J Intell Robot Syst (2006) 45: 369–394
DOI 10.1007/s10846-006-9050-8

Decentralized Control of Autonomous Swarm Systems Using Artificial Potential Functions: Analytical Design Guidelines

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Received: 20 August 2004 / Accepted: 2 May 2006

Abstract This paper presents a framework for decentralized control of self-organizing swarm systems based on the artificial potential functions (APFs). In this scheme, multiple agents in a swarm self-organize to flock and achieve formation control through attractive and repulsive forces among themselves using APFs. In particular, this paper presents a set of analytical guidelines for designing potential functions to avoid local minima for a number of representative scenarios. Specifically the following cases are addressed: 1) A non-reachable goal problem (a case that the potential of the goal is overwhelmed by the potential of an obstacle, 2) an obstacle collision problem (a case that the potential of the obstacle is overwhelmed by the potential of the goal), 3) an obstacle collision problem in swarm (a case that the potential of the obstacle is overwhelmed by potential of other robots in a group formation) and 4) an inter-robot collision problem (a case that the potential of the

Category (5) – Intelligent Systems/Intelligent Control/Fuzzy Control/Prosthetics/
Robot Motion Planning

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robot in a formation is overwhelmed by potential of the goal). The simulation results showed that the proposed scheme can effectively construct a self-organized swarm system with the capability of group formation, navigation and migration in the presence of obstacles.

Key words group behavior · potential functions · swarm systems

1. Introduction

Potential field methods have been studied extensively for path planning of autonomous mobile robot in the past decade [3, 7, 9, 14, 17]. In this method, a robot is modeled as a moving particle inside an artificial potential field that is generated by superposing an attractive potential that pulls the robot to a goal configuration and a repulsive potential that pushes robot away from obstacles. The negative gradient of the generated global potential field is interpreted as an artificial force acting on the robot and dictating its motion. In [5], it was reported that the usage of a sequence of basic behavior such as random wandering, obstacle avoidance and light following was able to direct a single robot to achieve complicated behavior by using the potential function field generated from coupled oscillators. However, these behavior-based computational approaches are often limited in fundamental theoretical understanding and as a result sometimes exhibit unpredicted and undesirable performances. Moreover, these approaches need extensive training for the selection of proper parameter values for different working environments [5]. Since none of these approaches can present a general solution to the problem of designing cooperative mobile agents, it makes sense to combine these schemes with certain trade-offs so as to render a hierarchical architecture and a multi-strategy adaptive approach for swarm systems of inhomogeneous mobile agents.

More recently some of the studies have extended potential field methods to the maneuvering of group behaviors such as formation, migration and obstacle avoidance in distributed swarm systems consisting of a large number of autonomous agents [2]. A fundamental problem in the application of potential field method is how to deal with the local minima that may occur in a potential field environment. In this aspect, how to select scaling parameters in APFs representing the sizes of attractive and repulsive forces to avoid the local minima has remained a challenge. Many of the potential field-based methods are heuristically oriented and the lack of analytical design guidelines can be problematic in applications. For instance, if the attractive and repulsive potentials are defined as commonly used, the repulsive force will be much larger than the attractive one [4, 11]. In this case, if the goal is near the obstacle, the robot cannot reach the goal due to the larger repulsive force coming from the obstacle, i.e. the goal position is not the global minimum of the total potential. One approach to provide a solution to this problem is to increase the ratio of scaling parameters (attractive force *versus* repulsive one) beyond a certain threshold. However, in doing so, it will cause problem for the situation that the obstacle lies between the robot and the goal, i.e., the robot may collide with the obstacle due to the large attractive force of the nearby goal [9]. Therefore, the ratio of scaling parameters has to be between a lower bound and an upper bound. In this paper, we present a set of analytical design guidelines of this nature for representative scenarios.

Some of the more challenging scenarios have to do with multi-agent swarm systems in which the suitable scaling parameters for group formation are required in addition to the ratios of scaling parameters for group migration and obstacle avoidance. In the absence of judiciously chosen design parameters, collision may occur among the agents as well as between the agents and the obstacles. Two representative scenarios are that 1) a case that the potential of the obstacle is overwhelmed by potential of other robots in a group formation and 2) a case that the potential of the robot in a formation is overwhelmed by the potential of the goal. In these cases, the robotic agents collides with each other and run into the obstacle and there has been few analytical studies on these problems. There are, however, increased number of studies on stability analysis of swarms [3, 7]. The focus of these studies is on collective convergence and its bound. In most of these studies, there are some prevailing assumptions, for example, the goal position is set relatively far away from obstacles.

In this paper, we present a framework for decentralized control of self-organizing multi-agent swarm systems based on the APFs. Our framework for APFs alleviates the above assumptions by the help of multiplicative and additive configuration between APF for group migration and APF for obstacle avoidance. The goal is not to tackle all possible local minima and collision problems in APF configurations, instead we focus our attention on a set of analytical guidelines for designing APFs for a number of representative scenarios in swarm systems. The framework enables agents to maintain a flexible formation, while migrating as a group and avoiding any obstacles. Different from previous studies on target-following strategies [13, 20], the purpose of this study is to explore the global behaviors such as group formation and obstacle avoidance as well as group migration to target by using APFs for a number of representative scenarios. Specifically the following cases are addressed: 1) A case that the potential of the goal is overwhelmed by the potential of an obstacle, 2) a case that the potential of the obstacle is overwhelmed by the potential of the goal, 3) a case that the potential of the obstacle is overwhelmed by potential of other robots in a group formation and 4) a case that the potential of the robot in a formation is overwhelmed by potential of the goal.

This paper is organized as follows. Section 2 discusses the environment and agent model and introduces the problem statement. In Section 3, we study a progressive sequence of scenarios involving designing potential force laws to maintain group migration and avoid obstacles. Also, our path planning method is compared with a conventional additive configuration of potential functions. Section 4 describes an extension to the group formation, and design guideline for its corresponding scaling parameters are proposed. In Section 5 prey-pursuit mission simulations using the proposed method are carried out. Finally concluding remarks are collected in Section 6.

2. Swarm Model, Notation and Problem Statement

2.1. Environment and Agent Model

The formation and maintenance of coherent group movement has long been studied in natural systems, and more recently efforts have been made to reproduce this type of behavior in artificial systems. There has been extensively simulation studies [16] that has led to successful synthesis of birds' behaviors such as collision avoidance, velocity matching and flock centering. For instance, it is learned that to avoid collision with other birds and obstacles, a bird uses a steer-to-avoid rule. However, theoretical treatment or analysis of flocking behavior was not presented. Rather computer models of coordinated animal motion such as bird flocks and fish schools for simulating visually satisfying flocking and schooling behaviors were developed for the animation industry. The models were based on three dimensional computational geometry of the sort normally used in computer animation or computer aided design. The generic simulated flocking creatures are referred as boids. Other experiments by the author of [16] involved evolving groups of artificial creatures. In [15] it studied the evolving control system of a group of creatures placed in an environment with static obstacles and a manually programmed predator for the ability to avoid obstacles and predation. Though the results described in the paper were rather preliminary, evidences indicate that coordinated motion strategies began to emerge.

The phenomena of swarming in nature have inspired the interest to engineer large-scale artificial swarms. A typical artificial swarm system is a large-scale fleet of cooperative robots. Each robot in such a robotic swarm can be viewed as an agent. The omni-directional robot without nonholonomic constraints can be one of such prototype agent model [8, 18]. They will likely possess only basic capabilities and mission specific sensors. Direct communication between agents may or may not exist. The environment model is very 'object-oriented' in its approach to agent construction. Sensors and behaviors are encapsulated when possible. This approach allows individual components to be added and/or removed from the model as if the corresponding physical components were being added to or removed from a real agent. We restrict the workspace to two-dimensional space where each agent moves in $[xy]$ plane. A conceptional figure of distributed swarm agents in two-dimensional potential fields is shown in Figure 1. The proposed method can be extended to more complex three-dimensional space.

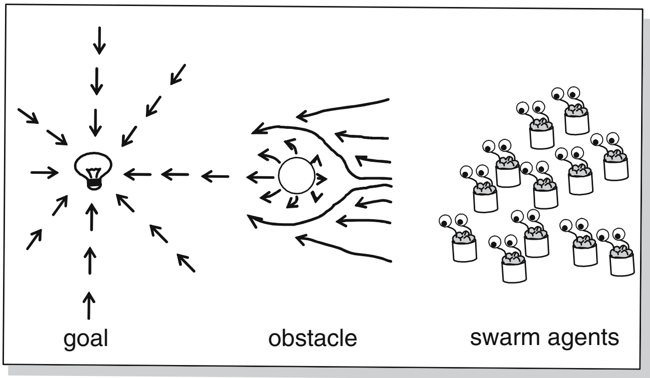


Figure 1 Distributed swarm agents in two-dimensional potential fields.

A swarm system may consist of anywhere from two to hundreds or more autonomous robots. The costs of production of robots are going down, and the robots are getting more capable in compact packages. Hence in the near future, many industrial and military applications of swarm systems in tasks such as hazardous inspection, patrolling, guarding and attacking are envisioned. In this paper, the model of a swarm agent is constructed by building upon an autonomous agent object. In abstract programming terms it may also be thought of as an object with some general capabilities. The basic agent possesses only locomotion as an innate capability. Neighbor position information may be used for group behaviors such as flocking and migration.

2.2. Notations

In this section, we introduce the notations used in this paper.

ψ	Relative position vector
U	Potential function
F	Force corresponding to potential function
c	Strength distance for exponential function
l	Correlation distance in exponential function
d	Positive constant for distance

Superscripts

g	Group migration
o	Obstacle avoidance
og	General configuration for group migration and obstacle avoidance
ogg	Proposed configuration for group migration and obstacle avoidance
f	Group formation
$oggf$	Proposed configuration for group formation, migration and obstacle avoidance

go	Relation between a goal and an obstacle
ro	Relation between a robot and an obstacle

Subscripts

i	Individual agent index
j	Obstacle index
k	Other individual agent index
$1x$	First index in x-axis
$1y$	First index in y-axis
m	Minimum value
g	Group migration
o	Obstacle avoidance
0	Zero total force
r	Repulsion between two agents
a	Attraction between two agents
f	Group formation

2.3. Problem Statement

We start with a point mass model in which an individual agent's motion is governed by Newton's law $m_i a_i = F_i$ where the subscript i denotes the ith agent, and m_i, a_i, and F_i are the mass and acceleration of the agent and the force acting on the agent, respectively. This gives rise to the following equations of motion

$$\dot{\mathbf{P}}_i = \mathbf{v}_i$$
$$m_i \dot{\mathbf{v}}_i = u_i \tag{1}$$

where $\mathbf{P}_i$ and $\mathbf{v}_i$ are the position and velocity of the ith agent, respectively. $u_i = F_i$ is total force acting on individual agent.

Now, suppose there is a velocity damping term of the form $-k_v \mathbf{v}_i$ in u_i, where $k_v > 0$. In other words, assume that we have

$$u_i = -k_v \mathbf{v}_i + \bar{u}_i. \tag{2}$$

$\bar{u}_i$ is the output of controller and described by

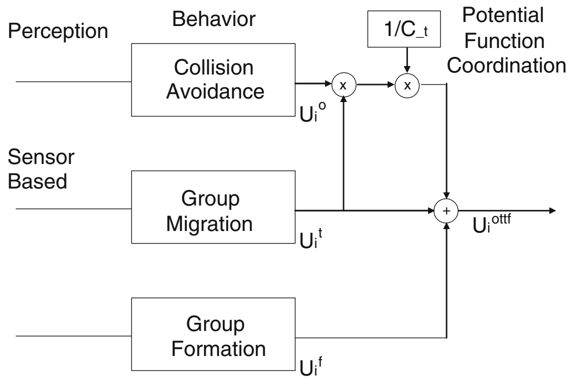
$$\bar{u}_i = -\nabla U_i \tag{3}$$

where U_i is the artificial potential energy in the system and is given by Section 3.

Now, note that for organism such as bacteria we have m_i very small (i.e. we have $m_i \approx 0$) and the viscosity of the environment for them is high. Therefore, we can take $m_i = 0$. Substituting this in the above system of equations we obtain

$$\dot{\mathbf{P}}_i = -\frac{1}{k_v} \nabla U_i. \tag{4}$$

Figure 2 Behavior architecture.



If we consider in the article with $k_v = 1$, we have the equation of motion of each individual i described by

$$\dot{\mathbf{P}}_i = -\nabla U_i. \tag{5}$$

Each of the individuals in the swarm moves so as to minimize the total artificial potential energy in the system. For a planar formation on multiple vehicles, similar dynamics are used in [12].

Once a set of individual behaviors has been developed, a framework or architecture must be constructed to initiate behavioral responses and coordinate multiple behaviors. The behavior of the swarm system in the proposed algorithm is largely divided into three parts: Group migration, collision avoidance and group formation as shown in Figure 2. We deal with global behaviors, not separate behaviors by subsumption coordination based on priority in [6]. We describe several artificial potential field techniques satisfying such behaviors. Path planning using artificial potential fields is based on an intuitive analogy. The robot is treated as a particle acting under the influence of a potential field U, which is modulated to represent the structure of free space [10]. Typically, obstacles are modeled as carrying electrical charges, and the resulting potential field is used to represent the free space. In this paper, localized distributed controls based on APFs are utilized throughout group behaviors such as group migration, formation and obstacle avoidance.

3. Path Planning

In this section, a self-organized swarm system controlled by the APFs is presented for the group migration and obstacle avoidance. The behavior of migration in this study is distinct from that of formation control (e.g. [1]), because the goal of migration issimply to achieve and maintain coherent group movement rather than to govern well organized inter-agent position relationships. Also, formation control is not an end in itself, but rather can be used as a component of a multi-agent system, organizing the nodes of a distributed sensing system. The formulation of the APFs is extended to group formation in Section 4.

3.1. APFs for Group Migration and Obstacle Avoidance

Before we describe artificial potential fields, relative position vectors between the robots and the goal are defined as

$$\psi_i^g = \mathbf{P}_i - \mathbf{P}_{goal} \tag{6}$$

where $\mathbf{P}_{goal}$ is the goal position.

This relative position vector physically means that the formation is independent of the absolute position of the group. That is why each robot controls its position based on its relative position to others and it never has any reference point in its working environment.

Attraction towards the goal is modeled by attractive fields, which in the absence of obstacles, draws the charged robot towards the goal. The simple APF for group migration is modeled as following.

$$U_i^g = c_g \left(1 - \; e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} \right) \tag{7}$$

where c_g and l_g are the strength and correlation distance for group migration. The second term c_g in the right side of (7) acts to make U_i^g zero when ψ_i^g=0.

Its corresponding force is then given by the negative gradient of (7).

$$F_i^g = -\nabla U_i^g = -\frac{2c_g \psi_i^g}{l_g{}^2} e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}}. \tag{8}$$

Relative position vectors between the robots and the obstacles are defined as

$$\psi_j^o = \mathbf{P}_i - \mathbf{O}_j \tag{9}$$

where $\mathbf{O}_j$ is the position of obstacle j which is a neighbor of agent i.

Collisions between the obstacles and the robot are avoided by the repulsive force between them, which is simply the negative gradient of the potential field. We employ the agorithm that prevents collisions with obstacles by calculating the repulsive potential, based on the shortest to an object. The simple APF for obstacle avoidance is modeled as following.

$$U_i^o = \sum_{j \in \mathcal{N}_{oi}} \left\{ c_o e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \tag{10}$$

where c_o and l_o are the strength and correlation distance for obstacle avoidance. $\mathcal{N}_{oi}$ denotes the set of labels of those obstacles which are neighbors of agent i.

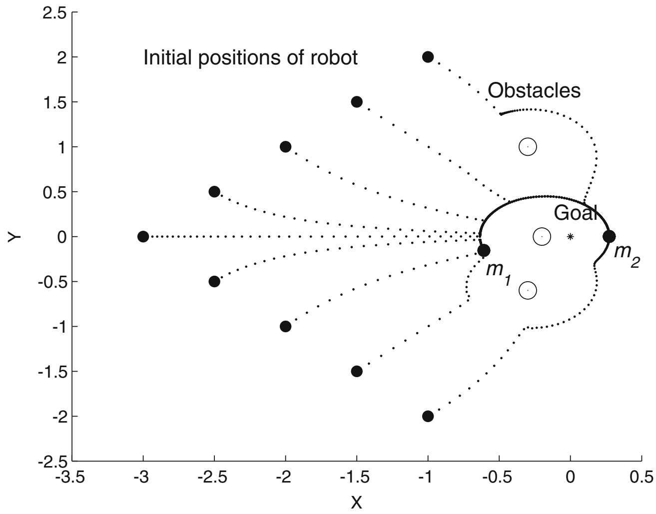


Figure 3 Group migration to a goal near obstacles using Equation (13).

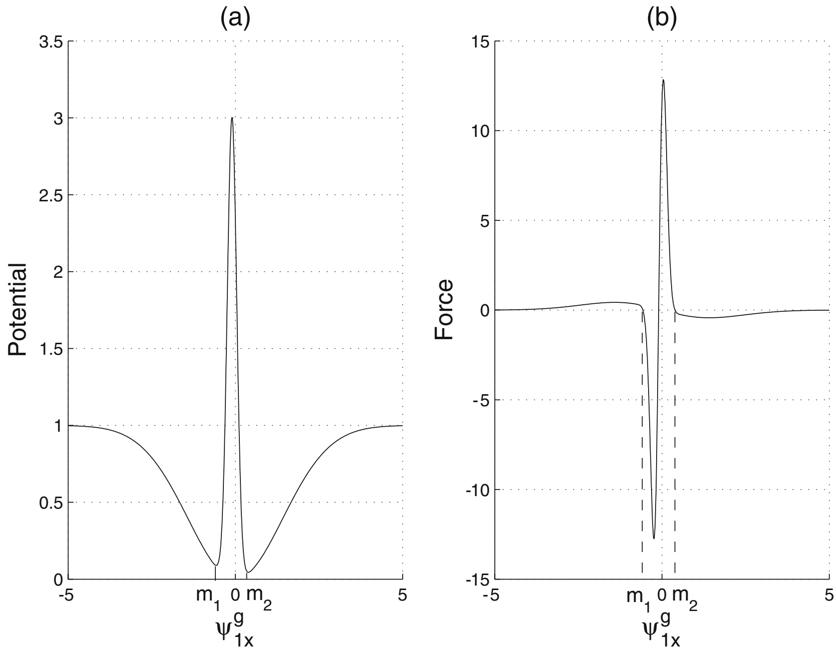


Figure 4 Potential and force for group migration to a goal near an obstacle using Equations (12) and (13), respectively, in a 1-D space.

Its corresponding force is then given by the negative gradient of (10).

$$F_i^o = -\nabla U_i^o = \sum_{j \in \mathcal{N}_{oi}} \left\{ \frac{2c_o \psi_j^o}{l_o{}^2} e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\}. \tag{11}$$

3.2. Total APFs for Path Planning

The total potential of conventional configuration that the potential for group migration and the potential for obstacle avoidance are combined together has an additive structure as following.

$$\begin{aligned} U_i^{og} &= U_i^o + U_i^g \\ &= \sum_{j \in \mathcal{N}_{oi}} \left\{ c_o e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} - c_g e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} + c_g. \end{aligned} \tag{12}$$

Its corresponding force is

$$\begin{aligned} F_i^{og} &= -\nabla U_i^o - \nabla U_i^g \\ &= \sum_{j \in \mathcal{N}_{oi}} \left\{ \frac{2c_o \psi_j^o}{l_o{}^2} e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} - \frac{2c_g \psi_i^g}{l_g{}^2} e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}}. \end{aligned} \tag{13}$$

If the above potential and force are used, each agent has common problems [11] such as a narrow passage between closely spaced obstacles and a non-reachable goal with obstacles nearby. Figure 3 shows such an example where $c_g = 1, l_g = 2, c_o = 3$, and $l_o = 0.2$ are used. Figure 4 illustrates its corresponding potential and force, respectively, when goal (0,0) is near the obstacle (−0.1,0). The potential plot in Figure 4 reveals two local minima as shown in Figure 3 whose cause is described in Section 3.2.1. One is due to a non-reachable goal problem with obstacles nearby, and the other is due to a narrow passage between closely spaced two obstacles.

For this reason, following configuration for total potential is proposed to overcome such local minimum problems. The total potential has a multiplicative and additive structure between the potential for group migration and the potential for obstacle avoidance.

$$\begin{aligned} U_i^{ogg} &= \frac{1}{c_g} U_i^o \cdot U_i^g + U_i^g \\ &= \sum_{j \in \mathcal{N}_{oi}} \left\{ c_o e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \left(1 - e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} \right) - c_g e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} + c_g. \end{aligned} \tag{14}$$

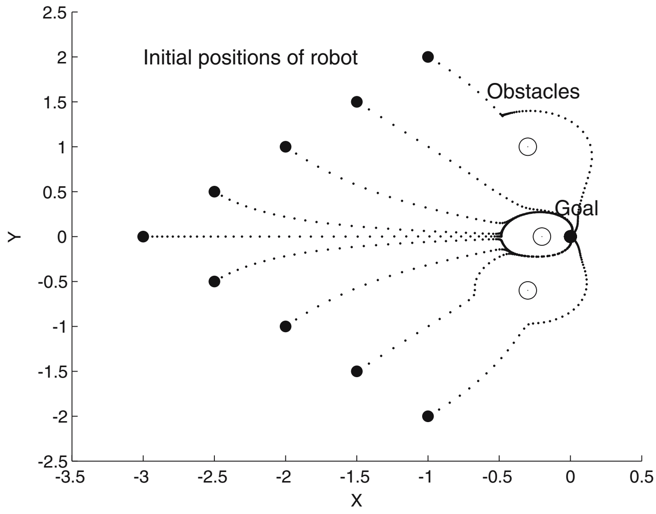


Figure 5 Group migration to a goal near obstacles using Equation (15).

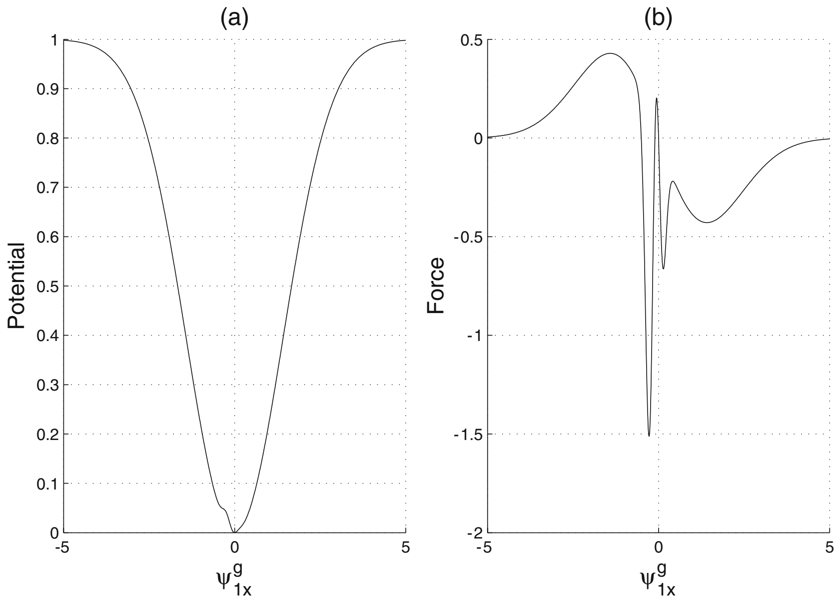


Figure 6 Potential and force for group migration to a goal near an obstacle using Equations (14) and (15), respectively, in a 1-D space.

Its corresponding force is

$$\begin{aligned} F_i^{ogg} &= -\nabla U_i^{ogg} \\ &= \sum_{j \in \mathcal{N}_{oi}} \left\{ \frac{2c_o \psi_j^o}{l_o{}^2} e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \left(1 - e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} \right) \\ &\quad + \sum_{j \in \mathcal{N}_{oi}} \left\{ c_o e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \left(-\frac{2\psi_i^g}{l_g{}^2} e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} \right) - \frac{2c_g \psi_i^g}{l_g{}^2} e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}}. \end{aligned} \tag{15}$$

Figure 5 shows that each robot starting from different initial points reaches the target near an obstacle while avoiding obstacles. Figure 6 illustrates the potential and force using Equations (14) and (15) when goal (0,0) is near an obstacle (−0.1, 0).

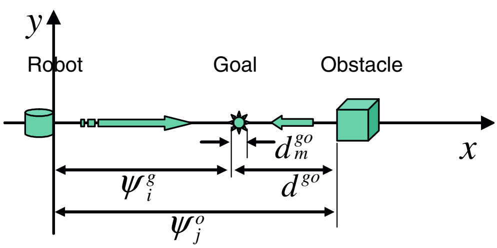
Now let us consider a number of problems that the proposed potential function may cause for group migration and obstacle avoidance, and then analyze how to design APFs' parameters to overcome such problems.

3.2.1. *A Non-Reachable Goal Problem: A Case that the Potential of the Goal is Overwhelmed by the Potential of the Obstacle*

First, consider the case that the potential of the goal is affected excessively by potential of the obstacle, i.e. when the robot and the goal are within the distance of the obstacle.

If we use the force of standard configuration (13), the robot will be trapped at the minimum where the total force becomes zero, not a goal position. Even though there is no obstacle in its way, the robot cannot reach the goal as shown in Figure 3. Suppose that the robot, obstacle and goal are collinear, with the robot lying on a different side of the obstacle and the goal as shown in Figure 7 where we do not need to consider the variables in y-axis due to $\psi_{1y}^g = \psi_{1y}^o = 0$. When the robot reaches the goal, ψ_{1x}^g becomes zero. Thus, the second component of the force in (13) is zero. However, the first component $\frac{2c_o\psi_j^o}{l_o^2} e^{-\frac{|\psi_j^o|^2}{l_o^2}}$ is not zero. Therefore, the conventional configuration for APFs followed by an additive structure does not enable the robot to reach the goal if

Figure 7 When the robot, goal and obstacle are collinear in order.



the goal is near obstacles. For this reason, goal (0,0) near the obstacle has no lowest potential as shown in Figure 4.

In order to overcome this problem, a new configuration for APFs is proposed in (14) and (15). As the robot reaches the goal, i.e. ψ_{1x}^{g} becomes zero, all the terms in (15) are also zero, which drives the robot to the goal. Thus, the proposed method can overcome the non-reachable goal problem appealing from the conventional potential configuration followed by an additive structure.

Now let us consider the guideline of design parameters for the above local minimum problem. Suppose that the robot, goal and obstacle are collinear, with the obstacle lying on a different side of the robot and goal as shown in Figure 7 where $\psi_{1x}^{o} = -\left|\psi_{1x}^{o}\right|$ and $\psi_{1x}^{g} = -|\psi_{1x}^{g}|$ until the robot reaches the goal. Following conditions are needed so that the robot reaches the goal. First, until the robot reaches the goal, F_1^{ogg} should be larger than zero because the robot moves from 0 to (+) direction in x-axis. Second, if the robot reaches the goal, i.e. $\psi_{1x}^{g} = 0$, F_1^{ogg} should be zero. For latter one, we already mentioned. Now, we prove that $F_1^{ogg} \geq 0$ until the robot reaches the goal. By Equation (15), we obtain

$$
\begin{aligned}
F_1^{ogg} &= \frac{2c_o\left|\psi_{1x}^{o}\right|}{{l_o}^2} e^{-\frac{\left|\psi_{1x}^{o}\right|^2}{{l_o}^2}} \left(e^{-\frac{\left|\psi_{1x}^{g}\right|^2}{{l_g}^2}} - 1 \right) + c_o e^{-\frac{\left|\psi_{1x}^{o}\right|^2}{{l_o}^2}} \left(\frac{2\left|\psi_{1x}^{g}\right|}{{l_g}^2} e^{-\frac{\left|\psi_{1x}^{g}\right|^2}{{l_g}^2}} \right) \\
&\quad + \frac{2c_g\left|\psi_{1x}^{g}\right|}{{l_g}^2} e^{-\frac{\left|\psi_{1x}^{g}\right|^2}{{l_g}^2}} \\
&= 2c_o \left(\frac{\left|\psi_{1x}^{o}\right|}{{l_o}^2} + \frac{\left|\psi_{1x}^{g}\right|}{{l_g}^2} \right) e^{-\frac{\left|\psi_{1x}^{o}\right|^2}{{l_o}^2} - \frac{\left|\psi_{1x}^{g}\right|^2}{{l_g}^2}} - \frac{2c_o\left|\psi_{1x}^{o}\right|}{{l_o}^2} e^{-\frac{\left|\psi_{1x}^{o}\right|^2}{{l_o}^2}} \\
&\quad + \frac{2c_g\left|\psi_{1x}^{g}\right|}{{l_g}^2} e^{-\frac{\left|\psi_{1x}^{g}\right|^2}{{l_g}^2}}.
\end{aligned}
\tag{16}
$$

To eliminate the free path local minima, F_1^{ogg} should be pointing the goal, i.e. $F_1^{ogg} \geq 0$. So the question is whether $F_1^{ogg} \geq 0$, i.e.

$$
c_o \left(\frac{\left\|\psi_j^{o}\right\|}{{l_o}^2} + \frac{\left\|\psi_i^{g}\right\|}{{l_g}^2} \right) e^{-\frac{\left\|\psi_j^{o}\right\|^2}{{l_o}^2} - \frac{\left\|\psi_i^{g}\right\|^2}{{l_g}^2}} - \frac{c_o\left\|\psi_j^{o}\right\|}{{l_o}^2} e^{-\frac{\left\|\psi_j^{o}\right\|^2}{{l_o}^2}} + \frac{c_g\left\|\psi_i^{g}\right\|}{{l_g}^2} e^{-\frac{\left\|\psi_i^{g}\right\|^2}{{l_g}^2}} \geq 0. \tag{17}
$$

PROPOSITION 1. *For* $\left|\psi_{1x}^{g}\right|$ *in the force* (16), *there exist positive constants* c_g, l_g, c_o *and* l_o *satisfying following inequality.*

$$
c_{g/o} \geq \frac{{l_g}^2}{\left|\psi_{1x}^{g}\right|} e^{-\frac{\left|\psi_{1x}^{o}\right|^2}{{l_o}^2}} \alpha\left(\left|\psi_{1x}^{g}\right|\right) \tag{18}
$$

where $c_{g/o} = c_g/c_o$ *and* $\alpha\left(\left|\psi_{1x}^{g}\right|\right)$ *is given in* (20) *afterward.*

Sketch of proof: For $F_{1x}^{ogg} > 0$ and $F_{1y}^{ogg} = 0$, the following inequality can be obtained.

$$
\begin{aligned}
c_{g/o} &\geq \left[\frac{|\psi_{1x}^{o}|}{{l_o}^2} e^{-\frac{|\psi_{1x}^{o}|^2}{{l_o}^2}} - \left(\frac{|\psi_{1x}^{o}|}{{l_o}^2} + \frac{|\psi_{1x}^{g}|}{{l_g}^2} \right) e^{-\frac{|\psi_{1x}^{o}|^2}{{l_o}^2} - \frac{|\psi_{1x}^{g}|^2}{{l_g}^2}} \right] \Bigg/ \left(\frac{|\psi_{1x}^{g}|}{{l_g}^2} e^{-\frac{|\psi_{1x}^{g}|^2}{{l_g}^2}} \right) \\
&\geq \frac{{l_g}^2}{{l_o}^2} \frac{|\psi_{1x}^{o}|}{|\psi_{1x}^{g}|} e^{-\frac{|\psi_{1x}^{o}|^2}{{l_o}^2} + \frac{|\psi_{1x}^{g}|^2}{{l_g}^2}} - \frac{{l_g}^2}{|\psi_{1x}^{g}|} \left(\frac{|\psi_{1x}^{o}|}{{l_o}^2} + \frac{|\psi_{1x}^{g}|}{{l_g}^2} \right) e^{-\frac{|\psi_{1x}^{o}|^2}{{l_o}^2}} \\
&\geq \frac{{l_g}^2}{|\psi_{1x}^{g}|} e^{-\frac{|\psi_{1x}^{o}|^2}{{l_o}^2}} \alpha\left(\psi_{1x}^{g}, \psi_{1x}^{o}\right)
\end{aligned} \tag{19}
$$

where $\alpha\left(\psi_{1x}^{g}, \psi_{1x}^{o}\right) = \left[\frac{|\psi_{1x}^{o}|}{{l_o}^2} e^{\frac{|\psi_{1x}^{g}|^2}{{l_g}^2}} - \left(\frac{|\psi_{1x}^{o}|}{{l_o}^2} + \frac{|\psi_{1x}^{g}|}{{l_g}^2} \right) \right]$.

Let d^{go} be a positive constant for the distance between the goal and the obstacle. We have $|\psi_{1x}^{o}| = |\psi_{1x}^{g}| + d^{go}$. Let us denote d_m^{go} as the minimum distance of d^{go}, i.e. the diameter of permissible goal region. Since (19) demands large $c_{g/o}$ for small d^{go}, we can replace d^{go} by d_m^{go} after substituting $|\psi_{1x}^{g}|$ for $|\psi_{1x}^{o}|$ in (19). Through some simple algebraic manipulations, we have

$$
\alpha\left(|\psi_{1x}^{g}|\right) = \frac{|\psi_{1x}^{g}| + d_m^{go}}{{l_o}^2} e^{\frac{|\psi_{1x}^{g}|^2}{{l_g}^2}} - \left(\frac{|\psi_{1x}^{g}| + d_m^{go}}{{l_o}^2} + \frac{|\psi_{1x}^{g}|}{{l_g}^2} \right) \cdot \begin{cases} \leq 0, & if\ |\psi_{1x}^{g}| \leq d_o^{go} \\ > 0, & if\ d_o^{go} < |\psi_{1x}^{g}| \end{cases} \tag{20}
$$

where d_o^{go} is the distance when the total of repulsive force from the obstacle and attractive force from the goal is zero.

Since $\alpha\left(|\psi_{1x}^{g}|\right) \leq 0$ when $|\psi_{1x}^{g}| \leq d_o^{go}$, the lower bound of $c_{g/o}$ is chosen when $d_o^{go} < |\psi_{1x}^{g}|$. The details of the choice for the lower bound of $c_{g/o}$ are dealt with in Section 3.2.3. □

Consider the repulsive force coming from the obstacle *versus* the distance of the goal and obstacle. Since the first term $-e^{-\frac{\|\psi_i^{g}\|^2}{{l_g}^2}} + 1$ in (14) reduces to zero, the agent is not under the influence of the potential term by the obstacle. Thus, even though the obstacle is near to the goal, the robot is relatively affected less by the repulsive force coming from the obstacle. When the robot and the goal are out of the distance of the obstacle, the inequality of (18) is guaranteed for any c_g, l_g, c_o and l_o, that is, $F_i^{ogg} > 0$ until the robot reaches the goal. If the robot reaches the goal, $F_i^{ogg} = 0$ and $U_i^{ogg} = 0$.

In [9], the non-reachable goal problem with obstacles nearby has been presented for the path planning of mobile robot for the first time. Design guideline for scaling parameters of the attractive and repulsive potential functions are proposed. However, since only the lower boundary of the scaling parameters are considered, the robot may collide with its obstacle nearby due to large attractive force coming from the goal when the obstacle lies between the robot and its goal. In Section 3.2.2

we deal with such a problem and propose the design guideline for the upper bound of the scaling parameters for the attractive and repulsive forces.

3.2.2. *An Obstacle Collision Problem: A Case that the Potential of the Obstacle is Overwhelmed by the Potential of the Goal*

Suppose that the robot, obstacle and goal are collinear, with the obstacle lying between the robot and its own goal as shown in Figure 8 where $\psi_{1x}^{o} = -\left|\psi_{1x}^{o}\right|$ and $\psi_{1x}^{g} = -\left|\psi_{1x}^{g}\right|$ until the robot reaches the point S. In Figure 8, d_0^{ro} is the distance between the obstacle and the robot when the total of repulsive force from the obstacle and attractive force from the goal is zero. There exists a point (denoted the point S here) where the forces coming from repulsive force by the obstacle and attractive force by the goal are zero. Following conditions are needed so that the robot avoids collision with the obstacle. Until the robot starting from (0,0) reaches the point near the obstacle, F_1^{ogg} should be larger than zero. If the robot reaches the point, F_1^{ogg} should be zero. And then the robot can turn around the obstacle and go toward the goal by the negative gradient of the generated potential field.

Now, we will prove that $F_1^{ogg} \leq 0$ after the robot reaches the point S., i.e.

$$c_o\left(\frac{\left\|\psi_j^o\right\|}{l_o{}^2}+\frac{\left\|\psi_i^g\right\|}{l_g{}^2}\right)e^{-\frac{\left\|\psi_j^o\right\|^2}{l_o{}^2}-\frac{\left\|\psi_i^g\right\|^2}{l_g{}^2}}-\frac{c_o\left\|\psi_j^o\right\|}{l_o{}^2}e^{-\frac{\left\|\psi_j^o\right\|^2}{l_o{}^2}}+\frac{c_g\left\|\psi_i^g\right\|}{l_g{}^2}e^{-\frac{\left\|\psi_i^g\right\|^2}{l_g{}^2}}\leq 0 \quad (21)$$

Otherwise the robot may collide with the obstacle due to stronger attractive force by the goal than repulsive force by the obstacle. Thus, using the same procedure as 3.2.1, we propose a guideline for the upper bound of $c_{g/o}$ so that the robot does not collide with the obstacle.

PROPOSITION 2. *For $d^{ro}{}_m < \left|\psi_{1x}^{o}\right| \leq d_0^{ro}$ in the force (16), there exist positive constants c_g, l_g, c_o and l_o satisfying following inequality.*

$$c_{g/o} \leq \frac{l_g{}^2}{d^{ro}{}_m + d^{go}{}_m}e^{-\frac{d^{ro}{}_m{}^2}{l_o{}^2}}\left[\frac{d^{ro}{}_m}{l_o{}^2}e^{\frac{(d^{ro}{}_m+d^{go}{}_m)^2}{l_g{}^2}}-\left(\frac{d^{ro}{}_m}{l_o{}^2}+\frac{d^{ro}{}_m+d^{go}{}_m}{l_g{}^2}\right)\right]. \quad (22)$$

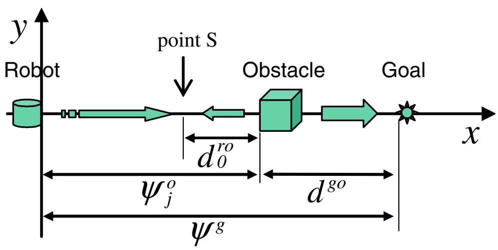


Figure 8 When the robot, obstacle and goal are collinear in order.

where d^{ro} is a positive constant for the distance between the robot and the obstacle, and d^{ro}_m is the minimum distance of d^{ro}, i.e. the minimum distance avoiding collision between the robot and the obstacle.

Sketch of proof: After the robot reaches the point S where the total of repulsive force from the obstacle and attractive force from the goal is zero, F_1^{ogg} should be less than zero for $d^{ro}{}_m < \left|\psi^o_{1x}\right| < d^{ro}_0$ so that the robot does not approach the obstacle any more.

The closer the goal is to the obstacle, the higher the probability that the robot collides with the obstacle is. We have $\left|\psi^g_{1x}\right| = \left|\psi^o_{1x}\right| + d^{go}$. Thus we can replace d^{go} by $d^{go}{}_m$ and substitute $\left|\psi^o_{1x}\right| + d^{go}{}_m$ for $\left|\psi^g_{1x}\right|$.

$$c_o\left(\frac{\left|\psi^o_{1x}\right|}{{l_o}^2} + \frac{\left|\psi^o_{1x}\right| + d^{go}{}_m}{{l_g}^2}\right)e^{-\frac{\left|\psi^o_{1x}\right|^2}{{l_o}^2}-\frac{\left(\left|\psi^o_{1x}\right|+d^{go}{}_m\right)^2}{{l_g}^2}} - \frac{c_o\left|\psi^o_{1x}\right|}{{l_o}^2}e^{-\frac{\left|\psi^o_{1x}\right|^2}{{l_o}^2}}$$
$$+\frac{c_g\left(\left|\psi^o_{1x}\right| + d^{go}{}_m\right)}{{l_g}^2}e^{-\frac{\left(\left|\psi^o_{1x}\right|+d^{go}{}_m\right)^2}{{l_g}^2}} \leq 0 \tag{23}$$

Using the same procedure as Proposition 3.2.1, similarly we have

$$c_{g/o} \leq \frac{{l_g}^2}{\left|\psi^o_{1x}\right| + d^{go}{}_m}e^{-\frac{\left|\psi^o_{1x}\right|^2}{{l_o}^2}}\left[\frac{\left|\psi^o_{1x}\right|}{{l_o}^2}e^{\frac{\left(\left|\psi^o_{1x}\right|+d^{go}{}_m\right)^2}{{l_g}^2}} - \left(\frac{\left|\psi^o_{1x}\right|}{{l_o}^2} + \frac{\left|\psi^o_{1x}\right| + d^{go}{}_m}{{l_g}^2}\right)\right]. \tag{24}$$

The smaller $\left|\psi^o_{1x}\right|$ is, the smaller $c_{g/o}$ is required in (24). Thus, replacing $\left|\psi^o_{1x}\right|$ by d^{ro}_m in (24) we obtain (22). The details of the choice for the upper bound of $c_{g/o}$ are dealt with in Section 3.2.3. □

REMARK 1. The purpose of Proposition 2 let the robot not to collide with the obstacle by excessive potential fields from the goal. The robot may be trapped in the point S in the case that the robot, obstacle and goal are collinear exactly. [14] suggests for a single robot to escape local minimum using escape-force function if a local minimum is identified when some conditions are satisfied. In dynamic situation where a target is moving, the problem is solved easier than the stationary environment [8]. Such a problem can also be solved by group formation followed by next section. The vector that robot trapped at local minimum directs can be changed by the attractive or repulsive forces coming from neighboring robots. Thus, the trapped robot can be easily escaped by the help of neighboring robots in self-organization problem of swarm robots which is called a 'waiting time' strategy.

3.2.3. *Design Guideline of Potential Functions for Path Planning*

To derive the lower bound of $c_{g/o}$ from a given distance d^{go}_m in (20), we choose l_g and l_o first. Since l_g and l_o are design parameters related with the distance of influence from the goal and obstacle, respectively, they are chosen first. Then depict the graph

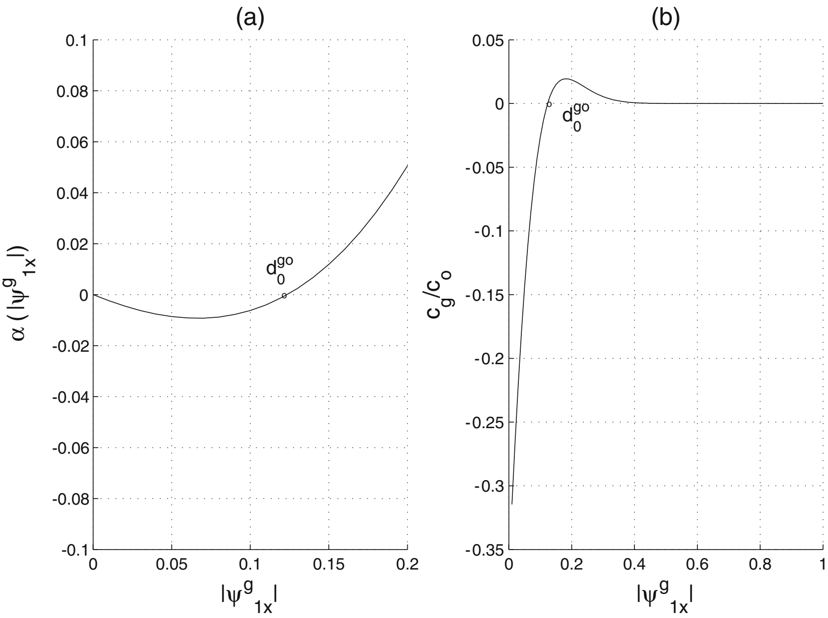


Figure 9 $\alpha\left(\psi^g_{1x}\right)$ and $c_{g/o}$ when $l_g = 2$ and $l_o = 0.2$.

of $c_{g/o}$ using a computer software such as MATLAB [19] and find the maximum value of $c_{g/o} = c_g/c_o$. For example, with $d^{go}_m = 0.2$, $l_g = 2$ and $l_o = 0.2$, the graphs of $\alpha\left(\psi^g_{1x}\right)$ and $c_{g/o}$ are illustrated in Figure 9. We can get $d^{go}_o = 0.122$ and $c_{g/o} > 0.022$. If there is no permissible region of the goal, i.e. $d^{go}_m = 0$, we can get $c_{g/o} > 0.151$. If the robot is far from the obstacle, the lower bound of $c_{g/o}$ is zero due to $e^{-\frac{\left|\psi^o_{1x}\right|^2}{l_o{}^2}}$ in (19).

For the higher bound of $c_{g/o}$, we can get $d^{ro}_0 = 0.281$ and $c_{g/o} < 0.383$ with $d^{ro}_m = 0.2$ and $d^{go}_m = 0.2$ in (22). Thus we can choose c_o and c_g satisfying to $0.022 < c_g/c_o < 0.383$. Finally, we choose $c_g = 1$ and $c_o = 3$.

REMARK 2. This study suggests that the lower and upper bounds for design parameters to solve the non-reachable goal problem and its corresponding obstacle avoidance. The selection of optimal design parameters within the bounds depends on physical plants or missionary tasks. Thus, the optimization problem for design parameters within the bounds is beyond the intention of this paper.

4. An Extension to Group Formation

Artificial potential methods have been previously used for obstacle-avoidance path planing [9, 14, 17]. In the last decade, they have been extended to group behaviors such as swarming or aggregation of autonomous mobile agents. The purpose of this section proposes the formulation of the APFs for group formation and then solve possible local minimum and collision problem in potential function configuration. Thus, our focus is different from previous studies [2, 3, 5] and [21].

4.1. APF for Group Formation

The group formation behavior seeks to establish a specific relationship between adjacent neighbors. A swarm of N robots(agents) are considered. Relative position vectors among the robots are defined as

$$\psi_k^f = \mathbf{P}_i - \mathbf{P}_k. \tag{25}$$

Robots flock together and arrange their formation through attractive and repulsive forces among themselves using APFs. The potential function of each robot for group formation is designed as following.

$$U_i^f = \sum_{k \in \mathcal{N}_{fi}} \left\{ c_r e^{-\frac{\|\psi_k^f\|^2}{l_r{}^2}} - c_a e^{-\frac{\|\psi_k^f\|^2}{l_a{}^2}} + c_a^{'} \left\| \psi_k^f \right\|^2 + c_f \right\} \tag{26}$$

where $\mathcal{N}_{fi}$ denotes the set of labels of those agents which are neighbors of agent i. c_r, c_a, l_r, and l_a are the strengths and correlation distances of the repulsive and attractive force, respectively. $c_a^{'}$ is the strength of the auxiliary attractive force.

$$c_f = -c_r e^{-\frac{c_f'}{l_r{}^2}} + c_a e^{-\frac{c_f'}{l_a{}^2}} - c_a^{'} c_f' \tag{27}$$

where $c_f' = \frac{l_r{}^2 l_a{}^2}{l_r{}^2 - l_a{}^2} ln \frac{c_a c_a^{'} l_r^2}{c_r l_a^2}$. c_f acts to make the minimum of the potential function zero. The distance between two agents at the point where $U_i^f(k)$ is minimum is $d^f = \sqrt{c_f'}$.

The corresponding force is then given by the negative gradient of (26)

$$F_i^f = -\nabla U_i^f = \sum_{k \in \mathcal{N}_{fi}} \left\{ \frac{2 c_r \psi_k^f}{l_r{}^2} e^{-\frac{\|\psi_k^f\|^2}{l_r{}^2}} - \frac{2 c_a \psi_k^f}{l_a{}^2} e^{-\frac{\|\psi_k^f\|^2}{l_a{}^2}} - 2 c_a^{'} \psi_k^f \right\}. \tag{28}$$

Let us analyze the cohesive behavior for the above potential function and force.

PROPOSITION 3. *For formation force* (28), $\mathbf{P}_i(t) \to \mathbf{B}_\epsilon\left(\bar{\mathbf{P}}_i(t)\right)$ *as* $t \to \infty$, *where*

$$\mathbf{B}_\epsilon\left(\bar{\mathbf{P}}_i(t)\right) = \left\{\mathbf{P}_i : \left\|\mathbf{P}_i - \bar{\mathbf{P}}\right\| \le \epsilon\right\} \tag{29}$$

and the center of swarm agents is defined as $\bar{\mathbf{P}}_i = \frac{1}{N}\sum_{i=1}^N \mathbf{P}_i$.

Sketch of proof: The distance between $\mathbf{P}_i$ and the center of swarm is defined as $e_i = \mathbf{P}_i - \bar{\mathbf{P}}$. From the definition of the center of swarm, we have $\sum_{i=1}^N \mathbf{P}_i = N\bar{\mathbf{P}}_i$. Subtracting from both sides $N\mathbf{P}_i$, we obtain

$$\sum_{k=1}^N (\mathbf{P}_i - \mathbf{P}_k) = N\left(\mathbf{P}_i - \bar{\mathbf{P}}\right) = N e_i. \tag{30}$$

The error equation can be written as

$$\dot{e}_i = -\nabla U_i^f + \frac{1}{N}\sum_{i=1}^{N} \nabla U_i^f . \tag{31}$$

Defining a Lyapunov function as $V_i = \frac{1}{2}\left\|e_i\right\|^2 = \frac{1}{2}{e_i}^T e_i$ and using (31), we obtain

$$\dot{V}_i = -\left[\nabla U_i^f - \frac{1}{N}\sum_{i=1}^{N}\nabla U_i^f\right]^T e_i = -\left[\nabla U_i^f\right]^T e_i \tag{32}$$

since we obtain $\sum_{i=1}^{N} \nabla U_i^f = 0$ which follows from the fact that ∇U_i^f are odd functions. Using (28) and (30), we obtain

$$\dot{V}_i = -2c_a^{'} N\left\|e_i\right\|^2 + \sum_{k\in\mathcal{N}_{fi}} \left\{ \frac{2c_r\psi_k^f}{{l_r}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_r}^2}} - \frac{2c_a\psi_k^f}{{l_a}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_a}^2}} \right\} \left\|e_i\right\| . \tag{33}$$

For the second term to be negative semi-definite, note that $\left\|\psi_k^f\right\| e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_r}^2}}$ and $\left\|\psi_k^f\right\| e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_a}^2}}$ are bounded functions whose maximum are given by $\frac{l_r}{\sqrt{2}} e^{-\frac{1}{2}}$ and $\frac{l_a}{\sqrt{2}} e^{-\frac{1}{2}}$, respectively. Substituting this in the above equation we obtain

$$\begin{aligned} \dot{V}_i &\leq -2c_a^{'} N\left\|e_i\right\|^2 + \sqrt{2}(N-1)e^{-\frac{1}{2}}\left(\frac{c_r}{l_r} + \frac{c_a}{l_a}\right)\left\|e_i\right\| \\ &\leq -2c_a^{'} N\left\|e_i\right\|\left(\left\|e_i\right\| - \epsilon\right) \end{aligned} \tag{34}$$

where $\epsilon = \frac{(N-1)}{\sqrt{2}c_a' N} e^{-\frac{1}{2}}\left(\frac{c_r}{l_r} + \frac{c_a}{l_a}\right)$.

There exists a constant ϵ such that for $\left\|e_i\right\| > \epsilon$ we obtain $\dot{V}_i < 0$. Thus, it is guaranteed that in that region $\left\|e_i\right\|$ is decreasing and eventually $\left\|e_i\right\| \leq \epsilon$ is achieved. □

4.2. APFs for Group Formation, Migration, and Obstacle Avoidance

Total potential for group formation, migration, and obstacle avoidance is

$$\begin{aligned} U_i^{oggf} &= \frac{1}{c_g} U_i^o \cdot U_i^g + U_i^g + U_i^f \\ &= \sum_{j\in\mathcal{N}_{oi}} \left\{ c_o e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \cdot \left(-e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} + 1 \right) - \; c_g e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} + c_g \\ &\quad + \sum_{k\in\mathcal{N}_{fi}} \left\{ c_r e^{-\frac{\|\psi_k^f\|^2}{l_r{}^2}} - c_a e^{-\frac{\|\psi_k^f\|^2}{l_a{}^2}} + c_a' \left\| \psi_k^f \right\|^2 + c_f \right\}. \end{aligned} \tag{35}$$

Its corresponding force is

$$\begin{aligned} F_i^{oggf} &= -\nabla U_i^{oggf} \\ &= \sum_{j\in\mathcal{N}_{oi}} \left\{ \frac{2c_o\psi_j^o}{l_o{}^2} e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \left(-\, e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} + 1 \right) \\ &\quad + \sum_{j\in\mathcal{N}_{oi}} \left\{ c_o e^{-\frac{\|\psi_j^o\|^2}{l_o{}^2}} \right\} \left(-\frac{2\psi_i^g}{l_g{}^2} e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} \right) - \frac{2c_g\psi_i^g}{l_g{}^2} e^{-\frac{\|\psi_i^g\|^2}{l_g{}^2}} \\ &\quad + \sum_{k\in\mathcal{N}_{fi}} \left\{ \frac{2c_r\psi_k^f}{l_r{}^2} e^{-\frac{\|\psi_k^f\|^2}{l_r{}^2}} - \frac{2c_a\psi_k^f}{l_a{}^2} e^{-\frac{\|\psi_k^f\|^2}{l_a{}^2}} - 2c_a'\psi_k^f \right\}. \end{aligned} \tag{36}$$

Now let us consider and analysis the collision problems that the proposed potential function may cause for group formation, migration, and obstacle avoidance.

4.2.1. *An Obstacle Collision Problem in Swarm: A Case that the Potential of the Obstacle is Overwhelmed by the Potential of the Other Robots in Formation*

In the existence of an obstacle, if c_a and c_r are excessively larger than c_o, the robot might collide with obstacles. Suppose that a robot is located in different side of the obstacle and the other robots, and P_1 and P_2 are closest to the obstacle for flocking as shown in Figure 10. To define maximum attractive force by P_2, we suppose P_2 is the closest to the obstacle. The robot P_1 is affected by repulsive force F_1 by the obstacle, attractive force F_2 by the robot P_2 and the other attractive forces F_3 by the robots

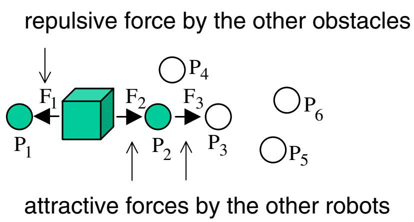


Figure 10 The robot P_1 affected by repulsive force F_1 from the obstacle, attractive force F_2 from the robot P_2 and the other attractive forces F_3 from the robot $P_3 - P_6$.

$P_3 - P_6$. F_1 should be larger than $F_2 + F_3$ so that the robot does not collide with the obstacle. This case appears in no goal situation, because the robots $P_2 - P_6$ migrate to the goal if there is a goal.

PROPOSITION 4. *For* $\left\|\psi_j^o\right\| \geq \frac{l_o}{\sqrt{2}}$, *there exist positive constants* $c_o, l_o, c_a, l_a, c_g, c_a^{'}$ *and* c_p *satisfying following inequality.*

$$\frac{c_o}{l_o} > c_p\left(\frac{c_a}{l_a} + c_a^{'} l_a e^{\frac{1}{2}}\right) \tag{37}$$

where c_p *is a positive constant and chosen afterward.*

Sketch of proof: From (11) and (28), we obtain

$$\frac{2c_o\left\|\psi_j^o\right\|}{{l_o}^2} e^{-\frac{\left\|\psi_j^o\right\|^2}{{l_o}^2}} > \sum_{k\in\mathcal{N}_{fi}(k)}\left\{\frac{2c_a\left\|\psi_k^f\right\|}{{l_a}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_a}^2}} + 2c_a^{'}\left\|\psi_k^f\right\|\right\} \tag{38}$$

since the repulsive force term $\frac{c_r\psi_k^f}{{l_r}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_r}^2}}$ in (28) is a negative term in the right term of (38).

In order to maximize the right term in (38), the robots can be positioned as shown in Figure 11 where $\mathcal{N}_p$ denotes the set of labels of those robots which are collinear to the obstacle. Let us denote n_p as the number of robots in $\mathcal{N}_p$, i.e. the number of columns that compose $2p - 1$ agents in sequence. Denoting $c_p = n_p + 1 + \frac{N-n_p-1}{2}$ to maximize the right term in (38) gives

$$\frac{c_o\left\|\psi_j^o\right\|}{{l_o}^2} e^{-\frac{\left\|\psi_j^o\right\|^2}{{l_o}^2}} > c_p\left(\frac{c_a\left\|\psi_k^f\right\|}{{l_a}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{{l_a}^2}} + c_a^{'}\left\|\psi_k^f\right\|\right). \tag{39}$$

Let us design the APFs' parameters so that the repulsive force by the obstacle should be larger than the attractive force by agents on the basis of the distance between the agent $\mathbf{P}_1$ and the obstacle decreasing to $\|\psi^o\| = \frac{l_o}{\sqrt{2}}$. At this distance, maximum repulsive force is induced. To maximize the right term in (39), maximum attractive force is applied when $\left\|\psi^f\right\| = \frac{l_a}{\sqrt{2}}$. Thus, replacing $\|\psi^o\|$ and $\left\|\psi^f\right\|$ by $\frac{l_o}{\sqrt{2}}$ and $\frac{l_a}{\sqrt{2}}$, respectively, and simple manipulation give (37). □

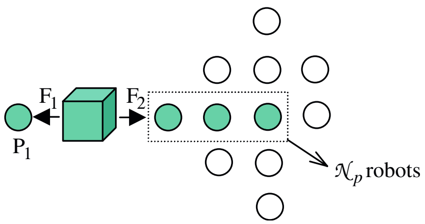


Figure 11 The structure for maximizing the right term in (38).

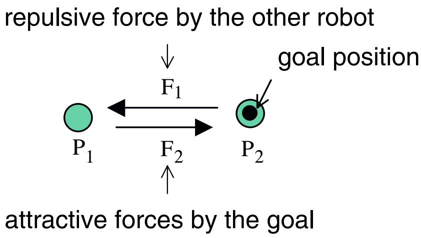


Figure 12 The robot P_1 is affected by the attractive force F_2 from the goal and the repulsive force F_1 from the robot P_2.

4.2.2. *An Inter-Robot Collision Problem: A Case that the Potential of the Robot in Formation is Overwhelmed by the Potential of the Goal*

When the robots migrate, if the attractive force from the goal is larger than the repulsive force among the robots, the robots might collide with each other. Suppose that the robot P_2 is located in the goal position, and the other robot P_1 is affected by the attractive force F_2 from the goal and the repulsive force F_1 from the robot P_2 as shown in Figure 12. F_1 should be larger than F_2 so that the robot P_1 does not collide with the other robot P_2.

PROPOSITION 5. *For* $\left\|\psi_k^f\right\| > d_m^f$, *there exist positive constants* c_r, l_r, c_g *and* l_g *satisfying following inequality.*

$$c_r > \frac{c_g d^r l_r{}^2}{d_m^f l_g{}^2} e^{-\frac{(d^r)^2}{l_g{}^2}+\frac{d_m^f}{l_r{}^2}} \tag{40}$$

where d^r *is the radius of the robot.* d_m^f *is a positive constant and chosen afterward.*

Sketch of proof: To guarantee $F_1 > F_2$ from (8) and (28), we obtain

$$\frac{c_r \left\|\psi_k^f\right\|}{l_r{}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{l_r{}^2}} > \frac{c_g d^r}{l_g{}^2} e^{-\frac{(d^r)^2}{l_g{}^2}} \tag{41}$$

since the attractive force term $\frac{c_a \psi_k^f}{l_a{}^2} e^{-\frac{\left\|\psi_k^f\right\|^2}{l_a{}^2}} + c_a^{'} \psi_k^f$ in (28) is a positive term in the left term of (41).

Replacing $\left\|\psi_k^f\right\|$ by d_m^f and simple manipulation give (40). Since the robot collides with the other robot when $\left|\psi_{1x}^f\right| = 0$, we choose $d_m^f = \frac{1}{10} d^f$ as the minimum distance that maintains a safe distance between the robots. As an example, if we take $l_o = 1/5, l_g = 2, l_a = 1/2, l_r = 1, c_o = 3, c_g = 1, c_a = 1/2, c_r = 1/3, c_a^{'} = 0.1$ and $N = 10$, both (37) in Section 4.2.1 and (40) are effective. □

Proposition 5 covers the case that the robot P_1 is affected by the repulsive forces from more than two robots because the repulsive forces are larger than F_1 in (41).

Use of the APFs to keep a formation has a lot of flexibility. While maintaining the characteristic of swarm, each agent wanders about flexibly, i.e. it has a nature of self-organized flocking that each agent makes a formation dynamically without explicit reorganization contrary to [1]. Since the proposed approach does not explicitly use the alignment of other group members, individual agents were not commanded to be located to any positions for alignment. Also if they encounter with obstacles, they reorganize their formation to avoid the obstacle without external command. For example, if their formation encounter a tunnel, they change their maintenance to a kind of line as themselves while keeping a formation. Moreover, this approach has a good scalability which adds or removes any number of agents easily. In prey-pursuit scenario of next Section, these characteristics are shown specifically.

5. Prey-Pursuit Simulation

As the well-known collective behavior of ants attacking a larger insect than them with cooperation, self-organized swarm robots are designed as agents which migrate to a designated place or follow a moving target while keeping a formation. The

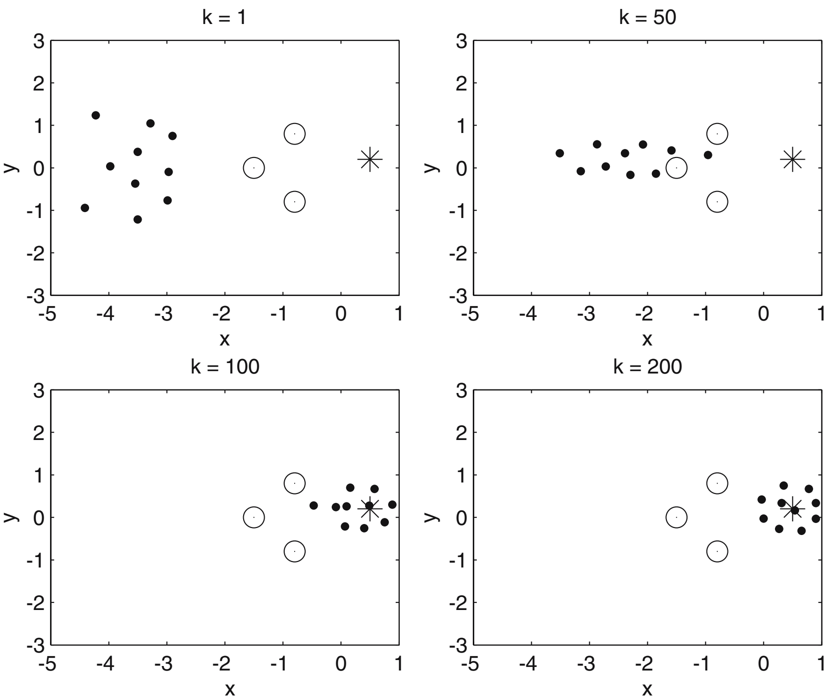


Figure 13 Snap shots of prey-pursuit for a static goal (*dot*: agent, *astral mark*: static goal, *circle*: obstacle).

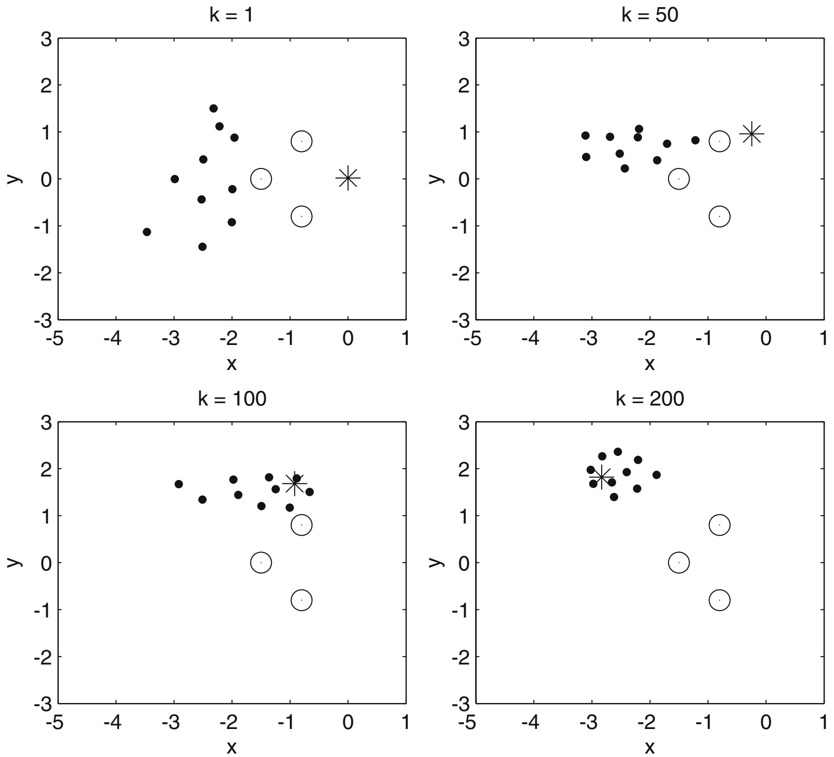


Figure 14 Snap shots of prey-pursuit for a moving goal (*dot*: agent, *astral mark*: moving target, *circle*: obstacle).

task is due to motivations related to the biological inspirations behind cooperative systems. Each agent in this task is to migrate to a static goal or follow a moving target, while avoiding obstacles, keeping away from colliding with other agents and maintaining a formation. The agents used in the proposed approach would also have the following characteristics: All agents are physically and functionally identical. Therefore, they can be manufactured inexpensively in large numbers, which would be the case. Furthermore, new agents can be added to the team whenever necessary. They can be adapted to various tasks with minimal structural changes. Individually, agents have limited capabilities and limited knowledge of the environment. However, as a swarm, they can exhibit 'intelligent behavior'. Simple individual behavior will result in an intelligent swarm behavior provided that some type of direct or indirect communications between agents exists.

Figure 13 and 14 illustrate the different snap shots of a migration process of 10 agents for a static goal and a moving target, respectively. Each agent is randomly initialized on the left side of $x = -2$, and the static goal and the moving target are initialized on (0,0). In Figure 14, it is assumed that the target moves slower than the maximum speed of the agent. Design parameters are set to $l_o = 1/5$, $l_g = 2$, $l_a = 1/2$, $l_r = 1$, $c_o = 3$, $c_g = 1$, $c_a = 1/2$ and $c_r = 1/3$. In Figures 13 and 14, the swarm agents spontaneously divide into several parts by themselves to surpass the blocking area when meeting the obstacle, and finally form a certain kind of group pattern at the neighborhood of the static goal and the moving target, respectively.

6. Conclusions

This paper presents a design framework based on APFs for group formation, migration and obstacle avoidance in autonomous swarm systems. One of the main contributions is to solve the non-reachable goal problem, caused by excessive potential of the obstacles, and the obstacle collision problem, caused by excessive potential of the goal, together to render the lower and upper bounds for selecting the scaling parameters of potential functions. Moreover, collision problems in formation, that is, the obstacle collision problem caused by excessive potential of the other agents, and the agent collision problem caused by excessive potential of the goal, are also proposed and addressed. The results form a set of analytical guidelines for designing APFs for swarm systems. The framework enables the agents in a swarm to maintain a flexible formation, while migrating as a group and avoiding any obstacles. In the presence of obstacles the formation of migrating agents can change shape, split and merge. Although in this paper we have focused on cooperative behaviors in swarm systems in 2D environment, the underlying method can be extended to scenarios in 3D setting.

Ackowledgment This work is supported by the Kyungnam University Research Fund, 2005.

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