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Survey of Motion Planning Literature in the Presence of Uncertainty: Considerations for UAV Guidance

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Abstract This paper provides a survey of motion planning techniques under uncertainty with a focus on their application to autonomous guidance of unmanned aerial vehicles (UAVs). The paper first describes the primary sources of uncertainty arising in UAV guidance and then describes relevant practical techniques that have been reported in the literature. The paper makes a point of distinguishing between contributions from the field of *robotics and artificial intelligence*, and the field of *dynamical systems and controls*. Mutual and individual contributions for these fields are highlighted providing a roadmap for tackling the UAV guidance problem.

Keywords Survey · Motion planning · Autonomous guidance · Uncertainty · Sensing · UAV

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1 Introduction

Autonomous vehicles are candidates for a broad variety of applications ranging from reconnaissance, target acquisition, search and rescue, surveillance, environmental monitoring, disaster area surveying and mapping. An important capability needed to broaden the range of applications under autonomous control is to be able to operate in the presence of various sources of uncertainty.

The problem of motion planning under uncertainty has been a topic of interest in both the *Artificial Intelligence and Robotics* community as well as the *Dynamical Systems and Controls* community. In the *Artificial Intelligence and Robotics community* this problem is often referred to as *decision-theoretic planning under uncertainty*, where decision making–which provides a way to select among multiple plans with uncertain state outcomes–is the central problem [1, 2]. In *Dynamical System and Control*, however, it is referred to as *control-theoretic planning under uncertainty*, where the emphasis is on the dynamical response, including taking full advantage of vehicle dynamical capabilities and accounting for state and control constraints. Here, safety and stability are usually the main issues.

The main focus of this paper is to survey planning algorithms in the presence of uncertainty with an emphasis on methods that can be applied to autonomous aerial vehicles. UAVs, due to their faster speed, more complex dynamics and stricter payload limitations tend to pose more challenges in obstacle avoidance and planning than ground robots. UAVs have to deal with the typical types of uncertainties that have already been classified by LaValle and Sharma [3, 4]:

- Uncertainty in vehicle dynamics and limited precision in command tracking.
- Uncertainty in the knowledge of the environment (e.g., obstacle locations).
- Disturbances in the operational environment (e.g., wind, atmospheric turbulence).
- Uncertainty in pose information.

Goerzen et al. [5] reviewed deterministic motion planning algorithms in the literature from the perspective of UAV guidance. This survey extends the existing literature in the following directions. First, the paper focuses on the identification and understanding of the different sources of uncertainty arising in the UAV guidance problem. Second, the paper describes practical methods that have been reported in the literature for handling these uncertainties in the formulation of motion planning problems.

In the following sections, we briefly review the relevant literature for each source of uncertainty given above and describe them separately making a point of distinguishing between the robotics and controls approach. The last section deals with reactive planning methods necessary for obstacle avoidance using on-board exteroceptive sensors. Reactive planning was included since it is an integral part of the approach that provides a 'last resort' solution for dealing with uncertainty. Throughout this article we tried to keep the level of mathematics to a minimum, focusing instead on the concepts behind the different techniques.

2 Uncertainty in Vehicle Dynamics

Under uncertainty in vehicle dynamics, the future robot configuration cannot be predicted accurately. This could be due to the inherent characteristics of the vehicle dynamics itself or limited precision in the system's command tracking performance. Disturbances, which are extraneous effects, will be considered separately in Section 4.

2.1 Robotics and Artificial Intelligence Approach

In robotics literature this type of uncertainty is referred to as the *uncertainty in action effect* (see e.g., [6]). The mathematical framework used to tackle this type of uncertainty is mainly the *Markov Decision Processes* (MDPs). In the MDP framework, the mapping from state s to stochastic action a is known as the control policy. A reward function specifies the instantaneous reward $R(s, a)$ that the robot derives from taking each action a at each state s. Good actions receive positive reward, bad actions are punished with negative reward. The Markov property entails that the next state s_{t+1} only depends on the previous state s_t and action a_t, i.e.,

$$p(s_{t+1}|s_t, s_{t-1}, \ldots, s_0, a_t, a_{t-1}, \ldots a_0) = p\left(s_{t+1}|s_t, a_t\right). \tag{1}$$

The goal of the agent is to act in such way that it maximizes the long term reward. This can be achieved by maximizing $E\left\{\sum_{t=0}^{\infty} \gamma^t R(s_t, a_t)\right\}$, which is the expected value of the long term reward (γ is a discount factor, $0 \leq \gamma < 1$, which controls the influence of rewards in the planning horizon). One way to characterize an MDP policy is to consider its value function $V_\pi(s)$, which is for every state s the amount of reward the agent can accumulate when it starts in s and acts according the policy π [7],

$$V_\pi(s) = R(s, \pi(s)) + E\left[\sum_{t=0}^{\infty} \gamma^t R\left(s_t, \pi(s_t)\right)\right]. \tag{2}$$

Applying the stochastic transition model, it leads to Bellman recursion [8]

$$V_\pi(s) = R(s, \pi(s)) + \gamma \sum_{s' \in S} p\left(s'|s, \pi(s)\right) V_\pi\left(s'\right). \tag{3}$$

This can be turned into an optimality principle, which specifies the following optimal value function known as a Bellman equation

$$V^*(s) = \max_{a \in A}\left[R(s, a) + \gamma \sum_{s' \in S} p\left(s'|s, a\right) V^*\left(s'\right)\right], \tag{4}$$

Solving this equation for each state s yields the optimal value function, and the corresponding optimal policy $\pi^\star$.

The basic dynamic programming approaches to solved MDPs are referred to as value iteration and policy iteration (see e.g., [9, 10]). In Cassandra et al. [11] some heuristic control strategies were proposed to navigate a mobile robot with model uncertainty.

However, it is well-known that many real-world problems modeled by MDPs tend to have huge state/action spaces, leading to the curse of dimensionality making the solution of the resulting models intractable [12]. MDPs were shown to be P-hard by Papadimitriou and Tsisiklis [13]. Chang et al. [14] presented various sampling and population-based numerical algorithms to overcome the computational difficulties of computing an optimal solution in terms of a policy and/or value function.

2.2 Optimal Control Based Approaches

In the controls community focus is placed on the system's dynamic with an emphasis on the physical properties. The planning algorithms for a dynamic system can be cast as an *Optimal Control Problem* (OCP). The standard nonlinear OCP can be formulated as follows

$$\inf_{u(.)} \int_0^\infty q(x,u)\, d\tau \tag{5}$$

$$s.t. \quad \dot{x} = f(x,u).$$

These OCP methods have been the primary techniques to plan trajectories in early aerospace applications [15, 16]. One popular approach to solve the OCP, is by converting it into a parameter optimization problem [17, 18], i.e., a Nonlinear Programing Problem (NLP). However, the corresponding NLP that represents the guidance problem is NP-hard and cannot be solved in real-time [19]. Therefore, *approximate* OCP is typically used for real-time implementation. The subsections below give an overview of these techniques.

2.2.1 Model Predictive Control (MPC) or Receding Horizon Control (RHC)

Among the techniques that have been used for real-time optimization one of the most popular is the family of *Model Predictive Control* (MPC) (also known as *Receding Horizon Control* (RHC)). While used mainly for slow processes, like those found in process engineering [20], the availability of faster and cheaper computers as well as efficient numerical algorithms, has made these techniques popular in robotic's and UAV applications.

MPC is essentially a feedback control scheme in which the OCP is solved over a finite horizon $[t, t+T]$, where at each time t step the future states are predicted over the horizon length T based on the current measurements. The first control input of the optimal sequence is applied to the system and the optimization is repeated.

The *closed-loop* implementation provides robustness against modeling uncertainties and disturbances. However, because of the finite horizon (discarding the system's future state history after $t+T$) if no special precautions are taken in the design and implementation, the closed-loop stability cannot be guaranteed [21].

One way to address this issue is to use terminal constraints or cost-to-go functions. In Primbs et al. [22] the global stability of RHC is achieved by combining a *Control Lyapunov Function* (CLF) and including an additional state constraint on the CLF ($V(x)$) at the end of the horizon. The proposed Constrained RHC+CLF is stated as follows

Constrained RHC+CLF [22]

$$\inf_{u(.)} \int_t^{t+T} \left(q(x) + u^T u\right) d\tau \tag{6}$$

$$s.t. \quad \dot{x} = f(x,u)$$

$$\frac{\partial V}{\partial x}\left[f(x,u)\right] \leq -\epsilon\sigma(x(t))$$

$$V(x(t+T)) \leq V(x_\sigma(t+T)).$$

where $0 < \epsilon \leq 1$, $\sigma(x(t)$ is a continuous positive definite function and T is the horizon length.

An alternative approach proposed by Jadbabaie et al. [23], is through the use of an *a priori* CLF as terminal cost of the optimization (or cost-to-go) rather than imposing additional constraint to the problem. The proposed Unconstrained RCH+CLF is computationally faster and also the stability is guaranteed as long as CLF is an upper bound on the cost-to-go.

Unconstrained RHC+CLF [23] *The finite horizon OCP*

$$\inf_{u(.)} \int_0^T q(x,u)\, d\tau + V(x(T)) \tag{7}$$
$$s.t.\ \dot{x} = f(x,u),$$

where $V(.)$ *is a non-negative* C^2 *function with* $V(0) = 0$ *satisfying* $V(x) \geq c\|x\|^2$ *such that*

$$\min_u \left(\dot{V} + q\right)(x,u) \leq 0, \tag{8}$$

is exponentially stable.

With uncertainties in the system behavior, the state evolution will not match the prediction. As mentioned earlier, the MPC framework has some inherent, implicit robustness to uncertainties due to the closed-loop implementation. In order to explicitly handle uncertainties, the MPC framework has been extended into *Robust Model Predictive Control* (see [24] for a survey in 1999). Uncertainty can be considered as either plant variability (cf. [25–27]) or affine disturbances (cf. [28, 29]). Considering uncertainty in the MPC formulation alleviates the need for accurate a-priori knowledge of the model.

2.2.2 Finite-state Approximate Optimal Control Techniques

In an effort to reduce the size of the state-space, finite-state approximations of the dynamics such as *Maneuver Automaton* (MA) (see [30]) have been proposed. With a finite-state representation of the vehicle dynamics, the trajectory optimization becomes a sequential decision problem, which can be solved as a dynamic program [9]. The approach was simulated for a miniature X-Cell helicopter [31]. To address uncertainties in the dynamics, Schouwenaars et al. [32] propose a robust MA, which takes the uncertainty in the maneuver outcome explicitly into account. They considered uncertainty in the duration of action itself and the uncertainty in the time-to-go resulting from uncertainty in the end state of action. The solution involves applying dynamic programming considering the expected value of the outcome.

2.2.3 Stochastic Predictive Control Techniques

Predictive stochastic control considers probabilistic uncertainty in dynamic systems and aims to control the predicted distribution of system state over a finite planning horizon in some optimal way. This is done by optimizing over the space of possible future distributions. In this regard Calafiore and El Ghaoui [33, 34] formulate a linear program in which the data that specify the constraints are subject to random uncertainty. Blackmore et al. [35] used this idea and proposed chance constrained predictive control under stochastic uncertainty. The chance constraints specify the probability of collision with an obstacle or failure to reach a goal region.

It was shown in the literature that approximating the probability distribution of random variables using samples or particles can lead to tractable algorithms for estimation and control. Methods have been developed by which a distribution of particles can be controlled in an optimal manner. These methods, appearing under the names of Monte Carlo filters and particle filters, have made it possible to numerically solve many complex problems that were previously intractable (see [36]). Shapiro [37] showed that stochastic programming problems can be solved with a reasonable accuracy by Monte Carlo sampling techniques. Blackmore et al. [35] formulated a so-called *chance constrained model predictive control* using particle method and solved it using Mixed Integer Linear Programming (MILP). Blackmore et al. [38] uses a probabilistic representation of the uncertainty in vehicle motion and determines the control based on the distribution of the vehicle state such that the probability of collision with obstacles is below a pre-specified threshold. The problem was posed as a Disjunctive Linear Program (DLP) and solved using constrained optimization methods to generate a finite sequence of optimal inputs. In the same vein, Richards [39] uses norm-bounded uncertainties. The advantage of the former is that vehicle localization techniques often provide probabilistic distribution over the possible location. This is also the case of disturbances, such as wind, which are often modeled as random processes.

Goerzen and Whalley [40] describe the probabilistic modeling of the inner-loop tracking error for a UAV helicopter. They proceeded by performing in-flight obstacle field navigations experiments in a variety of atmospheric conditions under varying speed and acceleration levels. To minimize the risk of obstacle collision, the estimated probability distribution of the inner-loop tracking error is incorporated into the navigation function.

3 Uncertainty in Environment Knowledge

Under environment knowledge uncertainty, the robot has incomplete or imperfect information about its environment. This could be due to inaccuracy in the a-priori map or imperfect and noisy exteroceptive sensory information that is provided to the robot in order to map the environment.

3.1 Planning Techniques for an Uncertain Environment

Weiß et al. [41] proposed a real-time path planning method for UAVs in an uncertain environment. In their approach, the kinematic system of the vehicle is reduced to a set of feasible trim trajectories and maneuvers (similar to the maneuver automaton in Frazzoli [42]). In addition, the environment is partitioned based on low-risk points given by the risk map. After computing the cost of all branches between the low risk points, the Dijkstra algorithm is used to find a best sequence of points. Finally a local, finite horizon planner is used to plan the trajectory between the important waypoints of the initial path. The local planner iteratively determines the best trim trajectory by minimizing a cost function that considers the time integral of the risk and the path length.

Randomized motion planning techniques such as Probabilistic Roadmap (PRM) [43] and Rapidly-exploring Random Trees (RRT) [44] have been successful in dealing with the high-dimensional configuration space arising in many real-world applications (see e.g., Pettersson and Doherty [45] for an application to unmanned helicopter). However they rely on accurate models of the environment. Missiuro and Roy [46] proposed an extension of PRM that computes motion plans that are robust to environment uncertainty by incorporating uncertainty in PRM sampling as well as modeling the cost of collision in traveling through uncertain regions of the map.

In the aforementioned approaches, the uncertain area of the map is globally known and the vehicle has to determine a plan to reach the goal as safely as possible. However, many urban UAV applications require the vehicle to have sufficient on-board situational awareness to avoid collision with unanticipated obstacles in the immediate environment while fulfilling the global planning requirements. This can be achieved by environment sensing, mapping and fast re-planning in real-time.

To accommodate plan updates when operating in partially known or unknown environments, mainly incremental graph search algorithm have been proposed [47]. Ramalinam and Reps [48] proposed an incremental algorithm for a generalization of the shortest path problem in a graph with an arbitrary edge insertion, edge deletion and edge-length changes. Stenz [49] proposed the D* algorithm (Dynamic A*) for optimal and efficient re-planning in partially known environments. In this approach, rather than re-calculating the optimal path for the entire map when changes in the map are detected, a reduced set of cells are checked and the optimal path to the robot's pose is updated incrementally. Focused D* [50], focuses the repairs using heuristics to reduce the total time for re-planning. Koeng and Likhachev [51] proposed D* Lite, which implements the same navigation strategy as Focused D* but is algorithmically different. These incremental planners, which make use of the results of the previous plans to generate a new plan, can substantially speed up the planning cycles. However, finding an optimal plan within the available time might not be possible. Anytime algorithms [52], in contrast, try to find the best plan within the given available time, Likhachev et al. [53] propose the Anytime

D* planning algorithm, which is both anytime and incremental.

3.2 Environment Mapping Techniques

Robot mapping is an active research area in Robotics. It addresses the problem of acquiring spatial models of the environment through robot's on-board sensors. Thrun [54] has presented a survey of robot mapping algorithm with a focus on indoor environments. Since both the robot pose and the map are uncertain, a vast body of literature has focused on solving the mapping problem and the induced problem of localizing the robot with respect to the map. This process is referred to as *Simultaneous Localization and Mapping* (SLAM) (see e.g., [55]). The fundamental mathematical principles used in robot mapping and localization is Bayesian filtering. The filter is used to calculate the robot pose and map posterior probability distribution, given all the control and measurements via the following recursion [54]

$$p\left(s_t, m|z^t, u^t\right) = \eta p\left(z_t|s_t, m\right) \times \int p\left(s_t|u_t, s_{t-1}\right) p\left(s_{t-1}, m|z^{t-1}, u^{t-1}\right) ds_{t-1}, \quad (9)$$

where m and s denote the map and robot pose, respectively. Also z is the sensor measurements and η is a normalizing factor. The superscript t refers to all data up to time t and subscript t refers to current time. In addition, the probability $p(s_t|u_t, s_{t-1})$ specifies the effect of control u on the state s. The probability $p(z_t|s_t, m)$ is referred to as the perceptual model. A fundamental issue with this formulation is that the posterior probability distribution (Eq. 9) has infinitely many dimensions and cannot be implemented. Hence most practical localization and mapping algorithms in the literature, such as the *Kalman filter* [56], *Expectation Maximization* (EM) [57] and *particle filter* [36], resort to simplifying assumptions on the original Eq. 9 such as assuming normal probability distribution or representing the probability distributions by samples in order to provide a working mapping algorithm.

Many localization and mapping algorithms focus on indoor and cyclic environments (such as office space) that provide many landmarks. Therefore most of these algorithms cannot be directly applied to aerial vehicles flying in 3D outdoor environments. The problem of mapping with known pose alleviates some of the practical challenges and received much attention in the literature. One of these mapping algorithms, known as occupancy grid map (originating from Moravec and Elfes [58]), is an approach to model the world and robot perception by using a probabilistic representation of spatial information. This approach employs a multidimensional tessellation of space into cells, where each cell stores a probabilistic estimate of its state. The probabilistic environment cell estimates are obtained by interpreting the spatial sensor data using the probabilistic sensor model [59]. Improvements to the original occupancy grid algorithms have been proposed recently (see e.g., [60, 61]).

3.3 Integration of Planning and Mapping

Many of the UAV planning methods that have been proposed in the literature to operate in uncertain environments use a combination of sensing, mapping and re-planning. Shim et al. [62] presented an exploration method in unknown environments using MPC-based obstacle avoidance for a UAV helicopter that carries a laser scanner to build obstacle maps. Sinopoli et al. [63] applied stereo vision to autonomous UAV navigation in partially known environments. Off-line computation, which is based on wavelet transformation of the map and Dijkstra's algorithm, exploits the a-priori knowledge of the environment and provides an initial guess of the optimal route. The local planner exploits the sensory information to update the occupancy grid map and plans the path based on dynamic programming principle. The results were validated in simulation. Hrabar [64] represented the 3D environment using occupancy maps obtained from stereo-based measurements. For path planning of the UAV rotorcraft, a combination of D* Lite and PRM was used. Scherer et al. [65] used the *plan globally and react locally* approach to control a UAV helicopter that combined a slower path planning layer based on Laplacian (which is a from of potential function inspired from fluid motion model [66]) and a high-frequency reactive obstacle avoidance algorithm. In their approach, a 3D laser scanner is used to sense the environment and provide the information to populate the occupancy grid. Andert and Adolf [67] presented an approach for 3D environment perception and global planning for unmanned helicopters. In their work, a 3D occupancy grid is built incrementally. In order to have a compact representation, an approximate polygonal prism shape world model is created from the occupancy grid data. Finally, sampling-based planning methods (such as PRM and QRM (Quasi Random Roadmap)) were used for path planning. Davis and Chakravorty [68] posed motion planning of a UAV helicopter in an uncertain environment as the adaptive optimal control of an uncertain Markov decision process and use the *certainty equivalence principle* [69] to compute the control policy based on the current estimate of the environment. In the previously discussed risk minimization motion planning technique proposed and flight tested on a UAV helicopter by Goerzen and Whalley [40], the environmental sensing error as well as inner-loop tracking error are combined into a risk map. The risk map is then used to determine a navigation function to guide the helicopter.

4 Environmental Disturbances

Disturbances in the operational environment make the true trajectory deviate from the planned trajectory and therefore limit the effectiveness of deterministic path planning techniques. This is especially true for miniature UAVs. Their slower speed and limited propulsion and control forces make them less capable to directly reject the effect of atmospheric disturbances.

A number of researchers have addressed the problem of optimal path planning in the presence of wind with known magnitude and direction. In the optimal control field this problem is know as Zermelo's navigation problem [15]. Jennings et al. [70] proposed a dynamic programming method to find the minimum-time waypoint path for a UAV flying in known wind. McGee et al. [71] explored the problem of generating optimal path from an initial position and orientation to a final position and orientation in the 2D plane for an aircraft with bounded turning radius in the presence of a constant known wind. In the absence of wind, this problem is known as a Dubin car problem (see [72, 73]). The original problem of finding an optimal path in the presence of a constant wind can be re-expressed as one of finding the optimal path with no wind to a final orientation over a moving virtual target whose velocity is equal and opposite to the velocity of the wind [71]. The approach was extended by McGee and Hedrick [74] to account for multiple point surveillance in constant wind. Ceccarelli et al. [75] address the path planning problem of MAV for the purpose of obtaining video footage of a set of known ground targets with preferred azimuthal viewing angles, using fixed on-board cameras. The approach is based on planning a finite sequence of waypoints such that the resulting trajectory of the MAV allows reconnaissance of each of the targets. Ketema and Zhao [76] formulated the problem of UAV trajectory optimization in an obstacle-free environment in the presence of wind as a nonlinear optimal control problem. Constraints on state and control variables arising from operational and performance limitations are included in the optimization problem.

In many real scenarios, the direction of wind is not known a-priori or it changes from time to time. Therefore it is more relevant to design path planners that are robust to wind disturbances. Nelson et al. [77] presented an approach based on overlaying a vector field of desired headings and then command the UAV to follow the vector field. They showed through Lyapunov stability criteria that the ground track heading error and lateral following error approaches zero asymptotically in the present of constant wind or disturbances. In Kuwata et al. [78], an RH controller was used to generate trajectories for an aerial vehicle operating in an environment with disturbances. The proposed algorithm modifies the on-line RH optimization constraints (such as turn radius and speed limits) to ensure that it remains feasible even when the vehicle is acted upon by unknown, but bounded, disturbances.

5 Uncertainty in Pose

With uncertainties in pose, the robot's location is uncertain with respect to the environment map. Most aerial robots nowadays are equipped with a Global Positioning System (GPS), which enables sufficient accuracy in the pose estimate. The pose estimation problem is primarily an issue for operation in GPS-denied environments (see e.g., [79–81]). In this case, the vehicle has to localize itself using environmental landmarks and features through on-board sensors (laser scanner or stereo-vision cameras).

In robotics literature, this type of uncertainty is sometimes referred to as *uncertainty in perception* [6]. The underlying mathematical framework to address this uncertainty is known as *Partially Observable Markov Decision Process* (POMDP) which is a general form of fully observable MDP discussed earlier. The term "partial" indicates that the state of the world cannot be sensed directly i.e., the robot is likely to suffer from uncertainty (such as noise and limited view of the environment) in its sensors.

POMDP is a stochastic decision making approach based on probabilistic estimates of the state of the environment. A POMDP uses an a priori model of the world together with the past history of the control actions and observations in order to infer a probability distribution, or *belief*, over the possible states of the world [82]. A *belief state* is a probability distribution over all states. It essentially summarizes the information of the past. The planner chooses actions, based upon the current belief as it maximizes the reward it expects to receive over time.

POMDP is essentially a belief-state MDP in which the agent summarizes all information about its past using a belief vector $b(s)$ [7]. Every time the agent takes an action a and makes an observation o, its belief is updated according the Bayes' rule

$$b^{ao}\left(s'\right) = \frac{p\left(o|s',a\right)}{p(o|b,a)} \sum_{s\in S} p\left(s'|s,a\right) b(s). \tag{10}$$

In POMDP framework, the value function is defined as the expected future reward $V^b(\pi)$ that the agent can gather by following π starting from belief b_0

$$V^b(\pi) = E_\pi \left[\sum_{t=0}^{h} \gamma^t R\left(b_t, \pi\left(b_t\right)\right) \mid b_0 \right], \tag{11}$$

where $R(b_t, \pi(b_t)) = \sum_{s\in S} R(s, \pi(b_t))\, b_t(s)$. The value of an optimal policy is defined by the optimal value function V^*, that satisfies the Bellman's optimality equation

$$V^*(b) = \max_{a\in A} \left[\sum_{s\in S} R(s,a) b(s) + \gamma \sum_{o\in O} p\left(o|b,a\right) V^*\left(b^{ao}\right) \right]. \tag{12}$$

Equation 12 is equivalent to Eq. 4 for MDPs in the fully observable case.

Unfortunately, solving POMDPs optimally has been proven PSPACE-hard (see [13]). A vast body of literature has focused on various approximate and heuristic solution techniques (see e.g., a survey by Aberdeen in 2003 [83]), but nonetheless, computation time remains the primary limiting factor for using POMDPs in realistically large domains arising in practical applications.

In the area of mobile robot path planning, Roy and Thrun [84] proposed an approximation to POMDP by representing the high dimensional belief space as the entropy of the belief space. The results suggest that by navigating sufficiently close to the areas of the map that have high information gain, the likelihood of getting lost is minimized. The belief space variant of the PRM algorithm [43] known as the *Belief Roadmap* (BRM) was proposed by Prentice and Roy [85] to cope with pose uncertainty where the belief space is tracked using a type of Kalman filter.

A variety of probabilistic estimation techniques were proposed in the literature for mobile robot localization (See e.g., [86, 87]). For an indoor or GPS-denied environment, the robot's ability to keep track of its position can vary considerably with the current robot's position with respect to the environment. Parts of the environment may lack proper features for localization. In Takeda and Latombe [88], given a model for a robot's environment a *Sensory Uncertainty Field* (SUF) is computed over the robot configuration space. At every point in the configuration space, SUF is the expected uncertainty in the sensed configuration that is computed by matching the sensory data against the environment model if the robot was at that point. The planner can then use of the computed SUF to generate paths that minimize expected uncertainty.

6 Reactive Planning

Even if successful global planning algorithms for uncertain environments were available, a perfectly up-to-date map of the environment may not be available and the exteroceptive sensors carried by the UAV cannot reach far enough, or behind obstacles, to account for a priori unknown information within the planning process. In these situations, the robot can only deal with these unknown elements using a reactive planning algorithm. The main difference between reactive planners is how tightly they are integrated with the global planner. Many reactive planning algorithms use only local knowledge of obstacles to plan trajectories, i.e., they do not take the global planning problem into consideration. The proposed algorithms also vary based on the type of sensors that can be carried by the robot. In this section we review two different techniques for reactive planning: vision based methods and depth sensor based methods. While vision sensors are relatively cheap and light weight, incorporating vision sensors in planning algorithms require more computation to extract obstacle locations from the raw sensor data. Many mobile robots carry depth sensors such as scanning laser or flash laser sensors. They provide an accurate depth map, which is a format that can more readily be incorporated in the planning algorithm. The shortcoming, however, is that the laser scanner sensors are often heavier and more expensive than vision sensors. Flash laser sensors that have sufficient range are still in the making. Finally, reactive planning algorithms can be combined with global path planning algorithms to provide a complete navigation solution (see, e.g., [89]).

6.1 Vision Based Reactive Methods

Reactive planning using vision sensors mostly relies on two methods: *Stereo Vision* and *Optic Flow*.

6.1.1 Stereo Vision Based Techniques

Stereo vision is perhaps one of the oldest and most established topics in computer vision. It has served the robotics community as a reliable means for recovering depth in the visual field. Inspired by human vision, two images are taken from slightly different perspectives. By comparing the two images and exploiting the known transformation between the two image planes, it is possible to recover the depth of objects in the field of view. Many approaches have been proposed ranging from correlation, edge-based to corner based methods. Stereo vision has proved useful for robotics because of its reliability in measurement and its robustness to environment types.

It is also possible to use monocular vision to estimate the depth map through *Structure-from-Motion* (SFM) techniques [90]. The aim of SFM is to infer the geometry of the visual field. Using consecutive images taken at different camera positions, the relative motion of the scene is calculated which can then be used to estimate the depth of objects in the field of view. It is often implemented using the optical flow [79] or a feature tracking method [91]. The velocity and position of the vehicle is needed to project this map into real world coordinates. For this reason, most SFM approaches rely on IMU and GPS data provided by sensor packages on-board the aircraft [91].

Prazenica et al. [92] used a SFM algorithm to estimate the depth of environment features. Then, an adaptive multi-resolution based learning algorithm is applied to estimate the geometry of the environment. The adaptive learning algorithm solves the problem of learning a depth surface from a collection of independent feature points. The receding horizon path planning algorithm is then used to generate obstacle free paths. Yu et al. [93] developed a path planning and obstacle avoidance method for MAVs using depth map information from forward looking on-board cameras. Higher resolution was given to the areas in the depth map that are closer to the MAV and lower resolution to areas that are farther away. Dijkstra's algorithm was used to plan the obstacle free path. Watanabe et al. [94] proposed a collision avoidance strategy using a vision sensor for a UAV helicopter. In their approach, an *Extended Kalman Filter* (EKF) is used to estimate the relative position of the obstacle from the vision-based measurements. The collision avoidance strategy called *minimum-effort guidance* is then designed by minimizing the helicopter lateral accelerations.

6.1.2 Optic Flow (OF) Based Techniques

As mentioned earlier *Optical Flow* (OF) is another technique for reactive planning. OF is the measurement of apparent motion in the visual field, resulting from camera motion or the motion of objects. It is a fundamental measurement in computer vision motion analysis. Many other branches of computer vision, such as feature detection, begin by calculating OF. OF gained interest in UAV research as a bio-inspired sensing method, similar to the sensing used by flies or bees [95]. For this reason, the focus of optical flow UAV research is often on reactionary tasks, such as avoiding obstacles, maintaining height above ground or landing [96]. Since optical flow is a measurement of apparent visual motion, it is related to both the relative velocity of the object and the distance to the object. As the UAV is flying, objects far away appear to move slowly in the visual field, while objects nearby appear to move quickly. For obstacle avoidance the common approach is to move away from areas of high optical flow. The standard way to estimate OF is by using cameras and computer vision processing. However, the need for optic flow sensors for MAVs motivated the research for small OF sensor chips. Barrows et al. [95] designed micro OF sensors that weigh 10 grams. The same sensor was used by Green et al. [97, 98] to perform autonomous take off, obstacle avoidance, and landing of a fixed-wing indoor aircraft. Beyeler et al. [99] designed an autopilot for MAV for near obstacle flight based on an optic mouse sensor to detect the OF. The OF has two components: Rotational OF and Translational OF. The Rotational OF is induced by the lateral or rotational motion of the aircraft. It is necessary to remove the Rotational OF component in order to estimate the proximity of the obstacles. This can be done using rate gyro measurements [100]. Without this measurement, a sudden aircraft turn will produce a spike in the optical flow measurement. In fact, to calculate the distance to surrounding obstacles, both the optical flow and velocities must be measured. Assuming constant translational speed, the Translational OF can be interpreted as proximity signal [99].

To provide a more reliable obstacle avoidance solution, optic flow measurements can be combined with other sensors. Hrabar et al. [101] used a combination of optic flow and stereo vision for obstacle avoidance through urban canyons. The optic flow from a pair of sideways-looking camera is used to stay centered in a canyon while stereo vision from a forward facing cameras is used to avoid obstacles in the front. The proposed approach was tested on a UAV helicopter.

As with all computer vision problems, stereo vision and SFM algorithms have difficulty in low contrast scenarios. As a result, 2D laser scanners, which can be thought of as two dimensional depth map sensors, are widely used in ground and aerial robots which will be discussed next.

6.2 Depth Map Based Reactive Methods

A number of reactive planning algorithms have been developed that directly utilize range sensor data. Among the earliest is *Virtual Force Field* (VFF) developed by Borenstein and Koren [102]. As the mobile robot navigates in the environment, range reading from a depth sensor is used to form a 2D certainty grid histogram. At the same time, the algorithm considers a small window of environment cells around the robot (called *active histogram*). Each occupied cell inside the window exerts a repulsive force pushing the robot away from the cell, while a constant magnitude attracting force pulls the robot toward the target. The authors improved this method in their later work [103] by proposing the *Vector Field Histogram* (VFH) approach. In VFH, a transformation maps the 2D active histogram into a 1D polar histogram whose sector holds a value that represents the obstacle density in the direction of that sector. The algorithm can then steer the robot toward the sector with a low polar obstacle density. Minguez and Montano [104] developed *Nearness Diagram* (ND) navigation algorithm for driving a robot in free space. By using a nearness diagram, the proximity of obstacles and areas of free spaces are constructed from the depth map sensor information and the best collision free path is chosen. These algorithms (VVF, VFH and ND), all find a set of steering angles for a robot to follow. There is another family of reactive planning methods that compute command in terms of velocity. *Curvature-Velocity Method* (CVM), developed by Simmons [105] and later extended to *Lane-Curvature Method* (LCM) by Ko and Simmons [106], are in this category. CVM selects a point in translational-rotational velocity space that satisfies some constraints and maximizes an objective function. The LCM approach improves some of the shortcomings by supplying a proper local collision-free heading to CVM.

7 Conclusions

In this paper we surveyed motion planning algorithms that deal with the primary sources of uncertainty arising in real world missions. Emphasis was placed on algorithms that can be applied to autonomous UAV guidance. Throughout the paper we compared the relative weakness and strength of the two main schools of thoughts in this field: Robotics and Controls. The main challenges in UAV guidance problems are a 3D partially known environment, limited payload capacity, limited on-board computation power, differential constraints, environmental disturbances and uncertainty in state and measurements. While efficient algorithms addressing each sub-problem exist, the solution to general planning in the presence of uncertainty is still intractable in real time. Therefore, for practical applications the algorithm must be chosen based on the mission scenario and characteristics of the problem. For instance, examples of criteria that narrow down the selection of possible motion planning algorithms include: accurate knowledge of vehicle's dynamics, availability of up-to-date global maps, possible GPS dropouts, and level of disturbances in the operational environment. Finally, most of the practical UAV guidance methods are based on some modular organization of the control system, mapping process, obstacle avoidance module and trajectory planning system. The modularity makes it possible to select the modules based on mission specifications and conditions and also allows to segregate the sources of uncertainties and address them with dedicated approaches. Given the maturity of the theory, the improvement in sensors will likely have a big impact on the applications. This progress will enable applications of autonomous UAVs under broader operational conditions and will also increase the operational flexibility and reduce the needs for a human operator. Empirical data from these applications are needed to help further unify and consolidate the field.

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