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Robust Formation Control of Multiple Wheeled Mobile Robots

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Abstract This paper considers formation control of a group of wheeled mobile robots with uncertainty. Decentralized cooperative robust controllers are proposed in two steps. In the first step, cooperative control laws are proposed for multiple kinematic systems with the aid of results from graph theory such that a group of robots comes into a desired formation. In the second step, cooperative robust control laws for multiple uncertain dynamic systems are proposed with the aid of backstepping techniques and the passivity properties of the dynamic systems such that multiple robots comes into a desired formation. Since communication delay is inevitable in cooperative control, its effect on the proposed controllers is analyzed. Simulation results show the effectiveness of the proposed controllers.

Keywords Formation control · Wheeled mobile robots · Cooperative control · Decentralized control · Robust control

Mathematics Subject Classifications (2010) 70E60 · 37N35

1 Introduction

Formation control of wheeled mobile robots has been extensively studied in recent years. Cooperative control laws have been proposed based on different methods. For example, in [1–4] leader–follower based cooperative controllers were proposed. The leaders tracked predefined reference trajectories, and the followers tracked transformed variables of the states of their nearest neighbors. In [5, 6], behavior-based cooperative controllers were proposed. In this method, different desired behaviors were prescribed for each vehicle and the cooperative controllers were

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calculated from a weighting of the relative importance of each behavior. In [7–9], virtual structure based controllers were proposed. In [10], a distributed smooth time-varying feedback control law was proposed with the analysis based on the averaging theory for coordinating the motion of multiple nonholonomic mobile robots to capture/enclose a target. In [11], formation control of several mobile robots was considered with the aid of the dynamic feedback linearization technique. In [12], the authors used decentralized control theory to propose and analyze controllers for multiple cooperating robotic vehicles. In [13], steering control laws were proposed for mobile robots to achieve both rectilinear and circular formations with the aid of Lie group. In [14], decentralized control laws were proposed based on non-smooth Lyapunov theory and graph theory. In [15], cooperative control laws with collision avoidance were proposed based on Lyapunov-type analysis. In [16], formation control of multiple mobile robots was considered. Control laws were proposed with the aid of backstepping techniques and neural networks. For cooperative control of multiple general nonholonomic kinematic systems, consensus based control laws were proposed in [17, 18].

In the literature, most of the existing results in formation control of wheeled mobile robots are based on the kinematics of the systems. The dynamics of wheeled mobile robots and possible uncertainty in the dynamics were not considered. In practice, wheeled mobile robots are dynamic systems. The dynamics usually cannot be neglected in the control when high performance of the closed system is required. In addition, the control laws which are of the generalized velocities designed based on the kinematic models cannot be directly used to control the practical dynamic systems which require generalized forces as their inputs. Considering the practical applications of the research on the formation control of wheeled mobile dynamic systems, paper [19] considered the cooperative control of multiple nonholonomic chained systems and proposed cooperative adaptive control laws such that the states of a group of systems converge to a desired trajectory. In this paper, we consider formation control of multiple wheeled mobile robots with *dynamics* and *uncertainty* and propose cooperative robust control laws such that a group of robots converge to a desired geometric pattern. To solve the formation control problem, we propose cooperative controllers in two steps. In the first step, decentralized cooperative control laws are proposed for multiple kinematic systems with the aid of results from graph theory. In the second step, cooperative robust control laws are proposed with the aid of backstepping techniques and passivity properties of dynamic systems. It is shown that the proposed control laws make a group of robots converge to a desired formation if the communication graph is connected. Since communication delays are inevitable between neighboring systems, we analyze the effects of communication delays on the stability of the closed-loop systems with the proposed cooperative control laws. It is shown that our proposed cooperative control laws solve the defined problem if the communication delays are constants. To verify effectiveness of the proposed cooperative control laws, simulation results are included. The contributions of this paper are that decentralized robust control laws are proposed for formation control of multiple wheeled mobile robots with *dynamics* and *uncertainty* and the stability of the closed-loop systems with respect to communication delays are analyzed.

The rest of the paper is organized as follows. In Section 2, we formally state the control problem. In Section 3, cooperative control laws are proposed for the defined control problem. In Section 4, robustness of the proposed control laws with respect to the communication delays is analyzed. Section 5 includes simulation results. Section 6 concludes this paper.

2 Problem Statement

Consider a group of m wheeled mobile robots. For robot j (see Fig. 1), its motion is defined in the following form [20, 21]

$$M_j(q_{*j})\ddot{q}_{*j} + C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j} + G_j(q_{*j}) = B_j(q_{*j})\tau_j + [\sin\theta_j, -\cos\theta_j, 0]^\top \lambda_j, \quad (1)$$

$$\dot{x}_j \sin\theta_j - \dot{y}_j \cos\theta_j = 0 \quad (2)$$

where $q_{*j} = [q_{1j}, q_{2j}, q_{3j}]^\top = [x_j, y_j, \theta_j]^\top$ is the state of robot j, $M_j(q_{*j})$ is a 3×3 bounded positive-definite symmetric inertia matrix, $C_j(q_{*j}, \dot{q}_{*j})\dot{q}_{*j}$ presents centripetal and Coriolis torque, $G_j(q_{*j})$ is a gravitational torque vector, $B_j(q_{*j})$ is a 3×2 input transformation matrix, τ_j is a vector of control input. λ_j is the constraint force on robot j, and the superscript $^\top$ denotes the transpose.

Equation 1 has the following two properties [21, 22].

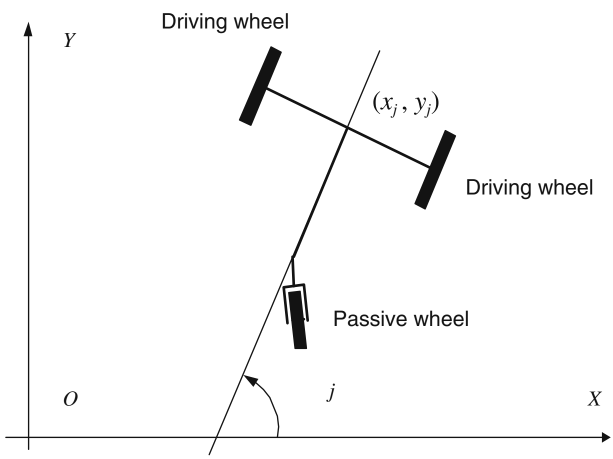
Property 1 Matrix $(\dot{M}_j - 2C_j)$ is skew-symmetric for a proper definition of C_j.

Property 2 For any differentiable vector $\xi \in R^3$,

$$M_j(q_{*j})\dot{\xi} + C_j(q_{*j}, \dot{q}_{*j})\xi + G_j(q_{*j}) = Y_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})a_j$$

where a_j is an inertia parameter vector, and the regressor matrix $Y_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})$ is a function of q_{*j}, $\dot{q}_{*j}$, ξ, and $\dot{\xi}$.

Fig. 1 Configuration of robot j



For each system, we assume that the regressor matrix $Y_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})$ is a known function of q_{*j}, $\dot{q}_{*j}$, ξ, and $\dot{\xi}$. For a_j, we assume its estimate is $\bar{a}_j$ and $\|a_j - \bar{a}_j\| \le \rho_j$ where ρ_j is a known constant.

In the control each robot knows it's own state and the states of its neighbors by communication. For simplicity, we assume that the communication between robots are bidirectional. If each robot is considered as a node, the communication between robots can be described by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{1, 2, \ldots, m\}$ is a node set, $\mathcal{E}$ is an edge set with element (i, j) which describes the communication from node i to node j. If the state of robot i is available to robot j, there will be an edge (i, j) in $\mathcal{E}$. We call robot i a *neighbor* of robot j if the state of robot i is available to robot j. Since communication is bidirectional, $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. For robot j, the indexes of its neighbors form a set which is denoted by $\mathcal{N}_j$. Therefore, the available states to robot j for the control are the state of robot j and the state of robot i for all $i \in \mathcal{N}_j$. We call a graph $\mathcal{G}$ *connected* if for any two different nodes i and j in $\mathcal{V}$ there exist a series of nodes $l_1(= i), l_2, \ldots, l_k(= j)$ such that $(l_s, l_{s+1}) \in \mathcal{E}$ for $1 \le s \le k-1$. For more terminology on graph theory, readers may refer to the references [23–25].

In this paper, we make the following assumption on the communication graph.

Assumption 1 *The communication graph $\mathcal{G}$ is bidirectional, fixed, and connected.*

Given a desired geometric pattern $\mathcal{P}$ described by constant vectors (p_{jx}, p_{jy}) $(1 \le j \le m)$. The control problem discussed in this article is defined as follows.

Formation Control Problem Design a control law τ_j for robot j using its own state $(q_{*j}, \dot{q}_{*j})$ and its neighbor's state $(q_{*i}, \dot{q}_{*i})$ for $i \in \mathcal{N}_j$ such that the group of robots comes into formation $\mathcal{P}$ and the centroid of the group of robots is stationary, i.e., design control laws for system (1–2) such that

$$\lim_{t\to\infty}\left(\begin{bmatrix} x_l - x_j \\ y_l - y_j \end{bmatrix} - \begin{bmatrix} \cos\psi & \sin\psi \\ -\sin\psi & \cos\psi \end{bmatrix}\begin{bmatrix} p_{lx} - p_{jx} \\ p_{ly} - p_{jy} \end{bmatrix}\right) = 0, \tag{3}$$

$$\lim_{t\to\infty}(\theta_l - \theta_j) = 0, \quad 1 \le l \ne j \le m \tag{4}$$

$$\lim_{t\to\infty}\sum_{j=1}^{m} x_j = \sum_{j=1}^{m} p_{jx}, \quad \lim_{t\to\infty}\sum_{j=1}^{m} y_j = \sum_{j=1}^{m} p_{jy}. \tag{5}$$

where ψ is a free variable.

Remark 1 In the formation control problem, Eq. 3 means that the group of robots comes into the desired formation. In Eq. 3, ψ denotes the rotation angle of the desired geometric pattern. Equation 4 means that the group of robots has the same orientation. Equation 5 means that the centroid of the group of robots converges to the center of the given geometric pattern. Since the inertia parameter vector a_j is unknown, cooperative robust control laws will be proposed in this paper.

To solve the cooperative control problem, we convert Eqs. 1–2 into a suitable form. Let the vector fields

$$g_1(q_{*j}) = [\cos\theta_j, \sin\theta_j, 0]^\top, \quad g_2 = [0, 0, 1]^\top,$$

then, by Eq. 2, there exists $v_{*j} = [v_{1j}, v_{2j}]^\top$ such that

$$\dot{q}_{*j} = g(q_{*j})v_{*j} = g_1(q_{*j})v_{1j} + g_2 v_{2j} \tag{6}$$

where $g(q_{*j}) = [g_1(q_{*j}), g_2] \in R^{3\times2}$. Differentiating both sides of Eq. 6 and substituting it into Eq. 1 and multiplying both sides of Eq. 1 by $g^\top(q_{*j})$, we have

$$\tilde{M}_j(q_{*j})\dot{v}_{*j} + \tilde{C}_j(q_{*j}, \dot{q}_{*j})v_{*j} + \tilde{G}_j(q_{*j}) = \tilde{B}_j(q_{*j})\tau_j \tag{7}$$

where

$$\begin{aligned}
\tilde{M}_j(q_{*j}) &= g^\top(q_{*j})M_j(q_{*j})g(q_{*j}),\\
\tilde{C}_j(q_{*j}, \dot{q}_{*j}) &= g^\top(q_{*j})M_j(q_{*j})\dot{g}(q_{*j}) + g^\top(q_{*j})C_j(q_{*j}, \dot{q}_{*j})g(q_{*j}),\\
\tilde{G}_j(q_{*j}) &= g^\top(q_{*j})G_j(q_{*j}),\\
\tilde{B}_j(q_{*j}) &= g^\top(q_{*j})B_j(q_{*j}).
\end{aligned}$$

The reduced systems in Eqs. 6–7 describes the motion of the original systems in Eqs. 1–2. Therefore, the formation control problem can be considered based on the reduced systems in Eqs. 6–7 instead of the systems in Eqs. 1–2.

3 Cooperative Controller Design

In order to solve the defined problem, we introduce the following variables.

$$\begin{cases}
z_{1j} = -\theta_j - \int_0^t w_1(s)ds,\\
z_{2j} = (x_j - p_{jx})\cos\theta_j + (y_j - p_{jy})\sin\theta_j + \beta w_1 z_{3j},\\
z_{3j} = -(x_j - p_{jx})\sin\theta_j + (y_j - p_{jy})\cos\theta_j\\
u_{1j} = -v_{2j}\\
u_{2j} = v_{1j} + \zeta_{3j}v_{2j}
\end{cases} \tag{8}$$

where constants $\beta > 0$, w_1 is a design variable and satisfies the following condition.

Assumption 2 w_1 *is bounded and satisfies the following condition:*

$$\int_t^{t+T} w_1^2(s)ds \geq \epsilon \textit{ for some } T > 0,\ \epsilon > 0,\ \textit{and for all } t \geq 0.$$

The definitions in Eq. 8 yield the following dynamic equations,

$$\begin{cases}
\dot{z}_{1j} = u_{1j} - w_1\\
\dot{z}_{2j} = u_{2j} + \beta w_1(-\beta z_{3j}w_1^2 + w_1 z_{2j}) + \beta z_{3j}\dot{w}_1 + (u_{1j} - w_1)\beta w_1 \zeta_{2j}\\
\dot{z}_{3j} = -\beta z_{3j}w_1^2 + z_{2j}w_1 + (u_{1j} - w_1)\zeta_{2j}
\end{cases} \tag{9}$$

$$\bar{M}_j(q_{*j})\dot{u}_{*j} + \bar{C}_j(q, \dot{q}_{*j})u_{*j} + \bar{G}_j(q_{*j}) = \bar{B}_j(q_{*j})\tau_j \tag{10}$$

where

$$\begin{aligned}
\bar{M}_j(q_{*j}) &= \bar{g}^\top(q_{*j})M_j(q_{*j})\bar{g}(q_{*j}),\\
\bar{C}_j(q_{*j}, \dot{q}_{*j}) &= \bar{g}^\top(q_{*j})M_j(q_{*j})\dot{\bar{g}}(q_{*j}) + \bar{g}^\top(q_{*j})C_j(q_{*j}, \dot{q}_{*j})\bar{g}(q_{*j}),\\
\bar{G}_j(q_{*j}) &= \bar{g}^\top(q_{*j})G_j(q_{*j}),\\
\bar{B}_j(q_{*j}) &= \bar{g}^\top(q_{*j})B_j(q_{*j})\\
\bar{g}(q_{*j}) &= \begin{bmatrix} z_{3j}\cos\theta_j & \cos\theta_j \\ z_{3j}\sin\theta_j & \sin\theta_j \\ -1 & 0 \end{bmatrix}.
\end{aligned} \tag{11}$$

Based on Properties 1 and 2, the following two properties can be easily proved.

Property 3 Matrix $(\dot{\bar{M}}_j - 2\bar{C}_j)$ is skew-symmetric.

Property 4 For any differentiable vector $\xi \in R^{2\times 2}$

$$\bar{M}_j(q_{*j})\dot{\xi} + \bar{C}_j(q_{*j}, \dot{q}_{*j})\xi + \bar{G}_j(q_{*j}) = \bar{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi})a_j$$

where

$$\bar{Y}_j(q_{*j}, \dot{q}_{*j}, \xi, \dot{\xi}) = \bar{g}^\top(q_{*j})Y_j\left(q_{*j}, \dot{q}_{*j}, \bar{g}(q_{*j})\xi, \frac{d}{dt}(\bar{g}(q_{*j})\xi)\right).$$

With the aid of the defined variables, we have the following results whose proof is omitted.

Lemma 1 *Under Assumption 2, if* $\lim_{t\to\infty}(z_{*j} - z_{*i}) = 0$ *for* $1 \leq i \neq j \leq m$, *then Eqs. 3–4 holds, where* $z_{*j} = [z_{1j}, z_{2j}, z_{3j}]^\top$. *Furthermore, if* $\lim_{t\to\infty} z_{*j} = 0$ *for* $1 \leq j \leq m$, *then Eqs. 3–5 hold.*

With the aid of the results in Lemma 1, in order to solve the formation control problem it is sufficient to design cooperative controllers for the systems in Eqs. 9–10 such that $\lim_{t\to\infty} z_{ij} = 0$. Noting the structure of Eq. 9, we have the following results.

Lemma 2 *For the systems in Eq. 9, under Assumption 2,*

1. *if*

$$\lim_{t\to\infty}(u_{1j} - w_1)\zeta_{2j} = 0, \quad \lim_{t\to\infty}(z_{2j} - z_{2l}) = 0, \quad \textit{for} \quad 1 \leq l \neq j \leq m \tag{12}$$

then $\lim_{t\to\infty}(z_{3j} - z_{3l}) = 0$.

2. *If*

$$\lim_{t\to\infty}(u_{1j} - w_1)\zeta_{2j} = 0, \quad \lim_{t\to\infty} z_{2j} = 0, \quad \textit{for} \quad 1 \leq j \leq m \tag{13}$$

then $\lim_{t\to\infty} z_{3j} = 0$.

Proof First, we prove that $\lim_{t\to\infty}(z_{3j} - z_{3l}) = 0$ if Eq. 12 are satisfied. Let $\sigma = z_{3l} - z_{3j}$, for $1 \le l \ne j \le m$, we have

$$\dot{\sigma} = -\beta w_1^2 \sigma + (z_{2l} - z_{2j})w_1 + (u_{1l} - w_1)\zeta_{2l} - (u_{1j} - w_1)\zeta_{2j}. \tag{14}$$

Since $\lim_{t\to\infty}[(z_{2l} - z_{2j})w_1 + (u_{1l} - w_1)\zeta_{2l} - (u_{1j} - w_1)\zeta_{2j}] = 0$, noting Assumption 2, $\lim_{t\to\infty}\sigma = 0$. If Eq. 13 is satisfied, noting Assumption 2, $\lim_{t\to\infty} z_{3j} = 0$. □

Based on the results in Lemma 2, the formation control problem can be solved by designing cooperative controllers for the systems in Eqs. 9–10 such that the requirements in Lemma 2 are satisfied. Noting the interconnection between systems (9) and (10), we apply backstepping techniques in this paper. We first design the cooperative controllers for the systems in Eq. 9. Then, we propose the cooperative controllers for the systems in Eqs. 9–10.

3.1 Cooperative Control Laws for Eq. 9

Assume that u_{1j} and u_{2j} are virtual control inputs of system j in Eq. 9, we design cooperative control laws u_{1j} and u_{2j}. Before proposing the controllers, we present some results on the algebraic graph theory.

Given an $m \times m$ symmetric constant matrix $B = [b_{ji}]$ with $b_{ji} > 0$, let $\mathcal{G}$ be the communication graph among m systems, the Laplacian matrix $L = [L_{ji}]$ of the graph $\mathcal{G}$ with weight matrix B is defined by

$$L_{ji} = \begin{cases} -b_{ji}, & \text{if } i \ne j \text{ and } i \in \mathcal{N}_j \\ 0, & \text{if } i \ne j \text{ and } i \notin \mathcal{N}_j \\ \sum_{l \ne j, l \in \mathcal{N}_j} b_{jl}, & \text{if } i = j. \end{cases}$$

Obviously, L is a symmetric matrix and has real eigenvalues. Without loss of generality, we assume that its eigenvalues $\lambda_l(L)$ $(1 \le l \le m)$ are ordered as $\lambda_1(L) \le \lambda_2(L) \le \cdots \le \lambda_m(L)$.

Lemma 3 *Given an $m \times m$ symmetric constant matrix $B = [b_{ji}]$ with $b_{ji} > 0$, under Assumption 1, the eigenvalues $\lambda_l(L)$ $(1 \le l \le m)$ of the Laplacian matrix L corresponding to the graph $\mathcal{G}$ with the weight matrix B satisfy $\lambda_m(L) \ge \lambda_{m-1}(L) \ge \cdots \ge \lambda_2(L) > \lambda_1(L) = 0$. Furthermore, for any bounded function vector $\xi(t) \in R^m$, if $\lim_{t\to\infty} \xi^\top(t) L \xi(t) = 0$, then $\lim_{t\to\infty}\left[\xi(t) - \left(\sum_{l=1}^m \frac{\xi_l(t)}{m}\right)\mathbf{1}\right] = \mathbf{0}$ where $\mathbf{1} = [1, \ldots, 1]^\top$ and $\mathbf{0} = [0, \ldots, 0]^\top$.*

Proof Noting the definition of L, by the Gerschgorin Circle Theorem [26], each $\lambda_i(L)$ is contained in the union of the m Gerschgorin circles $|z - L_{jj}| \le L_{jj}$ for $1 \le j \le m$. Therefore, either $\lambda_j(L) > 0$ or $\lambda_j(L) = 0$ for $1 \le j \le m$. Since $\mathcal{G}$ is connected, there is only one zero eigenvalue [27]. Therefore, $\lambda_1 = 0$ and $\lambda_m \ge \cdots \ge \lambda_3 \ge \lambda_2 > 0$.

Since L is symmetric and $\lambda_1 = 0$, there exists an orthogonal matrix $Q = [Q_{ij}]$ with its first column being $\mathbf{1}/\sqrt{m}$ such that $Q^\top L Q = \text{diag}[0, \lambda_2, \ldots, \lambda_m]$. So,

$$\lim_{t\to\infty} \xi^\top L \xi = \lim_{t\to\infty} (Q^\top \xi)^\top \text{diag}[0, \lambda_2, \ldots, \lambda_m](Q^\top \xi) = 0.$$

Let $\varrho = [\varrho_1, \varrho_2, \ldots, \varrho_m]^\top = Q^\top \xi$, then $\lim_{t\to\infty} \varrho_i = 0$ for $2 \le i \le m$. Noting $\varrho_1 = \frac{1}{\sqrt{m}} \sum_{l=1}^m \xi_l$,

$$\begin{aligned}\lim_{t\to\infty} \left(\xi - \left(\sum_{l=1}^m \frac{\xi_l}{m} \right) \mathbf{1} \right) &= \lim_{t\to\infty} \left(Q\varrho - \frac{1}{\sqrt{m}} \varrho_1 \mathbf{1} \right) \\ &= \lim_{t\to\infty} \left[\sum_{l=2}^m Q_{1l}\varrho_l, \ldots, \sum_{l=2}^m Q_{ml}\varrho_l \right]^\top = \mathbf{0}.\end{aligned}$$

Therefore, the lemma is proved. □

Noting the special structure of Eq. 9, we have the following result.

Theorem 1 *Consider the systems in Eq. 9, under Assumptions 1–2, the controllers*

$$u_{1j} = \eta_{1j} \tag{15}$$

$$u_{2j} = \eta_{2j} \tag{16}$$

for $1 \le j \le m$ *make Eqs. 3–4 hold, where*

$$\eta_{1j} = -\sum_{i\in\mathcal{N}_j} b_{ji} \left(z_{1j} - z_{1i} + \Delta_j - \Delta_i \right) + w_1 \tag{17}$$

$$\eta_{2j} = -\sum_{i\in\mathcal{N}_j} b_{ji} \left(z_{2j} - z_{2i} \right) - \beta w_1 \left(-\beta z_{3j} w_1^2 + w_1 z_{2j} \right) - \beta z_{3j} \dot{w}_1 \tag{18}$$

constants $b_{ji} = b_{ij} > 0$, *and*

$$\Delta_j = -z_{2j}\beta w_1 \zeta_{2j} - z_{3j}\zeta_{2j}. \tag{19}$$

Proof Apply the control laws in Eqs. 15–16 to Eq. 9, we have

$$\begin{cases} \dot{z}_{1j} = u_{1j} - w_1 \\ \dot{z}_{2j} = -\sum_{i\in\mathcal{N}_j} b_{ji} \left(z_{2j} - z_{2i} \right) + (u_{1j} - w_1)\beta w_1 \zeta_{2j} \\ \dot{z}_{3j} = -\beta z_{3j} w_1^2 + z_{2j} w_1 + \left(u_{1j} - w_1 \right) \zeta_{2j} \end{cases} \tag{20}$$

where

$$u_{1j} - w_1 = -\sum_{i\in\mathcal{N}_j} b_{ji} (z_{1j} - z_{1i} + \Delta_j - \Delta_i). \tag{21}$$

Let the positive definite Lyapunov function

$$V = \frac{1}{2} \sum_{j=1}^{m} \sum_{i=1}^{3} z_{ij}^2. \tag{22}$$

Differentiating V along the solutions of Eq. 20, we have

$$\dot{V} = -\sum_{j=1}^{m} \beta w_1^2 z_{3j}^2 - z_{2*}^\top L z_{2*} - (z_{1*} + \Delta)^\top L (z_{1*} + \Delta) \leq 0$$

where $z_{1*} = [z_{11}, \ldots, z_{1m}]^\top$, $z_{2*} = [z_{21}, \ldots, z_{2m}]^\top$, $\Delta = [\Delta_1, \ldots, \Delta_m]^\top$. Therefore, V is bounded. Furthermore, z_{ij} are bounded. By Barbalat's Lemma [28], $\lim_{t\to\infty} \dot{V} = 0$. So

$$\lim_{t\to\infty} \beta w_1^2 z_{3j}^2 = 0, \quad 1 \leq j \leq m \tag{23}$$

$$\lim_{t\to\infty} z_{2*}^\top L z_{2*} = 0, \qquad \lim_{t\to\infty} (z_{1*} + \Delta)^\top L (z_{1*} + \Delta) = 0. \tag{24}$$

By Lemma 3 and Eq. 24, we have $\lim_{t\to\infty}(z_{2*}(t) - c_2(t)\mathbf{1}) = 0$ and $\lim_{t\to\infty}(z_{1*}(t) + \Delta(t) - c_1(t)\mathbf{1}) = 0$ where c_1 and c_2 are bounded and defined as

$$c_2 = \frac{1}{m} \sum_{l=1}^{m} z_{2l}, \quad c_1 = \frac{1}{m} \sum_{l=1}^{m} (z_{1l} + \Delta_l). \tag{25}$$

Therefore, $\lim_{t\to\infty}(z_{2j} - z_{2l}) = 0$ and $\lim_{t\to\infty}(z_{1j} + \Delta_j - z_{1l} - \Delta_l) = 0$ for $1 \leq j \neq l \leq m$. We see $\lim_{t\to\infty}(u_{1j} - w_1)\zeta_{2j} = 0$ from Eq. 21 for $1 \leq j \leq m$. By Lemma 2, $\lim_{t\to\infty}(z_{3j} - z_{3l}) = 0$ for $1 \leq l \neq j \leq m$. By the definition of Δ_l, it can be proved that $\lim_{t\to\infty}(z_{1j} - z_{1l}) = 0$ for $1 \leq j \neq l \leq m$. By Lemma 1, Eqs. 3–4 hold. □

Remark 2 In Eqs. 17–18, the first term is a weighted sum of the relative state information between system j and its neighbors, and the other terms are used to cancel the terms induced by the variable transformation. The motion of the systems is driven by the relative information between neighbors.

Remark 3 In the control laws, the control parameters are b_{jl}, β, and w_1. w_1 can be a sine function or a constant. Generally, increasing β will increase the convergence rate of z_{3j}. The value of b_{ji} and the topology of the communication graph determine $\lambda_2(L)$. Large $\lambda_2(L)$ means that $(z_{1j} - z_{1i})$ and $(z_{2j} - z_{2i})$ converge to zero fast. The rate in which $(z_{*j} - z_{*i})$ converge to zero is called the *cohesion rate*. The value $\lambda_2(L)$ depends on the topology of the graph $\mathcal{G}$ and the weights b_{jl}.

Remark 4 The control laws (15–16) are decentralized because for each system the control laws depend only on its own state and its neighbors' states.

The controllers in Theorem 1 cannot make Eq. 5 hold. To make Eq. 5 hold, we introduce damping terms in the controllers and have the following result.

Theorem 2 *Consider the systems in Eq. 9, under Assumptions 1–2, the decentralized controllers (15–16) make Eqs. 3–5 hold, where*

$$\eta_{1j} = -\sum_{i\in\mathcal{N}_j} b_{ji}\left(z_{1j} - z_{1i} + \Delta_j - \Delta_i\right) - \mu_j(z_{1j} + \Delta_j) + w_1 \tag{26}$$

$$\eta_{2j} = -\sum_{i\in\mathcal{N}_j} b_{ji}\left(z_{2j} - z_{2i}\right) - \mu_j z_{2j} - \beta w_1(-\beta z_{3j} w_1^2 + w_1 z_{2j}) - \beta z_{3j}\dot{w}_1 \tag{27}$$

constants $b_{ji} = b_{ij} > 0$*, constants* $\mu_j \geq 0$ *and* $\sum_{l=1}^{m}\mu_l > 0$*, and* Δ_j *is defined in Eq. 19.*

Proof Let the positive definite Lyapunov function V be defined in Eq. 22. Differentiate V along the solutions of Eq. 9 with the control laws (15–16), we have

$$\dot{V} = -\sum_{j=1}^{m}\beta w_1^2 z_{3j}^2 - \sum_{j=1}^{m}\mu_j[z_{2j}^2 + (z_{1j}+\Delta_j)^2] - z_{2*}^\top L z_{2*} - (z_{1*}+\Delta)^\top L(z_{1*}+\Delta) \leq 0$$

Therefore, V is bounded. Furthermore, z_{ij} are bounded. By Barbalat's Lemma [28], $\lim_{t\to\infty}\dot{V} = 0$. So, Eqs. 23–24 hold and

$$\lim_{t\to\infty}\sum_{j=1}^{m}\mu_j z_{2j}^2 = 0, \qquad \lim_{t\to\infty}\sum_{j=1}^{m}\mu_j(z_{1j}+\Delta_j)^2 = 0 \tag{28}$$

By Lemma 3, $\lim_{t\to\infty}(z_{2*}(t) - c_2(t)\mathbf{1}) = 0$ and $\lim_{t\to\infty}(z_{1*}(t) + \Delta(t) - c_1(t)\mathbf{1}) = 0$ where c_1 and c_2 are defined in Eq. 25. Since at least one of μ_j is greater than zero, say $\mu_p > 0$, then by Eq. 28 we have $\lim_{t\to\infty} z_{2p} = 0$ and $\lim_{t\to\infty}(z_{1p} + \Delta_p) = 0$. Noting that z_{2j} converges to c_2, we see $c_2 = 0$. Since $(z_{1j} + \Delta_j)$ converges to c_1, we see $c_1 = 0$. So, $\lim_{t\to\infty}(u_{1j} - w_1)\zeta_{2j} = 0$ for $1 \leq j \leq m$. By Lemma 2, $\lim_{t\to\infty} z_{3j} = 0$ $(1 \leq j \leq m)$. Noting the definition of Δ_j, we can prove that $\lim_{t\to\infty} z_{1j} = 0$ $(1 \leq j \leq m)$. By Lemma 1, Eqs. 3–5 hold. □

Remark 5 Terms $\mu_j(z_{1j} - \Delta_j)$ and $\mu_j z_{2j}$ in Eqs. 26–27 are called the damping terms which are used to make $\mu_j(z_{1j} - \Delta_j)$ and $\mu_j z_{2j}$ converge to zero. Large μ_j means that z_{1j} and z_{2j} converge to zero quickly.

Remark 6 If the communication graph $\mathcal{G}$ is connected, z_{1*} and z_{2*} converge to zero if one of μ_j is greater than zero. In fact, if $\mathcal{G}$ is not connected, we can make z_{1*} and z_{2*} converge to zero by choosing some μ_j to be positive. In the worst case, if there is no communication between any two systems, we can make z_{1*} and z_{2*} converge to zero by choosing $\mu_j > 0$ for all j.

3.2 Cooperative Control Laws for Dynamics (9–10)

With the aid of the controllers for the systems in Eq. 9, we propose decentralized cooperative controllers for dynamics (10) using backstepping techniques.

With the aid of the results in Theorem 1, we have the following result.

Theorem 3 *For the systems in Eqs. 9–10, under Assumptions 1–2, the control laws*

$$\tau_j = \bar{B}_j^{-1}\left(-K_j(u_{*j} - \eta_{*j}) + \bar{Y}_j(\zeta_{*j}, \dot{\zeta}_{*j}, \eta_{*j}, \dot{\eta}_{*j})\left(\bar{a}_j - \Pi_j\right) - \Lambda_j\right) \tag{29}$$

for $1 \leq j \leq m$ *make Eqs.* 3–4 *hold, where constant matrix* K_j *is symmetric and positive definite,* $\eta_{*j} = [\eta_{1j}, \eta_{2j}]^\top$, η_{1j} *and* η_{2j} *are defined in Eqs.* 17–18, $u_{*j} = [u_{1j}, u_{2j}]^\top$,

$$\Lambda_j = [z_{1j} + \Delta_j, z_{2j}]^\top, \tag{30}$$

$$\Pi_j = \frac{\rho \bar{Y}_j^\top(\zeta_{*j}, \dot{\zeta}_{*j}, \eta_{*j}, \dot{\eta}_{*j})\tilde{u}_{*j}}{\|\bar{Y}_j^\top(\zeta_{*j}, \dot{\zeta}_{*j}, \eta_{*j}, \dot{\eta}_{*j})\tilde{u}_{*j}\| + e^{-\gamma_j t}} \tag{31}$$

constants $\gamma_j > 0$ *and the other control parameters are defined in Theorem* 1.

Proof Let $\tilde{u}_{*j} = u_{*j} - \eta_{*j}$, with the control laws in Eq. 29, we have

$$\begin{cases} \dot{z}_{1j} = u_{1j} - w_1 \\ \dot{z}_{2j} = -\sum_{i \in \mathcal{N}_j} b_{ji}(z_{2j} - z_{2i}) + \tilde{u}_{2j} + (u_{1j} - w_1)\beta w_1 \zeta_{2j} \\ \dot{z}_{3j} = -\beta z_{3j} w_1^{2n-4} + z_{2j} w_1 + (u_{1j} - w_1)\zeta_{2j} \end{cases} \tag{32}$$

$$\bar{M}_j \dot{\tilde{u}}_{*j} + \bar{C}_j \tilde{u}_{*j} = -K_j \tilde{u}_j - \bar{Y}_j(\zeta_{*j}, \dot{\zeta}_{*j}, \eta_{*j}, \dot{\eta}_{*j})(a_j - \bar{a}_j + \Pi_j) - \Lambda_j \tag{33}$$

where $u_{1j} - w_1 = -\sum_{j \in \mathcal{N}_j} b_{ji}(z_{1j} - z_{1i} + \Delta_j - \Delta_i) + \tilde{u}_{1j}$. Let the Lyapunov function

$$V = \frac{1}{2} \sum_{j=1}^{m} \left(\sum_{i=1}^{3} z_{ij}^2 + u_{*j}^\top \bar{M}_j u_{*j} \right). \tag{34}$$

Differentiating V along the solutions of Eqs. 32–33, we have

$$\begin{aligned} \dot{V} &= \sum_{j=1}^{m} \left(-\beta z_{3j}^2 w_1^2 - \tilde{u}_{*j}^\top K_j \tilde{u}_{*j} \right) - (z_{1*} + \Delta)^\top L (z_{1*} + \Delta) - z_{2*}^\top L z_{2*} \\ &\quad - \tilde{u}_{*j}^\top \bar{Y}_j (a_j - \bar{a}_j) - \frac{\rho \|\bar{Y}_j^\top \tilde{u}_{*j}\|^2}{\|\bar{Y}_j^\top \tilde{u}_{*j}\| + e^{-\gamma_j t}} \\ &\leq \sum_{j=1}^{m} \left(-\beta z_{3j}^2 w_1^2 - \tilde{u}_{*j}^\top K_j \tilde{u}_{*j} \right) - (z_{1*} + \Delta)^\top L (z_{1*} + \Delta) - z_{2*}^\top L z_{2*} + \rho e^{-\gamma_j t} \\ &= \sum_{j=1}^{m} \left(-\beta z_{3j}^2 w_1^2 - \tilde{u}_{*j}^\top K_j \tilde{u}_{*j} \right) - [Q(z_{1*} + \Delta)]^\top [Q(z_{1*} + \Delta)] \\ &\quad - (Q z_{2*})^\top (Q z_{2*}) + \rho e^{-\gamma_j t} \end{aligned} \tag{35}$$

where we use the facts that $(\dot{\bar{M}}_j - 2\bar{C}_j)$ are skew symmetric and that $L = Q^\top Q$ where Q is a symmetric constant matrix. Therefore, V is bounded. Furthermore, z_{ij} and u_{*j} are bounded for $1 \leq i \leq 3$ and $1 \leq j \leq m$. Also, $Q(z_{1*} + \Delta)$ and Qz_{2*} are bounded. By integrating both sides of Eq. 35, we know $w_1 z_{3j}$, $\tilde{u}_{*j}$, $Q(z_{1*} + \Delta)$, and Qz_{2*} are square integrable. Noting Eqs. 32–33, the derivative of $w_1 z_{3j}$, $\tilde{u}_{*j}$, $Q(z_{1*} + \Delta)$, and Qz_{2*} are bounded. By Barbalat's Lemma, $w_1 z_{3j}$, $\tilde{u}_{*j}$, $Q(z_{1*} + \Delta)$, and Qz_{2*} converge to zero. So, Eqs. 23–24 hold and

$$\lim_{t \to \infty} \tilde{u}_{*j} = 0 \ \ (1 \leq j \leq m), \tag{36}$$

By Assumptions 1–2, following the proof of Theorem 1, we can prove that $\lim_{t\to\infty}(z_{*j} - z_{*l}) = 0$ for $1 \leq \neq j \leq m$. By Lemma 1, Eqs. 3–4 hold. □

Remark 7 The control law τ_j for system j consists of its own state and the relative state information with its neighbors. Therefore, it is decentralized. Noting the structure of Eqs. 32–33, the motion of the closed-loop system is driven by the relative information between neighbors. The control parameters are b_{ji}, K_j, γ_j, β, and w_1. Increasing K_j and β will increase the convergence rate of the closed-loop system.

In Theorem 3, Eq. 5 does not hold. To make Eqs. 3–5 hold, we introduce damping terms in η_{*j}. With the aid of the results in Theorem 2, we have the following result.

Theorem 4 *For the systems in Eqs.* 9–10*, under Assumptions* 1–2*, the control laws in Eq.* 29 *make Eqs.* 3–5 *hold, where constant matrix* K_j *is symmetric positive definite,* $\eta_{*j} = [\eta_{1j}, \eta_{2j}]^\top$*,* η_{1j} *and* η_{2j} *are defined in Eqs.* 26–27*,* $u_{*j} = [u_{1j}, u_{2j}]^\top$*,* Λ_j *is defined in Eq.* 30*, and the other control parameters are defined in Theorem* 2*.*

Proof Let the Lyapunov function V be defined in Eq. 34. Differentiating it along the solutions of the closed-loop system, we have

$$\begin{aligned}\dot{V} &= \sum_{j=1}^{m}\left(-\beta z_{3j}^2 w_1^2 - \tilde{u}_{*j}^\top K_j \tilde{u}_{*j} - \mu_j z_{2j}^2 - \mu_j (z_{1j} + \Delta_j)^2\right) - (z_{1*} + \Delta)^\top L (z_{1*} + \Delta) \\ &\quad - z_{2*}^\top L z_{2*} - \tilde{u}_{*j}^\top \bar{Y}_j (a_j - \bar{a}_j) - \frac{\rho \|\bar{Y}_j^\top \tilde{u}_{*j}\|^2}{\|\bar{Y}_j^\top \tilde{u}_{*j}\| + e^{-\gamma_j t}} \\ &\leq \sum_{j=1}^{m}\left(-\beta z_{3j}^2 w_1^2 - \tilde{u}_{*j}^\top K_j \tilde{u}_{*j} - \mu_j z_{2j}^2 - \mu_j (z_{1j} + \Delta_j)^2\right) - (z_{1*} + \Delta)^\top L (z_{1*} + \Delta) \\ &\quad - z_{2*}^\top L z_{2*} + \rho e^{-\gamma_j t} \end{aligned} \tag{37}$$

where we use the facts that $(\dot{\bar{M}}_j - 2\bar{C}_j)$ are skew symmetric and that L is symmetric. Following the proof of Theorem 3, we can prove that Eqs. 23–24, 28, and 36 hold. By Assumptions 1–2, following the proof of Theorem 2, we can prove that $\lim_{t\to\infty} z_{3j} = 0$ for $1 \leq j \leq m$. By Lemma 1, Eqs. 3–5 holds. □

Remark 8 Cooperative controllers (29) are decentralized and make Eqs. 3–5 hold. The difference between the control laws in Theorems 3 and 4 is that the damping terms are introduced in the control laws in Theorem 4. For the relationship between the closed-loop system performance and the control parameters, readers may refer to the remarks after Theorems 1–3. In this paper, we do not consider collision between robots. Discussion on this topic is our future work.

4 Closed-loop System Stability With Communication Delays

In the previous controller design, we did not consider communication delays. In practice, there are always time delays due to communication and other factors. For simplicity, in this paper we assume that communication delays only appear in the neighbor's states and are constants.

Corresponding to Theorem 2, we have the following result.

Theorem 5 *Consider the systems in Eq. 9, under Assumptions 1–2, the controllers (15–16) make Eqs. 3–5 hold, in Eqs. 15–16*

$$\eta_{1j}(t) = -\mu_j[z_{1j}(t) + \Delta_j(t)] - \sum_{i\in\mathcal{N}_j} b_{ji}\left[z_{1j}(t) - z_{1i}(t-\delta_i) + \Delta_j(t) - \Delta_i(t-\delta_i)\right] + w_1(t) \tag{38}$$

$$\eta_{2j}(t) = -\mu_j z_{2j}(t) - \sum_{i\in\mathcal{N}_j} b_{ji}[z_{2j}(t) - z_{2i}(t-\delta_i)] - \beta\left[-\beta z_{3j}(t)w_1^3(t) + w_1^2(t)z_{2j}(t)\right] - \beta z_{3j}(t)\dot{w}_1(t) \tag{39}$$

where constants $b_{ji} = b_{ij} > 0$, $\mu_j \geq 0$ and $\sum_{l=1}^m \mu_l > 0$, $\Delta_j(t)$ is defined in Eq. 19, and constants $\delta_i \geq 0$ ($1 \leq i \leq m$).

Proof Let the nonnegative function

$$V(t) = \frac{1}{2}\sum_{j=1}^m\left(\sum_{i=1}^3 z_{ij}^2(t) + \sum_{i\in\mathcal{N}_j}\int_{t-\delta_i}^t b_{ji}\left((z_{1i}(s)+\Delta_i(s))^2 + z_{2i}^2(s)\right)ds\right). \tag{40}$$

Differentiate it along the solutions of Eq. 9 with the control laws (15–16), we have

$$\dot{V}(t) = -\sum_{j=1}^m \beta z_{3j}^2(t)w_1^2(t) - \sum_{j=1}^m \mu_j(z_{1j}(t)+\Delta_j(t))^2 - \sum_{j=1}^m \mu_j z_{2j}^2(t) - \frac{1}{2}\sum_{j=1}^m\sum_{i\in\mathcal{N}_j} b_{ji}\left[(\bar{\Delta}_j(t) - \bar{\Delta}_i(t-\delta_i))^2 + (z_{2j}(t) - z_{2i}(t-\delta_i))^2\right] \leq 0 \tag{41}$$

where we use the fact that the communication graph $\mathcal{G}$ is bidirectional, and

$$\bar{\Delta}_j(t) = z_{1j}(t) + \Delta_j(t).$$

Therefore, V is bounded. Furthermore, z_{ij} are bounded for $1 \leq i \leq 3$ and $1 \leq j \leq m$. By Barbalat's Lemma [28], $\lim_{t\to\infty}\dot{V} = 0$. So, Eqs. 23 and 28 hold and

$$\lim_{t\to\infty}(\bar{\Delta}_j(t) - \bar{\Delta}_i(t-\delta_i)) = 0,\ \lim_{t\to\infty}(z_{2j}(t) - z_{2i}(t-\delta_i)) = 0,\ i\in\mathcal{N}_j,\ 1 \leq j \leq m. \tag{42}$$

Since at least one of μ_j, say μ_p, is greater than zero, we see from Eq. 28 that $\lim_{t\to\infty}\bar{\Delta}p(t) = 0$ and $\lim_{t\to\infty} z_{2p}(t) = 0$. Since the graph $\mathcal{G}$ is connected, from Eq. 42 we can prove that $\lim_{t\to\infty}\bar{\Delta}_j(t) = 0$ and $\lim_{t\to\infty} z_{2j}(t) = 0$ ($1 \leq j \leq m$). Following the proof of Theorem 2, we can prove that $\lim_{t\to\infty} z_{1j}(t) = 0$ and $\lim_{t\to\infty} z_{3j}(t) = 0$ for $1 \leq j \leq m$. By Lemma 1, Eqs. 3–5 hold. □

Corresponding to Theorem 4, we have the following result.

Theorem 6 *For the systems in Eqs. 9–10, under Assumptions 1–2, the control laws (29) make Eqs. 3–5 hold, where η_{1j} and η_{2j} are defined in Eqs. 38–39, constants*

$b_{ji} = b_{ij} > 0$, constants $\mu_j \geq 0$ and $\sum_{l=1}^{m} \mu_l > 0$, the communication delays $\delta_i (\geq 0)$ are constants, and the other variables and control parameters are defined in Theorem 4.

Proof Let the nonnegative function

$$V(t) = \frac{1}{2}\sum_{j=1}^{m}\left(\sum_{i=1}^{3} z_{ij}^2(t) + u_{*j}^{\top}(t)\bar{M}_j(t)u_{*j}(t)\right.$$
$$\left. + \sum_{i\in\mathcal{N}_j}\int_{t-\delta_i}^{t} b_{ji}\left((z_{1i}(s)+\Delta_i(s))^2 + z_{2i}^2(s)\right) ds\right), \tag{43}$$

differentiating V along the closed-loop system, we have

$$\begin{aligned}
\dot{V}(t) = &\sum_{j=1}^{m}\left(-\beta z_{3j}^2(t) w_1^{2n-4}(t) - \tilde{u}_{*j}^{\top}(t)K_j\tilde{u}_{*j}(t)\right)\\
&-\sum_{j=1}^{m}\mu_j(z_{1j}(t)+\Delta_j(t))^2 - \sum_{j=1}^{m}\mu_j z_{2j}^2(t)\\
&-\frac{1}{2}\sum_{j=1}^{m}\sum_{i\in\mathcal{N}_j} b_{ji}\left[(\bar{\Delta}_j(t)-\bar{\Delta}_i(t-\delta_i))^2 + (z_{2j}(t)-z_{2i}(t-\delta_i))^2\right]\\
&-\tilde{u}_{*j}^{\top}\bar{Y}_j(a_j-\bar{a}_j) - \frac{\rho\|\bar{Y}_j^{\top}\tilde{u}_{*j}\|^2}{\|\bar{Y}_j^{\top}\tilde{u}_{*j}\| + e^{-\gamma_j t}}\\
\leq &\sum_{j=1}^{m}\left(-\beta z_{3j}^2(t) w_1^{2n-4}(t) - \tilde{u}_{*j}^{\top}(t)K_j\tilde{u}_{*j}(t)\right) - \sum_{j=1}^{m}\mu_j(z_{1j}(t)+\Delta_j(t))^2 - \sum_{j=1}^{m}\mu_j z_{2j}^2(t)\\
&-\frac{1}{2}\sum_{j=1}^{m}\sum_{i\in\mathcal{N}_j} b_{ji}\left[(\bar{\Delta}_j(t)-\bar{\Delta}_i(t-\delta_i))^2 + (z_{2j}(t)-z_{2i}(t-\delta_i))^2\right] + \rho e^{-\gamma_j t}
\end{aligned} \tag{44}$$

where we use the facts that $(\dot{\bar{M}}_j - 2\bar{C}_j)$ is skew symmetric and that L is symmetric. Following the proof of 4, we can prove that Eqs. 23, 28, 42, and 36 hold. Since at least one of μ_j, say μ_p, is greater than zero, we see from Eq. 28 that $\lim_{t\to\infty}\bar{\Delta}p(t) = 0$ and $\lim_{t\to\infty} z_{2p}(t) = 0$. Since the graph $\mathcal{G}$ is connected, from Eq. 42 we can prove that $\lim_{t\to\infty}\bar{\Delta}_j(t) = 0$ and $\lim_{t\to\infty} z_{2j}(t) = 0$ $(1 \leq j \leq m)$. Following the proof of Theorem 4, we can prove that $\lim_{t\to\infty} z_{1j}(t) = 0$ and $\lim_{t\to\infty} z_{3j}(t) = 0$ for $1 \leq j \leq m$. By Lemma 1, Eqs. 3–5 hold. □

Remark 9 In [16], formation control of multiple robots was also considered. Control laws were proposed with the aid of the leader–follower approach, backstepping techniques, and neural networks. In this paper, formation control of multiple robots was solved with the aid of results from algebraic graph theory and backstepping techniques. For a group of systems in this paper, there is no leader and the control input for each system is generated based on its own information and its neighbor's information. If one or more systems fail the other systems still can maintain desired

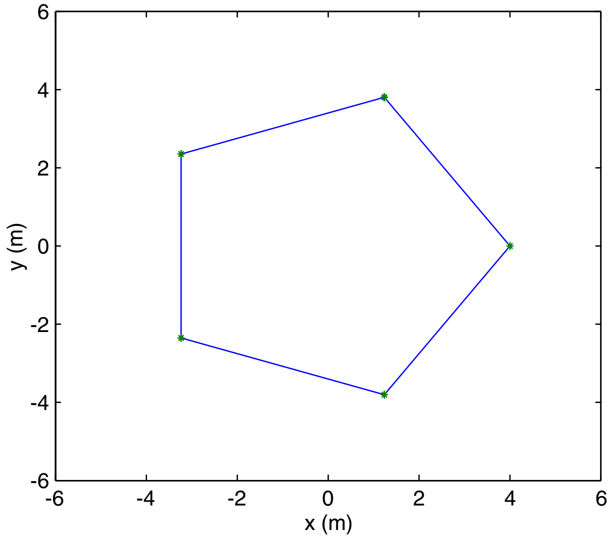


Fig. 2 Desired geometric pattern

distances between the remaining systems if the communication graph of the remaining systems is connected. While for leader–follower based formation control in [16] if one system fails the remaining systems cannot maintain desired distances between the remaining system because the followers of the failing system has no information of its leader. Furthermore, in this paper robustness of the stability of the closed-loop systems with respect to communication delays is discussed and it is shown that the proposed control laws in this paper work if the communication delays are constant. In [16], communication delay was not considered.

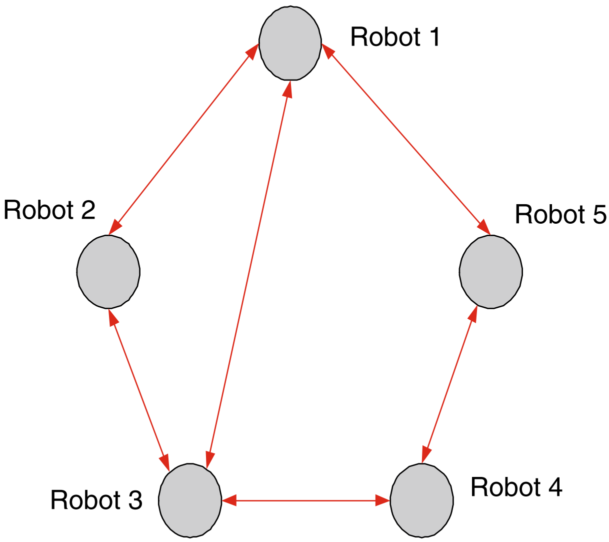


Fig. 3 Communication graph $\mathcal{G}$

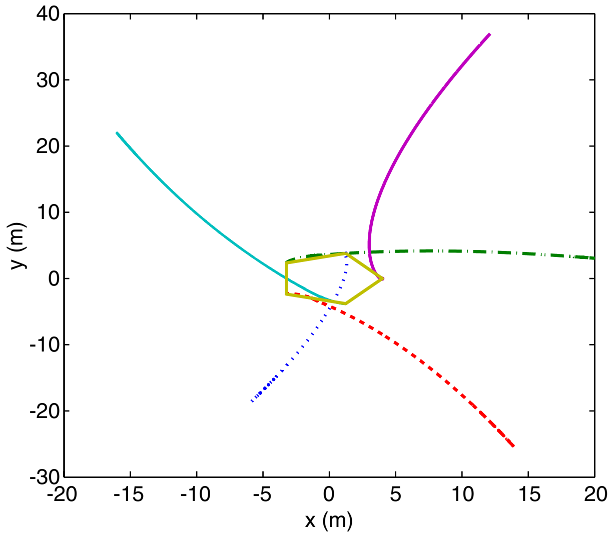


Fig. 4 Paths of the five robots and the final geometric pattern

5 Simulations

To verify effectiveness of the proposed control laws, we present some simulation results.

Let $m = 5$ and the initial conditions of the five robots be $(-3.5, -13.8, -2.0)$, $(18.1, 3.3, 3.0)$, $(13.9, -25.3, 2.0)$, $(-14.5, 18.7, 2.0)$, and $(12.1, 37.0, -2.0)$. Assume that the desired formation $\mathcal{P}$ is defined by $(p_{1x}, p_{1y}) = (1.24, 3.8)$, $(p_{2x}, p_{2y}) = (-3.24, 2.35)$, $(p_{3x}, p_{3y}) = (-3.24, -2.35)$, $(p_{4x}, p_{4y}) = (1.24, -3.8)$, and $(p_{5x}, p_{5y}) = (4, 0)$ (Fig. 2). Assume that the communication graph $\mathcal{G}$ is shown in Fig. 3. The

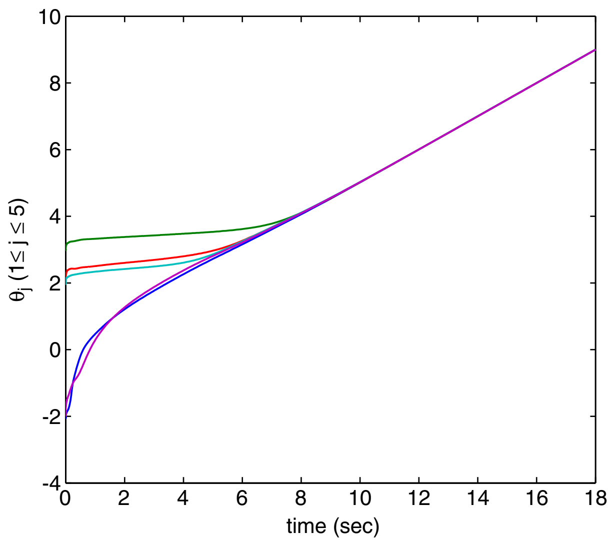


Fig. 5 Responses of θ_j $(j = 1, 2, 3, 4, 5)$

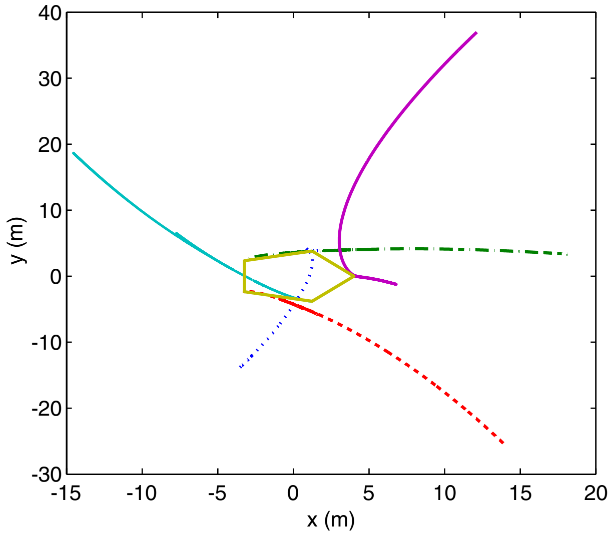


Fig. 6 Paths of the five robots and the final geometric pattern with communication delay $\delta_j = 0.2$ s

cooperative controllers can be obtained by Theorem 4. In the simulation, we assume $L_j = 0.5m$, $R_j = 0.2m$, and the real inertia parameters $m_j = 1$ and $I_j = 1$. We choose the control parameters $b_{ji} = 2$, $\beta = 10$, $K_j = 10$, $w_1 = 0.5$, $\Gamma_j = 0.1$, $\mu_1 = 4$, $\mu_j = 0$ for $j \neq 1$, and the estimates $\bar{m}_j = 2$ and $\bar{I}_j = 2$. Figure 4 shows the paths of the five robots. It can be seen that they come into the desired formation. The geometric pattern of the formation is stationary. The orientations of the five robots converge to the same value (see Fig. 5). If there are constant communication delays in the control, the control laws achieve the same objectives according to Theorem 6. To simplify the simulation, we assume all the communication delays are the same and

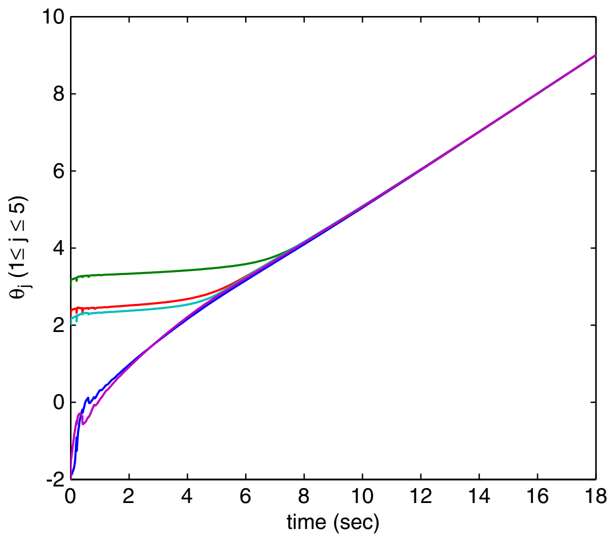


Fig. 7 Responses of θ_j $(j = 1, 2, 3, 4, 5)$ with communication delay $\delta_j = 0.2$ s

$\delta_j = 0.1$ s. Figure 6 shows the paths of the five robots. It's shown that the five robots come into the desired formation. The orientations of the five robots converge to the same value (see Fig. 7).

6 Conclusion

This paper has considered the formation control of multiple wheeled mobile robots with uncertainty. Cooperative control laws have been proposed with the aid of results from graph theory. The robustness of the control laws with respect to communication delays are also analyzed. Simulation results show effectiveness of the proposed control laws.

References

1. Desai, J.P., Ostrowski, J.P., Kumar, V.: Modeling and control of formations of nonholonomic mobile robots. IEEE Trans. Robot. Autom. **17**, 905–908 (2001)
2. Fierro, R., Das, A., Kumar, V., Ostrowski, J.: Hybrid control of formations of robots. In: Proc. of the IEEE Int. Conf. on Robotics and Automation, pp. 157–162 (2001)
3. Mesbahi, M., Hadaegh, F.Y.: Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching. AIAA J. Guid. Control Dyn. **24**, 369–377 (2001)
4. Tanner, H.G., Pappas, G.J., Kumar, V.: Leader-to-formation stability. IEEE Trans. Robot. Autom. **20**, 443–455 (2004)
5. Balch, T., Arkin, R.C.: Behavior-based formation control for multirobot teams. IEEE Trans. Robot. Autom. **14**(6), 926–939 (1998)
6. Parker, L.E.: Alliance: an architecture for fault tolerant multirobot cooperation. IEEE Trans. Robot. Autom. **14**, 220–240 (1998)
7. Beard, R.W., Lawton, J., Hadaegh, F.Y.: A coordination architecture for spacecraft formation control. IEEE Trans. Control Syst. Technol. **9**(6), 777–790 (2001)
8. Kang, W., Xi, N., Sparks, A.: Formation control of autonomous agents in 3d workspace. In: Proc. IEEE Int. Conf. Robotics and Automation, pp. 1755–1760 (2000)
9. Lewis, M., Tan, K.-H.: High precision formation control of mobile robots using virtual structures. Auton. Robots **4**, 387–403 (1997)
10. Yamaguchi, H.: A distributed motion coordination strategy for multiple nonholonomic mobile robots in cooperative hunting operations. Robot. Auton. Syst. **43**, 257–282 (2003)
11. Lawton, J., Beard, R.W., Young, B.: A decentralized approach to formation maneuvers. IEEE Trans. Robot. Autom. **19**(6), 933–941 (2003)
12. Feddema, J.T., Lewis, C., Schoenwald, D.A.: Decentralized control of cooperative robotic vehicles: theory and application. IEEE Trans. Robot. Autom. **18**(5), 852–864 (2002)
13. Justh, E.W., Krishnaprasad, P.S.: Equilibrium and steering laws for planar formations. Syst. Control. Lett. **52**, 25–38 (2004)
14. Dimarogonas, D.V., Kyriakopoulos, K.J.: On the rendezvous problem for multiple nonholonomic agents. IEEE Trans. Automat. Contr. **52**, 916–922 (2007)
15. Mastellone, S., Stipanovic, D.M., Graunke, C., Intlekofer, K., Spong, M.W.: Formation control and collision avoidance for multi-agent non-holonomic systems: theory and experiments. Int. J. Rob. Res. **27**, 107–126 (2008)
16. Dierks, T., Jagannathan, S.: Neural network output feedback control of robot formations. IEEE Trans. Syst. Man Cybern., Part B, Cybern. **40**(2), 383–399 (2010)
17. Dong, W., Farrell, J.A.: Decentralized cooperative control of multiple nonholonomic systems. In: Proc. of Conference of Decision and Control (2007)
18. Dong, W., Farrell, J.A.: Cooperative control of multiple nonholonomic mobile agents. IEEE Trans. Automat. Contr. **53**(7), 1434–1448 (2008)
19. Dong, W., Farrell, J.A.: Decentralized cooperative control of multiple nonholonomic dynamic systems with uncertainty. Automatica **45**, 706–710 (2009)

20. Bloch, A.M., Reyhanoglu, M., McClamroch, N.H.: Control and stabilization of nonholonomic dynamic systems. IEEE Trans. Automat. Contr. **37**, 1746–1757 (1992)
21. Dong, W., Xu, W.L.: Adaptive tracking control of uncertain nonholonomic dynamic system. IEEE Trans. Automat. Contr. **46**(3), 450–454 (2001)
22. Lewis, F., Abdallah, C., Dawson, D.: Control of Robot Manipulators. Macmillan, New York, (1993)
23. Chung, F.R.K.: Spectral graph theory. In: Regional Conf. Series in Mathematics of Amer. Mathematical Soc., vol. 92 (1997)
24. Merris, R.: A survey of graph laplacians. Linear and Multilinear Algebra **39**, 19–31 (1995)
25. Merris, R.: Laplacian graph eigenvectors. Linear Algebra Appl. **278**, 221–236 (1998)
26. Horn, R.A., Johnson, C.R.: Matrix Analysis. Cambridge University Press (1987)
27. Olfati-Saber, R., Murray, R.M.: Consensus problems in networks of agents with switching topology and time-delays. IEEE Trans. Automat. Contr. **49**, 101–115 (2004)
28. Slotine, J.E., Li, W.: Applied Nonlinear Control. Prentice Hall, Englewood Cliffs, N.J. (1991)