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System Identification for a Miniature Helicopter at Hover Using Fuzzy Models

Ioannis A. Raptis · Kimon P. Valavanis · Abraham Kandel · Wilfrido A. Moreno

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Abstract The objective of this paper is to present a system identification method suitable for miniature rotorcrafts under hovering. The proposed model to be identified is a Takagi-Sugeno fuzzy system, representing translational and rotational velocity dynamics. For parameter estimation of the Takagi-Sugeno system a classical Recursive Least Squares (RLS) algorithm is used, which allows identification to take place on-line since parameter updates are produced whenever a new measurement becomes available. The validity of this approach is tested using the *X-Plane*© flight simulator. Data obtained offer justification for the applicability of the approach in real-time applications.

Keywords Estimation · Identification · Takagi-Sugeno fuzzy systems · Unmanned helicopter

I. A. Raptis (✉) · W. A. Moreno
Department of Electrical Engineering, University of South Florida, Tampa, FL 33620, USA
e-mail: iraptis@mail.usf.edu

W. A. Moreno
e-mail: moreno@eng.usf.edu

K. P. Valavanis
Department of Electrical and Computer Engineering, University of Denver, Denver, CO 80208, USA
e-mail: Kimon.Valavanis@du.edu

A. Kandel
Department of Computer Science and Engineering, University of South Florida, Tampa, FL 33620, USA
e-mail: kandel@cse.usf.edu

1 Introduction

Advantages of helicopter unique flight capabilities have drawn much attention through the years. Apart from the mechanical and aerodynamic design of such vehicles, interest is also focused on automation and control. The main characteristic attribute of the helicopter is the use of rotary wings to produce the thrust force that is necessary for motion. This thrust controls the vertical motion of the vehicle, and it can be tilted to produce forces and moments that control the rotational and translational motion. Therefore, a helicopter has an advantage relative to fixed wing aircrafts due to the vertical flight and due to the fact that helicopters do not need any translational velocity to produce aerodynamic flight forces [8].

However, helicopters are considered to be much more unstable than fixed wing aircraft and constant control action must be sustained at all times. Controller design requires knowledge of the dominating dynamics of the system. Miniature helicopters are coupled nonlinear systems of high order dynamics. Due to the higher order and complicated dynamic structure there is much difficulty in developing a consistent model of minimum order, being at the same time accurate enough. Although sophisticated control algorithms have been designed, system identification from Input/Output data remains a challenge.

A detailed study in aircraft system identification is presented in [28] and adopted in [16] for the case of miniature rotorcraft. This approach is based on frequency response identification. A linear model is produced which describes sufficiently the dynamic behavior of the rotorcraft around an operating condition. The complete behavior of the vehicle is obtained by switching between linear models depending on the flight condition. The validity of the linear models in most cases cover a relative wide area of the operating condition [16].

In [18] Prediction Error Method is preformed to identify a linear small scale helicopter model placed in a 3 DOF stand. In [9] the nonlinear system parameters are identified from linearized transfer functions, obtained by isolating the effect of each input to each output. Moreover, special attention is given to the modeling of the flybar dynamics. Modeling of the helicopter dynamics by Neural Networks for control purposes, is reported in [2, 24]. An additional study in the system identification techniques for large scale helicopters is given in [6].

The novelty of this paper is the application of a classical system identification method for fuzzy systems, which is specially configured to derive a discrete time nonlinear fuzzy model of the helicopter translational and angular velocity dynamics under hovering. The paper illustrates a time domain identification approach that can be implemented on-line in the sense that estimates can be made each time a new state measurement is available. Results illustrate that this method is successful in producing a nonlinear discrete model of relatively low complexity and high accuracy. Low order accurate helicopter models are suitable for the design of model based nonlinear controllers.

More specifically, a Takagi-Sugeno fuzzy system is developed based on the discretized dynamics of translational and angular velocity. The discrete velocity dynamics are obtained by the helicopter's nonlinear equations of motion. The derivation of the motion equations is based on widely adopted modeling assumptions regarding the helicopter dynamics. The helicopter is considered as a rigid body actuated by a simplified model of force and torque generation. This simplified model of the external wrench, incorporates the most dominant effects of the main and the tail rotor while keeping a low modeling complexity.

After the development of the Takagi-Sugeno system, a standard RLS algorithm is used to estimate its parameters. The resulting fuzzy system is an interpolator of nonlinear discrete systems which depends on the helicopters flight condition. Important role in the identification procedure plays the excitation signals used to obtain the experimental data. In this paper frequency sweeps are used which are common excitation signals in frequency domain identification techniques of flight vehicles. This type of excitation was proven to be adequate for the time domain identification technique provided in this paper.

The experimental data and verification results were obtained using *X-Plane*©, a commercially available flight simulator. The use of *X-Plane*© for the evaluation of the approach was significant since it provides a good indication of the applicability of the approach to real flight applications. The verification results illustrate the success of the approach. The calculation time and the algorithm complexity are not an issue, therefore the proposed approach can provide a simple and yet accurate method of identifying the system dynamics which can easily compete the well established frequency domain techniques. Furthermore, the verification results have illustrated that the use of a fuzzy system to model the helicopter dynamics, is capable of producing accurate results in a very wide area around the trim flight mode.

The paper is organized as follows. In Section 2 the basic dynamic equations of the helicopter are described, which include the rigid body dynamics and force generation by the main and tail rotor. In Section 3 the discrete translational and angular velocity dynamics are derived from the continuous equivalent. Section 4 illustrates how a Takagi-Sugeno fuzzy system can be used for parameter identification of dynamic systems with the aid of the standard RLS algorithm. In Section 5 the proposed Takagi-Sugeno fuzzy system used to represent the helicopter model is discussed. In Section 6 a description of the control commands used to excite the system is given. Finally, simulation results used to verify the model are given in Section 7.

2 Helicopter Model

A helicopter is a nonlinear model of high order. The first approach to dynamic modeling is derivation of the basic equations of motion treating the helicopter as a rigid body with 6 Degrees Of Freedom (DOF). Therefore, in order to describe the rigid body dynamics 12 state space variables are required [22]. The equations of motion are directly derived by implementing Newton's second law. For rotational motion, the analysis can be greatly simplified if the motion is described relatively to a body-fixed reference frame attached to the rigid body.

Forces and moments are produced by the main and tail rotors. The pilot posses four control commands, which include control of the collective pitch of the rotor's blade required for vertical motion, the collective pitch of the tail rotor's blade for controlling the yaw and two additional cyclic commands necessary for longitudinal and lateral motion. Since the helicopter has less number of control commands than DOF it is characterized as an underactuated system [7]. A major difficulty associated with the description of the helicopter's forces and moments is the significant coupling between those [11].

A more complete and accurate description of the helicopter dynamics [11] requires rotor dynamics [13], the rotor's Tip-Path-Plane (TPP) dynamics [5, 16, 17, 26, 27] and the actuator dynamics [16]. However, for miniature rotorcraft the identification results illustrated in this paper, show that for the model miniature rotorcraft developed in *X-Plane*©, the rigid body dynamics and the force and torque generation model are sufficient to capture the motion of the system.

2.1 Rigid Body Dynamics

The first step towards derivation of the rigid body's equation of motion is the definition of two reference frames. Each frame is fully characterized by its center and three mutually orthonormal vectors. The first one is spatial (inertia) frame defined as $\mathcal{F}_s = \{O_s, \mathbf{i}_s, \mathbf{j}_s, \mathbf{k}_s\}$. The second is the body fixed reference frame defined as $\mathcal{F}_b = \{O_b, \mathbf{i}_b, \mathbf{j}_b, \mathbf{k}_b\}$ where the center O_b is located at the Center of Gravity (CG) of the helicopter. In general the orientation of the orthonormal set of vectors $\{\mathbf{i}_b, \mathbf{j}_b, \mathbf{k}_b\}$ is standard, related to aerodynamics of air vehicles [14, 21]. Instead of the standard orientation, the orientation considered by *X-Plane*© is adopted. According to that $\mathbf{i}_b$ is pointing at the right side of the helicopter, $\mathbf{k}_b$ is pointing backwards aligned with the tail, and $\mathbf{j}_b$ is facing upwards and it is normal to both $\mathbf{i}_b, \mathbf{k}_b$.

The linear velocity expressed in the body coordinate frame is $v^b = [v_x^b \; v_y^b \; v_z^b]^T \in \mathbb{R}^3$. This is the velocity of the helicopter's CG measured with respect to the inertia frame and expressed in the coordinates of the body frame. The angular velocity with respect to the body frame is $\omega^b = [q \; r \; p]^T \in \mathbb{R}^3$. Positive direction of the angular velocity components refers to the right-hand rule of the respective axis.

Following the analysis in [11], denote $F^b = [f^b \;\; \tau^b]^T \in \mathbb{R}^6$ to be the external wrench acting on the CG of the helicopter, expressed in the body frame coordinates. The Newton-Euler equations of motion, relatively to the body fixed reference frame are given by [19]:

$$\begin{bmatrix} mI_{3\times3} & 0 \\ 0 & \mathcal{I} \end{bmatrix} \begin{bmatrix} \dot{v}^b \\ \dot{\omega}^b \end{bmatrix} + \begin{bmatrix} \omega^b \times mv^b \\ \omega^b \times \mathcal{I}\omega^b \end{bmatrix} = \begin{bmatrix} f^b \\ \tau^b \end{bmatrix} \tag{1}$$

where $\mathcal{I}$ denotes the inertia matrix of the helicopter with respect to the body fixed reference frame, and m the mass of the helicopter. The rotation matrix R is parametrized with respect to the three Euler angles yaw (ψ), pitch (θ) and roll (ϕ) and maps vectors from the the body fixed frame $\mathcal{F}_b$ to the inertia frame $\mathcal{F}_s$. The rotation matrix is produced by three consecutive rotations by the roll-pitch-yaw angles where the order of rotation is important. From standard results the rotation matrix is:

$$R = \begin{bmatrix} C_\phi C_\psi + S_\phi S_\theta S_\psi & S_\phi C_\psi - C_\phi S_\theta S_\psi & -C_\theta S_\psi \\ -S_\phi C_\theta & C_\phi C_\theta & -S_\theta \\ C_\phi S_\psi - S_\phi S_\theta C_\psi & S_\phi S_\psi + C_\phi S_\theta C_\psi & C_\theta C_\psi \end{bmatrix}$$

The orientation vector is given by $\Theta = [\psi \;\; \theta \;\; \phi]^T$ and from standard results the associated orientation dynamics are governed by $\dot{\Theta} = \Psi(\Theta)\omega^b$, where:

$$\Psi(\Theta) = \begin{bmatrix} S_\phi / C_\theta & C_\phi / C_\theta & 0 \\ C_\phi & -S_\phi & 0 \\ S_\phi T_\theta & C_\phi T_\theta & 1 \end{bmatrix}$$

The translational dynamics can also be expressed with respect to the inertia frame. Considering $p^s = [p^s_x\ p^s_y\ p^s_z]^T \in \mathbb{R}^3$ the position vector of the CG of the helicopter with respect to the spatial coordinates, and $v^s = [v^s_x\ v^s_y\ v^s_z]^T \in \mathbb{R}^3$ the linear velocity vector in spatial coordinates then the complete dynamic equations of the rigid body can be written as:

$$\dot{p}^s = v^s \tag{2}$$

$$\dot{v}^s = \frac{1}{m} R f^b \tag{3}$$

$$\dot{\omega}^b = \mathcal{I}^{-1}\left(\mathcal{I}\omega^b \times \omega^b\right) + \mathcal{I}^{-1}\tau^b \tag{4}$$

$$\dot{\Theta} = \Psi(\Theta)\omega^b \tag{5}$$

The translational dynamics are expressed in $\mathcal{F}_s$ while the rotational dynamics are derived with respect to $\mathcal{F}_b$. The advantage of working in the body fixed reference frame for the rotational motion is that the inertia matrix is constant since $\mathcal{I}' = R\mathcal{I}R^T$, where $\mathcal{I}'$ is the instantaneous inertia tensor with respect to the inertia frame.

2.2 External Wrench Model

The aerodynamic forces and moments are nonlinear functions of motion characteristics and controls. Due to the complexity and the uncertainty associated with the aerodynamic phenomena a detailed model of the external wrench would be of high order and completely impractical for control design. As indicated in [11], the main force production sources for the helicopter are the thrust vectors produced by the main and tail rotor, the damping forces of the stabilizers, the drug produced by the fuselage and the gravitational force. The main torques generation is provided by the torques due to the main rotor's gyroscopic effects and the torques produced by the forces of the main and tail rotor.

There are four control commands associated with helicopter piloting. The control input is defined as $u = [u_{col}\ u_{ped}\ u_{lon}\ u_{lat}]^T$ where u_{col} and u_{ped} are the collective control of the main and tail rotor correspondingly. The rest two control commands u_{lon}, u_{lat} are the cyclic control of the helicopter which control the inclination of the tip-path-plane (TPP) on the longitudinal and lateral direction.

Regarding the main and tail rotor force generation a simplified approach if followed that can be found in [11, 16, 17]. According to that the thrust vector produced by the rotor disk is perpendicular to the rotors disk plane. The disk plane or the rotors TPP is the plane which the tips of the blades lie and it is used to provide a simplified representation of all the rotors blade effects [16].

The main rotor blades apart from rotating about the shaft axis, they also exhibit a flapping motion normal to the plane of rotation. This flapping motion is facilitated my mechanical means (such as hinges) or by structural bending of the blade root. This flapping motion is needed to relief the large moments to the roots of the blade which are created by the generation of aerodynamic forces in the blades. Since the thrust vector is normal to the TPP, controlling the TPP the pilot indirectly controls the generated thrust. The TPP is characterized by two angles, a and b which represent the tilt of the TPP at the longitudinal and lateral axis respectively. The TPP is itself a dynamic system. The work presented in [16] and in [27] provide a simplified model of the TPP dynamics which is augmented to the rigid body model in order to conclude to what they called as the "hybrid model". The simplified TPP first order dynamic equations can be found in [5] as:

$$\tau_c \dot{a} = -a - \tau_c q + A_c u_{lon} \tag{6}$$

$$\tau_c \dot{b} = -b + \tau_c p + B_c u_{lat} \tag{7}$$

where τ_c is a time constant of the rotor dynamics which includes the effect of the stabilizer bar, and A_c, B_c are just gains. In this paper the TPP dynamics are going to be consider very fast in comparison with the rigid body dynamics and only their steady state effect will be regarded (also no angular motion is assumed). Then, regarding the TPP angles:

$$a = K_a u_{lon} \tag{8}$$

$$b = K_b u_{lat} \tag{9}$$

where K_a, K_b are constant parameters. The magnitude of the main and tail rotor thrust will be consider proportional of the cyclic controls, therefore:

$$T_M = K_M u_{col} \tag{10}$$

$$T_T = K_T u_{ped} \tag{11}$$

where T_M, T_T are the magnitude of the main and tail rotor respectively while K_M, K_T are constant parameters. As mentioned earlier the thrust vector is normal to the TPP. By simple geometry the following equations hold regarding the thrusts of the two rotors:

$$\mathbf{T}_M = \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} \cos a \ \sin\ b \\ \cos a \ \cos\ b \\ \sin a \ \cos\ b \end{bmatrix} T_M \approx \begin{bmatrix} b \\ 1 \\ a \end{bmatrix} T_M \tag{12}$$

The above equation is simplified by assuming small angle approximation ($\cos(\cdot) \approx 1$ and $\sin(\cdot) \approx (\cdot)$)for the flapping angles. The assumption of small flapping generally holds true both for miniature and full scale helicopters. For the tail rotor:

$$\mathbf{T}_T = \begin{bmatrix} X_T \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} T_T \tag{13}$$

Therefore by including the helicopters weight the complete force vector will be:

$$f^b = \begin{bmatrix} X_M + X_T \\ Y_M \\ Z_M \end{bmatrix} + R^T \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} \tag{14}$$

A common simplification practice followed in [7, 11, 13] is to neglect the effect of the lateral and longitudinal forces produced by the TPP tilt and the effect of the tail rotor thrust. In this case the only two forces applied to the helicopter are the main rotor's thrust vector at the direction $\mathbf{j}_b$ of the body frame and the weight force. Therefore:

$$f^b = \begin{bmatrix} 0 \\ T_M \\ 0 \end{bmatrix} + R^T \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} \tag{15}$$

Let $\mathbf{h}_M = [x_m\ y_m\ z_m]^T$ and $\mathbf{h}_T = [x_t\ y_t\ z_t]^T$ be the position vectors of the main and tail rotors shafts respectively (expressed in the body coordinate frame). The generated torques are the result of the above forces with moment arms $\mathbf{h}_M$,$\mathbf{h}_M$ and the rotors moments. Let $\tau^b_M = \mathbf{h}_M \times \mathbf{T}_M$ and $\tau^b_T = \mathbf{h}_T \times \mathbf{T}_T$ be the torques generated by $\mathbf{T}_M$ and $\mathbf{T}_T$ correspondingly, the complete torque vector will be:

$$\tau^b = \begin{bmatrix} R_M \\ M_M \\ N_M \end{bmatrix} + \begin{bmatrix} y_m Z_M - z_m Y_M \\ z_m X_M - x_m Z_M + z_t X_T \\ x_m Y_M - y_m X_M - x_t X_T \end{bmatrix} \tag{16}$$

The first column in the right side of the above equation includes parasitic moments associated with the rotor torque and the stiffness of the main rotor. Those moments are not going to be included in the identification model since they have secondary importance and they increase the complexity of the model. However, a description can be found in [11, 13]. Substituting Eqs. 8, 9, 10, and 11 to 16 a more compact form of the torque can be given as:

$$\tau^b = A v_c + B u_{col} \tag{17}$$

where

$$v_c = \left(u_{lon} u_{col}\ \ u_{ped}\ \ u_{lat} u_{col}\right)^T \tag{18}$$

with $A \in \mathbb{R}^{3\times3}$ and $B \in \mathbb{R}^{3\times1}$ being parameter matrices.

3 Discrete System Dynamics

The proposed identification method is designed for the nonlinear discretized equations of motion. By using Euler's implicit method for the approximation of the continuous derivatives, the following equations are obtained:

$$v^s(k+1) = v^s(k) + \alpha_1 R(k) e_2 u_{col}(k) + \alpha_2 e_2 \tag{19}$$

$$\omega^b(k+1) = \omega^b(k) + \Pi\left(\omega^b(k)\right)\mathbb{I}(\mathcal{I}, \Delta T) + A' v_c(k) + B' u_{col}(k) \tag{20}$$

where $e_2 = [0\ 1\ 0]^T$ and ΔT denotes the sampling period. In Eq. 20 $\Pi(\omega^b(k))$ is a matrix of $\mathbb{R}^{3\times p}$ composed only by nonlinear functions of the angular velocities while $\mathbb{I}(\mathcal{I}, \Delta T)$ is a vector of $\mathbb{R}^{p\times1}$ composed by inertia terms and multiplied by the sampling period ΔT. Both of them satisfy:

$$\Pi(\omega^b(k))\mathbb{I} = \Delta T \mathcal{I}^{-1}\left[\mathcal{I}\omega^b(k) \times \omega^b(k)\right] \tag{21}$$

Regarding the rest of the terms in Eqs. 19, 20 the following holds:

$$\alpha_1 = \frac{\Delta T K_M}{m} \tag{22}$$

$$\alpha_2 = -\Delta T m g \tag{23}$$

$$A' = \Delta T \mathcal{I}^{-1} A \tag{24}$$

$$B' = \Delta T \mathcal{I}^{-1} B \tag{25}$$

4 Takagi-Sugeno Fuzzy Models

This section illustrates how RLS can be used to identify the parameters of a Takagi-Sugeno fuzzy model [25] used to represent the discrete dynamics of a single state model. This approach will be modified to identify the complete helicopter dynamics. The identification of the Takagi-Sugeno system proposed in this paper is based on the method described in [20].

The Takagi-Sugeno fuzzy systems are characterized as "functional fuzzy systems" [20] since their output is a function rather than a membership function center. The fuzzy system is a static nonlinear mapping between the inputs and the outputs and they are composed by R rules of the form **If-Then**. It will be illustrated how the Takagi-Sugeno system can be used to adjust its parameters in order to provide the best estimate $\hat{y}(k+1)$ of the state $y(k)$ given the inputs to the fuzzy system $(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, the state vector $Y(k) = [y(k), y(k-1), \ldots, y(k-m)] \in \mathbb{R}^m$ and the inputs of the plant $U(k) = [u_1(k), u_2(k), \ldots, u_p(k)] \in \mathbb{R}^p$. Following similar notation of [23] the i^{th} rule of the rule base can be written as:

If $(F_{x_1}^j$ and $F_{x_2}^w$ and ... and $F_{x_n}^l)$ **Then**

$$\hat{y}_i(k+1) = \alpha_{i,1}\Delta_1(Y(k), U(k)) + \cdots + \alpha_{i,d}\Delta_d(Y(k), U(k))$$

where $\hat{y}_i(k+1)$ is the the estimate of $y(k+1)$ given by the i^{th} rule. Moreover, F_a^b is a fuzzy set defined as:

$$F_a^b := \{a, \mu_{F_a^b}(a) : a \in \mathbb{R} \text{ and } \mu_{F_a^b}(a) \in [0\ 1]\}$$

As mentioned in [20, 23] the membership function $\mu_{F_a^b}(a)$ describes the certainty that the value of a represented by the linguistic variable $\tilde{a}$ can be described by the linguistic value $\tilde{F}_a^b$. The membership functions considered in this paper are belled shaped Gaussians with or without a saturation portion. Their form can be seen in Table 1. The functions $\Delta_s(Y(k), U(k)) : \mathbb{R}^{m+p} \to \mathbb{R}$ with $s = 1, 2, \ldots, d$ are used to indicate that the parameter identification can be used for nonlinear dynamic systems which are linear in the parameters. The inference mechanism used to calculate the premise of each rule for this paper will be the dot product. Therefore, the membership function representing the premise of the above i^{th} rule will be:

$$\mu_i(x_1, x_2, \ldots, x_n) = \mu_{F_{x_1}^j}(x_1)\mu_{F_{x_2}^w}(x_2)\cdots\mu_{F_{x_n}^l}(x_n)$$

After-center average defuzzification the estimated output of the identifier will be:

$$\hat{y}(k+1) = \frac{\sum_{i=1}^R \hat{y}_i(k+1)\mu_i}{\sum_{i=1}^R \mu_i}$$

where μ_i denotes the premise of i^{th} rule $\mu_i(x_1, x_2, \ldots, x_n)$ for convenience. Let:

$$\xi_i = \frac{\mu_i}{\sum_{i=1}^R \mu_i}$$

and:

$$\xi^T(k) = \left[\Delta_1(k)\xi_1 \cdots \Delta_1(k)\xi_R \cdots \Delta_d(k)\xi_1 \cdots \Delta_d(k)\xi_R\right]$$
$$\theta^T = \left[\alpha_{1,1} \cdots \alpha_{R,1} \cdots \alpha_{1,d} \cdots \alpha_{R,d}\right]$$

Table 1 Gaussian membership functions

Left	$\mu^l(x) = \begin{cases} 1 & \text{if } x \leq c^l \\ \exp\left(-\frac{1}{2}\left(\frac{x-c^l}{\sigma^l}\right)^2\right) & \text{otherwise} \end{cases}$
Centers	$\mu(x) = \exp\left(-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2\right)$
Right	$\mu^r(x) = \begin{cases} 1 & \text{if } x \geq c^r \\ \exp\left(-\frac{1}{2}\left(\frac{x-c^r}{\sigma^r}\right)^2\right) & \text{otherwise} \end{cases}$

where $\xi(k)$ and θ are vectors of $\mathbb{R}^{Rd}$. From the above the estimated state can be written as:

$$\hat{y}(k+1) = \xi^T(k)\theta$$

By establishing the above form, an on-line parameter identification algorithm can be used to identify the θ vector. Suggestions for the on-line algorithms [20] are the RLS algorithm or the Gradient Descent method. In this paper a standard RLS algorithm will be used. The form of the RLS algorithm can be found in most textbooks related with parameter identification. From [15] the estimates of the parameter vector using RLS are provided by the following algorithm:

$$Kw(k+1) = P(k)\xi(k)[\xi^T(k)P(k)\xi(k)+1]^{-1}$$
$$P(k+1) = [I_{dR\times dR} - Kw(k+1)\xi^T(k)]P(k)$$
$$\hat{\theta}(k+1) = \hat{\theta}(k) + Kw(k+1)[y(k+1) - \xi^T(k)\hat{\theta}(k)]$$

The series of calculations for the above RLS algorithm as indicated by [15] is $P(k) \rightarrow K_w(k+1) \rightarrow P(k+1) \rightarrow \hat{\theta}(k+1)$. The initialization of the algorithm is suggested to be $P(0) = \alpha I_{dR\times dR}$ where α is a very large number and for the $\hat{\theta}(0)$ a good initial guess of the parameters or just a zero vector.

At this point it should be mentioned that the inputs to the fuzzy system $(x_1, x_2, \ldots, x_n)$ could be a subset of the state vector. In general the choice of the inputs to the fuzzy system should be descriptive values of the operational condition of the system to be identified.

5 Proposed Takagi-Sugeno System for Rotorcraft in Hovering Mode

As previously stated,the main objective of this paper is to identify a Takagi-Sugeno fuzzy system that best describes the discrete dynamic behavior of the actual helicopter. Based on the system equations presented in Eqs. 19 and 20 a Takaki-Sugeno system will be developed with the dual objective of minimal complexity and satisfactory results. The key feature is insert the terms that have a dominant effect in the rotorcraft's dynamics and at the same time exclude those that deteriorate or do not effect the identifier. Those key dynamics are obtained from the helicopters dynamic equations of linear and angular velocity by substituting the force and torque generation described in Eqs. 15 and 16 respectively.

5.1 Simplification Assumptions

After working back and forth between the system equations and the verification of the experimental results several assumptions are made in order to simplify the Takagi-Sugeno fuzzy system and at the same time provide satisfactory verification results. Those simplification assumptions are considered relative to the discrete difference Eqs. 19 and 20 of the translational and angular velocity dynamics, to which the Takagi-Sugeno fuzzy system is based on.

5.1.1 Translational Velocity Dynamics

The first simplification assumption is associated with the translational velocity dynamics provided by Eq. 15. Neglecting of those secondary forces produced by the tilt of the TPP is suggested by [3, 7, 11–13] to facilitate the control design. Neglecting those forces does not effect the identification procedure since the main rotor's thrust is the dominant producting force of the vehicle while those secondary forces are of parasitic nature.

5.1.2 Angular Velocity Dynamics

The translation velocity dynamics are straightforward based on Eq. 19. The actual interest and complications are associated with the identification of the angular velocity dynamics. Symmetry to the principal axes is assumed. This assumption simplifies significantly the angular velocity dynamics. Therefore $\Pi(\omega^b(k)) = diag\,(r(k)p(k), q(k)p(k), q(k)r(k))$ and $\mathbb{I}(\mathcal{I}, \Delta T) = (\mathbb{I}_1\ \mathbb{I}_2\ \mathbb{I}_3)$. The second simplification assumes that the position vectors $\mathbf{h}_M$ and $\mathbf{h}_T$ are aligned with the unitary vectors $\mathbf{j}_b$ and $\mathbf{k}_b$ respectively. Therefore, $\mathbf{h}_M = \left[0\ y_m\ 0\right]^T$ and $\mathbf{h}_T = [0\ 0\ z_t]^T$. Let $\gamma = (\gamma_1\ \gamma_2\ \gamma_3)$ be the parameters associated with the control commands. Incorporating the above simplification assumptions to the angular velocity discrete dynamics the following equations are provided:

$$\begin{aligned} q(k+1) &= q(k) + \mathbb{I}_1 r(k)p(k) + \gamma_1 u_{lon}(k)u_{col}(k) \\ r(k+1) &= r(k) + \mathbb{I}_2 q(k)p(k) + \gamma_2 u_{ped}(k) \\ p(k+1) &= p(k) + \mathbb{I}_3 q(k)r(k) + \gamma_3 u_{lat}(k)u_{col}(k) \end{aligned} \tag{26}$$

5.2 Takagi-Sugeno Fuzzy System

As indicated by Eq. 3 the velocity dynamics depend on the orientation of the helicopter and the force vector. The proposed Takagi-Sugeno system representing the translational dynamics will have as input the translational velocity vector $v^s(k)$. Let the system be composed by R_1^1 fuzzy rules then the i^{th} will be:

If $(F^j_{v^s_x}$ and $F^w_{v^s_y}$ and $F^\epsilon_{v^s_z})$ **Then**

$$\begin{aligned} \hat{v}^s_x(k+1)_i &= v^s_x(k) + \alpha^i_1 \left[\sin\phi(k)\cos\psi(k) - \cos\phi(k)\sin\theta(k)\sin\psi(k)\right] u_{col}(k) \\ \hat{v}^s_y(k+1)_i &= v^s_y(k) + \alpha^i_1 \left[\cos\phi(k)\cos\theta(k)\right] u_{col}(k) + \alpha^i_2 \\ \hat{v}^s_z(k+1)_i &= v^s_z(k) + \alpha^i_1 \left[\sin\phi(k)\sin\psi(k) + \cos\phi(k)\sin\theta(k)\cos\psi(k)\right] u_{col}(k) \end{aligned} \tag{27}$$

where $F^{j}_{\tilde{v}^s_x}$, $F^{w}_{\tilde{v}^s_y}$ and $F^{\epsilon}_{\tilde{v}^s_z}$ are fuzzy sets representing the linguistic values of the linguistic variables $\tilde{v}^s_x$, $\tilde{v}^s_y$ and $\tilde{v}^s_z$. For the angular velocities, lets assume that the fuzzy systems representing the outputs $\hat{q}$,$\hat{r}$ and $\hat{p}$ are composed by R^1_2, R^2_2 and R^3_2 rules correspondingly with the i^{th} rule for each output being:

If (F^{ϵ}_q and F^{γ}_r and F^{λ}_p) **Then**

$$\hat{q}(k+1)_i = q(k) + \mathbb{I}^i_1 r(k)p(k) + \gamma^i_1 u_{lon}(k)u_{col}(k) \tag{28}$$

If (F^{ν}_q and F^{π}_r and F^{ρ}_p) **Then**

$$\hat{r}(k+1)_i = r(k) + \mathbb{I}^i_2 q(k)p(k) + \gamma^i_2 u_{ped}(k) \tag{29}$$

If (F^{σ}_q and F^{τ}_r and F^{ξ}_p) **Then**

$$\hat{p}(k+1)_i = p(k) + \mathbb{I}^i_3 q(k)r(k) + \gamma^i_3 u_{lat}(k)u_{col}(k) \tag{30}$$

where $F^{(\cdot)}_q$, $F^{(\cdot)}_r$ and $F^{(\cdot)}_p$ are fuzzy sets representing the linguistic values of the linguistic variables $\tilde{q}$, $\tilde{r}$ and $\tilde{p}$ respectively. In the case of the angular velocity dynamics three independent fuzzy models are considered (one for each state). The angular velocity dynamics are affected by greater parameter and model uncertainty in contrast with the translational velocity dynamics. By considering each state as an individual Many-Input Single-Output fuzzy system, will provide greater design flexibility during the identification procedure since there will be more tunning membership functions. The parameters of the fuzzy system are unknown. The RLS algorithm can be used so the above equation in order to provide an estimate of the Takagi-Sugeno parameters at each time step that a new measurement is available.

6 Time History Data and Excitation Inputs

One issue of primary concern is the design of the excitation inputs used to collect experimental data. The quality of the experimental data is crucial to the final outcome of the identifier. The excitation signal must be capable of exciting the system modes that are needed to appear in the identified model.

A description of several excitation signals specially designed for aircraft identification can be found in [10]. In this paper the excitation that was used is the frequency sweep, suggested in [28]. The frequency sweep is a sinusoidal signal with variable frequency. More specifically, the frequency of the signal increases logarithmically over time. Frequency sweeps are commonly used in frequency identification techniques where the model is identified within specific frequency bands. Results have shown that frequency sweeps are adequate for the proposed time identification technique.

Frequency sweeps are not required to have a constant amplitude. The symmetry of those signals allows the rotorcraft to sustain its position around a certain operating condition. One of the problems encountered by the frequency sweeps is the excitation of the rotorcraft by the low frequency portion of the signal. For those longer periods the helicopter might be drifted away from the desired operating condition until the symmetrical control is applied. To this extend for the low frequencies the amplitude of the signal should be significantly minimized.

When the frequency sweep is applied to one of the helicopter's control commands the rest controls should be implemented in such a way to keep the helicopter around the reference flight condition. At the system identification procedure sweep data

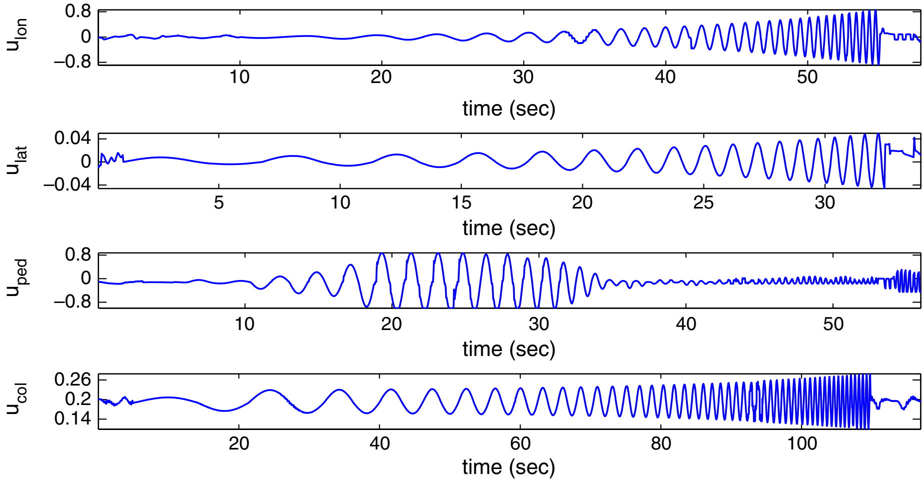

Fig. 1 Excitation signals used to produce the identification data. The control values lie in the interval [−1 1]

collected by several maneuvers can be concatenated so it is very important that the data start and end at the trim condition. A 3 sec period in trim at the beginning and at the end is suggested.

The design of the frequency sweeps require to determine a priori the frequency bandwidth. In general, a suggested bandwidth for rotorcraft identification lies between 0.3-12 rad/sec [28]. The recorded length of the data for each sweep following a rule of thump should be four to five times the period that corresponds to the minimum frequency. Let $[\omega_{min}\ \omega_{max}]$ be the desired frequency interval that the excitation signal should contain. Then, the period that corresponds to the smallest frequency will be $T_{max} = 2\pi/\omega_{min}$. The suggested recorded length should be $T_{rec} \geq 4T_{max}$. The proposed excitation signal is given by $u = A \sin[f(t)]$ where A is the amplitude of the signal and:

$$K(t) = C_2[exp(C_1 t/T_{rec}) - 1] \tag{31}$$

$$v(t) = \omega_{min} + K(t)(\omega_{max} - \omega_{min}) \tag{32}$$

$$f(t) = \int_0^{T_{rec}} v(t)dt \tag{33}$$

From [28] the proposed parameters of Eq. 31 are $C_1 = 4.0$ and $C_2 = 0.0187$. Examples of excitation sweep for each control command can be seen in Fig. 1. Indicative values of the frequency parameters and the recorded length of the excitation of each control, used in this experiment can be seen in Table 2.

Table 2 Parameters used configuring the excitation signals

	ω_{min} (rad/s)	ω_{max} (rad/s)	T_{rec} (s)
u_{lon}	1	15	45
u_{lat}	1	12	31
u_{ped}	2	23	50
u_{col}	0.3	12	105

Apart from the pedal control u_{ped} the amplitude of the excitations is adjusted in such a manner that the helicopter will not drift away significantly from the hover trimmed operation. The pedal control which essentially controls the yaw of the helicopter was a special case. Since the model helicopter installed in *X-Plane*© does not include a yaw dumper or a gyro, the behavior of the helicopters heading was much more sensitive than the one accounted in actual miniature rotorcraft. The design of the excitation signal was much more challenging than the rest of the controls since for the long period of the sweep the yaw velocity increases significantly. The excitation signal applied was based on the frequency sweeps and at the beginning of each sinusoidal waiving the amplitude was determined to preserve the yaw velocity between some bounds.

7 Simulation and Verification

7.1 Simulation

Experimentation has been done using the *X-Plane*© a realistic and powerful flight simulator. *X-Plane*© apart from simulating flights, also provides a plethora of flight data which were used identification purposes. The helicopter used for the experiments was designed in *X-Plane*© in such a way that the behavior of the latter will resemble the behavior of an actual miniature model. However in the software model the heading velocity presents some additional sensitivity from actual helicopters and therefore it requires high frequency control to adjust it in the trim position. This sensitivity in the yaw is resulted from the fact that the software model does not include a gyro mechanism which inserts additional feedback control for controlling the heading.

The control excitations was generated from code designed in *SIMULINK*©. The communication between *SIMULINK*© and *X-Plane*© took place through a User Datagram Protocol (UDP) connection. The communication of the two software packages is based on the work presented in [4]. The sampling rate is variable with an average value of $50Hz$.

The excitation signals initialized when the helicopter was in hover operation mode and after their effect the helicopter was set back to hover again.

7.2 Tunning of the Membership Functions Parameters

The centers and the spreads of the Gaussian membership functions of the helicopter's Takagi-Sugenano fuzzy system, described by Eqs. 27, 28–30, are given in Table 3. The $(*)$ symbol indicates that the specific linguistic variable does not participate in the rule base. The choice of these parameters has been based on intuitive criteria rather than an optimizing method over the training set. The main idea is that the linguistic values corresponding to hover operation should have a wide spread in order to dominate over the linguistic variables that correspond to other flight operations. The left and right membership functions are used as supportive means to describe the behavior of the system when the helicopter operates outside the bounds of the hover mode. Instead of this intuitive approach there are many optimizing methods to determine the membership function parameters over the training set. A gradient

Table 3 Gaussian centers and spreads

Output	Linguistic	Left		Centers		Right	
	Variables	c^l	σ^l	c	σ	c^r	σ^r
$\hat{v}^s$	$\tilde{v}^s_x$	-0.5	0.01	0	1	0.5	0.01
	$\tilde{v}^s_y$	-1	0.03	0	3	1	0.03
	$\tilde{v}^s_z$	-1	0.3	0	0.3	1	0.3
$\hat{q}$	$\tilde{q}$	-1.5	0.01	0	6	1.5	0.01
	$\tilde{r}$	-4	0.01	0	8	4	0.01
	$\tilde{p}$	-0.5	1	$*$	$*$	0.5	1
$\hat{r}$	$\tilde{q}$	$*$	$*$	$*$	$*$	$*$	$*$
	$\tilde{r}$	-0.5	0.01	0	8	0.5	0.01
	$\tilde{p}$	-1.5	0.03	0	6	1.5	0.03
$\hat{p}$	$\tilde{q}$	-2	0.03	0	6	2	0.03
	$\tilde{r}$	-0.5	0.01	0	8	0.5	0.01
	$\tilde{p}$	$*$	$*$	$*$	$*$	$*$	$*$

descent tuning method for determining the membership function parameters, is given in [20], however gradient descent should be used to tune the fuzzy model parameters as well. More advance methods for updating the rule base and the parameters of the fuzzy system, by supervised and unsupervised learning, is presented in [1]

7.3 Verification

In order to verify the model, the actual helicopter is set to hover mode. The applied control commands are periodically perturbing the helicopter to a new hover state until a new excitation occurs. Those excitations take place for all the control inputs.

The comparison between the actual and estimated translational and rotational velocities can be seen in Figs. 2 and 3 correspondingly. The estimated error for

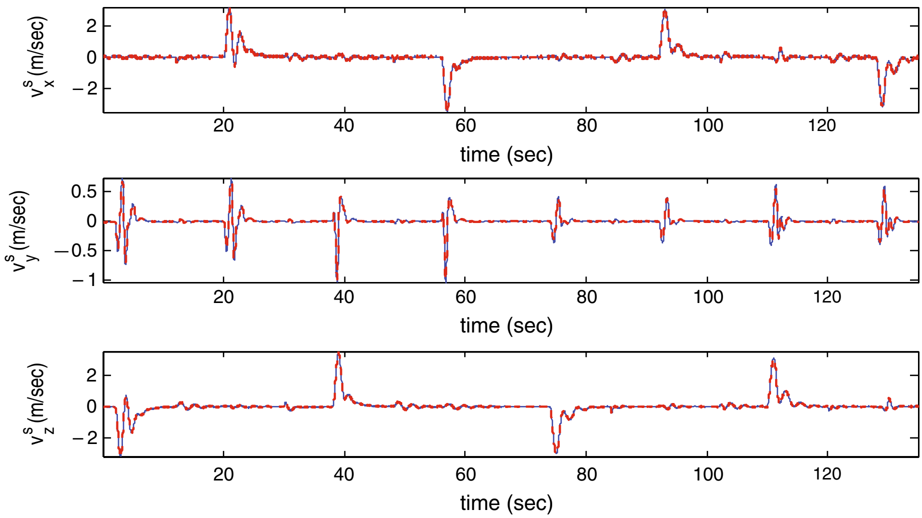

Fig. 2 Comparison between the actual (*solid line*) and estimated (*dotted line*) linear velocities using the verification data

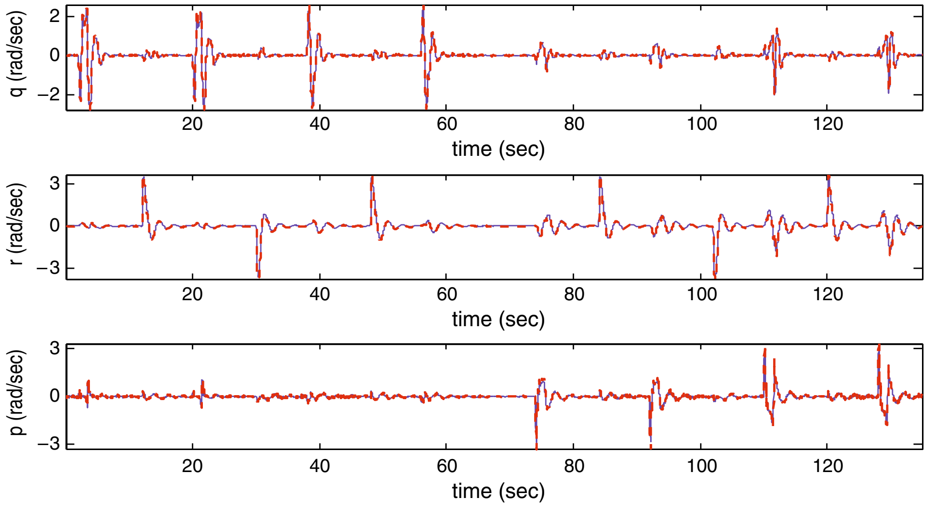

Fig. 3 Comparison between the actual (*solid line*) and estimated (*dotted line*) angular velocities using the verification data

those are illustrated in Figs. 4 and 5. The mean error over the identification data is illustrated in Table 4. The same table presents the mean error of the RLS identification procedure using the straight forward model of Eqs. 19, 20 instead of a Takagi-Sugeno fuzzy model. The fuzzy model has a significant improvement in the angular velocity dynamics, which are the biggest identification challenge. The verification results show the success of the approach since the associated error are small and bounded even in the case of high excitations. Based on the data it can

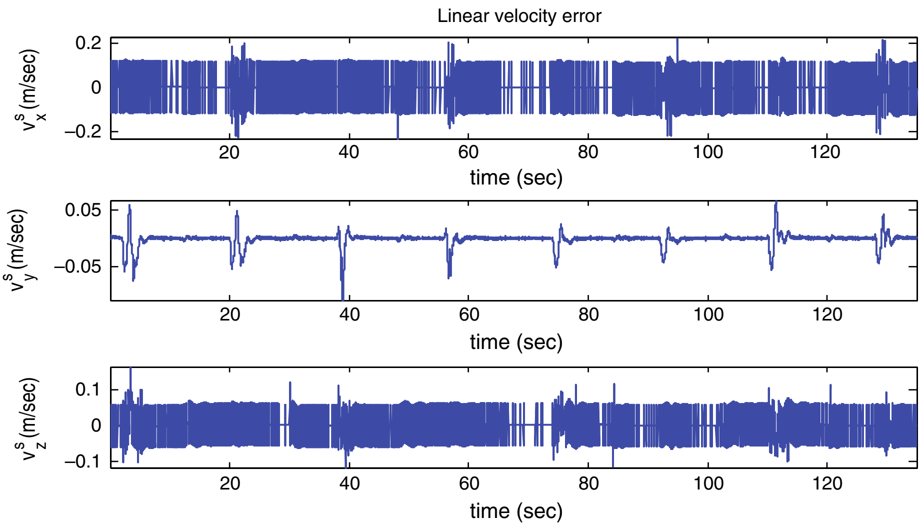

Fig. 4 Errors between the actual and estimated linear velocities using the verification data

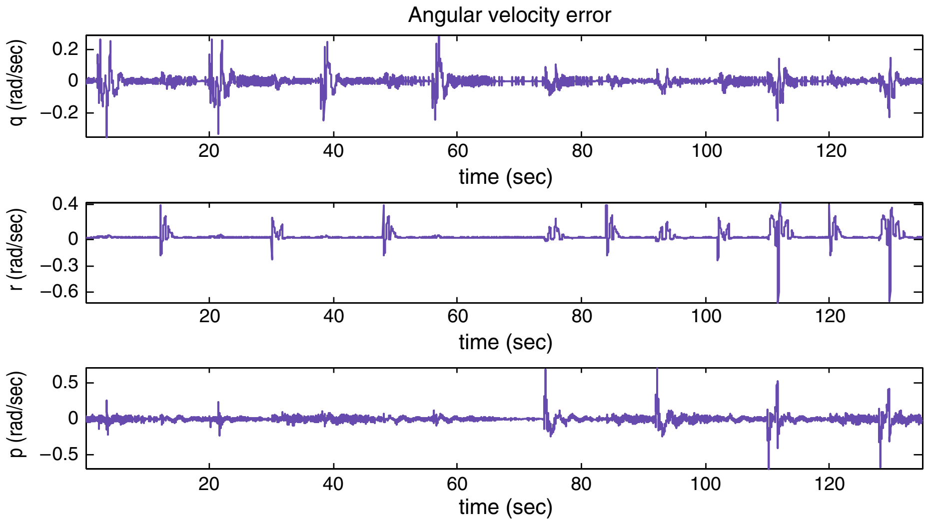

Fig. 5 Errors between the actual and estimated angular velocities using the verification data

be seen that the model also provides sufficient estimates for large variations in the velocities.

8 Conclusions and Future Work

This paper presents a time domain identification method for a miniature helicopter at hover. One of the objectives was to investigate whether a time domain approach can be successful by using the same excitation signal that are commonly used in frequency domain estimation approaches. The proposed Takagi-Sugeno model was proven to be capable of encapsulating the dynamic behavior of the helicopter at hover. Furthermore, the verification results show that the identified model can also capture the behavior of the rotorcraft for significant variations from the operating point. This approach was very promising for providing a more global model of the helicopter just by using excitation data in the hover mode. The computation time and the complexity were not an issue in the design of the identifier and could easily compete with frequency domain approaches. The produced model is a relative simple nonlinear discrete system, which facilitates the control design. More specifically the

Table 4 Mean error of the Takagi-Sugeno RLS in comparison with RLS identification over the verification data

State estimate		Mean error		Improvement
		Fuzzy RLS	RLS	
$\tilde{v}_x^s$	m/s	0.0456	0.0457	0.2%
$\tilde{v}_y^s$	m/s	0.0049	0.0052	5.7%
$\tilde{v}_z^s$	m/s	0.0253	0.0255	0.7%
$\tilde{q}$	deg/s	1.0432	1.2050	13.4%
$\tilde{r}$	deg/s	2.2671	4.0852	43.7%
$\tilde{p}$	deg/s	1.5554	1.8629	16.5%

fuzzy system succeed to produce an interpolator between systems that represent the helicopter dynamics in hovering mode. By providing a rich excitation in the identification step the model will be able to include sufficient information for a wider range of operation of the flight envelope.

Future work involves the design of a fuzzy controller to stabilize the identified model in trimmed flight conditions such as hovering and cruising in low velocities. For the type of system that was identified in this paper an interesting approach would be the implementation of a the Parallel Distributed Compensator [20]. The success of the model based control design can be reasonably expected due to the good fidelity of the identified model.

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