boldsymbol{x},t) \equiv 0)\}$. Then, if one of the assumptions A or B holds, all bounded trajectories ultimately reach the domain Ω. In particular, equilibrium points and limit cycles belong to the restricted domain Ω_i* [28, 30].

Note also that this work emphasizes the identity relation $\Omega_i = \{\boldsymbol{x}|\dot{V}(\boldsymbol{x},t) \equiv 0\}$ instead of simple equality relation $\Omega = \{\boldsymbol{x}|\dot{V}(\boldsymbol{x},t) = 0\}$. Examples illustrate the practicality and usefulness of defining the limit set by the identity relation instead of simple equality relation, because it results in sharper conclusions.

The example $\dot{e}(t) = -e(t) + \theta w(t)$ $\dot{\theta}(t) = -e(t)w(t)$ of a very simple adaptive control system was used in [158] *for an illustration of Barbalat's Lemma application to stability analysis. Here $e(t)$ is a tracking error while $\theta(t)$ is the gain. Selecting the Lyapunov function $V(t) = e^2(t) + \theta^2(t)$ results in the derivative $\dot{V}(t) = -2e^2(t) \leq 0$. Because, as stated in* [158] *"one cannot conclude the convergence of e(t) because the dynamics is nonautonomous,"* [158] *imposes conditions on the input command w(t) that would guarantee uniform continuity of Lyapunov derivative and therefore, application of Barbalat's Lemma indicates that $e \to 0$ as $t \to \infty$.*

Because the system satisfies (the fairly mild) Assumption A for any bounded w(t), LaSalle's Invariance Principle directly tells us that the state-vector $\{e, \theta\}$ ends within the domain defined by $W(e(t)) = -2e^2(t) = 0$ which immediately results in same result $e(t) = 0$ as above without requiring that $W(e(t))$ necessarily be uniformly continuous.

Still, this result is more restricted than it may look at first sight. First, in both cases there is no clear conclusion that can be drawn with respect to θ. Moreover, it does not say whether the set includes all points on the $e(t) = 0$ axis, just some points that the trajectories pass when they occasionally cross the axis, or if ultimately an entire motion occurs along this axis.

Instead, a full response is provided by the new Invariance Principle. The clear satisfaction of Assumption A for any bounded w(t) directly tells us that the state-vector $\{e, \theta\}$ ends within the domain defined by $W(e(t)) = -2e^2(t) \equiv 0$ which immediately results in same result $e(t) = 0$ as above. However, because $W(e(t)) \equiv 0$ implies that the derivatives of $W(e(t))$ are also zero at $e(t) = 0$, we differentiate the function $W(e(t))$. The first derivative does not add much, because $\dot{W}(e(t)) = -4e(t)\dot{e}(t) = 0$ at $e(t) = 0$ for any value of $\dot{e}(t)$. However, the second derivative $\ddot{W}(e(t)) = -4(\dot{e}(t))^2 - 4e(t)\ddot{e}(t) = -4(\dot{e}(t))^2 = 0$ implies $\dot{e}(t) = 0$. Substituting in both equations also leads to $\dot{\theta}(t) = 0$ and $\theta w(t) = 0$. In other words, the fairly straightforward result of the Invariance

Principle for this nonautonomous system is that the tracking error ends at zero and stays at zero, while the adaptive gain $\theta(t)$ ends at some constant value. If the input command $w(t)$ ends at a nonzero value, then the gain $\theta(t)$ ends at zero, yet even if $w(t)$ ends at a zero, the gain $\theta(t)$ still ends at some constant value, without requiring any of the "customary" persistent excitation conditions [28, 30].

Moreover, even LaSalle's original formulation restricted the discussion to those systems where one could write $\dot{V}(\boldsymbol{x}, t) \le W(\boldsymbol{x}) \le 0$. However, many real-world examples [28] *show that in many practical situation one cannot define Ω as above, yet nonetheless Ω can be defined as $\Omega = \{\boldsymbol{x} | \dot{V}(\boldsymbol{x}, t) \equiv 0\}$. As one can prove* [28]*, the new definition is legitimate and is covered by the new Invariance Principle and allows extending the stability analysis to large classes of systems that were not covered before.*

In other words, under fairly mild conditions, the Invariance Principle extension to nonautonomous systems guarantees that all trajectories ultimately reach the domain Ω. However, as examples illustrate [28, 30]*, its significance and efficiency and sometimes even its mere existence seems to have remained unknown to a large section of potential users.*

Towards a new Theorem of Stability Note that stability theory based on the new Invariance Principle approach eliminates the previous requirement for *uniform* continuity of the Lyapunov derivative and actually any other requirement, except for the guarantee that any bounded trajectory **x**(t) *cannot* pass an *infinite* distance in *finite* time.

The many examples of [28] showed that the use of Ω_i was very efficient in locating equilibrium points and limit cycles. Also, as far as equilibrium points and limit cycles are concerned, no one of the prior assumptions above was actually needed, because the way the trajectory reaches the isolated equilibrium point or the limit cycle (i.e., in finite or infinite time) is immaterial. Assumptions A and B were needed only in order to show that limit points of type *rosette*, i.e., those isolated rosette-type limit points, that the trajectories might reach, leave, and then come back to them an infinite number of times, must belong to $\Omega_e = \{\mathbf{x} | \dot{V}(\mathbf{x}, t) = 0\}$. However, already in [28] the usefulness of the equality relation was already considered to be very doubtful, because without supplementary knowledge about a particular system, it would be hard to differentiate between rosette-type limit points and all other points of the trajectory, with no special meaning whatsoever, where at one time or other the Lyapunov derivative just occasionally happens to be zero.

For a very simple illustration, assume that the Lyapunov function for a system in R_{20} is $V(\mathbf{x}) = V(x_1, x_2, ..., x_{20}) = x_1^2 + x_2^2 + ... + x_{20}^2$, while the derivative *along the trajectories* of the system is $\dot{V}(\mathbf{x}) = -x_1^2$. All limit points must satisfy $\dot{V}(\mathbf{x}) = -x_1^2 = 0$, which in turn results in $x_1 = 0$. However, the converse is not necessarily relevant. Although at first look the "solution" $x_1 = 0$ may look satisfactory, a second look shows that it does not have much relevance in terms of system stability. The result $x_1 = 0$ not only contains all system trajectories that keep moving within R_{19} but also all points and trajectories that *all* trajectories of the *entire* space R_{20} may form when, at this time or other, they occasionally cross $x_1 = 0$.

Here, it could be useful to recall that in the case of the original Lyapunov Stability Theorem, where the Lyapunov derivative is negative *definite*, the conclusion $\dot{V}(\mathbf{x}) = 0$ is *equivalent* to $\mathbf{x} = 0$. Furthermore, because the vector **x** contains the *entire* dynamics of the system, there was no need to even mention that the result $\mathbf{x} = 0$ actually is *equivalent* to $\mathbf{x} \equiv 0$. However, although it may be difficult to accept it and even though some points that satisfy the simple equality $\dot{V}(\mathbf{x}) = 0$ in the *semi*definite case $\dot{V}(\mathbf{x}) \le 0$ could also have relevance with respect to stability, most have no relevance at all and it is almost impossible to separate those that may have any relevance. In this context, although the result "trajectories ultimately end within the domain defined by $\dot{V}(\mathbf{x}) = 0$" seemed as a good result, it should have been clear that no "end of motion" is guaranteed by $\dot{V}(\mathbf{x}) = 0$ *unless* its next derivative is also zero, $\ddot{V}(\mathbf{x}) = 0$, and then next derivative and so on, or in other words unless one requires that at least the dynamics of the Lyapunov derivative vanish, or in other words that $\dot{V}(\mathbf{x}) \equiv 0$.

Nevertheless, although the great effort to guarantee the mere $\dot{V}(\mathbf{x}) = 0$ was motivated by the fear of missing those "special" rosette-type limit points, now one can see [30] that, although most probably a necessary step in the development of a complex idea, the special treatment that rosette-type limit points have received might have been exaggerated and that the eventual use of the assumptions and of $\Omega_e = \{\mathbf{x}|\dot{V}(\mathbf{x}, t) = 0\}$ could be redundant.

In retrospect, it is amazing that, while so much thought and effort had been invested to investigate what must happen *at* those particular rosette-type point locations, not much room or thought was left for all those segments that a trajectory must pass *after* leaving the limit point and *before* coming back to it. If the trajectory happens to only pass a rosette-type point and then come back to it a *finite* number of times, then this rosette-type point is not a limit point at all. Only if the trajectory revisits the point an *infinite* number of times would the rosette-type point become a limit point. In this case, however, except for those moments when the trajectory coincides with the rosette-type point and when indeed $\dot{V}(\mathbf{x}, t) = 0$, at other times and along most sections of the trajectory we are supposed to have $\dot{V}(\mathbf{x}, t) < 0$ and this *strict* negativity of the derivative situation is repeated again and again, for all times up to infinity. In such a case, assuming that the trajectory first reaches the rosette-type limit point at time $t = t_1$, then $V(\mathbf{x}(t), t) = V(\mathbf{x}(t_1), t_1) + \int_{t_1}^{t} \dot{V}(\mathbf{x}(\tau), \tau)d\tau$ and therefore $\lim_{t\to\infty} (V(\mathbf{x}(t), t))$ would tend to $-\infty$ *unless* for any ϵ positive and arbitrarily small there exists some *finite* time t_2 such that $|\dot{V}(\mathbf{x}, t)| \leq \epsilon$ for any $t \geq t_2$. Thus, ultimately, $\dot{V}(\mathbf{x}, t)$ tends to zero *all along* the trajectory or, in other words, even what might have started looking as a rosette-type limit point *ultimately* must also belong to Ω_i, as part of a limit cycle or even as an equilibrium point.

Another point to ruminate about before going on to the theorem of next section is the very definition of limit points. A limit point, or point of accumulation, of a trajectory is such a point that any neighborhood, arbitrarily small, around it contains an infinite number of points of the trajectory. When one first hears this definition, one could be confused, because it seems pretty clear that any point of a continuous curve satisfies this condition. Therefore, when one deals with trajectories, one defines a *discrete* time sequence $\{t_k\}$ and, correspondingly, discrete-time *points* on the trajectory $\{\mathbf{x}(t_k)\}$. In this context, a limit point is that point of the *discrete* sequence $\{\mathbf{x}(t_k)\}$ that any neighborhood, arbitrarily small, around it contains an infinite number of *discrete* points of the trajectory. Next, any *bounded* trajectory that leads to the creation of such an infinite number of discrete points, *must* contain at least one such accumulation. Therefore, as we only want to know where these discrete-time limit points are located, even though the ever diminishing distances between points around a limit point may sound similar to the definition of continuity, they have nothing in common with, neither do they need to even mention any continuity.

The new Theorem of Stability As explained above, *all* limit points of any bounded trajectory must ultimately either become equilibrium points or belong to a limit cycle. In order to formulate the new Theorem of Stability in its most general form, we define the domain Ω_i as follows:

$$\Omega_i = \left\{\mathbf{x}|\lim_{t\to\infty}\left(\dot{V}(\mathbf{x}, t)\right) \equiv 0\right\}. \tag{75}$$

Now we can write the new Theorem of Stability in the following simple formulation:

Theorem 5 *(The new Theorem of Stability) Consider the nonlinear non-autonomous system (74). Let $V(\boldsymbol{x})$ be a differentiable function bounded from below. (Note that $V(\boldsymbol{x})$ is not required to be Positive Definite.) Assume that its derivative $\dot{V}(\boldsymbol{x}, t)$ along the trajectories of (74) is Negative Semidefinite, i.e., satisfies $\dot{V}(\boldsymbol{x}, t) \leq 0$. Then, all limit points of any bounded trajectory $\boldsymbol{x}(t)$ belong to the domain Ω_i* [27, 30].

Because limit points of trajectories are those points that the trajectory reaches as time approaches infinity, it is important to emphasize that is sufficient if the identity relation that defines Ω_i is also only satisfied as time approaches infinity. In special cases, though, as examples of [28] *and* [30] *show, the identity could be satisfied after some finite time and even for any time, implying that for some limit points, if the trajectory starts there or reaches there at some finite time, it stays there thereafter. Note also that, for convenience, because $V(\boldsymbol{x})$ is a selected function, we assume that both $V(\boldsymbol{x})$ and $\dot{V}(\boldsymbol{x}, t)$ are continuous functions of $\boldsymbol{x}$. However, as shown in* [28] *and* [30]*, their differentiability with respect to t implies the Dini derivatives (see for example* [50, 134]*). In other words, while it is nice to have continuous functions that are also differentiable in the classical sense, stability is not affected if eventual discontinuity of $\boldsymbol{x}(t)$ leads to discontinuity of $V(\boldsymbol{x}(t))$. In this context, a piece-wise continuous* *function may still have a derivative everywhere, even if its derivatives at the points of discontinuity are δ-functions.*

In its most general form, the new Theorem of Stability does not require that $V(\boldsymbol{x})$ be positive definite and, therefore, it does not guarantee that all trajectories are bounded. Of course, when special selections, such as positive definite functions, functions of class K, etc. [63, 148]*, are available, boundedness of either some trajectories or of all trajectories is guaranteed.*

It is also important to again explain the use of the identity relation $\dot{V}(\boldsymbol{x}, t) \equiv 0$. Because it implies that higher order derivatives must also be zero, which in turn seems to imply that the function must be infinitely differentiable, it is easy to think about counterexamples. Nevertheless, before thinking of counterexamples in the general context of mathematical functions, the reader is encouraged to check the various examples of [28] *and* [30] *where, again in the context of systems of equations that are entirely defined by the first derivative $\boldsymbol{f}(\boldsymbol{x}, t)$ of the n-dimensional state-vector and, because only differentiation along the trajectories is concerned and only as time tends to infinity, when the Lyapunov derivative reaches zero and comes to rest there, the conditions are satisfied in most relevant cases and allow solving situations that would seem unsolvable otherwise.*

8 Brief Review and Summary of Results

1. Although the name "Adaptive" got various interpretations over the years, it was the attractive idea of fitting the right gain to the right situation that was behind the first Adaptive Control approaches. In particular, the MIT Adaptive Control rule, based on solid Engineering experience and intuition, looked very promising in fulfilling the promises. Its ending in disaster showed that in the nonstationary world, intuition alone is not enough.
2. The first basic MRAC concepts attempted to extend the LTI Model Following approach from the fully deterministic world to the world of uncertainty and thus, to use adaptive gains in order to supply the Plant with appropriate signals from a "good" Model Reference that would ultimately force the Plant state to reproduce the behavior of the Model state. Even though it ended with a first proof of stability, the nonstationary world proved to me much more demanding that the stationary world and so, in order to reach this proof, the Plant itself was required to be SPR, condition that proved to be virtually nonexistent in real-wold applications.
3. The move to "classical" MRAC was based on assuming full-state availability and using and extending the state-feedback Stabilization (or Optimization) idea into the adaptive world. The "classical" MRAC algorithm basically uses the appropriate Stabilization (or Optimization) signal $b'Px$ and then multiplies it by the state-following error e in order to build the adaptive gain vector $\dot{\theta}(t) = \gamma \mathbf{e}^T(t)\mathbf{Pbx}(t)$ (of the order of the plant). Along with assuming other mitigating assumptions, this approach managed to use state feedback and fulfill an SPR-like condition that managed to end with a proof of asymptotically perfect tracing under ideal conditions. The developers immediately observed serious problems when the conditions are less than ideal.
4. As this paper noted, *except for a scalar factor*, the signal $b'Px$ already *is* the state feedback signal needed to stabilize (or "optimize") the plant. Therefore, although in the uncertain world this scaling factor is not known, the entire MRAC adaptive gain vector actually can be replaced by just *one single scalar* adaptive gain that multiplies *one single scala*r tracking error. Not only that the proof of stability is not affected, but all "inherent" problems of MRAC seem to vanish. One of the important properties of the special signal $b'Px$ is the guarantee of high-gain stability. In other words, the adaptive algorithm that will compute the unknown scale factor must only be such that increases if something tends to go wrong. This does not imply that high-gains are or have to be used. It only adds the guarantee that, if the adaptive gain has to increase because of some temporary disturbance or other issue, this does not lead to any bursting, etc, not to mention divergence. This guarantee, in turn, ultimately allows obtaining superior performance without requiring high gains or bandwidth from the internal control-loop.
5. Moreover, a new approach has been developed, which first of all attempts to do control of large real-world systems without relying on

high-order adaptive controllers and/or on full-state availability. Instead of assuming availability of full-state feedback, this so-called Simple Adaptive Control (SAC) approach started looking for special systems that may directly provide the desired signal $b'Px$ at their output. Such systems would then be stabilized by just some simple output gain feedback. In other words, there exists some PDS matrices, P and Q, and a gain $\widetilde{K}_e$ such that the stability Lyapunov equation relation (52) of the fictitious closed loop is satisfied. Because it also satisfies a relation of the form $b'P = c$, the closed-loop system is Strictly Passive (or Strictly Positive Real). Because the open-loop Plant itself is only at the distance of a constant gain, it got the name Almost SP (ASP) or Almost SPR (ASPR).

6. Even though, like the SPR concept before, the ASPR concept was also initially received as another obscure term, it was finally shown that any minimum-phase plant $\{A, B, C\}$ with $CB > 0$, i.e. PDS (which makes the system to be of relative degree 1), is ASPR. Also, any minimum-phase plant $\{A, B, C, D\}$ with D nonsingular is ASPR. Therefore, ALMOST SPR (ASPR) brings the ideal SPR property from a better world down to our Earth.
7. Nevertheless, in spite of the successful application of the ASPR concept to Large Flexible Structure with collocated sensors and actuators (even if later on mitigated to "almost" collocation), not many real systems have been found which inherently satisfy even the ASP property. Here, the Simple Adaptive Control (SAC) made an important observation. Given the tremendous progress of Classical Control and, on the other hand, given the reluctance of practitioners to use adaptive control methods, because of their apparent complexity and 'inherent' problems, SAC is not called to solve all problems of totally unknown systems. On the contrary, it assumes that some basic Classical Control design is available and that this does guarantee some degree of stability. Therefore, SAC only intends to use the information already available in order to improve performance using adaptive techniques.
8. The important property that allows implementation of safe adaptive control is stabilizability. In particular, assume that a basic design ends with the Open-Loop system G. In order to improve performance, the designer would want to add another Controller C, yet this would require expensive supplementary velocity sensors or derivatives (and improper controllers) and high gains, with all problems. Instead, it was observed that if C maintains stability, then the fictitious augmented system $G_a = G+D$, where $D = C^{-1}$ is minimum-phase. If then C is such that the relative degree of G_a is 1 (or 0), then the augmented system is ASP.
9. Important note: even though the term "parallel" was introduced for convenience of the augmented system and, in practice, the addition D is only another feedback around the adaptive gain $K_e(t)$, the concept of "parallel feedforward," which makes the adaptive gain 'see' an augmented ASP plant, is important. In order to guarantee stability with the adaptive controller, even though the PFC is connected in feedback, the error signal that must be used for adaptation is not the actual error e_y, but rather the augmented error e_{ya} of the ASP system (Fig. 3).
10. The big surprise even for the developers of SAC: As we mentioned, the developer may think about some eventually better performance, with some supplementary ideal controller, $C(s)$, maybe improper (more zeros than poles), with some new sensors, high gains, etc., maybe a good idea to pursue towards next generation controllers. Instead, one just implements some simple feedback $D(s) = C^{-1}(s)$ and the Simple Adaptive Control ultimately provides this *ideal* performance. Because the use of PFC makes possible to supply the exact feedforward signal from the model, *the performance is also obtained without having to stress the gain and the bandwidth of the internal closed-loop*.
11. OT. Aristotle's Paradigm: Aristotle, arguably the greatest mind in the history of mankind, teaches us why the moon is larger at the horizon than up in the middle of the sky. The explanation is clear and convincing and based on best reasoning. Nevertheless, the real explanation is that Aristotle never felt the need to stick a finger out and actually *measure* and see that the moon is the same in both positions. This does not take anything off from our estimation of Aristotle. It only

shows the power of Paradigm that may affect even the best of minds.

12. The problem is that we all live within some Paradigm and it is difficult to accept that things could be too different from what we have been taught, in particular *after* we also start teaching them. "Terrible" objections and accusations: "I believe that, from the adaptive control viewpoint, all that can be done has already been done. Adding feedforward to the plant, is an artificial trick of little practical interest." The problem here is not that someone may not like something. The issue is that people, who practically are in charge of deciding the fate of adaptive control, feel they already know much too much and do not feel the need to really read things that are not their own or, at least, inherited from their own Grandmaster. They may even just close their eyes and ignore the myriads of 'impractical' applications, and yet, this does not stop them from making decisions.
13. Fact is, if practitioners only managed to end with the "best possible" system $G(s)$ and would want to add the expensive system $C(s)$ in order to improve performance, at the price of new expensive parts, high gains, and high bandwidth, they could be surprised that same aim could be obtained by just changing and adding a few lines of the existent software, and without requiring the cost, the high gain and bandwidth. Sure, this sounds like pure bragging for someone who never tried it. Fortunately, many people around the world, most of them experienced classical Control designers, who did not have the "good luck" to hear that "this bad method doesn't work," decided it was easy enough to just try it and see and... it worked. They are now adaptive control developers.
14. Now, do things really work? This is nonlinear stuff, so what about the proofs of stability? Even when one accepts to have a brief look at some application, the answer could be "I can kill all your proofs of stability with even the tiniest discontinuity." This is scary, not because it threatens to 'destroy' the proof, but because potential applicants could be afraid that their robot may break apart, or that their flying plane may fall, just because they only tried a square-wave command, or just because of some occasional disturbance, not to even think of mentioning impulse response. Why is this? Because the Good Old Lyapunov required an appropriate Lyapunov function with a Negative Definite derivative, while, except for class-room examples, the derivative is at most Negative *Semi*definite. Here, all Hell seems to have broken loose and all kinds of "absolutely necessary" conditions (in particular uniform continuity of the Lyapunov derivative) and limitations seem to make the stability analysis of nonlinear systems not only complicated and incomplete, but also the stability itself to look fragile and unsafe.
15. In this context, it is peculiar how such important contributions like the *real* LaSalle's works of 1976-1980 have remained totally unknown for the vast majority of Nonlinear system users and developers, including the best books and publications on Nonlinear Control. Moreover, seing the name "LaSalle" in a new work makes the most respected Colleagues to not only reject and declare it 'routine' without any more reading, but they would actually want the author to be ashamed and apologize for calling "LaSalle" an Invariance Principle that should actually be called "Krasovskii-LaSalle."

    Nothing is wrong... except for the fact that they all talk prehistory (1950-1960) to the *real* LaSalle's Invariance Principle of 1976-1980.
16. In retrospect, as usually may happen, the 'bad' reviews may end being the best and harsh responses actually had a very positive effect, as they ultimately forced the author to not only explain and illustrate everything better, but actually to mitigate end even eliminate most remaining "conditions" left there even by LaSalle. The end result, the new Theorem of Stability, is a direct extension of the original Lyapunov theorem to the case of Negative *Semi*definite derivative. Indeed, one is still required to fit an appropriate Lyapunov function with a Negative Semidefinite derivative, yet then stability analysis is clear and straightforward, without requiring any uniform continuity or any of all those other "customary" ifs and maybes.
17. Bottom line, the forefathers' adaptive control ideas, their introduction of passivity and of its use towards first successful proofs of stability of the Adaptive Control algorithms are the good

basis on which safe and simple adaptive algorithms can be built, which are able to use the flexibility of nonstationary gains and thus, to improve upon the best classical Control design even when all parameters are known. Moreover, as many practitioners have observed, the performance is maintained in the presence of either stationary or nonstationary uncertainties (see, for example, the recent [32] and [31]). In this context, the Simple Adaptive Control methodology is the stable Model Reference.

9 Conclusion

This paper presents a Simple Adaptive Control approach that, while continuing and building on the important contributions of Model Reference Adaptive Control pioneers, it not only significantly simplifies and reduces the order of the Adaptive Controller, but it also avoids customary problems of Model Reference Adaptive Control, thus making Adaptive Control into a useful and reliable tool for continuous performance improvement of Control design.

Acknowledgment This is a modest tribute to the memory of the late Eugène Aisberg, author of 'La Radio? Mais c'est très simple!' and similar, which facilitated to so many generations of curious youngsters a first real understanding of the secrets of radio, television, transistor, etc. The author is grateful to the respected Colleague who awarded this approach the greatest honor of calling it 'trivial.'

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In 2002 has received the Benjamin Franklin Key Award from Philadelphia Chapter of IEEE for "advancing the theory and practice of adaptive control, and in so doing, making major contributions to improving the speed and accuracy of specialty robotics for the semiconductor industry." These techniques helped to extend the life of wire bonding technology. As a result of this work, the challenge of portability – the ability to repeatedly perform the same task on different machines – was solved. The time duration of bond table motion was reduced by more than 60%, dramatically increasing productivity, and the accuracy limit was lowered from more than 100 microns in the late 1990s to 35 microns in the current generation of bonders. The robustness of the algorithms has been proven over the last 17 years by its successful implementation in tens of thousands of machines representing various platforms. It is one of the major innovations that have made these wire bonders the world industry leaders.

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