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Comparison of fuzzy inference system (FIS), FIS with artificial neural networks (FIS + ANN) and FIS with adaptive neuro-fuzzy inference system (FIS + ANFIS) for inventory control

Prasert Aengchuan[1] · **Busaba Phruksaphanrat**[1]

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Abstract Conventional inventory models mostly cope with a known demand and adequate supply, but are not realistic for many industries. In this research, the fuzzy inference system (FIS) model, FIS with artificial neural network (ANN) model and FIS with adaptive neuro-fuzzy inference system (ANFIS) model in which both supply and demand are uncertain were applied for the inventory system. For FIS model, the generated fuzzy rules were applied to draw out the fuzzy order quantity continuously. The order quantity was adjusted according to the FIS model with the evaluation algorithm for the inventory model. The output of FIS model was also used as data for FIS + ANN and FIS + ANFIS models. The FIS + ANFIS model was studied with three membership functions; trapezoidal and triangular (Trap), Gaussian and bell shape. Inventory costs of the proposed models were compared with the stochastic economic order quantity (EOQ) models based on previous data of a case study factory. The results showed that the FIS + ANFIS_Gauss model gave the best performance of total inventory cost saving by more than 75 % compared to stochastic EOQ model.

Keywords Fuzzy inference system (FIS) · Artificial neural network (ANN) · Adaptive neuro-fuzzy inference system (ANFIS) · Inventory control · Lot-sizing

✉ Prasert Aengchuan
prasert.aengchuan@gmail.com

Busaba Phruksaphanrat
lbusaba@engr.tu.ac.th

[1] Industrial Statistics and Operational Research Unit (ISO-RU), Department of Industrial Engineering, Faculty of Engineering, Thammasat University, Rangsit Campus, Khlong Luang, Pathum Thani 12120, Thailand

Introduction

An inventory system controls the level of inventory by deciding how much to order (the level of replenishment) and when to order (reorder point). The purpose of an inventory system is to make decisions concerning the level of inventory that will effect in a desirable balance between holding inventories and the cost related with them (Meredith and Shafer 2011). The inventory level is difficult to deal with because of the number of factors concerned and uncertain events such as unpredictability of demand and supply. A appropriate policy and control system for each type of product is necessary.

Inventory lot-sizing problems are production planning problems with the purpose of deciding the periods when production should happen and the quantities to be made for meeting demand while reducing production and inventory costs. Since the original lot-sizing model presented by Harris in 1913 (Andriolo et al. 2014), most models focus mainly on deterministic static lot-sizing models. Further work (Sommer 1981; Samanta and Al-Araimi 2001) has developed fuzzy lot sizing models, followed by adaptive neuro-fuzzy inference system (ANFIS) (Samanta and Al-Araimi 2003) to fuzzy inventory lot-sizing models. Recently, several literature reviews of lot-sizing models have been presented (Andriolo et al. 2014; Aloulou et al. 2014; Glock et al. 2014).

Inventory lot-sizing models can be divided into three groups which are deterministic models, stochastic models and fuzzy models as illustrated in Fig. 1. Table 1 shows the contributions of the inventory lot-sizing models.

Deterministic lot-sizing models

All input data of deterministic lot-sizing models are supposed to be available. These models can be classified into two groups, static and dynamic models. For deterministic

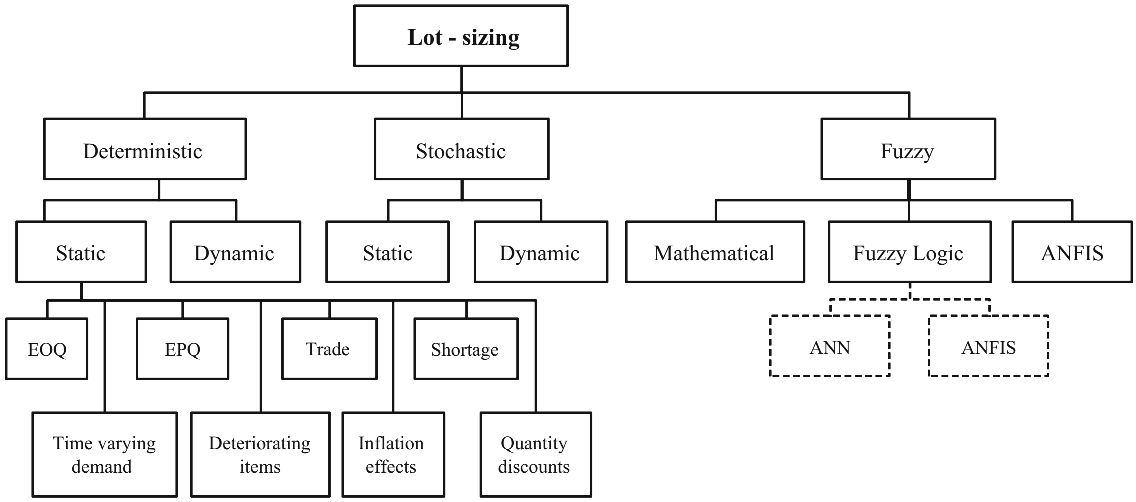

Fig. 1 Classification of inventory lot-sizing models. *EOQ* Economic order quantity, *EPQ* Economic production quantity, *ANN* Artificial neural network, *ANFIS* Adaptive neuro-fuzzy inference system, ⬚ Methods proposed in this research

static lot-sizing models, the original model was known as Economic Order Quantity (EOQ) or square root formula, with the objective to minimize the sum of inventory holding and ordering costs. The EOQ formula was modified (Taft 1918) by adding ratio between demand rate and production capacity and called Economic Production Quantity (EPQ). Since then many extended researches about EOQ and EPQ have been reported.

For deterministic dynamic lot-sizing models, the goal is to minimise the sum of inventory holding and set up costs, but it permits the demand for products to change over time.

Because the deterministic models assume known parameters, most of the existing literature tries to present an optimal solution of the problem while others present some heuristic approaches in order to achieve desirable results for pragmatic conditions. However, in the real world, there are some uncertain parameters that need to be considered.

Stochastic lot-sizing models

Some input data of stochastic lot-sizing models are defined as probability density functions. These models can be classified into two groups, static and dynamic models. Many stochastic static lot-sizing models are based on EOQ model but have different stochastic information such as lead time, demand, supplier capacity, cost, price etc. Heuristics method (Seneyigit and Erol 2010) for stochastic lot-sizing and EPQ models for deteriorating inventory (Chung et al. 2011; Wee and Widyadana 2012) have been proposed.

Dynamic stochastic lot-sizing models were presented to solve the problem of uncertain demand (Kamal and Sculfort 2007) and normally can be solved by the optimization models such as the Wagner-Whitin (WW) algorithm and the heuristic models such as Silver-Meal (SM) method, part period balancing (PPB), lot for lot (L4L) etc. According to fundamental of probability theory, stochastic inventory models are efficient when the input information of models is known precisely and is obtainable (Chen 2011). In a real world situation, supply data may not exist when required because of random capacity of suppliers, uncertain events or seasonal factors. Meanwhile, some of the uncertainties within the inventory system cannot be taken into account properly by using concepts of probability theory (Tanthatemee and Phruksaphanrat 2012).

Fuzzy lot-sizing models

Fuzzy set theory has been applied to unpredictable inventory problems in non-stochastic judgment. These models can be divided into three groups, mathematical, fuzzy logic and ANFIS models.

For fuzzy mathematical lot-sizing models, many models of fuzzy EOQ models have been proposed. Many researches applied fuzzy sets to demand, deterioration rate, defective rate, lead time, etc, but many of these methods are complex and arduous to accomplish.

Fuzzy logic lot-sizing models have been presented (Samanta and Al-Araimi 2001) for fuzzy demand. A fuzzy simulation of a single item inventory system with variable demand to determine the EOQ with uncertain lead time (Yimer and Demirli 2004) was developed. Other fuzzy logic models considered inventory control of fuzzy demand and stock (Rothstein and Rakityanskaya 2006; Chede et al. 2012) and also demand and lead time uncertainties by fuzzy logic (Kamal and Sculfort 2007). A fuzzy continuous inventory control system for a single item with both uncertain demand and supply has been presented (Tanthatemee and Phruksaphanrat 2012) and later determination of

Table 1 The contributions of the inventory lot-sizing models

	References	Lot sizing models							Parameters								Method/Keywords
		Det.		Sto.		Fuzzy			Demand		Supply		Leadtime		Other		
		S	D	S	D	MA	FL	AN	C	U	C	U	C	U	C	U	
1.	Harris (1913)	x							x		x		x		x		Original EOQ formula (square root formula)
2.	Taft (1918)	x							x		x		x		x		Add ratio between demand rate and production capacity (EPQ)
3.	Wagner and Whitin (1958)		x						x		x		x		x		Optimization method, dynamic programming, difficult to understand and more computation effort
4.	Silver and Meal (1973)	x							x		x		x		x		Heuristic for time varying demand rate
5.	Sommer (1981)					x			x		x		x			x	Fuzzy dynamic programming for production scheduling with capacity constraints
6.	Porteus (1986)			x					x		x		x			x	Process deterioration "out of control" uncertain of process
7.	Park (1987)					x			x		x		x			x	Fuzzy cost (holding and ordering cost)
8.	Parlar and Perry (1995)					x			x			x	x		x		Markov chain for uncertain supply
9.	Chen et al. (1996)					x				x	x		x			x	Fuzzy demand, fuzzy cost (ordering, inventory and backorder)
10.	Lee and Yao (1999)					x				x	x		x			x	Fuzzy demand rate, fuzzy production rate
11.	Samanta and Al-Araimi (2001)						x			x	x		x		x		Fuzzy logic control with fuzzy demand
12.	Mondal and Maiti (2002)					x				x	x		x			x	Genetic algorithm for multi-item fuzzy demand and inventory cost
13.	Samanta and Al-Araimi (2003)							x		x	x		x			x	ANFIS, Fuzzy logic control for fuzzy demand and inventory level
14.	Yimer and Demirli (2004)						x			x	x		x		x		Fuzzy simulation for variable demand and uncertain lead time
15.	Rothstein and Rakityanskaya (2006)						x			x	x		x			x	Fuzzy logic model for inventory control of fuzzy demand and stock
16.	Kamal and Sculfort (2007)						x			x	x			x	x		Demand and lead time uncertainties
17.	Handfield et al. (2009)					x				x	x			x		x	Fuzzy model for uncertain demand, lead time, supplier yield and penalty cost
18.	Rong (2010)		x						x		x		x			x	Stochastic cost (ordering cost, holding cost and purchasing cost)
19.	Khan et al. (2010)		x						x		x		x			x	Mathematical for imperfect quality items, defective rate with learning effects
20.	Wang (2010)					x			x			x	x		x		Mathematical for uncertain supply
21.	Seneyigit and Erol (2010)		x							x	x		x			x	Demand and price uncertainties with service-level constraint
22.	Feng (2010)				x				x			x				x	Dynamic pricing and decisions under supply capacity uncertainty
23.	Sana (2011)			x						x	x		x			x	Random sales price
24.	Chung et al. (2011)			x					x		x		x			x	Random unavailability of maintenance time
25.	Guan (2011)				x					x	x		x			x	Dynamic programming with time varying capacity constraint
26.	Wee and Widyadana (2012)			x					x		x		x			x	Stochastic preventive maintenance time
27.	Guillaume et al. (2012)					x				x	x		x			x	Fuzzy demand to find robust procurement plan
28.	Chede et al. (2012)						x			x	x		x			x	Fuzzy logic for inventory control with fuzzy demand and stock on hand
29.	Tanthatemee and Phruksaphanrat (2012)						x			x			x		x		Fuzzy logic for inventory control with fuzzy demand and supply

Table 1 continued

References	Lot sizing models							Parameters								Method/Keywords
	Det.		Sto.		Fuzzy			Demand		Supply		Leadtime		Other		
	S	D	S	D	MA	FL	AN	C	U	C	U	C	U	C	U	
30. Lenart et al. (2012)							×		×	×		×			×	ANFIS to find the optimal storage level and cost with fuzzy demand and feedback information
31. Kang and Lee (2013)				×					×	×		×			×	Mix integer programming and heuristic dynamic programming for multiple suppliers, quantity discount and safety stocks
32. Mahata and Goswami (2013)					×				×	×		×			×	Fuzzy models for items with imperfect quality and shortage backordering under crisp and fuzzy decision variables
33. Aengchuan and Phruksaphanrat (2013)						×			×		×	×		×		Fuzzy logic design range for inventory control with fuzzy demand and supply
34. Grubbstrom (2014)		×						×		×		×		×		Dynamic EPQ model with net present value (NPV)
35. Li and Thorstenson (2014)				×					×	×		×			×	Joint lot-sizing, pricing of stochastic demand and capacity constraints
36. Lee et al. (2014)			×						×	×		×		×		Random demand, transient shortage during production stage
37. Ullah and Kang (2014)	×							×		×		×		×		Mathematical model for work in process inventory

Lot sizing models *Det.* Deterministic, *Sto.* Stochastic, *S* Static, *D* Dynamic, *MA* Mathematical, *FL* Fuzzy Logic, *AN* ANFIS. Parameters *C* Certain, *U* Uncertain

the design range and the effect of trend demand (Aengchuan and Phruksaphanrat 2013). This model saved inventory costs greatly when compared with the conventional stochastic EOQ model, Silver Meal model and Wagner Whitin Model.

For ANFIS lot-sizing models, adaptive neuro-fuzzy inference system and fuzzy logic control have been proposed for fuzzy demand and inventory level (Samanta and Al-Araimi 2003). The ANFIS approach to adaptive inventory control has been applied to single input – single output (Lenart et al. 2012). The set of input values were determined by the expected values of the demand.

Fuzzy mathematical models are complicated and difficult for decision makers to implement in real life situations but fuzzy logic tools are not complicated to implement and modify. However, fuzzy tools should achieve the same as or better than other soft approaches (Azedegan et al. 2011). These characteristics have made fuzzy logic and tools associated with its use quite popular in tackling manufacturing related challenges. Inventory problem is a crucial problem in manufacturing system, which can cause a lot of wastes. Most models are complicated and difficult for the practical use. Many researches focus on a fuzzy mathematical for inventory lot-sizing problem, but there is limited published work regarding applications of the neuro-fuzzy approach to inventory based on FIS + ANN and FIS + ANFIS. Furthermore, consideration of both fuzzy demand and supply by ANN and ANFIS has not been taken into account. So, this research proposes the integrated methodologies of FIS + ANN model and FIS + ANFIS model for choosing of criteria and developing the model of the practical problem with the fuzzy inputs for both demand and supply to the inventory lot-sizing problem.

Inventory system

The relevant elements associated with how much to order are normally concerned with inventory costs and inventory lot-sizing models. The inventory cost consists of holding cost, ordering cost and shortage cost. All inventory models try to reduce the total inventory costs.

Inventory cost

In making any decision with respect to inventories, the following costs must be considered.

Ordering cost is the fixed costs usually related to the production of a lot inside or the placing of an order outside with a vendor.

Holding (or carrying) cost includes the costs for warehouse, transporting, insurances, pilferage, fragility, obsolescence.

Shortage cost. This is usually the sum of the lost profit. It occurs when customer demand cannot be met due to inadequate inventory. There is a trade-off between holding stock to fulfill demand and the costs effecting from stock out.

The case study model considers the total inventory costs as the summation of ordering cost, holding cost and shortage cost.

$$TC = mC_o + C_h Q_h + C_s Q_s \tag{1}$$

where TC is the total inventory cost. m is *the* number of ordering per period. C_o is ordering cost per time. C_h is the holding cost per unit per period. C_s is the shortage cost per unit per period. Q_h is the holding quantity per period and Q_s is the shortage quantity per period.

Static inventory lot-sizing

In a fixed-order-quantity system when inventory approaches a particular level, referred as the reorder point, a fixed quantity is ordered. The EOQ model is extended to the stochastic EOQ model to figure out the problem of unpredictable demand, and is applied when the uncertainties are considered as random that can coped with probability theory. Supposing that the demand is expressed by a normal distribution, determination of how much to order can be calculated (Kamal and Sculfort 2007) by the following equation.

$$Q^* = EOQ = \sqrt{\frac{2C_o H\overline{d}(C_h + C_s)}{C_h C_s}}, \tag{2}$$

where $\overline{d}$ is the average weekly demand. H is the total length of the planning horizon (number of weeks).

If demand is uncertain, safety stock must be added into the reorder point and the reorder point and the safety stock can be calculated.

$$R = \overline{d}L + SS, \tag{3}$$

$$SS = z\sigma_d\sqrt{L}, \tag{4}$$

where R is the unit of reorder point. *SS* is safety stock. L is lead time. σ_d is the standard deviation of weekly demand. z is the number of standard deviations according to the service level probability.

Fuzzy inference system (FIS)

Fuzzy inference system is a system that is applied to govern the connection between the input and output variables of a system as shown in Fig. 2. There are three distinct types of fuzzy inference systems: Mamdani-type, Sugeno-type and Tsukamoto-type (Castillo and Melin 2008). The main difference between Mamdani and Sugeno resides in the consequence of fuzzy rules. Mamdani-type uses fuzzy sets as rule consequence whereas Sugeno-type uses linear functions as rule consequence. For Tsukamoto-type, the consequent of each fuzzy rule uses a monotonical membership function. In this research Mamdani-type is used.

In a fuzzy inference system, the crisp inputs are converted into fuzzy inputs by using fuzzification interface. After fuzzification the rule bases are developed. The rule bases and the database are mutually referred to as the knowledge base. Defuzzification is applied to transform the fuzzy value to the real life value which is the output. FIS is implemented in various applications for both management and manufacturing (Kovac et al. 2013; Nasrollahzadeh and Basiri 2014; Guner and Yumuk 2014; Camastra et al. 2015; Kocygit 2015).

In this research, fuzzy logic toolbox of MATLAB was applied to the Fuzzy Inventory System (FIS) model to compute order quantity in any time period. The flow chart of all parameters of the inventory system model is illustrated in Fig. 3. The two fuzzy input variables are demand (D_i) and supply (S_i). The output variable is order quantity (Q_i), and is described by linguistic variables. Then the output is entered to the evaluation algorithm as shown in Fig. 4.

From Fig. 4, the inventory levels, which are the beginning inventory (I_{b_i}) and the end inventory (I_{e_i}) can be determined by output variable (Q_i) and reorder point (R). Then the inventory costs, which are ordering cost (C_{o_i}), holding cost (C_{h_i}) and shortage cost (C_{s_i}) can be calculated by ordering quantity (Q_{o_i}), holding quantity (Q_{h_i}) and shortage quantity (Q_{s_i}), respectively. The total cost per period is determined by the summation of the inventory costs. This fuzzy logic model then generates for the next period and follows this flow chart for each period ($i = 1, 2, 3, \ldots, n$). Then the total inventory cost of the model is the summation of the total inventory costs of all periods.

Artificial neural networks (ANN)

Artificial neural networks consist of a number of interlinked cells as neurons with weights running coincidently to initiate artificial intelligence. ANN composes of three layers: input, hidden and output layers. The input and output layers consist of a set of neurons expressing input and output variables. The hidden layer passes the data it receives from the input layer, and transmits a response to the output layer. There is no speculative limit on the number of hidden layers but generally there is just one or two (Sumathi and Paneerselvam 2010). The output layer receives all responses from the hidden layer and generates an output vector. Each layer has a fixed number of processing elements (neurons) which are linked with adjustable weights. These weights are adjusted during the training process until the error is decreased greatly and is acceptable for a specific task. ANN is trained by an appropriate algorithm for a particular problem. Even though a

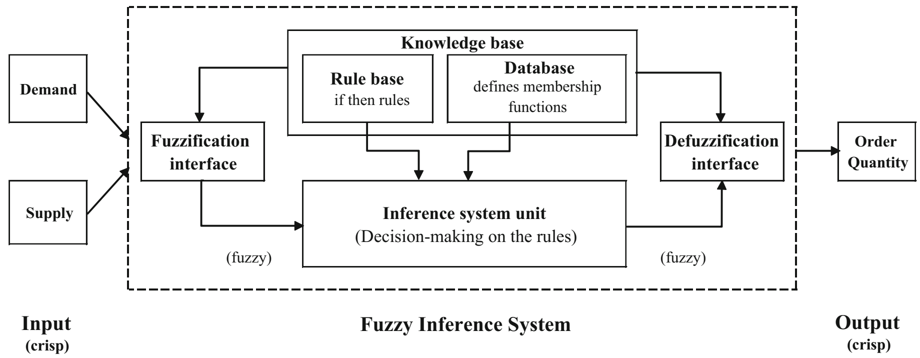

Fig. 2 A scheme of inference fuzzy inventory system

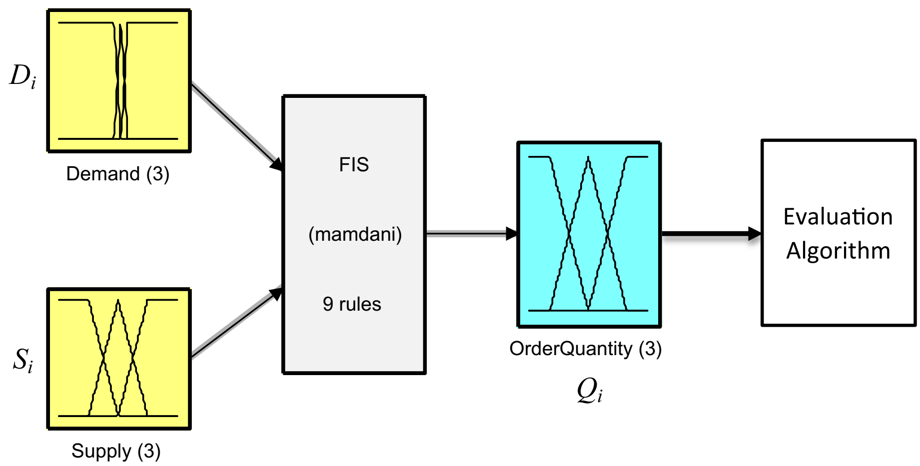

Fig. 3 The flow chart of the FIS model

number of training algorithms are convenient, the most well-known is feed-forward back propagation algorithm (Kiran and Rajput 2011). The output of each neuron is computed by multiplying its inputs by a weight vector, summing the results, and adding an activation function to the sum.

$$y = F\left(\sum_{k=1}^{l} w_k x_k + b_k\right) \tag{5}$$

where, F is the activation function, l is the number of neurons in the consecutive layer, w_k is the weight of the respective connection, and b_k is the bias for the neuron. F is ordinarily linear, step, threshold, logarithmic sigmoid (logsig) or hyperbolic tangent sigmoid (tansig) function (Razani et al. 2013). ANN is implemented in various applications such as forecasting of a ground-coupled heat pump performance (Esen et al. 2008a, b), modelling of a solar air heater (Esen et al. 2009), autoregressive control chart pattern recognition (Yang and Zhou 2013), and other applications (Kuo C. et al. 2014; Kuo R. et al. 2014; Tsai and Luo 2014; Jha et al. 2014; Kocyigit 2015; Wang et al. 2015).

Adaptive neuro-fuzzy inference system (ANFIS)

ANFIS unlike FIS, automatically creates sufficient rules concerning input and output data, and uses benefit of the learning capability of neural networks. It is currently one of the effective tools used for pattern recognition, system identification and can generate precise models of systems. This approach does not need expert opinion for modelling and training a system.

Although various applications of the ANFIS have been applied (Azizi et al. 2013; Guneri et al. 2011; Melin et al. 2012), there are few researches applying inventory control in production systems. Jang first initiated the ANFIS method by embedding the fuzzy inference system into the structure of adaptive networks (Jang 1993). An ANFIS provides the mapping relationship between the input and output data by utilizing hybrid learning method to find out the optimal distribution of membership functions (Ying and Pan 2008). In the ANFIS architecture, ANN learning algorithms are applied to define the parameters of fuzzy inference system. A typical architecture of ANFIS is shown in Fig. 5 for modeling of function $f(x, y)$. The round nodes describe nodes that are fixed, whereas the rectangular nodes are nodes that have parameters to be learnt or called adaptive nodes. For simplicity, consider a FIS with two inputs (x, y) and one output (f). In addition, the rule base of FIS includes two fuzzy if-then rules of Takagi-Sugeno type. The two rules can be represented as:

Rule 1: if x is A_1 and y is B_1, then $f_1 = p_1x + q_1y + r_1$

Rule 2: if x is A_2 and y is B_2, then $f_2 = p_2x + q_2y + r_2$

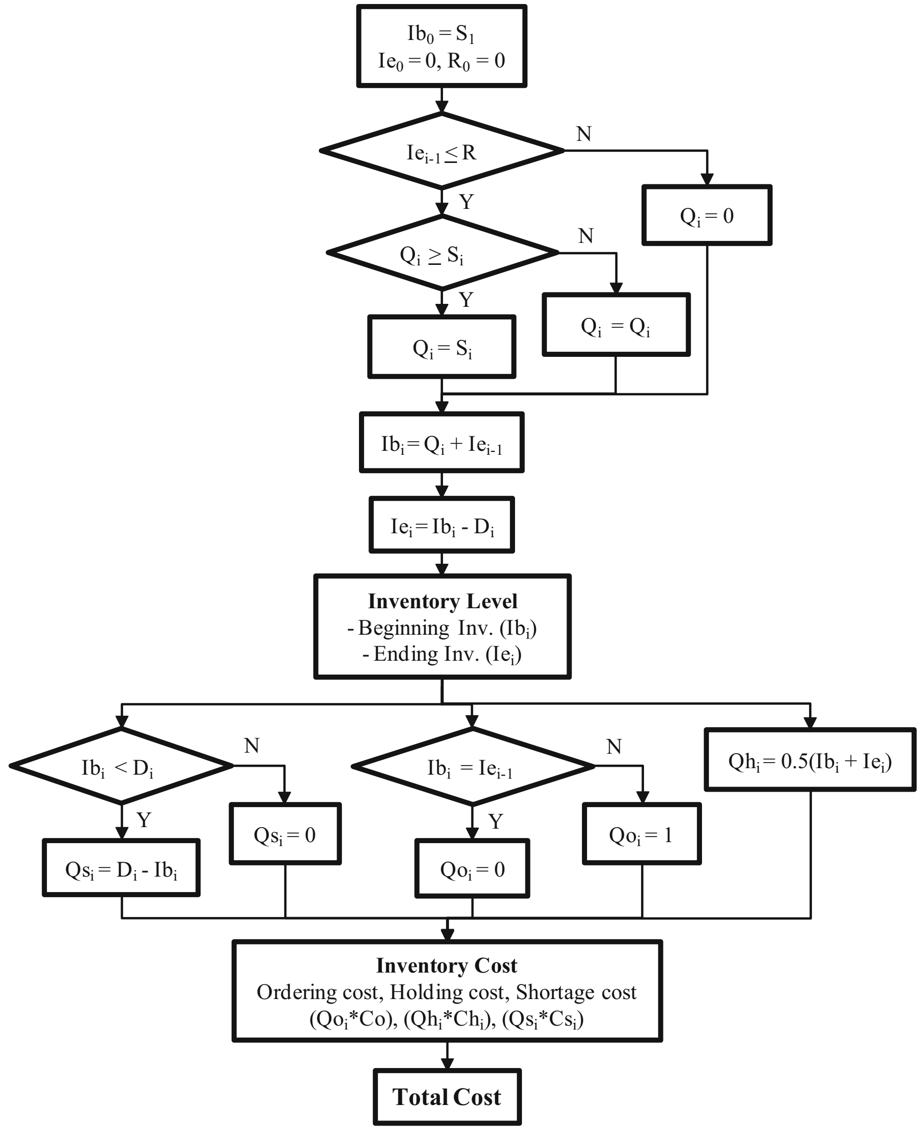

Fig. 4 The evaluation algorithm of the inventory system model. Ib_0 The beginning inventory at the end period of last year, Ie_0 The ending inventory at the end period of last year, R_0 Reorder point of last year, R Reorder point of this year, Ib_i The beginning inventory at period *i* of this year, Ie_i The ending inventory at period *i* of this year, D_i Demand at period *i* of this year, S_i Supply at period *i* of this year, Q_i Order quantity at period *i* of this year, Qs_i Shortage quantity at period *i* of this year, Qo_i Ordering quantity at period *i* of this year, Qh_i Holding quantity at period *i* of this year, Cs_i Shortage cost at period *i* of this year Co Ordering cost of this year, Ch_i Holding cost at period *i* of this year

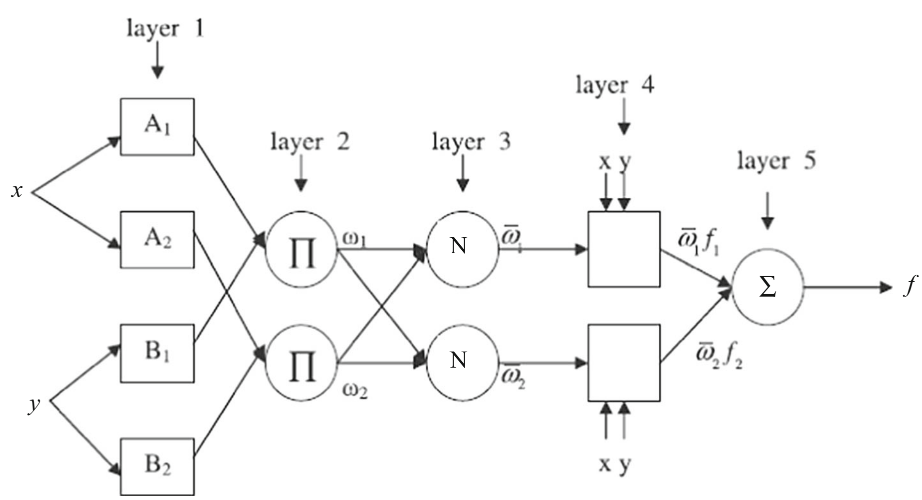

Fig. 5 ANFIS architecture with two rules

where A_i, B_i (i=1, 2) are fuzzy sets in the precursor, and p_i, q_i, r_i $(i = 1, 2)$ are the design parameters that are decided during the training process.

Layer 1: Input nodes. Every node i in this layer is rectangular node with a node function as Eq. (6):

$$O_i^1 = \mu_{A_i}(x), \quad i = 1, 2 \quad O_i^1 = \mu_{B_i}(y), \quad i = 1, 2 \tag{6}$$

where x, y are the crisp inputs of node i, and A_i, B_i are the linguistic labels identified by membership functions, $\mu_{A_i}(x)$ and $\mu_{B_i}(y)$, respectively.

Layer 2: Rule nodes. Every node in this layer expresses the firing strength of a rule by multiplying the entering signals and sending the product out as Eq. (7):

$$O_i^2 = \omega_i = \mu_{A_i}(x) \times \mu_{B_i}(y), \quad i = 1, 2. \tag{7}$$

Layer 3: Average nodes. The i-th node in this layer computed the average proportion of the i-th rule's firing strength.

$$O_i^3 = \varpi_i = \frac{\omega_i}{\omega_1 + \omega_2}, \quad i = 1, 2 \tag{8}$$

where ϖ_i is taken as the normalized firing strength.

Layer 4: Consequent nodes. The node function in this layer is expressed by Eq. (9):

$$O_i^4 = \varpi_i f_i = \varpi_i(p_i x + q_i y + r_i), i = 1, 2 \tag{9}$$

where ϖ_i is the output of layer 3, and $\{p_i; q_i; r_i\}$ is the parameter set. Parameters in this layer are specified to the consequent part of the Segeno fuzzy model.

Layer 5: Output nodes. The single node in this layer calculates the overall output as the total of all entering signals. Consequently, the defuzzification process converts each rule's fuzzy results into a crisp output in this layer.

$$O_i^5 = \sum_{i=1}^{2} \varpi_i f_i = \frac{\varpi_1 f_1 + \varpi_2 f_2}{\varpi_1 + \varpi_2}, \quad i = 1, 2 \tag{10}$$

It is noticed from the ANFIS structure that when the values of the premise parameters are fixed, the overall output can be represented as:

$$\begin{aligned} f &= (\varpi_1 x)p_1 + (\varpi_1 y)q_1 + (\varpi_1)r_1 \\ &\quad + (\varpi_2 x)p_2 + (\varpi_2 y)q_2 + (\varpi_2)r_2 \end{aligned} \tag{11}$$

ANFIS combines the gradient descent method and the least square methods to train parameters. Functional signals go onward until layer 4. Then, the resulting parameters are controlled by the least squares method to minimize the error. Furthermore, the assumption parameters are improved by the gradient descent in the backward pass. ANFIS is implemented in various applications such as modelling a ground-coupled heat pump system (Esen et al. 2008c, d, e), predicting the performance of a refrigeration system (Hosoz et al. 2011), applying for an industrial robot manipulator (Chaudhary et al. 2014), and other applications (Fragiadakis et al. 2014; Yang and Entchev 2014; Gokulachandran and Mohandas 2015; Phootrakornchai and Jiriwibhakorn 2015).

Performance parameters

The performance of the models can be corroborated with the following functions: the coefficient of determination (R^2), the root mean squared error (*RMSE*) and the mean absolute error (*MAE*) as described in Eqs. (12), (13) and (14).

$$R^2 = 1 - \frac{\sum_{i=1}^{n} (A_i - P_i)^2}{\sum_{i=1}^{n} \left(A_i - \overline{A}_i\right)^2} \tag{12}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (A_i - P_i)^2}{n}} \tag{13}$$

$$MAE = \frac{\sum_{i=1}^{n} |A_i - P_i|}{n}, \tag{14}$$

where P_i is the predicted values. A_i is the observed values. $\overline{A}_i$ is the average of observed set. n is the number of datasets.

R^2 represents how much the variability in dependent variables can be interpreted by independent variables, which have value between zero and one. A value for R^2 approach to one indicates a good fit of predicting model and a value approach to zero indicates a poor fit. *MAE* would expose if the results undergo from a bias between the actual and predicting datasets. *RMSE* is a measure applied to compute the deviation between values predicted by a model and the values observed. *RMSE* and *MAE* are non-negative numbers with no upper bound and can be zero only for an ideal model.

Industrial application

The problem of inventory control has been investigated by using a case study of a furniture company in Thailand. The company is a made-to-order manufacturer that produces three main products which are door frames, stairs and plywood doors. Supply and demand of their products are both uncertain. The materials are imported from neighbouring countries and consist of timber woods, shorea obtusa woods, rubber woods and hopea woods. The main materials are timber woods. Availability of these materials is uncertain due to the amount of timber woods based on climate, rainfall and proveniences of supply. Demand varies randomly but both demand and supply can be described by a normal distribution. Presently, a high stock level is maintained to secure against

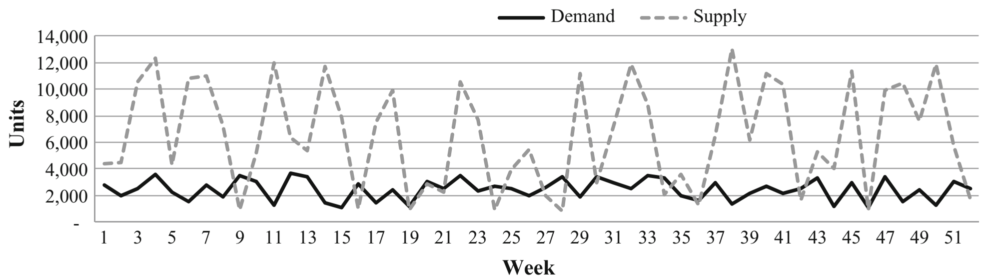

Fig. 6 Fluctuation of demand and supply in 52 weeks. *Note*: Demand: mean = 2452 units, SD = 776 units, Supply: mean = 6487 units, SD = 3921 units

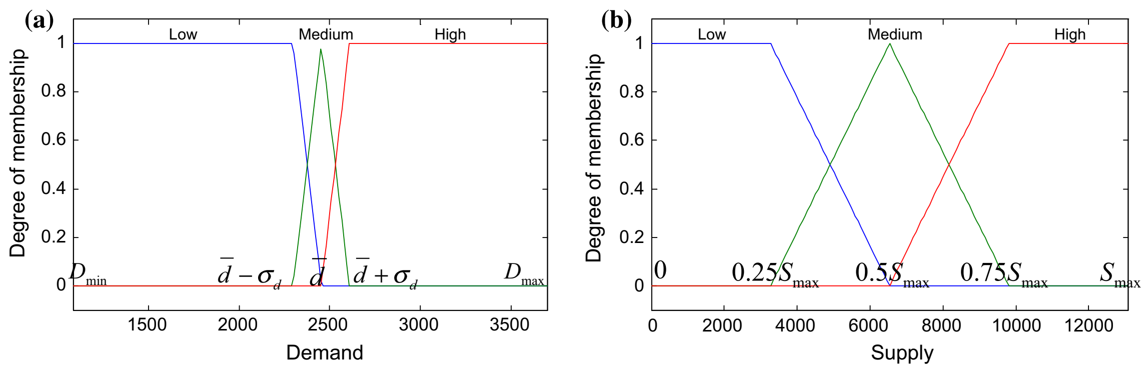

Fig. 7 Input membership functions. **a** Demand. **b** supply

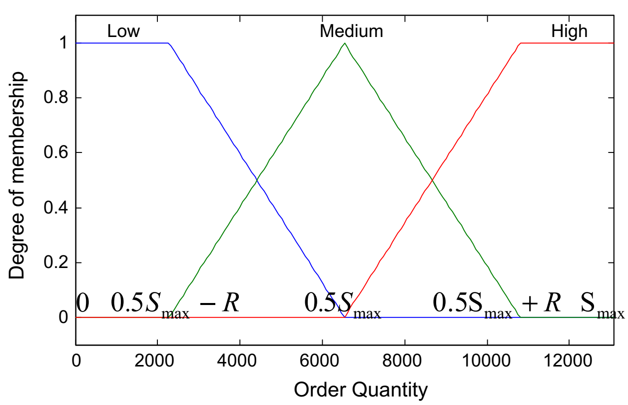

Fig. 8 Output membership functions, order quantity (μ_{Q_i})

Table 2 The relationship of membership functions for each fuzzy rule

Rule	x_1	x_2	y_1
1	Low	Low	Medium
2	Low	Medium	Low
3	Low	High	Medium
4	Medium	Low	Low
5	Medium	Medium	Medium
6	Medium	High	High
7	High	Low	Medium
8	High	Medium	High
9	High	High	High

insufficiency. The company guarantees to serve clients with over 95 % of service level efficiency. However, shortages still occur and the total inventory cost is high. These are serious problems for the company. So, FIS model, FIS + ANN model and FIS + ANFIS model were proposed to reduce the total inventory cost and inventory levels, and the results were compared with the conventional stochastic EOQ model. Fifteen data sets from the distribution of historical data of year 1999 to 2014 were investigated. The data sets of the year before 2010 were adjusted due to their tendency demand and for more realistic of the input data sets. From Fig. 6, it can be visualized that the supply of material fluctuated exceedingly compared with demand and caused a shortage in some periods. Ordering cost, holding cost and shortage cost per unit per period of the case study factory were $100, $0.05 and $59, respectively.

FIS for the lot-sizing problem

Fuzzy inputs

Fuzzy inputs are demand and supply. For systems with consequential dynamic variation in a short period of time, triangular or trapezoidal membership functions should be used (Bai and Wang 2006). Fuzzy demand and fuzzy supply, represented by membership functions, μ_{D_i} and μ_{S_i}, respectively, were determined based on inspection and

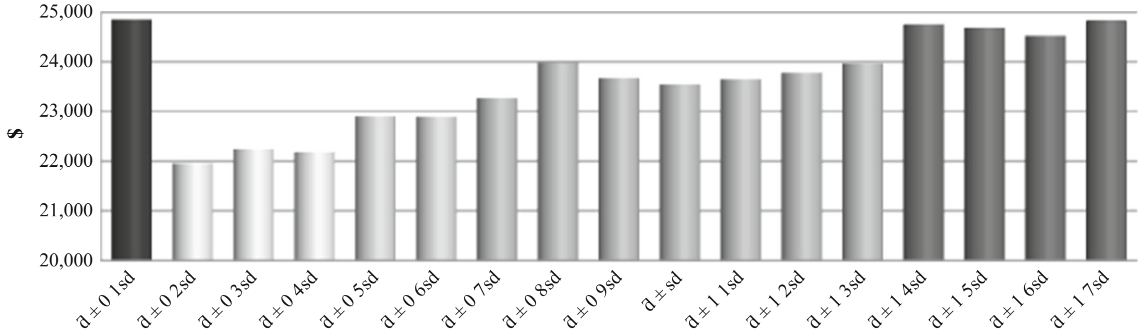

Fig. 9 The average total cost of 15 data sets of the proposed FIS model

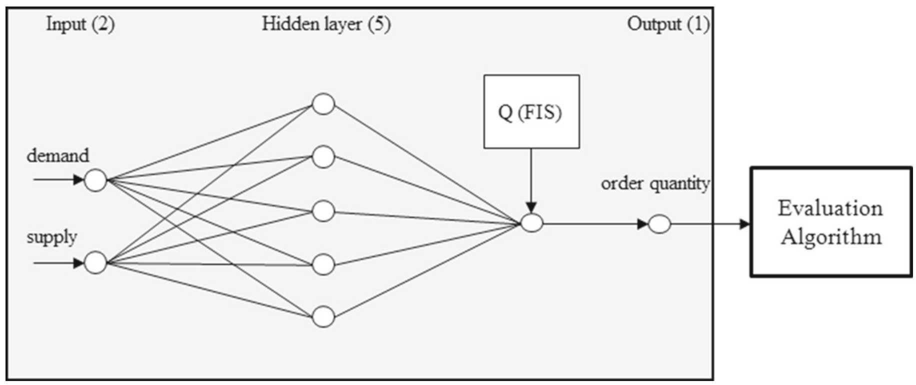

Fig. 10 The flow chart of FIS + ANN model

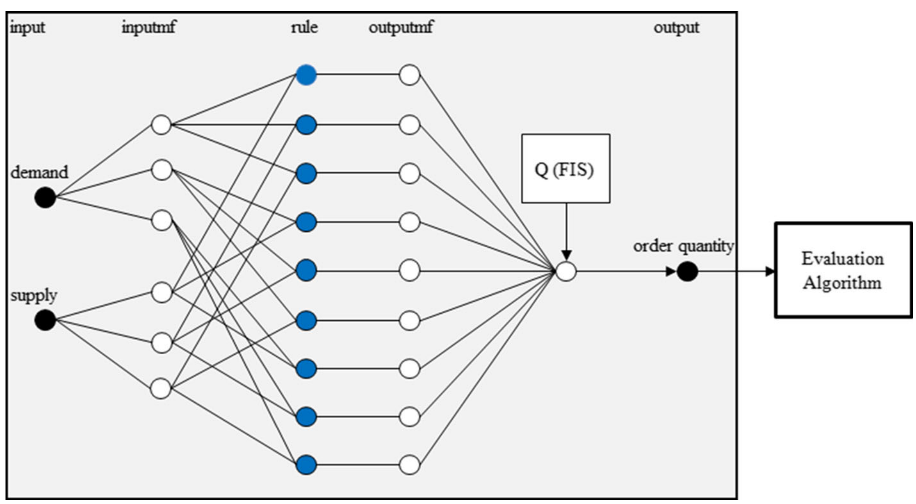

Fig. 11 The flow chart of FIS + ANFIS model

verifying of historical data. Both of them are supposed to be described by three linguistic values; low, medium, high.

The universe of discourse of demand input space was created within the range [D_{min}, D_{max}], where D_{min} and D_{max} are the minimum and maximum demand that had been ordered respectively. Demand membership functions rely on the parameters (D_{min}, $\bar{d}-\sigma_d$, $\bar{d}$, $\bar{d}+\sigma_d$, D_{max}) as presented in Fig. 7a. The parameters were created depending on the attributes of a normal distribution of unpredictable demand of the factory. Supply was created on actual data within the range [0, S_{max}], where S_{max} is the maximum supply from the recent suppliers. Membership functions are presented in Fig. 7b. The parameters (0, 0.25S_{max}, 0.5S_{max}, 0.75S_{max}, S_{max}) were applied for supply linguistic values.

Fuzzy outputs

The fuzzy output, order quantity, is constructed and represented by membership functions, μ_{Q_i}. Fuzzy order quantity is supposed to have three linguistic values; low, medium and high, represented by (0, 0.5S_{max}-R, 0.5S_{max}, 0.5S_{max}+R, S_{max}) as presented in Fig. 8, with universe of discourse interval [0, S_{max}].

Phase 1: ANFIS Input selections	
Step 1:	Determine MFs of inputs and outputs.
Step 2:	Split data into three parts as training, testing and checking.
Phase 2: Building and solving ANFIS Model	
Step 3:	Load training, testing and checking data.
Step 4:	Select grid partition method.
Step 5:	Determine type of MFs, number of MFs, and type of output MFs.
Step 6:	Choose MFs optimization method.
Step 7:	Set number of epochs.
Step 8:	Start train and get the training error.
Phase 3: Evaluating and analysing results of ANFIS model	
Step 9 :	Test the trained model with testing and checking data.
Step 10:	View result and adjusted the generated rules or MFs.
Step 11:	Apply to fuzzy inventory system model and calculate total costs.
Step 12:	Calculate predicted accuracy.

Fig. 12 Algorithms based on ANFIS for inventory system

Fuzzy rules

The fuzzy rule is described by a sequence of IF-THEN, directing to algorithms representing what activity or output should be selected in terms of the presently observed information, which involves both input and feedback if a closed-loop control system is used. The guidance to create or construct a set of fuzzy rules is originated on a human being's knowledge or experience, which depends on each veridical application. A fuzzy IF-THEN rule relates to a condition represented using linguistic variables and fuzzy sets to an output or a conclusion. This IF-THEN rule is extensively applied by the fuzzy inference system to calculate the degree to which the input data matches the stipulation of a rule. Since the outputs, order quantity and reorder point are fuzzy sets, a Mamdani- type inference system is chosen here for estimating and aggregating the fuzzy rules. The IF-THEN rule can be mathematically represented, as proposed by Mandani and Assilian (1975), by Cartesian product of the fuzzy inputs, $x_1 \times x_2$. The relationship between demand x_1, supply x_2 (IFs) and order quantity y_1 (THEN) are represented by 9 rules as shown in Table 2.

By using the max-min compositional operation, the fuzzy reasoning of these rules produces fuzzy outputs. Fuzzy order quantity ($\mu_{Q_i}(y_1)$) can be represented as

$$\mu_{Q_i}(y_1) = (\mu^1_{D_i}(x_1) \wedge \mu^1_{S_i}(x_2) \vee \ldots (\mu^n_{D_i}(x_1) \wedge \mu^n_{S_i}(x_2)), \quad (15)$$

where $\wedge$ is the minimum operation and $\vee$ is the maximum operation. D_{i}, S_i and Q_i are fuzzy subsets identified by the analogous membership functions, i.e., $\mu_{D_i}, \mu_{S_i}, \mu_{Q_i}$.

Actually, the fuzzy output is also a linguistic variable, and this linguistic variable needs to be transformed to the crisp variable through the defuzzification process. For this case study, the central of gravity method is chosen to convert the fuzzy inference output into non-fuzzy values of order quantity, y_1^*. Define rule number as n. The crisp values of order quantity are calculated as

$$y_1^* = \frac{\sum_{n=1}^{9} y_1(\mu^n_{Q_i}(y_1))}{\sum_{n=1}^{9} \mu^n_{Q_i}(y_1)}, \quad \text{for } i = 1, 2, \ldots, n \quad (16)$$

Designing of input parameters

The proposed FIS model focuses on demand variation by modifying the input membership function parameters with the designed universe of discourse of demand input space within the interval $[\mathrm{D_{min}}, \mathrm{D_{max}}]$. The demand membership function parameters are selected between $(\mathrm{D_{min}}, \bar{d} - 0.1\sigma_d, \bar{d}, \bar{d} + 0.1\sigma_d, \mathrm{D_{max}})$ to $(\mathrm{D_{min}}, \bar{d} - 1.7\sigma_d, \bar{d}, \bar{d} + 1.7\sigma_d, \mathrm{D_{max}})$. The first, the third and the fifth parameters are fixed because they are lower bound, midpoint and upper bound of the demand data. The second and the fourth parameters of the multiplier parameters of demand standard deviation (σ_d) are adjusted from 0.1 to 1.7. Fig. 9, compares the calculated average total costs of 15 data sets. The lowest average total cost is at $(\mathrm{D_{min}}, \bar{d} - 0.2\sigma_d, \bar{d}, \bar{d} + 0.2\sigma_d, \mathrm{D_{max}})$ or in abbreviated format denoted as $(\bar{d} \pm 0.2\sigma_d)$. Therefore in this case study, the recommended range of the membership function parameters for applying the FIS model should be between the range of $(\mathrm{D_{min}}, \bar{d} - 0.2\sigma_d, \bar{d}, \bar{d} + 0.2\sigma_d, \mathrm{D_{max}})$ and $(\mathrm{D_{min}}, \bar{d} - 0.4\sigma_d, \bar{d}, \bar{d} + 0.4\sigma_d, \mathrm{D_{max}})$.

FIS with ANN for the lot-sizing problem

In this research, the two layer feed-forward with a back propagation learning algorithm was used for the inventory model. The flow chart of FIS + ANN model is shown in Fig. 10. The input data consisted of 52 demand and supply quantities. The output data from FIS model was used as the target data to define the ANN output. To determine with ANN, 42 data

Table 3 The K-fold cross validation results of each model

	FIS + ANN	FIS + ANFIS_Trap	FIS + ANFIS_Gauss	FIS + ANFIS_Bell
R^2	0.975	**0.978**	0.946	0.914
RMSE	377	**339**	502	670

Best performance results are highlighted in bold

Table 4 The comparison of statistical values of 15 data sets for each model

Q	Data set	FIS + ANN	FIS + ANFIS Trap	FIS + ANFIS_Gauss	FIS + ANFIS Bell
R^2	D1	0.855	0.981	0.944	0.957
	D2	0.768	0.993	0.992	0.918
	D3	0.939	0.997	0.942	0.800
	D4	0.950	0.999	0.921	0.944
	D5	0.563	0.937	0.952	0.929
	D6	0.819	0.969	0.968	0.963
	D7	0.898	0.976	0.964	0.962
	D8	0.766	0.966	0.945	0.949
	D9	0.988	0.976	0.847	0.778
	D10	0.925	0.958	0.930	0.854
	D11	0.887	0.995	0.995	0.955
	D12	0.852	0.994	0.975	0.837
	D13	0.933	0.992	0.987	0.876
	D14	0.925	0.974	0.976	0.887
	D15	0.848	0.966	0.936	0.923
	Avg.	0.861	**0.978**	0.952	0.902
RMSE	D1	852	306	531	464
	D2	1284	222	232	758
	D3	582	137	570	1,087
	D4	579	94	728	609
	D5	1497	562	492	599
	D6	1022	419	425	461
	D7	639	308	375	389
	D8	1101	418	534	519
	D9	260	363	977	1,115
	D10	655	485	628	908
	D11	749	151	151	470
	D12	935	182	371	1,015
	D13	552	189	249	753
	D14	541	316	306	662
	D15	884	419	572	630
	Avg.	809	**305**	476	696
MAE	D1	625	146	332	308
	D2	978	60	155	567
	D3	396	52	417	730
	D4	390	61	592	412
	D5	1,015	308	321	443
	D6	994	235	232	281
	D7	379	76	146	259
	D8	780	236	418	369
	D9	183	92	345	894
	D10	471	280	490	670
	D11	518	72	101	332
	D12	755	108	191	530
	D13	397	111	191	586
	D14	408	144	161	441
	D15	619	214	450	498
	Avg.	594	**146**	303	488

Best performance results are highlighted in bold

Avg. is average

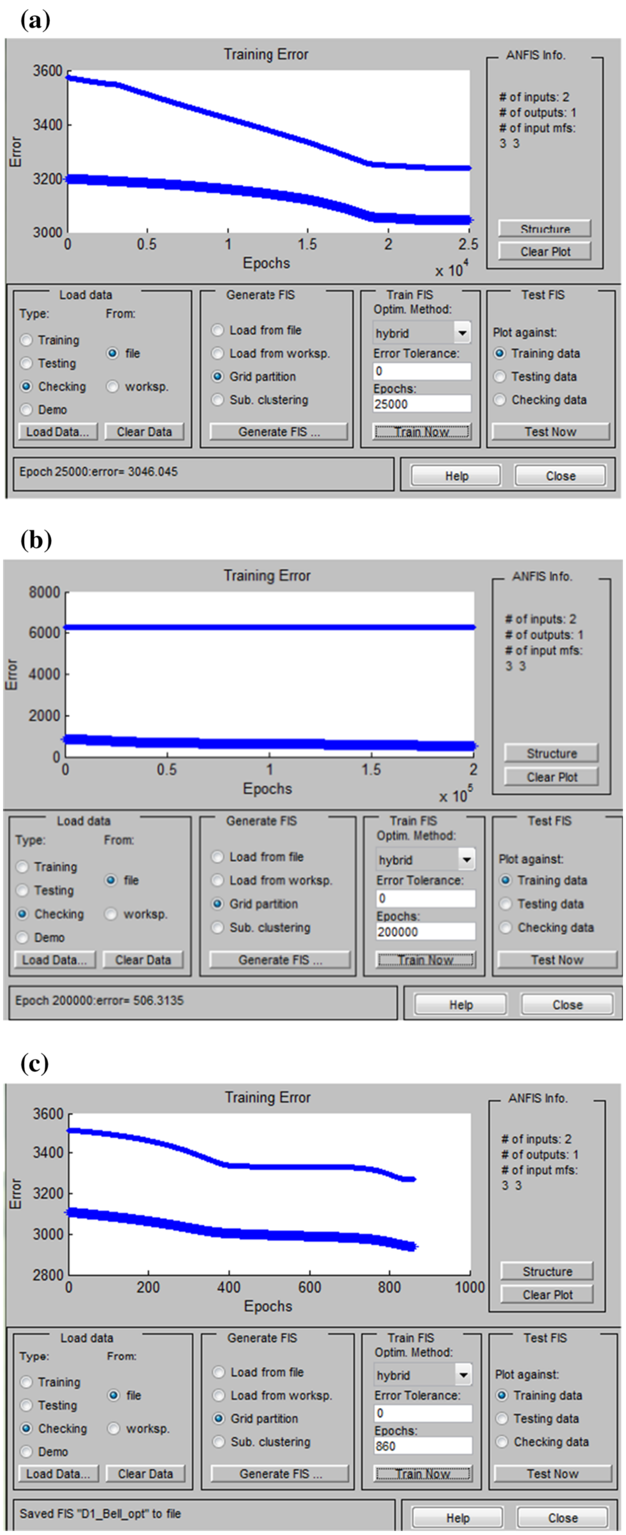

Fig. 13 Training and checking curves of data set 1. **a** FIS + ANFIS_Trap model. **b** FIS + ANFIS_Gauss model. **c** FIS + ANFIS_Bell model

were selected for training, 5 data for validation and 5 data for testing. The number of hidden neurons was defined as 5. The model was trained by using Levenberg-Marguardt with back propagation algorithm. Then the output from ANN model was entered into the evaluation algorithm to compute the total inventory cost of each time period. The total inventory cost of the model is the summation of inventory costs for all periods.

FIS with ANFIS for the lot-sizing problem

The flow chart of FIS + ANFIS model is shown in Fig. 11. The output from FIS model was applied as the training and testing data of ANFIS model. There are two inputs for each of the 3 MFs. Then the 9 rules were applied to normalize data and get the constant output for each data period. The result of ANFIS was entered to the evaluation algorithm to compute the total inventory cost for each time period, then summed to give the inventory cost of the model.

An algorithm of the model based on ANFIS for the inventory system is tabulated in Fig. 12 showing 3 phases. Similar to ANN, to determine with ANFIS, 42 data were selected for training, 5 data for checking or validation and 5 data for testing. Both demand and supply inputs consisted of three membership functions (MFs). The ANFIS models were developed by using the various shapes of input MFs, trapezoidal and triangular (Trap), Gaussian (Gauss), and bell shape (Bell). To determine ANFIS outputs, a constant order quantity (Q) was selected. In MFs optimization, a hybrid of the least-squares method and the back propagation gradient descent method was employed to imitate a given training data set.

K-fold cross validation method

K-fold validation is an assured method, presented to test generalization capability of ANN methods (Good 1999). This method was used for further estimation of the efficiency of the generated FIS + ANN model and FIS + ANFIS models. In this research, the total 15 data sets were separated into 5 even groups, and then the modelling training were implemented 5 times taking one group out at each time to check the model generality. By this method, the candidate model is examined by the all data. The average accuracy of the models was represented by R^2 and *RMSE* as showed in Table 3. The R^2 of all models have achieved greater than 0.9, which verify goodness of the model performance.

Table 5 The total inventory cost of 15 data sets for each model

Inventory cost	Data set	Stochastic EOQ	FIS	FIS + ANN	FIS + ANFIS_Trap	FIS + ANFIS_Gauss	FIS + ANFIS_Bell
Holding cost ($)	D1	16,093	12,775	14,906	12,806	14,852	14,578
	D2	15,944	13,975	15,207	14,191	13,615	13,923
	D3	16,413	15,585	13,581	14,094	13,601	14,755
	D4	18,043	13,679	15,255	13,991	14,609	15,561
	D5	16,617	12,398	13,488	12,943	11,895	13,458
	D6	18,314	15,864	13,708	15,433	16,430	14,926
	D7	17,153	14,676	15,174	14,996	14,955	15,467
	D8	17,269	13,977	14,174	13,589	14,086	14,559
	D9	16,710	14,248	14,839	14,182	14,627	14,447
	D10	15,720	14,587	13,974	14,910	13,813	14,408
	D11	16,275	14,814	13,164	14,903	14,854	14,553
	D12	16,272	14,071	14,894	14,088	13,989	13,737
	D13	16,289	13,771	15,242	13,781	13,922	15,227
	D14	16,441	13,093	14,299	13,139	13,062	13,124
	D15	16,434	14,761	14,778	15,658	15,879	15,880
	Avg.	16,666	**14,152**	14,445	14,180	14,279	14,574
Ordering cost ($)	D1	2000	2500	2300	2500	2600	2600
	D2	2100	2600	2300	2700	2700	2400
	D3	2000	2200	2600	2600	2500	2700
	D4	1800	2500	2000	2500	2400	2200
	D5	2100	3100	2700	2900	3000	2800
	D6	2100	2400	2700	2400	2400	2600
	D7	2000	2500	2600	2500	2500	2400
	D8	2000	2600	2300	2600	2600	2600
	D9	2100	2500	2500	2500	2500	2600
	D10	2300	2500	2600	2600	2500	2500
	D11	2300	2600	2800	2700	2600	2800
	D12	2000	2700	2400	2700	2700	2900
	D13	2000	2300	2200	2400	2500	2000
	D14	2300	2900	2700	2900	2900	2700
	D15	2100	2200	2300	2200	2400	2300
	Avg.	**2080**	2540	2467	2580	2587	2540

Table 5 continued

Inventory cost	Data set	Stochastic EOQ	FIS	FIS + ANN	FIS + ANFIS_Trap	FIS + ANFIS_Gauss	FIS + ANFIS_Bell
Shortage cost ($)	D1	60,593	46,492	37,052	44,722	0	0
	D2	0	0	10,797	0	12,331	0
	D3	0	0	46,728	0	531	0
	D4	0	1,062	7,906	0	0	0
	D5	118	0	120,124	0	0	44,840
	D6	61,242	0	89,857	0	0	0
	D7	0	0	0	0	0	0
	D8	0	0	0	34,338	0	0
	D9	402,498	354	0	6844	0	0
	D10	96,878	13,039	0	0	1239	0
	D11	5,841	0	0	0	0	0
	D12	250,160	17,995	23,305	18,526	18,113	0
	D13	0	0	0	0	10,856	0
	D14	41,182	0	0	0	0	0
	D15	23,069	0	3,599	0	0	0
	Avg.	62,772	5263	22,625	6962	**2871**	2989
Total cost ($)	D1	78,686	61,767	54,258	60,028	17,452	17,178
	D2	18,044	16,575	28,304	16,891	28,646	16,323
	D3	18,413	17,785	62,909	16,694	16,632	17,455
	D4	19,843	17,241	25,161	16,491	17,009	17,761
	D5	18,835	15,498	136,312	15,843	14,895	61,098
	D6	81,656	18,264	106,265	17,833	18,830	17,526
	D7	19,153	17,176	17,774	17,496	17,455	17,867
	D8	19,269	16,577	16,474	50,527	16,686	17,159
	D9	421,308	17,102	17,339	23,526	17,127	17,047
	D10	114,898	30,126	16,574	17,510	17,552	16,908
	D11	24,416	17,414	15,964	17,603	17,454	17,353
	D12	268,432	34,766	40,599	35,314	34,802	16,637
	D13	18,289	16,071	17,442	16,181	27,278	17,227
	D14	59,923	15,993	16,999	16,039	15,962	15,824
	D15	41,603	16,961	20,677	17,858	18,279	18,180
	Avg.	81,518	21,954	39,537	23,722	**19,737**	20,103

Best performance results are highlighted in bold

Table 6 The average total inventory cost of all data sets and cost saving of all models compared to stochastic EOQ model

Inventory cost	Holding cost		Ordering cost		Shortage cost		Total cost	
	($)	Saving (%)	($)	Saving (%)	($)	Saving (%)	($)	Saving (%)
Stochastic EOQ	16,666	–	2080	–	62772	–	81518	–
FIS	**14,152**	**15.1**	2540	−221	5263	91.6	21,954	73.1
FIS + ANN	14,445	13.3	2467	−18.6	22,625	64.0	39,537	51.5
FIS + ANFIS_Trap	14,180	14.9	2580	−24.0	6962	88.9	23,722	70.9
FIS + ANFIS_Gauss	14,279	14.3	2,587	−24.4	**2871**	**95.4**	**19,737**	**75.8**
FIS + ANFIS_Bell	14,574	12.6	2540	−22.1	2989	95.2	20,103	75.3

Best performance results are highlighted in bold

Results and discussion

The inventory models of both fuzzy demand and supply have been modelled analytically as well as with ANN and ANFIS approaches. The results derived from the developed models show that the FIS + ANFIS models outperformed FIS + ANN model. The comparison of statistical values of 15 data sets for each model is shown in Table 4. This comparison is based on FIS model that had been verified for its effectiveness with the conventional models; stochastic EOQ model, Silver Meal model and Wagner Whitin model.

The result of prediction performance showed that FIS + ANFIS_Trap model is outstanding, followed by FIS + AIS_Gauss model, FIS + ANFIS_Bell model and FIS + ANN model, respectively. For FIS + ANFIS models, the results of running times (epochs) showed that the lowest running time was FIS + ANFIS_Bell model, followed by FIS + ANFIS_Gauss model and FIS + ANFIS_Trap model, respectively. The training and checking curves of all proposed FIS + ANFIS models of data set 1 are shown in Fig. 13.

For implementation based on prediction performance, FIS + ANFIS_Trap model and FIS + ANFIS_Gauss model were suitable to use. However, based on running times, FIS + ANFIS_Bell model and FIS + ANFIS_Trap model were appropriate for the decision maker.

The results of each proposed model after entering the predicted values to the evaluation algorithm and calculating the total inventory costs is shown in Table 5. The ordering costs increased whereas the larger holding costs decreased for all models when compared with stochastic EOQ model.

The average total inventory cost of all data sets and cost saving of all models compared to stochastic EOQ model is shown in Table 6. All models performed well with total inventory cost saving. The FIS + ANFIS_Gauss model achieved the largest cost saving by more than 75 % compared to stochastic EOQ model, followed by FIS + ANFIS_Bell model, FIS model, FIS + ANFIS_Trap model and FIS + ANN model, respectively. Although FIS + ANFIS_Trap model outperformed stochastic EOQ prediction when applied as the inventory model it represented the lowest cost saving, because some predicted values were affected by shortages which caused the shortage cost to be more than 30 % of total cost.

Conclusion

Fuzzy Inventory System (FIS) model, FIS + ANN model and FIS + ANFIS models were proposed for solving a dynamic inventory lot-sizing problem with unpredictable conditions. Demand and supply were inputs and order quantity was output of the system. For FIS model, linguistic values were applied for both fuzzy inputs and outputs. Fuzzy rules were devised depending on the historical knowledge of a case study factory. Fifteen data sets originating from the distribution of the demand and supply of the case study factory were applied to evaluate the membership functions of the FIS model at different ranges of parameters. The appropriate ranges for the inputs of the FIS model were justified.

The output from FIS model was entered to the evaluation algorithm and calculated the total inventory cost. Then the output of FIS model was used as the input of the developed models, FIS + ANN model and FIS + ANFIS models. The FIS + ANFIS models were divided to 3 membership functions; trapezoidal and triangular, Gaussian and bell shape called the FIS + ANFIS_Trap model, FIS + ANFIS_Gauss model and FIS + ANFIS_Bell model, respectively.

The results from FIS + ANFIS models gave better values for prediction in terms of R^2, *RMSE* and *MAE*. The predicted values showed good fit, but when output data was entered to the evaluation algorithm of inventory model, the best total inventory cost saving compared to stochastic EOQ model was achieved by the FIS + ANFIS_Gauss model. The research emphasized that application of FIS with ANFIS was beneficial for the inventory system and that FIS + ANFIS with Gaussian membership function achieved the best performance.

In further extended studies, the output of the model should be considered with fuzzy reorder point or fuzzy lead time, and the ANFIS model studied with linear output. The evaluation algorithm should also be adjusted according to the realistic situation in the future study.

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References

Aengchuan, P., & Phruksaphanrat, B. (2013). Inventory system design by fuzzy logic control: A case study. *Advanced Materials Research*, *8*, 619–624.

Andriolo, A., Battini, D., Grubbstrom, R. W., & Persona, A. (2014). A century of evolution from Harris's basic lot size model: Survey and research agenda. *The International Journal of Production Economics*, *155*, 16–38.

Aloulou, M. A., Dolgui, A., & Kovalyov, M. Y. (2014). A bibliography of non-deterministic lot-sizing models. *International Journal of Production Research*, *52*(8), 2293–2310.

Azedegan, A., Porobic, L., Ghazinoory, S., Samouei, P., & Kheirkhah, A. S. (2011). Fuzzy logic in manufacturing: A review of literature and a specialized application. *International Journal of Production Economics*, *132*(2), 258–270.

Azizi, A., Ali, A. Y., & Ping, L. W. (2013). An adaptive neuro-fuzzy inference system for a dynamic production environment under uncertainties. *World Applied Sciences Journal*, *25*(3), 428–433.

Bai, Y., & Wang, D. (2006). Fundamentals of Fuzzy Logic Control–Fuzzy Sets, Fuzzy Rules and Defuzzifications. In *Advanced Fuzzy Logic Technologies in Industrial Applications* (pp. 17–36). Springer London.

Camastra, F., Ciaramella, A., Giovannelli, V., Lener, M., Rastelli, V., Staiano, A., et al. (2015). A fuzzy decision system for genetically modified plant environmental risk assessment using Mamdani inference. *Expert Systems with Applications*, *42*, 1710–1716.

Castillo, O., & Melin, P. (2008). *Type-2 fuzzy logic theory and applications*. Berlin: Springer.

Chaudhary, H., Panwar, V., Prasad, R., & Sukavanam, N. (2014). Adaptive neuro fuzzy based hybrid force/position control for an industrial robot manipulator. *Journal Of Intelligent Manufacturing*. doi:10.1007/s10845-014-0952-1.

Chede, B., Jain, C. K., Jain, S. K., & Chede, A. (2012). Fuzzy logic analysis based on inventory considering demand and stock quantity on hand. *Industrial Engineering Letters*, *2*(1), 13–21.

Chen, S. P. (2011). A membership function approach to lot size re-order point inventory problems in fuzzy environments. *International Journal of Production Research*, *49*(13), 3855–3871.

Chen, S. H., Wang, C. C., & Ramer, A. (1996). Backorder fuzzy inventory model under function principle. *Information Sciences*, *95*, 71–79.

Chung, C. J., Widyadana, G. A., & Wee, H. M. (2011). Economic production quantity model for deteriorating inventory with random machine unavailability and shortage. *International Journal of Production Research*, *49*(3), 883–902.

Esen, H., Inalli, M., Sengur, A., & Esen, M. (2008a). Forecasting of a ground-coupled heat pump performance using neural networks with statistical data weighting pre-processing. *International Journal of Thermal Sciences*, *47*(4), 431–441.

Esen, H., Inalli, M., Sengur, A., & Esen, M. (2008b). Performance prediction of a ground-coupled heat pump system using artificial neural networks. *Expert Systems with Applications*, *35*, 1940–1948.

Esen, H., Inalli, M., Sengur, A., & Esen, M. (2008c). Modelling a ground-coupled heat pump system using adaptive neuro-fuzzy inference systems. *International Journal of Refrigeration*, *31*(1), 65–74.

Esen, H., Inalli, M., Sengur, A., & Esen, M. (2008d). Artificial neural networks and adaptive neuro-fuzzy assessments for ground-coupled heat pump system. *Energy and Buildings*, *40*(6), 1074–1083.

Esen, H., Inalli, M., Sengur, A., & Esen, M. (2008e). Predicting performance of a ground-source heat pump system using fuzzy weighted pre-processing-based ANFIS. *Building and Environment*, *43*(12), 2178–2187.

Esen, H., Ozgen, F., Esen, M., & Sengur, A. (2009). Artificial neural network and wavelet neural network approaches for modelling of a solar air heater. *Expert Systems with Applications*, *36*(8), 11240–11248.

Feng, G. (2010). Integrating dynamic pricing and replenishment decisions under supply capacity uncertainty. *Management Science*, *56*(2), 2154–2172.

Fragiadakis, N. G., Tsoukalas, V. D., & Papazoglou, V. J. (2014). An adaptive neuro-fuzzy inference system (anfis) model for assessing occupational risk in the shipbuilding industry. *Safety Science*, *63*, 226–235.

Glock, C. H., Grosse, E. H., & Ries, J. M. (2014). The lot sizing problem: A tertiary study. *The International Journal of Production Economics*, *155*, 39–51.

Gokulachandran, J., & Mohandas, K. (2015). Comparative study of two soft computing techniques for the prediction of remaining useful life of cutting tools. *Journal of Intelligent Manufacturing*, *26*, 255–268. doi:10.1007/s10845-013-0778-2.

Good, P. I. (1999). *Resampling methods: A practical guide to data analysis*. Boston: Birkhauser.

Grubbstrom, R. W. (2014). Dynamic lot sizing with a finite production rate. *International Journal of Production Economics's*, *149*, 68–79.

Guan, Y. (2011). Stochastic lot-sizing with backlogging: Computational complexity analysis. *Journal of Global Optimization's*, *49*, 651–678.

Guillaume, R., Kobylanski, P., & Zielinski, P. (2012). A robust lot sizing problem with ill-known demands. *Fuzzy Sets and Systems*, *206*, 39–57.

Guner, H. A. A., & Yumuk, H. A. (2014). Application of a fuzzy inference system for the prediction of long shore sediment transport. *Applied Ocean Research*, *48*, 162–175.

Guneri, A. F., Ertay, T., & Yucel, A. (2011). An approach based on ANFIS input selection and modeling for supplier selection problem. *Expert Systems with Applications*, *38*, 14907–14917.

Handfield, R., Warsing, D., & Wu, X. (2009). (Q, r) Inventory policies in a fuzzy uncertain supply chain environment. *European Journal of Operational Research*, *197*, 609–619.

Harris, F. H. (1913). How many parts to make at once. *Factory, The Magazine of Management, 10*(2), 135–136, 152.

Hosoz, M., Ertunc, H. M., & Bulgurcu, H. (2011). An adaptive neuro-fuzzy inference system model for predicting the performance of a refrigeration system with a cooling tower. *Expert Systems with Applications*, *38*, 14148–14155.

Jang, J. S. R. (1993). ANFIS: Adaptive-network-based fuzzy inference system. *IEEE Transactions on Systems, Man, Cybernetics*, *23*, 665–685.

Jha, M. N., Pratihar, D. K., Bapat, A. V., Dey, V., Ali, M., & Bagchi, A. C. (2014). Knowledge-based systems using neural networks for electron beam welding process of reactive material (Zircaloy-4). *Journal of Intelligent Manufacturing*, *25*, 1315–1333. doi:10.1007/s10845-013-0732-3.

Kamal, L., & Sculfort, J.-L. (2007). Fuzzy modelling of inventory control system in uncertain environment. *International Symposium on Logistics and Industrial Informatics*, 53–57. doi:10.1109/LINDI.2007.4343512.

Kang, H.-Y., & Lee, A. H. I. (2013). A stochastic lot-sizing model with multi-supplier and quantity discounts. *International Journal of Production Research*, *51*(1), 245–263.

Khan, M., Jaber, M. Y., & Wahab, M. I. M. (2010). Economic order quantity model for items with imperfect quality with learning in inspection. *International Journal of Production Economics*, *124*(1), 87–96.

Kiran, T. R., & Rajput, S. P. S. (2011). An effectiveness model for an indirect evaporative cooling (IEC) system: Comparison of artificial neural networks (ANN), adaptive neuro-fuzzy inference system (ANFIS) and fuzzy inference system (FIS) approach. *Applied Soft Computing*, *11*, 3525–3533.

Kocyigit, N. (2015). Fault and sensor error diagnostic strategies for a vapor compression refrigeration system by using fuzzy inference systems and artificial neural network. *The International Journal of Refrigeration*, *50*, 69–79.

Kovac, P., Rodic, D., Pucovsky, V., Savkovic, B., & Gostimirovic, M. (2013). Application of fuzzy logic and regression analysis for modeling surface roughness in face milliing. *Journal of Intelligent Manufacturing*, *24*, 755–762. doi:10.1007/s10845-012-0623-z.

Kuo, C.-F. J., Hsu, C.-T. M., Liu, Z.-X., & Wu, H.-C. (2014). Automatic inspection system of LED chip using two-stages back-propagation neural network. *Journal of Intelligent Manufacturing*, *25*, 1235–1243. doi:10.1007/s10845-012-0725-7.

Kuo, R. J., Tseng, Y. S., & Chen, Z.-Y. (2014). Integration of fuzzy neural network and artificial immune system-based back-propagation neural network for sales forecasting using qualitative and quantitative data. *Journal Of Intelligent Manufacturing*. doi:10.1007/s10845-014-0944-1.

Lee, H., & Yao, J. (1999). Economic order quantity in fuzzy sense for inventory without backorder model. *Fuzzy Sets and Systems*, *105*(1), 13–31.

Lee, S.-D., Yang, C.-M., & Lan, S.-C. (2014). Economic lot sizing in a production system with random demand. *International Journal of Systems Science*. doi:10.1080/00207721.2014.915354.

Lenart, B., Grzybowska, K., & Cimer, M. (2012). Adaptive inventory control in production systems. In E. Corchado et al. (Eds.), HAIS 2012, Part II, LNCS. 7209. Springer: Berlin, Heidelberg (pp. 222–228).

Li, H., & Thorstenson, A. (2014). A multi-phase algorithm for a joint lot-sizing and pricing problem with stochastic demands. *International Journal of Production Research*, *52*(8), 2345–2362.

Mahata, G. C., & Goswami, A. (2013). Fuzzy inventory models for items with imperfect quality and shortage backordering under crisp and fuzzy decision variables. *Computer & Industrial Engineering*, *64*, 190–199.

Mamdani, E. H., & Assilian, S. (1975). An experiment in linguistic synthesis with fuzzy logic controller. *International Journal of Man-Machine Studies*, *7*, 1–13.

Melin, P., Soto, J., Castillo, O., & Soria, J. (2012). A new approach for time series prediction using ensembles of ANFIS models. *Expert Systems with Applications*, *39*, 3494–3506.

Meredith, J., & Shafer, S. (2011). *Operations management* (4th ed.). Hoboken: Wiley.

Mondal, S., & Maiti, M. (2002). Multi-item fuzzy EOQ models using genetic algorithm. *Computers & Industrial Engineering*, *44*(1), 105–117.

Nasrollahzadeh, K., & Basiri, M. M. (2014). Prediction of shear strength of FRP reinforced concrete beams using fuzzy inference system. *Expert Systems with Applications*, *41*, 1006–1020.

Park, K. S. (1987). Fuzzy set theoretic interpretation of economic order quantity. *IEEE Transactions on Systems, Man and Cybernetics*, *6*(17), 1082–1084.newpage

Parlar, M., & Perry, D. (1995). Analysis of a (Q, r, T) inventory policy with deterministic and random yields when future supply is uncertain. *European Journal of Operational Research*, *84*, 431–443.

Phootrakornchai, W., & Jiriwibhakorn, S. (2015). Online critical clearing time estimation using an adaptive neuro-fuzzy inference system (ANFIS). *Electrical Power and Energy Systems*, *73*, 170–181.

Porteus, E. L. (1986). Optimal lot sizing, process quality improvement and setup cost reduction. *Operations Research*, *34*(1), 137–144.

Razani, M., Yazdani-Chamzini, A., & Yakhchali, S. H. (2013). A novel fuzzy inference system for predicting roof fall rate in underground coal mines. *Safety Science*, *55*, 26–33.

Rong, L. (2010). Uncertain economic order quantity model with uncertain costs. *Journal of Information and Computational Science*, *7*(13), 2723–2730.

Rothstein, A. P., & Rakityanskaya, A. B. (2006). Inventory control as an identification problem based on fuzzy logic. *Cybernetics and Systems Analysis*, *42*(3), 411–419.

Samanta, B., & Al-Araimi, S. A. (2001). An inventory control model using fuzzy logic. *The International Journal of Production Economics*, *73*, 217–226.

Samanta, B., & Al-Araimi, S. A. (2003). Application of an adaptive neuro-fuzzy inference system in inventory control. *International Journal of Smart Engineering System Design*, *5*(4), 547–553.

Sana, S. S. (2011). Price-sensitive demand for perishable items—An EOQ model. *Applied Mathematics and Computation*, *217*(13), 6248–6259.

Seneyigit, E., & Erol, R. (2010). New lot sizing heuristics for demand and price uncertainties with service-level constraint. *International Journal of Production Research*, *48*(1), 21–44.

Silver, E. A., & Meal, H. A. (1973). A heuristic for selecting lot sizing requirements for the case of a deterministic time varying demand rate and discrete opportunities for replenishment. *Production and Inventory Management*, *14*(2), 64–74.

Sommer, G. (1981). Fuzzy inventory scheduling. In G. Lasker (Ed.), *Applied systems and cybernetics*. New York: Pergamon Press.

Sumathi, S., & Paneerselvam, S. (2010). *Computational intelligence paradigms: Theory and applications using MATLAB*. Boca Raton, FL: CRC Press, Inc.

Taft, E. W. (1918). The most economical production lot. *The Iron Age*, *101*, 1410–1412.

Tanthatemee, T., & Phruksaphanrat, B. (2012). Fuzzy inventory control for uncertain demand and supply. *Proceedings of the International MultiConference of Engineers and Scientists*, *2*, 1224–1229.

Tsai, K.-M., & Luo, H.-J. (2014). An inverse model for injection molding of optical lens using artificial neural network coupled with genetic algorithm. *Journal of Intelligent Manufacturing*. doi:10.1007/s10845-014-0999-z

Ullah, M., & Kang, C. W. (2014). Effect of rework, rejects and inspection on lot size with work-in-process inventory. *International Journal Production Research*, *52*(8), 2448–2460.

Wagner, H. M., & Whitin, T. M. (1958). Dynamic version of the economic lot size model. *Manangement Science*, *5*(1), 89–96.

Wang, B., Man, T., & Jin, H. (2015). Prediction of expansion behavior of self-stressing concrete by artificial neural networks and fuzzy inference systems. *Construction and Building Materials*, *84*, 184–191.

Wang, C.-H. (2010). Some remarks on an optimal order quantity and reorder point when supply and demand are uncertain. *Computers & Industrial Engineering*, *58*, 809–813.

Wee, H. M., & Widyadana, G. A. (2012). Economic production quantity models for deteriorating items with rework and stochastic preventive maintenance time. *International Journal Production Research*, *50*(11), 2940–2952.

Yang, L., & Entchev, E. (2014). Performance prediction of a hybrid microgeneration system using adaptive neuro-fuzzy inference system (ANFIS) technique. *Applied Energy*, *134*, 197–203.

Yang, W.-A., & Zhou, W. (2013). Autoregressive coefficient-invariant control chart pattern recognition in autocorrelated manufacturing processes using neural network ensemble. *Journal of Intelligent Manufacturing*. doi:10.1007/s10845-013-0847-6

Yimer, A.D., & Demirli, K. (2004). Fuzzy modeling and simulation of single item inventory system with variable demand. *IEEE annual meeting of the North American fuzzy information processing society Banff, 2, Alberta, Canada* (pp. 985–989).

Ying, L. C., & Pan, M. C. (2008). Using adaptive network based fuzzy inference system to forecast regional electricity loads. *Energy Conversation and Management*, *49*, 205–211.