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Cooperative Target-capturing with Incomplete Target Information

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Abstract This paper presents a distributed target-centric formation control strategy for multiple unmanned aerial vehicles (UAVs) in the presence of target motion uncertainty. The formation is maintained around a target using a combination of a consensus protocol and a sliding mode control law. Consensus helps in distributing the target information which is available only to a subset of vehicles. Sliding mode control compensates for the uncertainty in the target information. Hence, collectively the combined strategy enforces each of the vehicles to maintain its respective position in the formation. We show that if at least one vehicle in a group has target information with some uncertainty and the corresponding communication graph is connected, then a target-centric formation can be maintained. The performance of the proposed strategy is illustrated through simulations.

Keywords Formation control · Sliding mode control · Consensus

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1 Introduction

Recently, several researchers have studied multi-agent systems to explore their potential advantages over a single system including parallelism, robustness, and scalability [1]. Over the past few years multi-agent coordination and control solutions have been applied to many challenging civilian and defence applications: surveillance [2, 3], search and rescue [4, 5], and fire monitoring [6, 7].

In this paper, we focus on multi-agent target tracking, where a team of vehicles (or UAVs) must maintain a target-centric formation. In a target-centric formation, each vehicle in the group maintains a constant distance from the target with a specified angle. The target-centric formation can be used to restrict the motion of a hostile target or to safely escort a vehicle of importance to a desired location. In practise small UAVs have limited sensor and communication ranges and they may not be able to access complete target information. Maintaining a formation with these limitations is challenging. In this paper, we address the problem of formation control for multiple UAVs under these conditions.

Several researchers have investigated formation control using different approaches including leader-following [8–12] virtual structure approaches [13–16] and behavior-based methods [17, 18]. A concise review of several formation control techniques and stability issues can be found in [19]. In the leader-following approach, some of the vehicles act as leaders whereas the rest act as followers. Since, there is no information flow from follower vehicles to leaders, if any leader vehicle deviates from a desired trajectory, this may disturb the formation. In the virtual structure approach, there exists no hierarchy and it is assumed all vehicles are virtually connected. Then, the virtual structure (formation geometry) is maintained by generating reference trajectories for each agent and tracking them. The approach works in a centralized manner and hence is not suitable where agents cooperate in a distributed manner and do not scale with the number of UAVs. In the behavior-based approach, a particular (heading alignment) type of behavior is assigned to each vehicle and each agent tries to perform the assigned behavior. The approach can be useful where multiple competing objectives are involved and for large scale systems. However, it is difficult to carry out stability analysis limiting its applicability.

Kim et al. [20] proposed a distributive cooperative control method to maintain a target-centric formation based on a cyclic pursuit strategy. However, in this method, every vehicle in the group should have the target information without any uncertainties, which is not always possible. Consensus, as addressed in [21–23], can be used to propagate the target information from a subset of nodes to all the nodes in the network. Consensus is a method of reaching an agreement on some information of interest in a distributed manner. Ren [24] proposed a consensus based formation control strategy for a multi-vehicle system where the states of all vehicles approach a common time-varying reference state only with a subset of vehicles requiring knowledge of the reference state. Kawakami et al. [25] proposed a target-centric formation control strategy based on consensus seeking with a dynamic network topology. The authors show that a desired formation will be maintained if at least one vehicle in the group has the target information. Although, controllers developed using consensus in [24, 25] overcome the limitation of having information to all agents in a group, the controllers do not take into account the uncertainty in the target motion. If the upper bound in the uncertainty is known, then the sliding mode control can be used to drive the system states to a desired sliding manifold [26].

In this paper, we combine consensus protocol and sliding mode control to derive a target-centric formation controller. We use the sliding mode control to handle uncertainty of a target motion and show that the corresponding controller will produce desired system states, if the upper bound of the target inputs is known. We show that all vehicles maintain a target-centric formation in the presence of target motion uncertainty assuming that the vehicle communication network is connected and at least one of the vehicles has the target information, and upper bounds of the target motion input (velocity and acceleration) are available. Information of a vehicle (target/UAV) consists of vehicle position, orientation, velocity, and acceleration.

The paper is organized as follows. In Section 2, we formulate the problem, and in Section 3, we derive the target-centric formation controller using consensus and sliding mode control. We discuss numerical results in Section 4, and draw conclusions in Section 5.

2 Problem Statement

Let $\mathcal{A} = \{A_i \mid i \in I\}$ be a set of n UAVs, where $I = \{1, \cdots, n\}$. A directed graph $\mathcal{G}_n$ is used to model the communication topology and sensing topology among these UAVs. In $\mathcal{G}_n$, the i^{th} node represents the i^{th} agent A_i, and a directed edge from A_i to A_j denoted as (A_i, A_j)

represents a unidirectional information exchange link from A_i to A_j, which means that, agent j can receive or obtain information from agent i, $(i, j) \in I$. Two nodes A_i and A_j can communicate with each other if, and only if, the distance between two nodes ρ_{ij} is less than the sensing/communication range ρ_s. If there is a directed edge from A_i to A_j, A_i is defined as the parent node and A_j is defined as the child node. A directed path in graph $\mathcal{G}_n$ is a sequence of edges $(A_1, A_2), (A_2, A_3), \cdots, (A_{q-1}, A_q)$ where $q \leq n$. Graph $\mathcal{G}_n$ is called strongly connected if there is a directed path from A_i to A_j and A_j to A_i between pairs of distinct nodes A_i and A_j, $\forall\ (i, j) \in I$. A directed tree is a directed graph, where every node, except the root, has exactly one parent. A spanning tree of a directed graph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that, a graph has (or contains) a spanning tree if a subset of the edges forms a spanning tree. The adjacency matrix for a directed graph $\mathcal{G}_n$ is defined as $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} = 1$, $i \neq j$, $\forall\ (i, j) \in I$, if $\rho_{ij} < \rho_s$ and $a_{ij} = 0$ otherwise. The Laplacian of a directed graph G_n is defined as $L_n = [l_{ij}] \in \mathbb{R}^{n\times n}$

$$l_{ij} = \begin{cases} \sum_{j=1}^{n} a_{ij}, \ if\ i = j \\ -a_{ij}, \ if\ i \neq j \end{cases} \tag{1}$$

The objective of n UAVs is to encircle a target A_t as shown in Fig. 1. In the rest of the paper, we use target and evader interchangeably. The communication and sensing topology between the n UAVs and the evader can be modelled by a directed graph $\mathcal{G}_{n+1}$. In directed graph $\mathcal{G}_{n+1}$ the first n nodes represent n UAVs and $(n+1)^{th}$ represents an evader. An directed edge from A_t and A_i, denoted by (A_t, A_i), represents a unidirectional information exchange link from A_i to A_t; that is, agent j can receive or obtain information from evader A_t, $i \in I$. However, the evader does not receive any information from any UAV node A_i, and therefore, $a_{it} = 1$ and $a_{ti} = 0$ in the adjacency matrix of the directed graph G_{n+1}. We can write the Laplacian matrix L_{n+1} for the directed graph $\mathcal{G}_{n+1}$ as

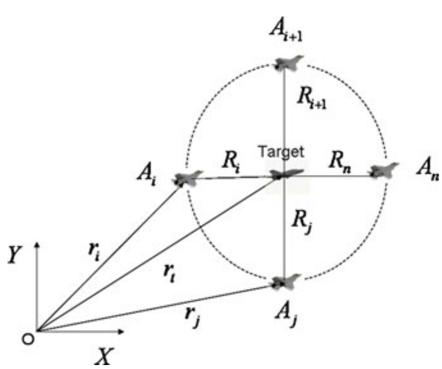


Fig. 1 Desired formation around the target. UAVs have to fly in formation with the target at relative distance $R_i = r_i - r_t = \xi[\cos\alpha_i \quad \sin\alpha_i]^T$. Angle of formation is $\alpha_i = \frac{2\pi i}{n}$, $i = 0, \cdots, n-1$

$$L_{n+1} = \begin{bmatrix} L_n + B & -b \\ 0_{1\times n} & 0 \end{bmatrix} \tag{2}$$

where $B \triangleq [b_{ij}] \in \mathbb{R}^{n\times n}$, $b_{ij} = \begin{cases} a_{it}, \ if\ i = j \\ 0, \ otherwise \end{cases}$, and $b = [a_{1t}, \cdots, a_{nt}]^\top$.

Lemma 1 *Rank of matrix $L_n + B$ is n if, and only if, $\mathcal{G}_{n+1}$ has a directed spanning tree.*

Proof From Eq. 2 we can write

$$\begin{aligned} \text{rank}(L_{n+1}) &= \text{rank}(L_n + B) \\ &= \text{rank}([L_n + B | -b]), \end{aligned}$$

and therefore, $rank(L_n + B) = n$, if and only if $\mathcal{G}_{n+1}$ has a spanning tree. □

Remark 1 In this paper, UAVs communicate their position, orientation, and control inputs (velocity and acceleration) with their neighboring UAVs. Therefore unless specified, information will mean position, orientation, and control inputs (velocity and acceleration).

In this paper, we assume that all UAVs and the evader fly at a constant altitude. The equations of the motion of the i^{th} UAV flying at a constant altitude are

$$\dot{r}_i = \begin{bmatrix} v_i \cos\psi_i \\ v_i \sin\psi_i \end{bmatrix} \tag{3}$$

$$\begin{aligned} \ddot{r}_i &= M_i u_i \qquad (4) \\ &= \begin{bmatrix} \cos\psi_i & -v_i \sin\psi_i \\ \sin\psi_i & v_i \cos\psi_i \end{bmatrix} \begin{bmatrix} a_i \\ \omega_i \end{bmatrix} \end{aligned}$$

where $r_i = [x_i \; y_i]^T$ is position vector, v_i is airspeed, ψ_i is heading angle, u_i is the control vector, a_i is acceleration, and ω_i is turn rate.

In a target-centric formation each UAV should maintain a constant distance ξ from the target at a constant angle α_i such that $\alpha_{i+1} - \alpha_i = \frac{2\pi}{n}$. To maintain target-centric formation, the following objective should satisfy

$$r_i(t) - r_t(t) \to R_i, \; \dot{r}_i(t) \to \dot{r}_t(t), \; \forall \, i \in I, \tag{5}$$

where $R_i = \xi \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \end{bmatrix}$. In order to satisfy the above control objective, we assume that the velocity of each UAV and the evader have upper and lower bounds, i.e, $v_{\min} < v_i \leq v_{\max}$, $\forall \, i \in I$ and $v_{\min} < v_t \leq v_{\max}$.

3 Target-centric Formation Controller

In this section, we detail control strategies to maintain a target-centric formation for two different cases. First, we derive a controller to maintain target-centric formation without any uncertainty in target motion. Secondly, we derive a target-centric controller using consensus and sliding mode control taking account of target motion uncertainty.

Remark 2 In this paper, incomplete information means that the exact target motion is not know but only the upper and lower bounds are known. We develop a controller which converges to a circular formation around a target if only upper bounds on the target inputs are available to UAVs.

3.1 Target-centric Formation Controller without Evader Motion Uncertainty

Given the evader state information with no uncertainty, we propose the following evader-centric formation strategy:

$$\begin{aligned} u_i = {} & M_i^{-1} \frac{1}{\sum_{j \in I} a_{ij} + a_{it}} \\ & \times \left(\sum_{j \in I} a_{ij} \left[\ddot{r}_j - (\dot{r}_i - \dot{r}_j) - \{\hat{r}_i - \hat{r}_j\} \right] \right) \\ & + M_i^{-1} \frac{1}{\sum_{j \in I} a_{ij} + a_{it}} \\ & \times \left(a_{it} \left[\ddot{r}_t - (\dot{r}_i - \dot{r}_t) - k_p \{\hat{r}_i - r_t\} \right] \right). \end{aligned} \tag{6}$$

where $\hat{r}_i = r_i - R_i$. This controller combines consensus protocol and target tracking information. The following theorem proves that the controller in Eq. 6 forms a target-centric formation given that at least one UAV can sense the target, and the communication network among the UAVs is connected.

Theorem 1 *Let each UAV in the group, with equation of motion given in Eq. 3, has the control vector as in Eq. 6. As $t \to \infty$, $r_i - r_t \to R_i$ and $\dot{r}_i \to \dot{r}_t$, $\forall \, i \in I$, if, and only if, the graph G_{n+1} has a spanning tree.*

Proof Let $r \triangleq [r_1^\top, \cdots, r_n^\top]^\top$ be the combined position vector of all UAVs in the group. Similarly, we define $\dot{r} \triangleq [\dot{r}_1^\top, \cdots, \dot{r}_n^\top]^\top$ and $\ddot{r} \triangleq [\ddot{r}_1^\top, \cdots, \ddot{r}_n^\top]^\top$. Using Eq. 6 we can write

$$\begin{aligned} [(L_n + B) \otimes \mathbf{I}_2]\ddot{r} = {} & -[L_n \otimes \mathbf{I}_2]\dot{r} - [L_n \otimes \mathbf{I}_2]\hat{r} \\ & -[B \otimes \mathbf{I}_2](\dot{r} - [\mathbf{1} \otimes I_2]r_t) \\ & -[B \otimes \mathbf{I}_2](\hat{r} - [\mathbf{1} \otimes I_2]r_t) \\ & +[B \otimes \mathbf{I}_2][\mathbf{1} \otimes I_2]r_t, \end{aligned} \tag{7}$$

where $\hat{r} = r - R$ and $R = [R_1^\top, \cdots, R_n^\top]^\top$. We use property of the Laplacian matrix, $(L_n + B)\mathbf{1} = B\mathbf{1}$, and write

$$\begin{aligned} & [(L_n + B) \otimes \mathbf{I}_2](\ddot{r} - [\mathbf{1} \otimes I_2]r_t) \\ & \quad = -[(L_n + B) \otimes \mathbf{I}_2](\dot{r} - [\mathbf{1} \otimes I_2]\dot{r}_t) \\ & \quad - [(L_n + B) \otimes \mathbf{I}_2](\hat{r} - [\mathbf{1} \otimes I_2]r_t) \end{aligned} \tag{8}$$

An unique solution of $\ddot{r}$ exists if and only if matrix $(L_n + B)$ is invertible, and from Lemma 1 matrix $(L_n + B)$ is invertible if and only if the $\mathcal{G}_{n+1}$ has a spanning tree. If matrix $(L_n + B)$ is invertible we can multiply $(L_n + B)^{-1}$ both sides of Eq. 8 and write

$$\ddot{r} - [\mathbf{1} \otimes I_2]r_t = -(\dot{r} - [\mathbf{1} \otimes I_2]\dot{r}_t) - (\hat{r} - [\mathbf{1} \otimes I_2]r_t), \tag{9}$$

and we can say that as $t \to \infty$, $r_i - r_t \to R_i$ and $\dot{r}_i \to \dot{r}_t$, $\forall\, i \in I$, if and only if the graph G_{n+1} has a spanning tree. □

3.2 Target-centric Formation Controller with Evader Motion Uncertainty

In the previous subsection, the controller needs exact information of the target to maintain a target-centric formation. In the presence of target uncertainty, follower vehicles may not be able to maintain the formation, especially when the target is executing evasive maneuvers. To tackle this issue, we use a sliding mode target tracking approach which compensates for uncertainty in target information and combine it with a consensus protocol to develop a robust target capturing strategy. Using sliding mode control a target can be tracked if an upper bound on the target input is known. For the given upper bound of target input, we propose the following evader-centric formation strategy, which takes into account uncertainty of the target motion.

$$u_i = M_i^{-1}\frac{1}{n_i}\left\{\sum_{j\,\in\, I} a_{ij}\left[\ddot{r}_j - (\dot{r}_i - \dot{r}_j) - k_p(\hat{r}_i - \hat{r}_j)\right] - a_{it}P_i\sigma(s_i,\epsilon)\right\} \tag{10}$$

where $P_i = diag(\varphi(\dot{r}_i - \dot{r}_t)) + \ddot{(}r_{\text{tmax}} + \beta_0)\mathbf{I}_2$, $n_i = \left(\sum_{j\,\in\, I} a_{ij} + a_{it}\right)$, $s_i = \dot{r}_i - \dot{r}_t + (\hat{r}_i - r_t)$, $\beta_0 > 0$, and $\ddot{r}_{\text{tmax}} = \begin{bmatrix}\ddot{r}_{tx}\\ \ddot{r}_{ty}\end{bmatrix}$.

$$\sigma\left(\begin{bmatrix}x_1\\ x_2\end{bmatrix},\epsilon\right) = \begin{bmatrix}sat(\frac{x_1}{\epsilon})\\ sat(\frac{x_2}{\epsilon})\end{bmatrix} \tag{11}$$

$$\varphi\left(\begin{bmatrix}x_1\\ x_2\end{bmatrix}\right) = \begin{bmatrix}|x_1|\\ |x_2|\end{bmatrix} \tag{12}$$

where $sat(\frac{s}{\epsilon}) = \begin{cases} s, & if\ |s| < \epsilon > 0\\ sign(s), & otherwise\end{cases}$.

$$[(L_n + B)\otimes I_2]\ddot{r} = -[L_n\otimes I_2]\dot{r} - [L_n\otimes I_2]\hat{r} - [B\otimes I_2]P\Sigma \tag{13}$$

where $P = \begin{pmatrix} P_1 & \dots & 0_{2\times 2}\\ \vdots & \ddots & \vdots\\ 0_{2\times 2} & \cdots & P_n\end{pmatrix}$ and $\Sigma = \left[\sigma^\top(s_1,\epsilon)\cdots\sigma^\top(s_n,\epsilon)\right]^\top$.

The following theorem proves that the controller in Eq. 10 maintains a target-centric formation in presence of target motion uncertainty; given that the upper bound on target input is known, the communication graph of UAVs is connected, and at least one of the UAVs has target information.

Theorem 2 *If each UAV in the group, with equation of motion given in Eq. 3, has the control vector as in Eq. 10 then as $t \to \infty$, $r_i - r_t \to R_i$ and $\dot{r}_i \to \dot{r}_t$, $\forall\, i \in I$, if and only if, the graph G_{n+1} has a spanning tree.*

Proof We choose a Lyapunov candidate function as,

$$V = \frac{1}{2}\bar{r}^\top(L_n\otimes I_2)\bar{r} + \frac{1}{2}S^\top((L_n + B)\otimes I_2))S \tag{14}$$

where $S = [s_1^\top, \cdots, s_n]^\top$ and $\bar{r} = \hat{r} - [\mathbf{1}\otimes I_2]r_t$. The time derivative of the Lyapunov function V along the trajectories satisfies

$$\begin{aligned}
\dot{V} &= (\hat{r} - [\mathbf{1}\otimes I_2]r_t)^\top[L_n\otimes I_2]\dot{\bar{r}}\\
&\quad + S^\top\{[(L_n + B)\otimes I_2]\}\dot{S}\\
&= \bar{r}^\top[L_n\otimes I_2](-\bar{r} + S)\\
&\quad + S^\top([(L_n + B)\otimes I_2]\ddot{r}\\
&\qquad - [(L_n + B)\otimes I_2])[\mathbf{1}\otimes I_2]\ddot{r}_t\\
&\quad + ([(L_n + B)\otimes I_2]\dot{r} - [(L_n + B)\otimes I_2])[\mathbf{1}\otimes I_2]\dot{r}_t)\\
&= -\bar{r}^\top[L_n\otimes I_2]\bar{r} + S^\top([L_n\otimes I_2]\hat{r} + [(L_n + B)\otimes I_2])\ddot{r}\\
&\quad - [(L_n + B)\otimes I_2][\mathbf{1}\otimes I_2]\ddot{r}_t\\
&\quad + [(L_n + B)\otimes I_2]\dot{r} - [(L_n + B)\otimes I_2][\mathbf{1}\otimes I_2]\dot{r}_t)\\
&= -\bar{r}^\top[L_n\otimes I_2]\bar{r} + S^\top([L_n\otimes I_2]\hat{r}\\
&\quad + [(L_n + B)\otimes I_2]\ddot{r} - [B\otimes I_2][\mathbf{1}\otimes I_2]\ddot{r}_t\\
&\quad + [(L_n + B)\otimes I_2]\dot{r} - [B\otimes I_2][\mathbf{1}\otimes I_2]\dot{r}_t).
\end{aligned}$$

Substituting the expression of $[(L_n + B) \otimes I_2]\ddot{r}$ from Eq. 13 we get

$$\begin{aligned}\dot{V} &= -\bar{r}^\top[L_n \otimes I_2]\bar{r} + S^\top([L_n \otimes I_2]\hat{r} \\ &\quad -[L_n \otimes I_2]\dot{r} - [L_n \otimes I_2]\hat{r} - [B \otimes I_2]P\Sigma \\ &\quad -[B \otimes I_2][\mathbf{1} \otimes I_2]\ddot{r}_t + [(L_n + B) \otimes I_2]\dot{r} \\ &\quad -[B \otimes I_2][\mathbf{1} \otimes I_2]\dot{r}_t), \\ &= -\bar{r}^\top[L_n \otimes I_2]\bar{r} + S^\top(-[B \otimes I_2]P\Sigma \\ &\quad -[B \otimes I_2][\mathbf{1} \otimes I_2]\ddot{r}_t \\ &\quad +[B \otimes I_2](\dot{r} - [\mathbf{1} \otimes I_2]\dot{r}_t)).\end{aligned}$$

Now we use inequality $S^\top[B \otimes I_2]P\Sigma - S^\top[B \otimes I_2]([\mathbf{1} \otimes I_2]\ddot{r}_t - (\dot{r} - [\mathbf{1} \otimes I_2]\dot{r}_t)) > \beta_0 S^\top[B \otimes I_2]\Sigma$. Therefore, we can write

$$\begin{aligned}\dot{V} &\leq -\bar{r}^\top[L_n \otimes I_2]\bar{r} - \beta_0 S^\top[B \otimes I_2]\Sigma, \\ &\quad [B \otimes I_2]\bar{S} > [B \otimes I_2][\mathbf{1} \otimes I_2]\epsilon. \end{aligned} \tag{15}$$

If G_{n+1} has a spanning tree, then we can write the above inequality as

$$\begin{aligned}\dot{V} &\leq -\bar{r}^\top[L_n \otimes I_2]\bar{r} - \beta_0 \sum_{i \in I} b_{ii}\varphi(s_i) \\ &< 0,\ s_i > \epsilon,\ \forall i \in I \end{aligned} \tag{16}$$

and therefore as $t \to \infty$, $r_i - r_t \to R_i$ and $\dot{r}_i \to \dot{r}_t$, $\forall i \in I$. □

4 Numerical Results

In this section, we present simulation studies to demonstrate the performance of the proposed distributed control laws to capture a target. Consider a team of fixed wing-UAVs attempting to capture a sinusoidally maneuvering target. The simulation parameters are

- Speed constraint of an UAV ($v_{\min} =$ 5m/s, $v_{\max} = 25$ m/sec).
- Desired distance from the target= 50 m.
- Desired angle from the target ($\alpha_i = \frac{2\pi i}{3}$).

Table 1 Initial conditions

UAV ID	x	y	v	ψ
1	50	50	8	0
2	−50	50	8.5	$\frac{\pi}{4}$
3	−50	−50	9	$\frac{\pi}{2}$
4	50	−50	9.5	$\frac{3\pi}{4}$

The limit on UAV speed represents a physical constraint. We assume that the target is executing a sinusoidal maneuver. This is because many complex maneuvers can be represented by a series of sinusoidal maneuvers. The target vehicle applies input $U = [0 \quad 0.5 \sin\left(\frac{2\pi}{50}t\right)]^T$ to execute such a maneuver from point $(0, 0)$ with airspeed of 10 m/sec and initial heading of 0^o.

The performance of the target-capturing cooperative strategies is demonstrated using the following example scenarios. The first example considers a target capturing problem with no uncertainty. However, it is assumed that at least one of UAVs in the group has complete target information and the graph including the target has a directed spanning tree. In the second example, the assumption of complete information is relaxed and the performance is verified.

4.1 Target-capturing with Complete Target Information

In this section, we check the performance of the cooperative strategy for target-capturing using the control law proposed in Eq. 6. The simulation is set up for four follower UAVs to capture a tar-

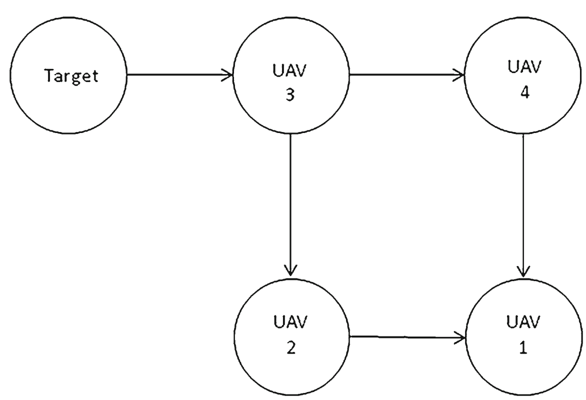
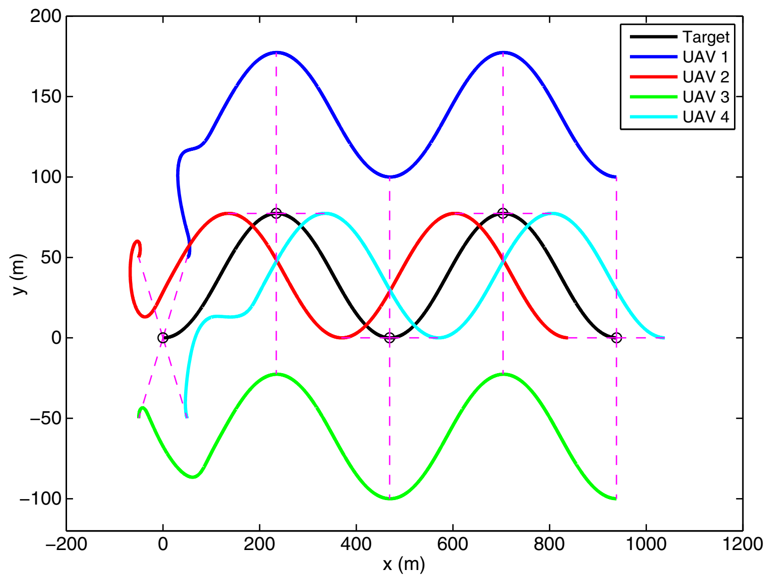


Fig. 2 Information exchange topology graph including the target

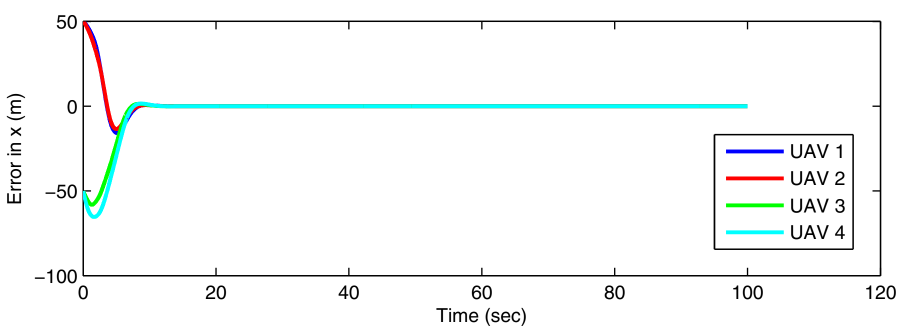
Fig. 3 Trajectory of the UAVs while maintaining aconstant separation with the target

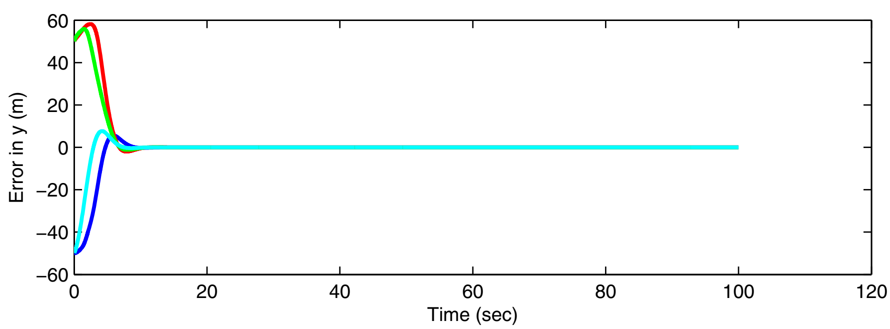


get. The initial conditions of the follower UAVs are given in Table 1. The information exchange topology is shown in Fig. 2 with $a_{ij} = 1$ if there is an information flow from j^{th} UAV to i^{th} UAV, otherwise $a_{ij} = 0$. We assume that only UAV 3 has complete target information. The Laplacian matrices of the graphs, excluding the target (denote as L_n) and including the target (denoted as L_{n+1}) are given by

$$L_n = \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad L_{n+1} = \begin{bmatrix} 2 & -1 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

Fig. 4 Tracking errors





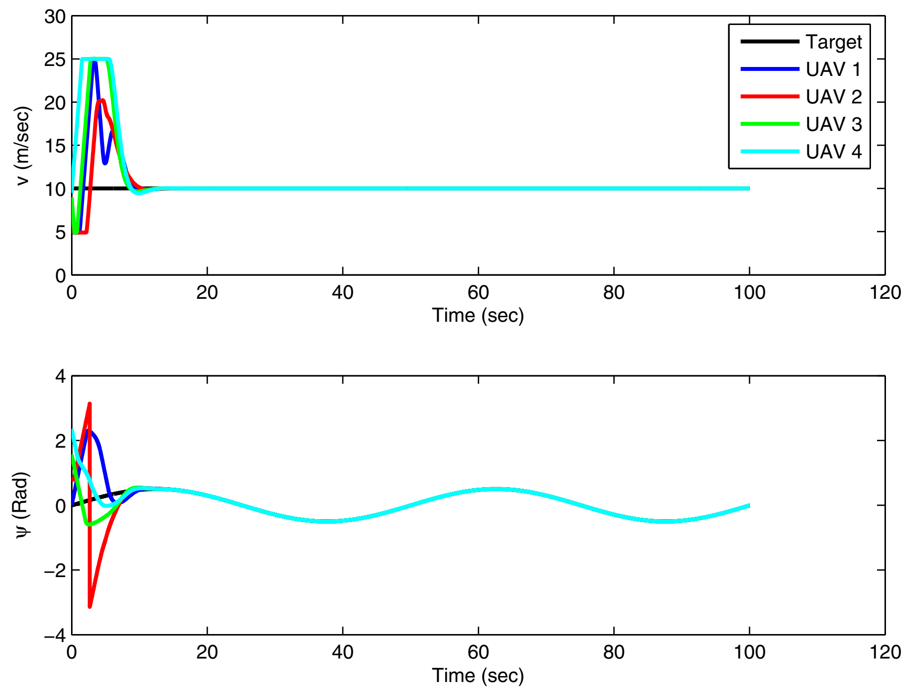


Fig. 5 Velocity and heading convergence

The rank of L_n is three, whereas the rank of L_{n+1} is four. The full rank of matrix L_{n+1} satisfies the condition of target-capturing in Lemma 1. As the topology of the UAVs has a directed spanning tree and the root of the graph has access to the target, the condition of target-capturing is met, as evaluated by the rank of L_{n+1} (which is full rank). It can also be observed from Fig. 2 that all the UAVs get information of the target through UAV 3. For example, UAV 1 gets information of the target through UAV 2. Note that here we do not mean that the target information is passing through the link as it does in a communication link.

As mentioned before, the target is maneuvering in a sinusoidal manner with a given initial condition. The objective is to maintain target centric formation which can be ensured by satisfying the control objective stated in Eq. 5. This is achieved by employing the distributed control law as in Eq. 6.

Figure 3 depicts the target and UAV trajectories around the target. The formation geometry is plotted in Fig. 3 at five instances in the color magenta. It can be observed that the initial formation does not satisfy the target-capturing conditions. Then, the cooperative strategy drives each UAV such that the desired separation at the given orientation with the target is maintained. The formation geometry at the second instance in Fig. 3 shows the desired formation around the

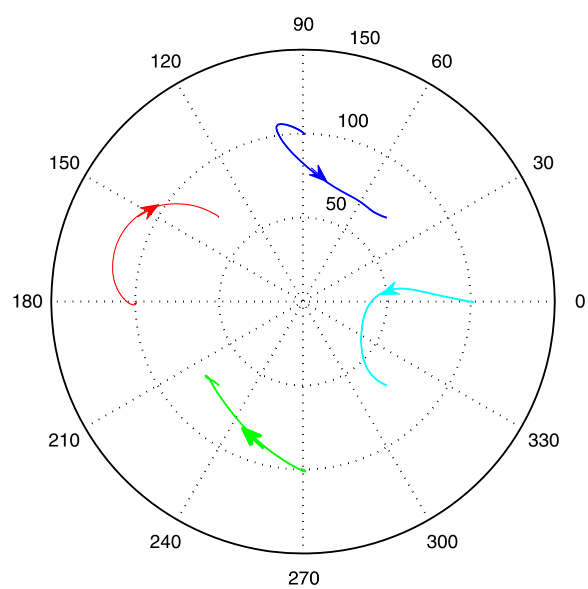


Fig. 6 Target centric trajectory in polar coordinates

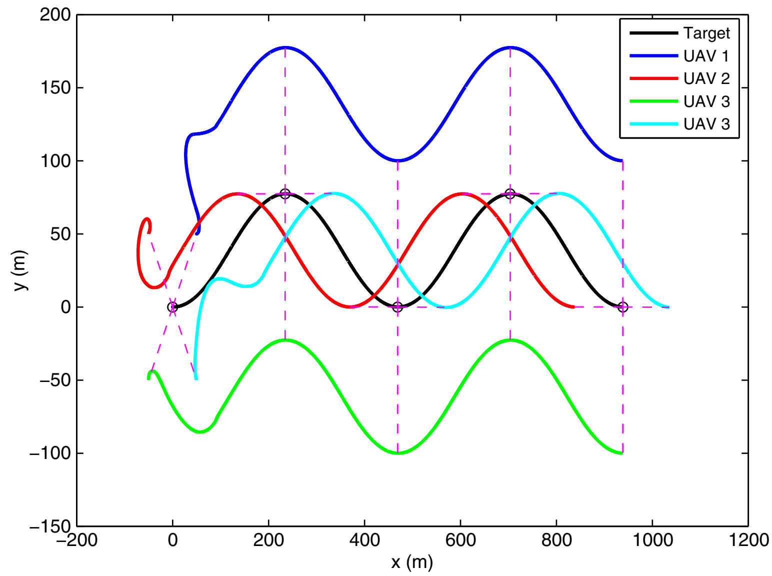


Fig. 7 Trajectory of the UAVs while maintaining a constant separation with the target

target. It can be further noted that the formation geometry is maintained throughout the mission. The associated formation error for each UAV is shown in Fig. 4. As the follower UAVs start from random positions, there is a relatively large formation error at the beginning. The formation error for each UAV quickly settles to zero which results in the desired formation around the target. The speed and heading convergence are shown in Fig. 5. Each UAV attains the same speed and heading with respect to the target and maintains these for all time. This means that they reach a consensus using the distributed law. Figure 6 shows the target centred trajectory for each UAV. It can

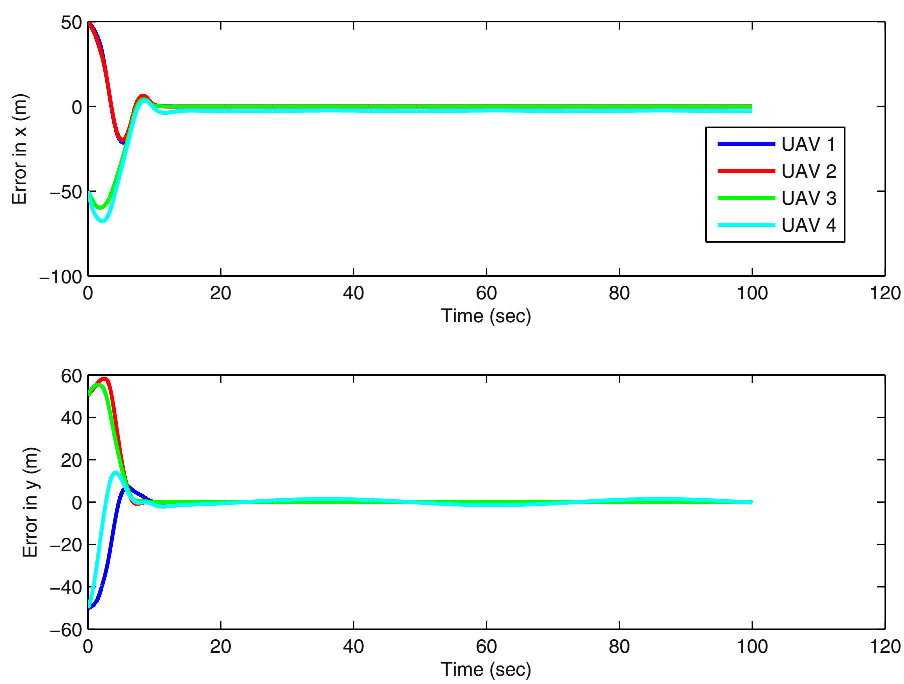
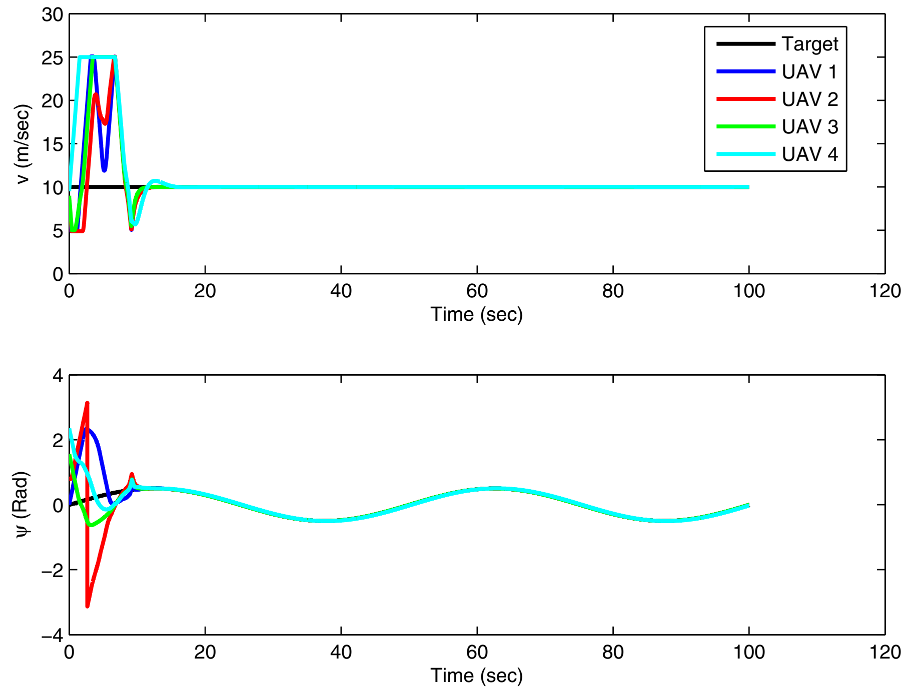


Fig. 8 Tracking errors

Fig. 9 Velocity and heading convergence



be noted that each UAV maintains the desired separation at the given orientation to capture the target.

4.2 Target-capturing with Incomplete Target Information

In the previous section, we assume that complete target information was available to each one of the UAVs to achieve the target centred formation. Now we relax this assumption and assume that the follower UAVs do not have complete access to the target input; instead, there is a bound on the target input (in this example only UAV 3). To capture the target, we employ the robust distributed control law proposed in Eq. 10, which is a combination of a linear policy (a dynamic inversion approach) and a nonlinear policy (sliding mode control). We perform simulations with the same initial conditions (both for the target and UAVs) and information exchange topology as considered in the previous section. Figure 7 shows trajectories of the target and follower UAVs around the target. The dashed magenta lines show the formation geometry around the target. UAVs start with a random formation and achieve the desired formation quickly. The formation tracking error for each UAV is shown in Fig. 8. It can be observed again that the formation errors settle to zero in a time similar to that in the previous example.

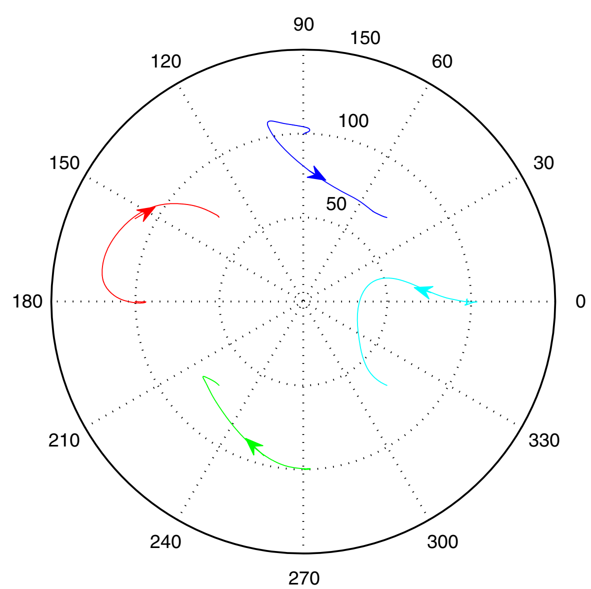


Fig. 10 Target centric trajectory in polar coordinates

The speed and heading convergence are shown in Fig. 9. The convergence in this case confirms that all UAVs arrive at a consensus quickly using the robust distributed control law (Eq. 10). The target centred formation is depicted in the polar plot in Fig. 10. It can be observed again that each vehicle maintains the desired separation at the given orientation to capture the target.

5 Conclusions and Future Work

In this paper, we developed a distributive controller for multiple UAVs to make a formation around a maneuvering target. We used consensus and sliding mode control theory to take into account uncertainty in target motion. We have shown both analytically and by simulations that UAVs can maintain a circular formation around the target if there exists a spanning tree in the communication graph with the target as the root node and upper bounds on target inputs (velocity and acceleration) are available to the UAVs.

Although the results presented in this paper show the advantage of using consensus and sliding mode control to form a formation around a maneuvering target without knowledge of exact target inputs, there are several important future directions which need attention. It is important to analyze: (i) the controller behavior in 3D (ii) the effect of delays in the information caused by communication lags and (iii) uncertainty in UAVs state.

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