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Reduced order modelling based control of two wheeled mobile robot

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Abstract This paper proposes a novel technique to design a pre-specified structure controller for balancing control of two wheeled mobile robot via reduced order modelling using cuckoo search algorithm. As the two wheeled mobile robot is an unstable system with various uncertainties and the controllers, available in the literature, comes up with higher order, the overall system becomes complex from analysis and manufacturing point of view. Therefore, in this paper, a lower order pre-specified structure controller is designed which is efficient enough to handle uncertain dynamics. The results of proposed controllers are compared with the results of controller designed by genetic algorithm, particle swarm optimization, Schur analysis, balanced truncation, modal truncation and conventional PD controllers. It is revealed that the proposed controller exhibit better performance comparatively. The performance of the higher and lower order controllers is also analysed with perturbed two wheeled mobile robot in terms of time response specifications and performance indices such as integral square error, integral absolute error and integral time absolute error.

Keywords Reduced order modelling · Optimization · Cuckoo Search · Two wheeled mobile robot

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Introduction

The balancing control problem of two wheeled mobile robot is not new in the field of robotics. This type of robot is in great demand nowadays because of their simple structure, simpler dynamics and applications in different fields such as transportation, security, search and rescue, entertainment and reducing the man power. As we know that two wheeled mobile robots are always unstable and also affected by external disturbances, therefore, robust control techniques are also in demand for proper and smooth balancing control and movement of such robots.

The basic control scheme for balancing control of two wheeled mobile robot using pole placement technique is developed by Grasser et al. (2002) in which a gain matrix has to be designed such that the desired poles should be in a linear plant. For designing the controller, various researches have used linear, quadratic regulation (LQR) design technique in which a cost function is considered for minimization (Ha and Yuta 1994; Kim et al. 2005; Akesson et al. 2006). The concept of mobile robots which are based on inverted-pendulum are suggested by Kim et al. (2006) and Takei et al. (2009) whereas the issues related to self-balancing control of two-wheeled robots are discussed by Butler and Bright (2008), Coelho et al. (2008) and Alarfaj and Kantor (2010). The modelling of manually controlled bicycle is suggested by Hess et al. (2012). Whereas, a navigation system for two wheeled mobile robot, to avoid obstacles in an amorphous environment, is presented by Kocaturk (2015). Recently, the Lyapunov function based control and stability region of two wheeled mobile robot is discussed by Kausar et al. (2012a, b, 2013). Another most widely used controllers in the different area of industries are proportional-integral (PI) (Liao and Ming 2010; Takahashi et al. 2001), proportional-derivative (PD) (Hatakeyama and Shimada 2008) and proportional-integral-derivative (PID) (Nasir et al. 2011). The simplicity of PID controller is that three parameters have to be tuned only. The PID controllers are suitable for two wheeled mobile robots because of their degree of freedom- one for tilt and one for speed and depending upon the applicability one for the yaw. Furthermore, numerous technique are available in the literature for balancing control of two wheeled mobile robot like flywheel balancing (Beznos et al. 1998; Gallaspy 1999; Suprapto 2006; Keo et al. 2011), mass balancing (Lee and Ham 2002) and steering balancing (Tanaka and Murakami 2004). The flywheel balancing is most widely used technique because of its short response time. Simultaneously, number of balancing control algorithm for two wheeled mobile robots have been developed by many researchers like nonlinear control algorithm is developed by Beznos et al. (1998). Gallaspy (1999) suggested compensator design using root locus technique whereas PD control algorithm was given by Suprapto (2006). In Martínez et al. (2009), unicycle mobile robot model has been considered for interval type-2 fuzzy logic controller design whereas in Das and Kar (2006), nonholonomic mobile robot has been considered for adaptive fuzzy logic-based controller design. Both of these papers consider mobile robots with two driving wheels mounted on the same axis and a front free wheel.

Further, the number of linear controllers came into the picture like H_2 and H_∞ controllers because of their robustness. They are more robust as compared to other controllers available in the literature because they are less sensitive to external disturbances and errors. The first technique to design robust controller for the system with various uncertainties is developed by Bernstein and Haddad (1989) which is based on a Riccati equation. Khargonekar and Rotea (1991) suggested a robust technique in which mixed H_2/H_∞ control is used for such type of systems. Rotea and Khargonekar (1991) suggested H_2 optimal control with a H_∞ constraint based on state feedback. Whereas multi objective H_2/H_∞ control technique is proposed by Scherer (1995). Recently, number of researches used nature inspired search algorithm for designing of robust controllers. Bui and Parnichkun (2008) suggested balancing control using particle swarm optimization whereas genetic algorithm based mixed H_2/H_∞ control scheme is used by Chen et al. (1995), Krohling (1998), Chang (2005), Ho et al. (2004, 2005). The complex design procedure and achieve higher order controller is the major drawback of such type of controllers. Furthermore, a large number of order reduction techniques have been suggested, recently, by several authors in the literature (Yamada and Ikeda 2014; Vishwakarma and Prasad 2014; Sikander and Prasad 2015a, b, c; Hummer and Szabo 2015; Sambariya and Gyanendra 2016). Therefore, an algorithm is required which generates a lower order controller which preserve all necessary properties of the higher order controller. The lower order controller may lead to less computation effort, reduce cost and simulation time.

So this paper contributes a novel technique to design prespecified structure controller for balancing of two wheeled mobile robot using reduced order modelling which is based on nature inspired search algorithm called cuckoo search. In this technique, cuckoo search algorithm is employed to achieve the coefficients of numerator and denominator polynomials of the reduced order controller by minimizing the integral square error between original and reduced order controller. The first, second and third order controllers designed by the proposed method are compared with higher order controller and other reduced order controllers available in the literature in terms of different performance criteria. The performance of the two wheeled mobile robot is analysed with higher and reduced order controllers in the presence of uncertainties also. It is found that the proposed third order controller exhibits excellent performance as compared to other controllers.

Mathematical model of two wheeled mobile robot

The model of two wheel mobile robot (TWMR) considered in this paper is a typical example of bicycle mobile robot as shown in Fig. 1, it consists of two driving wheels mounted on the different axis (Bui and Parnichkun 2008). The aim of this robot is to move, without falling down, forward, backward, left and right with or without load. The major issue with bicycle mobile robot is, balancing of this type of robot is not an easy task due to unstable nature of the robot and various uncertainties. Therefore, many authors suggested various control algorithms to tackle this issue (Bui and Parnichkun 2008; Chen et al. 1995; Krohling 1998; Chang 2005; Ho et al. 2004, 2005). Among all the techniques, the balancing using flywheel is used mostly in which a flywheel is put on the robot to balance the torque generated by the robot due to gravity. The dynamics model of the two wheeled mobile robot is derived using Lagrange equation as follows-

$$\frac{d}{dt}\left\{\frac{\partial K}{\partial \dot{f_i}}\right\} - \frac{\partial K}{\partial f_i} + \frac{\partial P}{\partial f_i} = F_i \tag{1}$$

where K is the system total kinetic energy, P is system total potential energy, F_i is external forces and f_i is generalised coordinate. K and P are calculated as follows-

$$P = m_r g h_r \cos\alpha + m_f g h_f \cos\alpha \tag{2}$$

$$K = \frac{1}{2}m_r(\dot{\alpha}^2 h_r^2) + \frac{1}{2}m_f(\dot{\alpha}^2 h_f^2) + \frac{1}{2}I_r\dot{\alpha}^2 + \frac{1}{2}\left[I_m\dot{\beta}^2 + I_f(\dot{\alpha}\sin\beta)^2 + I_m(\dot{\alpha}\cos\beta)^2\right] \tag{3}$$

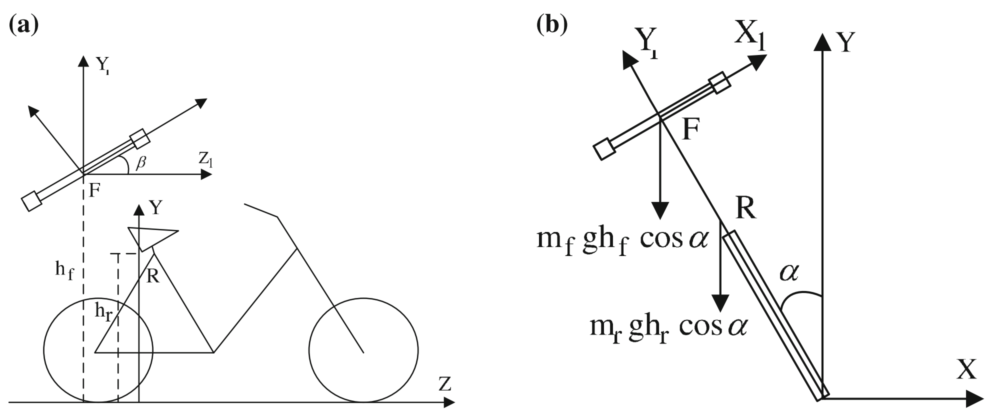

Fig. 1 Graphical representation of parameters of TWMR. **a** Side view, **b** front view

where m_r is mass of robot and m_f is mass of flywheel, α is lean angle of robot around Z axis, β is the angel of the flywheel along Z_1 axis, $\dot{\alpha}$ is angular velocity of the robot around Z axis, $\dot{\beta}$ is angular velocity of the flywheel along X_1 axis, h_r is the height of centre of gravity of robot whereas h_f is the height of flywheel centre of gravity, I_r, I_m, I_f are robot moment of inertia, flywheel radial moment of inertia and flywheel polar moment of inertia respectively.

For $f_i = \alpha$, using Eqs. 1–3, the following equation is obtained-

$$\ddot{\alpha}\left[m_r h_r^2 + m_f h_f^2 + I_r + I_f \sin^2\beta + I_m \cos^2\beta\right] + 2\sin\beta\cos\beta(I_f - I_m)\dot{\alpha}\dot{\beta} - g(m_r h_r + m_f h_f)\sin\alpha = I_f \omega\dot{\beta}\cos\beta \tag{4}$$

Similarly, for $f_i = \beta$ the following equation is obtained-

$$\ddot{\beta} I_m - \dot{\alpha}^2 (I_f - I_m)\sin\beta\cos\beta = T_m - I_f\omega\dot{\alpha}\cos\beta - V_m\dot{\beta} \tag{5}$$

T_m is torque developed by the motor and motor viscosity coefficient is V_m.

The dynamics of DC motor with 5 : 1 ratio is assumed for chain transmission system of the motor and the following relations are obtain-

$$T_m = 5K_m i \tag{6}$$

$$U = L\frac{di}{dt} + Ri + K_e\dot{\beta} \tag{7}$$

K_m, K_e are the torque and back emf constants of the motor respectively. R and L are armature resistant and inductance of the motor respectively.

By substituting Eq. (6) into (5) and linearising Eqs. (4) and (5) around the equilibrium point, the relations are achieved as follows-

$$\ddot{\alpha}[m_r h_r^2 + m_f h_f^2 + I_r + I_m] - g(m_r h_r + m_f h_f)\alpha - I_f\omega\dot{\beta} = 0 \tag{8}$$

$$\ddot{\beta} I_m + I_f\omega\dot{\alpha} + V_m\dot{\beta} - 5K_m i = 0 \tag{9}$$

Let us consider that $x = [\alpha \quad \dot{\alpha} \quad \dot{\beta} \quad i]'$, $y = \alpha$ and $u = U$. On combining Eqs. (7)–(9), the state space model of the system is represented as follows-

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{10}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g(m_r h_r + m_f h_f)}{m_r h_r^2 + m_f h_f^2 + I_r + I_m} & 0 & \frac{I_f\omega}{m_r h_r^2 + m_f h_f^2 + I_r + I_m} & 0 \\ 0 & -\frac{I_f\omega}{I_m} & -\frac{V_m}{I_m} & \frac{5K_m}{I_m} \\ 0 & 0 & -\frac{K_e}{L} & -\frac{R}{L} \end{bmatrix} \tag{11}$$

$$B = [0 \quad 0 \quad 0 \quad 1/L]', \quad C = [1 \quad 0 \quad 0 \quad 0], \quad D = [0] \tag{12}$$

The graphical representation of the parameters of two wheeled mobile robot and flywheel are depicted in Figs. 1 and 2 respectively where F and R denote flywheel and robot centre of gravity. The robot has one degree of freedom with rotation around Z-axis only whereas flywheel has three degree of freedom including rotation around X_1, Y_1 and Z axes as shown in Fig. 2.

Cuckoo search algorithm

Cuckoo search (CS) is an optimization technique/algorithm which is inspired by Nature (Yang and Deb 2008). This technique is based on the common behaviour of the cuckoo bird. All the cuckoo birds lay down their eggs into to the other bird's nest for fertilization. It is possible that the other birds

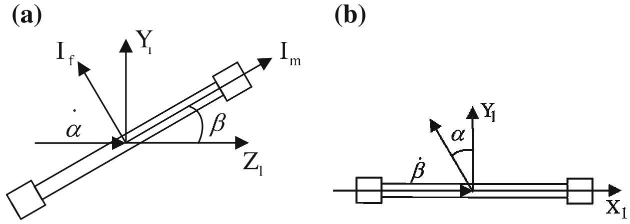

Fig. 2 Graphical representation of parameters of flywheel. **a** Side view, **b** front view

may recognize that it is not their eggs then either they throw the eggs or form a new nest at new place which results in the evolution of cuckoo eggs (Yang and Deb 2009).

A set of host nest show the cuckoo breading analogy. Each nest carries an egg which is considered as a solution. A new nest is formed using Lévy flight i.e. random walk (Brown et al. 2007; Viswanathan 2010). Success of resources random searches can be optimize using the Lévy flight movements (Humphries et al. 2012).

Yang and Deb (2008) gives the following three rules to combine the apply cuckoo species with the Lévy flight:

- For laying down the egg, the nest should be selected at random and dump by every cuckoo.
- The nest must be transferred to the next generation if good quality eggs are found in it.
- The probability of an alien egg which can be observed by the fixed number of host nest is $p_a \in [0, 1]$. In such cases the host nest either throw the alien egg or form a new nest at any other place.

For easiness, the fraction of p_a of n nests, which are interchanged new nests, is considered the approximation of previous assumption.

The Lévy flight can be represented b the following relation for the generation of new solution $Y(t+1)$ of cuckoo i (Yang and Deb 2009)

$$Y_i^{(t+1)} = Y_i^{(t)} + a \otimes \text{Lévy}(\lambda) \tag{13}$$

where a ($a > 0$) is a step size which is related to the level of the problem optimized by the technique. The random step size of the Lévy flights are calculated as follows-

$$\text{Lévy } \tilde{u} = t^{-\lambda}; \quad (1 < \lambda \leq 3) \tag{14}$$

Proposed methodology for controller design via reduced order modelling

The pre-specified structure controller is a controller whose order can be selected depending upon the requirement such as robust stability against plant perturbation, external disturbances and tracking error etc. In general the structure specified controller is represented as follows-

$$G_{cr}(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_0}{s^n + b_{n-1}s^{n-1} + \ldots + b_0} \tag{15}$$

where $a_0, a_1 \ldots a_m, b_0, b_1 \ldots b_{n-1}$ are unknown constants and the desired structure specified controller can be achieve by selecting the suitable values of m and n such as first order controller, second order controller and third order controller etc. The following steps are used to calculate the unknown constants of the structure specified controller by minimizing the integral square error using cuckoo search algorithm.

Step 1 Specify the fitness function and the number of chosen variables (say q) along with their range. Set the probability of the worst nests and step size also. Initialization of a population of p host nests then problem is summarised as-
Minimize fitness function F_α , subject to $a_{iL} < a_i < a_{iU}$ and $b_{iL} < b_i < b_{iU}$
Where, a_{iL}, b_{iL} and a_{iU}, b_{iU} are the lowest and highest values of the chosen variables respectively and $i = 0, 1 \ldots q$.
Step 2 Obtain the value of F_α for a randomly selected cuckoo (α) and select a nest (β) randomly among p.
Step 3 if ($F_\alpha > F_\beta$) then interchange β by the current obtained solution.
Step 4 Check whether the predefined stopping criterion is arrived or the maximum generation occurred or not if yes, then the solution obtained in the current generation would be the best solution otherwise go to next step.
Step 5 Abandon a fraction of worse nests with optimal value of probability p_a and step size a.
Step 6 Using Eq. (13) the obtained solution must be updated by calculating $Y_i^{(t+1)}$ and repeat this algorithm, until the predefined condition is arrived or the maximum generation occurred.

Furthermore, the efficacy of the controller obtained by the proposed method is evaluated by using the following performance indices-

$$ISE = \int_0^\infty [g_c(t) - g_{cr}(t)]^2 dt \tag{16}$$

$$IAE = \int_0^\infty |g_c(t) - g_{cr}(t)| \, dt \tag{17}$$

$$ITAE = \int_0^\infty t.|g_c(t) - g_{cr}(t)| \, dt \tag{18}$$

where ISE, IAE and ITAE are integral square error, integral of absolute error and integral of time multiplied by absolute

Table 1 Values of the parameters of two wheeled mobile robot

Parameter	Value	Parameter	Value
Mass of robot (m_r)	8.1 kg	DC motor viscosity coefficient (V_m)	0.000253 kg m^2/s
Mass of flywheel (m_f)	43.1 kg	Torque constants of the motor (K_m)	0.119 Nm/A
Heights of robot centre of gravity (h_r)	0.86 m	Back emf constants of the motor (K_e)	0.1184 V s
Height of flywheel centre of gravity (h_f)	0.8 m	Flywheel speed (ω)	157.08 rad/s
Robot moment of inertia (I_r)	27.584 kg m^2	Armature resistant of the motor (R)	0.41 Ω
Flywheel radial moment of inertia (I_m)	0.112304 kg m^2	Inductance of the motor (L)	0.0006 H
Flywheel polar moment of inertia (I_f)	0.215926 kg m^2	Gravity constant (g)	9.81 m/s^2

error respectively. $g_c(t)$ and $g_{cr}(t)$ are step responses of high and reduced order controller respectively.

Computational experiments

The values of the parameters of two wheeled mobile robot are depicted in Table 1. By substituting these values into Eq. 10 the transfer function of the two wheeled mobile robot is given as-

$$G(s) = \frac{\alpha(s)}{U(s)} = \frac{4887}{s^4 + 683.3s^3 + 1208s^2 + 109700s - 6949}$$

where $\alpha(s)$ is the output lean angle of robot and $U(s)$ is the input voltage to the DC motor that controls flywheel control axis. Let, following cases are there for perturbed two wheeled mobile robot.

Case1: Let us assume that the additional 10 kg load is added and the speed of flywheel is decreased to 147 rad/s then the perturbed mobile robot governs by the following transfer function.

$$G'(s) = \frac{3784}{s^4 + 683.3s^3 + 1162s^2 + 78290s - 6857}$$

Case2: If extra 10 kg load is added again and the speed of flywheel is boost up to 167 rad/s then the perturbed mobile robot is represented by the following transfer function.

$$G''(s) = \frac{4299}{s^4 + 683.3s^3 + 1197s^2 + 102300s - 6857}$$

The design algorithm for balancing control of robot is discussed by Bui and Parnichkun (2008) and the H_∞ controller is designed as follows-

$$G_c(s) = \frac{1275s^5 + 8.695\times10^5 s^4 + 5.151\times10^5 s^3 + 1.359\times10^8 s^2 + 2.435\times10^7 s + 1.091\times10^6}{s^6 + 715.7s^5 + 2.355\times10^4 s^4 + 2.789\times10^5 s^3 + 3.802\times10^6 s^2 + 6.519\times10^5 s + 2.872\times10^4}$$

This H_∞ controller is of sixth order. Therefore, practically this higher order controller is difficult to implement. In the following sections, implementation of lower order controllers such as first, second and third order controller designs are proposed and compared with other well-known controllers available in the literature. A comparison of the performance indices and time response specifications of closed loop system using different controllers is also included.

Design of first order controller

The first order controller using proposed method is obtained as follows.

$$G_{cr\_proposed1}(s) = \frac{150}{s + 4.367}$$

whereas the PD controller (Bui et al. 2010), first order controller based on GA (Bui and Parnichkun 2008) and based on PSO (Bui et al. 2010) are given as follows-

$$G_{cr\_PD}(s) = 2.5s + 30$$
$$G_{cr\_GA}(s) = \frac{197.33}{s + 4.91}$$
$$G_{cr\_PSO}(s) = \frac{135.2}{s + 4.63}$$

The closed loop step responses of two wheeled mobile robot using different first order controllers are shown in Fig. 3. The closeness of responses can be observed from enlarge view. It is found that the closed loop response of TWMR using proposed first order controller is closer to the response of TWMR using higher order controller as compared to the controllers obtained by other methods. The

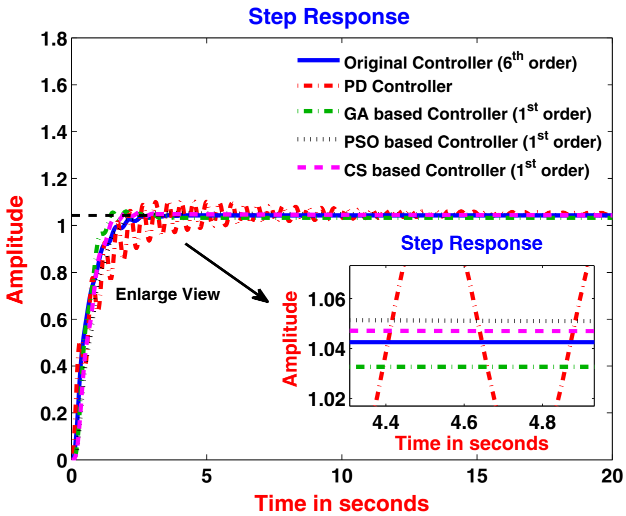

Fig. 3 Time response of original and reduced first order controllers with TWMR

closed loop step responses of perturbed TWMR for case-1 and case-2 are shown in Fig. 4. It observed that the proposed first order controller exhibits better performance in case of perturbation. Table 2 depicts the performance comparison of closed loop TWMR using first order controllers in terms of maximum overshoot (M_p) in percentage, settling time (t_s), rise time (t_r) in second and performance indices such as integral square error (ISE), integral absolute error (IAE) and integral time absolute error (ITAE). It is also clear from this table that the proposed controller also exhibits the lesser values of performance indices.

Design of second order controller

The second order controller using proposed method is obtained as follows.

$$G_{cr\_proposed2}(s) = \frac{266.5s + 595.6}{s^2 + 8.858s + 17.03}$$

whereas the transfer function of second order controller based on PSO (Bui et al. 2010) is given as follows-

$$G_{cr\_PSO}(s) = \frac{129.7s + 499.6}{s^2 + 6.835s + 16.18}$$

The closed loop step response of two wheeled mobile robot using different second order controllers is shown in Fig. 5. The closeness of responses can be observed from enlarge view. It is found that the closed loop response of TWMR using proposed second order controller is closer to the response of TWMR using higher order controller as compared to the controllers obtained by other methods. The closed loop step responses of perturbed TWMR for case-1 and case-2 are shown in Fig. 6. It observed that the proposed second order controller exhibits better performance in case of perturbation. Table 3 depicts the performance comparison of closed loop TWMR using second order controllers in terms of maximum overshoot (M_p) in percentage, settling time (t_s), rise time (t_r) in second and performance indices such as integral square error (ISE), integral absolute error (IAE) and integral time absolute error (ITAE). It is also clear from this table that the proposed controller provides the comparable performance.

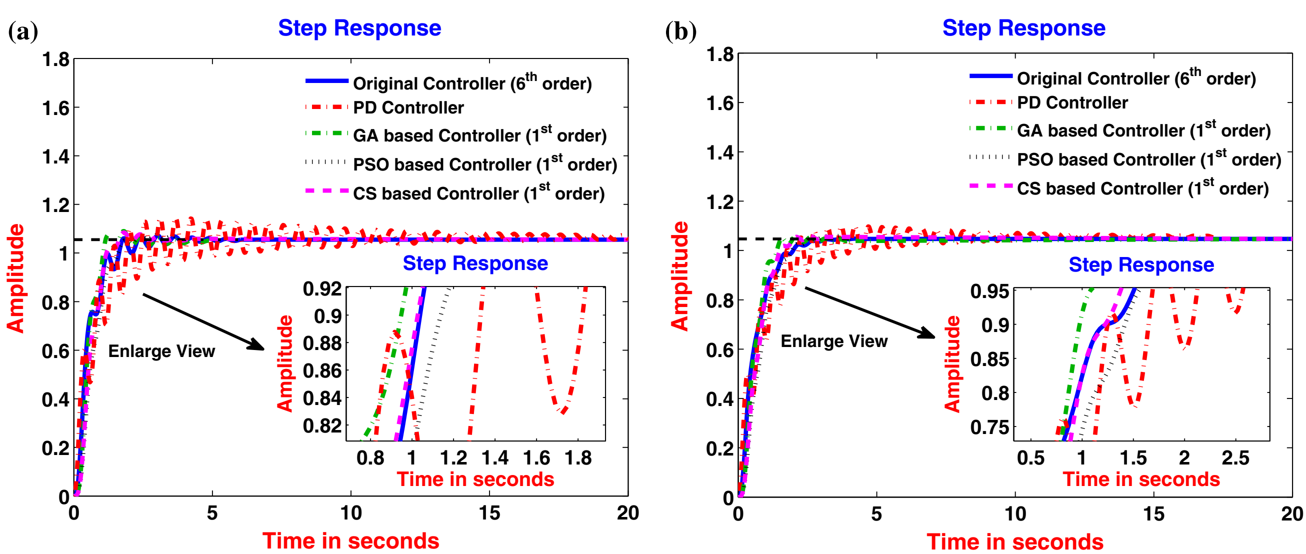

Fig. 4 Time response of original and reduced first order controllers with perturbed TWMR. **a** Case-1, **b** Case-2

Table 2 Performance comparison of closed loop TWMR using first order controllers

Controller	$M_p(\%)$	t_s	t_r	ISE	IAE	ITAE
Proposed controller	1.2922	1.7549	1.0245	0.0136	0.1507	0.1805
PD controller (Bui et al. 2010)	5.2968	10.1854	1.5138	0.0380	0.4065	1.1535
GA based controller (Bui and Parnichkun 2008)	2.1937	2.0569	0.7379	0.0100	0.1588	0.2988
PSO based controller (Bui et al. 2010)	0.2386	2.3130	1.1889	0.0239	0.2186	0.3881

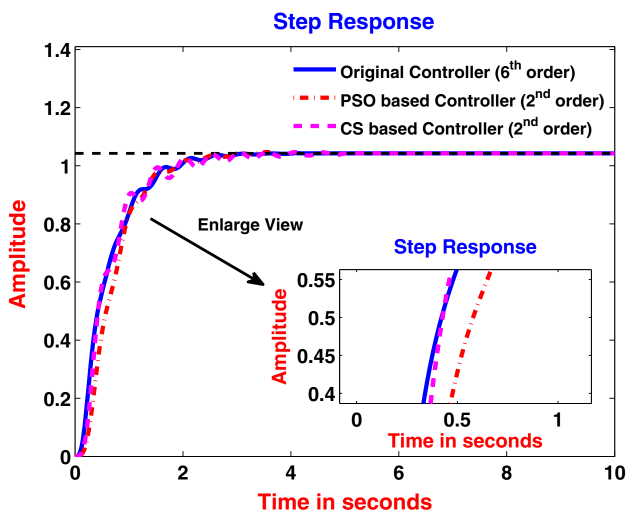

Fig. 5 Time response of original and reduced second order controllers with TWMR

Design of third order controller

Similarly, the reduced third order controller is obtained as follows-

$$G_{cr\_proposed}(s) = \frac{1241s^2 + 234.8s + 1.936 \times 10^5}{s^3 + 32.47s^2 + 395s + 5274}$$

whereas the third order reduced controllers designed by Schur Analysis (SA) (Nguyen et al. 2013), Balanced Truncation (BT) (Moore 1981) and Modal Truncation (MT) (Liu and Anderson 1989) are represented by the following transfer functions respectively.

$$G_{cr\_SA}(s) = \frac{1275s^2 + 234.8s + 1.993 \times 10^5}{s^3 + 33.78s^2 + 395s + 5506}$$

$$G_{cr\_BT}(s) = \frac{1275s^2 + 233.8s + 1.992 \times 10^5}{s^3 + 33.78s^2 + 395s + 5499}$$

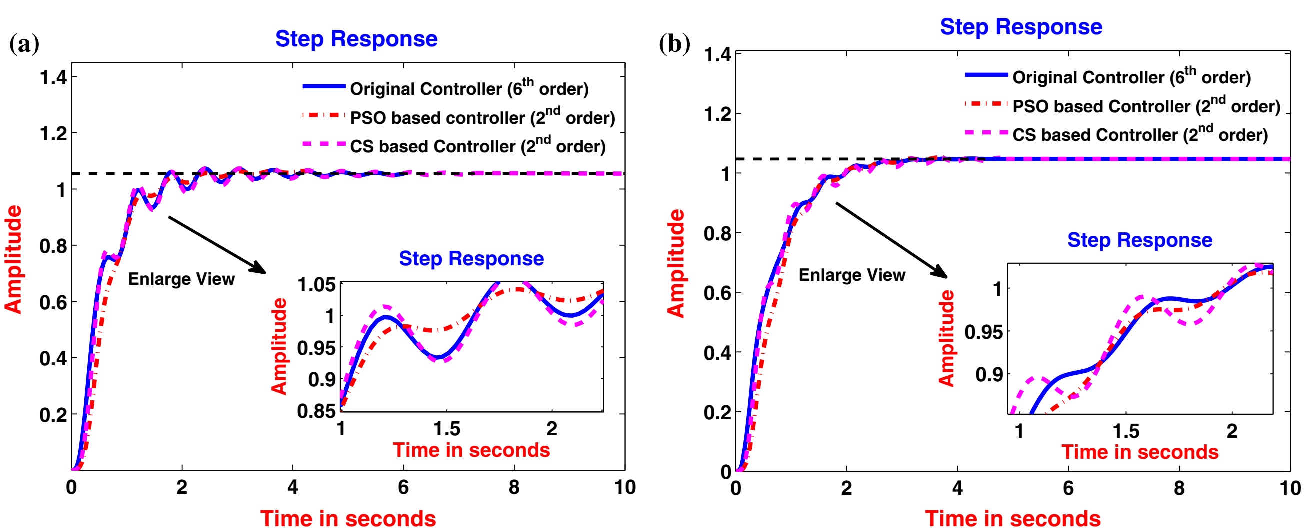

Fig. 6 Time response of original and reduced second order controllers with perturbed TWMR. **a** Case-1, **b** Case-2

Table 3 Performance comparison of closed loop TWMR using second order controllers

Controller	$M_p(\%)$	t_s	t_r	ISE	IAE	ITAE
Proposed controller	0.04436	2.8623	1.1467	0.0031	0.0842	0.1336
PSO based controller (Bui et al. 2010)	0.0468	2.4133	1.1200	0.0140	0.1434	0.2314

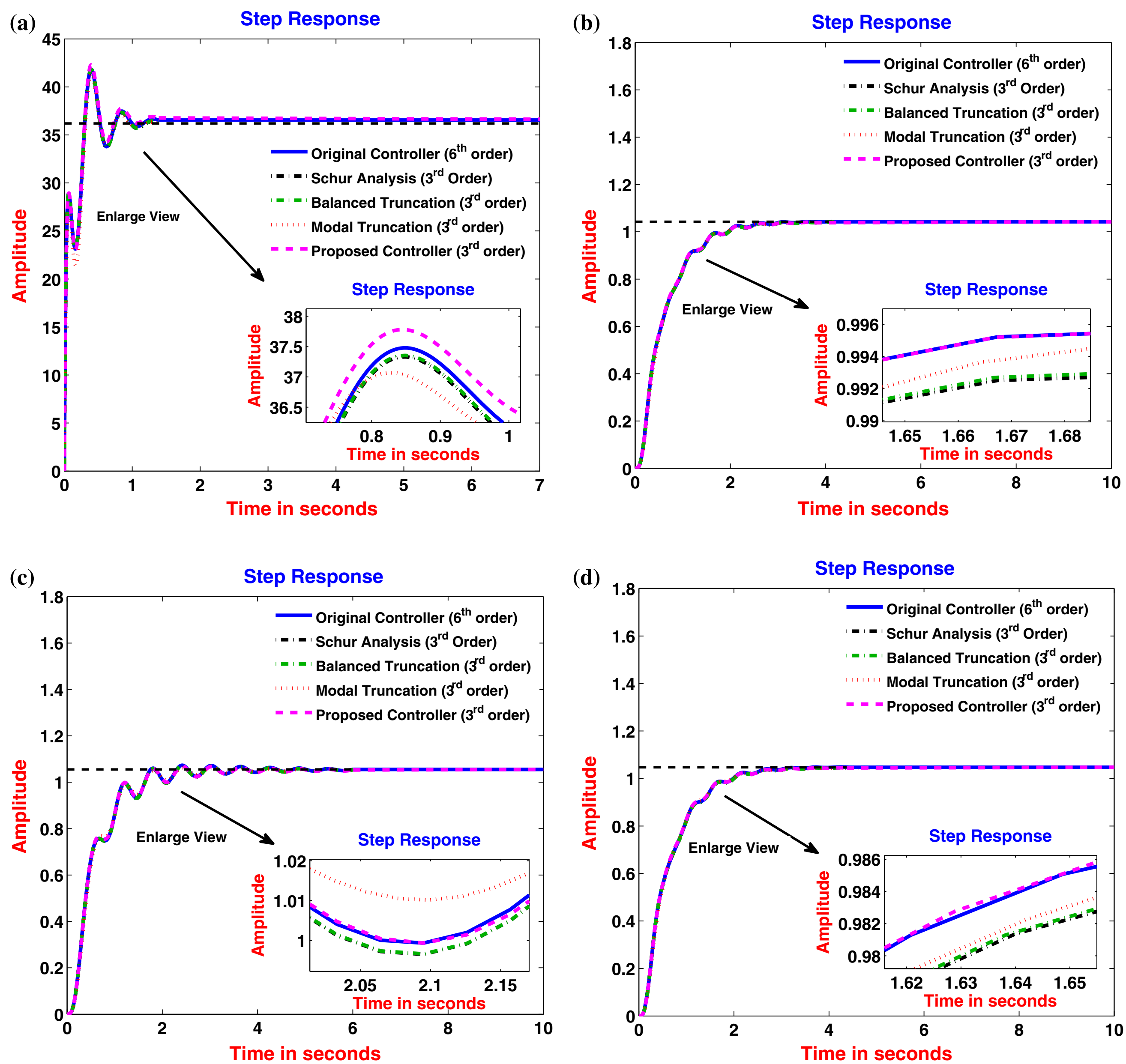

Fig. 7 Time response of original and reduced third order controllers. **a** Without TWMR, **b** with TWMR, **c** with perturbed TWMR Case-1, **d** with perturbed TWMR Case-2

$$G_{cr\_MT}(s) = \frac{1057s^2 + 226.5s + 1.638 \times 10^5}{s^3 + 27.99s^2 + 395.9s + 4521}$$

When subjected to unit step input, the time responses of full and reduced third order controllers are shown in Fig. 7a. It is observed that reduced third order controller obtained by proposed technique is much closer as compared to the controllers designed by other well-known techniques available in the literature. The closed loop step response of two wheeled mobile robot with full and reduced third order controllers is depicted in Fig. 7b. Hence the superiority of the proposed controller can be examined as it exhibits better performance as compared to other controllers.

The comparative analysis of reduced order controllers in terms of error indices is depicted in table 4 from which it is clear that the proposed reduced order controller gives lowest values of these error indices. Furthermore, to show the powerfulness of the proposed controller, its behaviour is analysed under two different cases of perturbed mobile robot as discussed above. The performance of perturbed mobile robot with full and reduced order controllers for case-1 and case-2 are depicted in Fig. 7c, d respectively. It is found that that

Table 4 Comparison of reduced third order controllers in terms of error indices

Reduction method	ISE	IAE	ITAE
Proposed	0.3531	1.2131	6.9234
Schur analysis (SA) (Nguyen et al. 2013)	1.8000	2.8051	16.009
Balanced truncation (BT) (Moore 1981)	1.6478	2.6514	15.132
Modal truncation (MT) (Liu and Anderson 1989)	2.6026	3.3232	18.966

Table 5 Poles of higher order and reduced order controllers

Poles of higher order controller	Poles of controller reduced by balanced truncation	Poles of controller reduced by model truncation	Poles of controller reduced by schur analysis	Poles of controller reduced by proposed method
-681.74	-26.6843	-19.565	-26.71	-25.1041
-26.71	$-3.539+13.912i$	$-4.211+14.606i$	$-3.535+13.916i$	$-3.6830+14.0186i$
$-3.535+13.916i$	$-3.539-13.912i$	$-4.211-14.606i$	$-3.535-13.916i$	$-3.6830-14.0186i$
$-3.535-13.916i$				
-0.09 and -0.08				

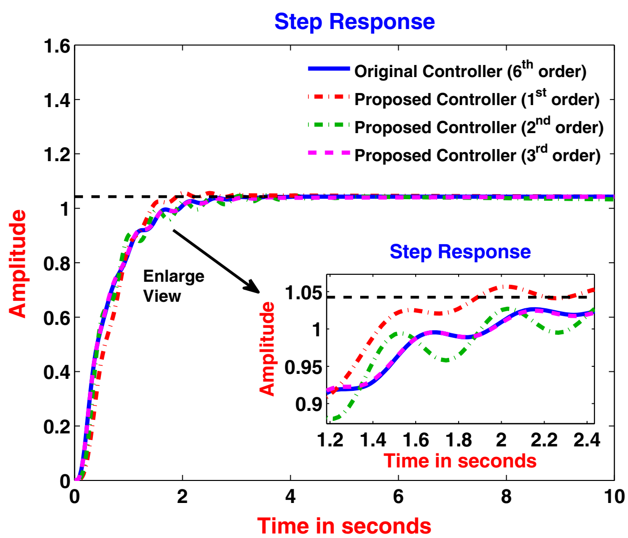

Fig. 8 Comparison of first, second and third order controller

proposed controller exhibits excellent performance for perturbed mobile robot also as compared to other well-known controllers. The poles of higher order and reduced order controllers are tabulated in Table 5 from which it is noticed that the proposed controller preserves the dominant poles of the full order controller. The step resonses of first, second and thrid order controllers obtained by proposed method is shown in Fig. 8. Furthermore, Table 6 depicts the performance comparison of closed loop TWMR using third order controllers in terms of time response specifications and performance indices. It is also clear from this table that the closed loop step response of TWMR with proposed third order controller is much better as compared to other controllers.

Conclusion

In this paper, the concept of reduced order modelling is utilised to design pre-specified structure controller for balancing control of two wheeled mobile robot. The three controllers are designed, namely, first, second and third order. Cuckoo search algorithm is employed to obtain the unknown parameters of the proposed reduced order controllers. The performance of the proposed/designed controllers is analysed in terms of time response specifications and performance indices such as ISE, IAE and ITAE. Two different cases of perturbed mobile robot are also considered to analyse the robustness and powerfulness of the proposed controllers. To show the efficacy of the proposed technique, the performance of the proposed controllers is compared with recently developed controllers such as Schur analysis based controller, GA based controller, PSO based controller and H_∞ controller

Table 6 Performance comparison of closed loop TWMR using third order controllers

Controller	$M_p(\%)$	t_s	t_r	ISE ($\times 10^{-5}$)	IAE	ITAE
Proposed controller	0.0248	2.4040	1.2443	2.9058	0.0138	0.0455
Schur analysis (SA) (Nguyen et al. 2013)	0.0193	2.4270	1.2636	3.1411	0.0143	0.0528
Balanced truncation (BT) (Moore 1981)	0.0198	2.4266	1.2630	3.0449	0.0140	0.0531
Modal truncation (MT) (Liu and Anderson 1989)	0	2.0835	1.2247	17.599	0.0209	0.0517

etc. It is found that the proposed third order controller perform well not only under normal conditions but also, in the presence of parameter uncertainty in the two wheeled mobile robot.

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