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# The use of improved TOPSIS method based on experimental design and Chebyshev regression in solving MCDM problems

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**Abstract** Decision makers today are faced with a wide range of alternative options and a large set of conflicting criteria. How to make trade-off between these conflicting attributes and make a scientific decision is always a difficult task. Although a lot of multiple criteria decision making (MCDM) methods are available to deal with selection applications, it's observed that in most of these methods the ranking results are very sensitive to the changes in the attribute weights. The calculation process is also ineffective when a new alternative is added or removed from the MCDM problem. This paper presents an improved TOPSIS method based on experimental design and Chebyshev orthogonal polynomial regression. A feature of this method is that it employs the experimental design technique to assign the attribute weights and uses Chebyshev regression to build a regression model. This model can help and guide a decision maker to make a reasonable judgment easily. The proposed methodology is particularized through an equipment selection problem in manufacturing environment. Two more illustrative examples are conducted to demonstrate the applicability of the proposed method. In all the cases, the results obtained using the proposed method almost corroborate with those derived by the earlier researchers which proves the validity, capability and potentiality of this method in solving real-life MCDM problems.

**Keywords** MCDM · TOPSIS · Experimental design · Chebyshev orthogonal polynomial regression

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**Abbreviations**

| | |
|---|---|
| AHP | Analytic hierarchy process |
| DM | Decision maker |
| DoE | Design of experiment |
| ELECTRE | Elimination et choice translating reality |
| IC | Integrated circuit |
| MCDM | Multiple criteria decision making |
| MM | Milling machine |
| MOORA | Multi-objective optimization on the basis of ratio analysis |
| PROMETHEE | Preference ranking organization method for enrichment evaluation |
| SAW | Simple additive weighting |
| TOPSIS | Technique for order preference by similarity to ideal solution |
| VIKOR | VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian |

**List of symbols**

| | |
|---|---|
| $a_i$ | Coefficient of the Chebyshev term $T_i$ |
| $A^+$ | The positive ideal solution |
| $A^-$ | The negative ideal solution |
| $A_i$ | The $i$th alternative |
| $C_j$ | The $j$th criterion |
| $D_i^+$ | The distance between the $i$th alternative and the positive ideal solution |
| $D_i^-$ | The distance between the $i$th alternative and the negative ideal solution |
| $m$ | The number of alternatives for a certain MCDM problem |
| $n$ | The number of criteria for a certain MCDM problem |

| | |
|---|---|
| $r_{ij}$ | The normalized value of $j$th criterion for the $i$th alternative |
| $T_i$ | Chebyshev orthogonal polynomial term |
| $v_{ij}$ | The weighted normalized value of $j$th criterion for the $i$th alternative |
| $w_j$ | The weight value assigned to the $j$th criterion |
| $\boldsymbol{W}$ | The weight set |
| $x_{ij}$ | The value of the $j$th criterion for the alternative $A_i$ |
| $\boldsymbol{X}$ | The decision matrix |
| $y$ | Response or TOPSIS score |

## Introduction

Multiple criteria decision making (MCDM) is all about making choices in the presence of multiple conflicting criteria (Köksalan et al. 2011). In MCDM problems, a decision maker (DM) has to choose the most appropriate alternative that satisfies the evaluation criteria among a set of candidate options. Typically, there does not exist a unique optimal solution for such problems, so it is necessary to use DM's preferences to differentiate between solutions. Good decision making requires a mixture of skills, like clarity of judgment, firmness of decision, and identification of options (Chakraborty 2011). However, it is not easy for a beginner to gain such skills in a short time. Thus, the problem of making a scientific decision becomes difficult.

MCDM occurs in a variety of actual situations, especially in manufacturing environment. Some of the important decision making situations in the manufacturing environment include: equipment selection (Dağdeviren 2008), facility location selection (Pasandideh et al. 2013), manufacturing process selection (Yu et al. 1993), machine tool selection (Önüt and Soner Kara 2008), rapid prototyping process selection (Frank and Fadel 1995), robot selection (Rao et al. 2011), vendor selection (Sharma and Balan 2013), etc. The selection decisions become more complex as the DMs in manufacturing organizations have to assess a wide range of alternative options based on a set of conflicting attributes. To solve these MCDM problems, several common methodologies have been developed. In 1968, MacCrimmon (1968) utilized the simple additive weighting (SAW) method to solve a simplified weapon system selection problem. In the same year, Roy (1968) and his colleagues carried out the ELECTRE (elimination et choice translating reality) method to deal with the problem of choosing, ranking and sorting alternatives. Based on mathematics and psychology, Saaty (1980) developed analytic hierarchy process (AHP) as a decision making method in the 1970s. Hwang and Yoon (1981) introduced the technique for order preference by similarity to ideal solution (TOPSIS) method in 1981. Brans et al. (1986) first presented PROMETHEE (preference ranking organization method for enrichment evaluation) method and later developed it into several versions. Opricovic (1998) originally proposed the compromise ranking method, also known as the VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje in Serbian) method, to solve decision problems with conflicting and noncommensurable criteria. In 2006, Brauers and Zavadskas (2006) proposed the multi-objective optimization on the basis of ratio analysis (MOORA) method and successfully applied it to solve various types of complex decision making problems. Although the earlier researchers have applied various MCDM methods to solve different selection problems, it is observed that in all these methods, the rankings of the alternatives are very sensitive to the changes in the attribute weights. Different attribute weights will produce different ranking results (Yurdakul and Tansel İÇ 2009). When weight changes, the whole mathematical calculation process has to do all over again, which may be impracticable and ineffective for DMs. Thus, there is a need for a simple, logical and systematic approach to solve the multiple criteria selection problems.

Due to its simplicity and practicality, TOPSIS method has gained popularity in the field of MCDM since it was introduced. There exists a large amount of literature involving TOPSIS theory and applications (Wang and Chang 2007; Dağdeviren et al. 2009; Chu 2002; Behzadian and Khanmohammadi Otaghsara 2012). Furthermore, different methods have been developed to refine the original TOPSIS idea such as those reported in Abo-Sinna and Amer (2005), Chen et al. (2009) and Shih et al. (2007). In classical TOPSIS method, the ratings and weights of criteria are known precisely. The weight coefficients are usually fixed using expert investigation or the AHP method, which both have subjectivity. Again, some of the improved TOPSIS methods are quite difficult to comprehend and complex to implement requiring extensive mathematical knowledge. In this paper we focus on an extension of the TOPSIS method combined with experimental design and Chebyshev orthogonal polynomial regression. Experimental design and Chebyshev regression have been well known in the literature. Experimental design is a statistical method used to study the effect of several factors simultaneously. This technique is used to determine independent and interaction effect of multiple factors on performance. Chebyshev regression is a kind of approximation techniques that uses orthogonal basis functions to model a functional relationship between the multiple factors and the response. These two methods are often combined to explore an unknown design space and build a statistically based mathematical model. Both experimental design and Chebyshev regression is already an effective tool for most intelligent manufacturing. For instance, in Dolgin (1996) paper, the Chebyshev approximation method was used for dynamic diagnostics of mechanical manufacturing systems.

The convenience and effectiveness of using DoE and Chebyshev regression tools had been confirmed in his paper. Zhang et al. (2013) implemented Chebyshev fitting to measure form accuracies of precision components. They pointed out that the Chebyshev fitting could obtain the correct parameter of the form error. Emenonye and Chikwendu (2014) applied Chebyshev polynomial approximation to solve stock allocation problems for manufacturing and distribution organizations. The use of Chebyshev polynomial approximation ensured the minimization of cost and maximization of profit. Wu et al. (2014) proposed a new design optimization framework for vehicle suspension systems using DoE and Chebyshev regression techniques. They proved that the Chebyshev meta-model was able to provide higher approximation accuracy.

Although the individual methods (experimental design and Chebyshev regression) of this study are not unique, the combination of these methods with TOPSIS technique has not ever been presented within the context of MCDM. In this paper, the experimental design of attribute weights overcomes the weakness of human subjectivity and decreases the sensitivity to the weight change. The experimental data is approximated using Chebyshev orthogonal polynomial regression model. This model can help and guide a DM to make a reasonable judgment without requiring professional skills or rich experience. Three decision making problems are considered and the whole selection procedure is finished using commercially available software Isight. This method is observed to be quite robust, comprehendible, and computationally easy, which may be useful and helpful to the DMs who may not have a strong background in mathematics.

The reminder of this paper is organized as follows. In "Backgrounds" section, a brief review of related background information involving TOPSIS method, experimental design and Chebyshev orthogonal polynomial regression is presented. A case study of equipment selection problem is proposed in "The proposed methodology" section to explain the detail applying process of the improved TOPSIS method. In "Applications and discussion" section, two more illustrative examples are conducted to demonstrate the capabilities of the proposed method. Conclusions and future research areas are discussed in the last section.

## Backgrounds

### TOPSIS method

The TOPSIS method is based on the concept that the best decision should be the closest to the ideal solution and farthest from the non-ideal solution (Karande and Chakraborty 2012). This method assumes that each attribute is monotonically increasing or decreasing. The ideal solution (also called positive ideal solution) is one that maximizes the benefit criteria and minimizes the cost criteria, whereas the non-ideal solution (also called negative ideal solution) maximizes the cost criteria and minimizes the benefit criteria. TOPSIS utilized Euclidean distances to measure the alternatives with their positive ideal solution and negative ideal solution. The preference order of alternatives is yielded through comparing the Euclidean distances.

Suppose a MCDM problem is based on $m$ alternatives $(A_1, A_2, \ldots, A_m)$ and $n$ criteria $(C_1, C_2, \ldots, C_n)$. $x_{ij}$ $(i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n)$ denotes the value assigned to the $j$th criterion of the $i$th alternative. $\boldsymbol{X} = [x_{ij}]_{mn}$ is the decision matrix. The related weight value of each criterion has been denoted by $\boldsymbol{W} = [w_1, w_2, \ldots, w_n]$, where $\sum_{j=1}^{n} w_j = 1$. The TOPSIS process (Mateo 2012) is carried out as follows:

Step 1: Normalize the decision matrix:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{m} x_{kj}^2}}, i = 1, \ldots, m;\ j = 1, \ldots, n \tag{1}$$

where $r_{ij}$ denotes the normalized value of $j$th criterion for the $i$th alternative $A_i$.

Step 2: Calculate the weighted normalized decision matrix:

$$v_{ij} = w_j r_{ij}, i = 1, \ldots, m;\ j = 1, \ldots, n \tag{2}$$

where $w_j$ is the weight of the $j$th criterion or attribute.

Step 3: Determine the positive ideal and negative ideal solutions:

$$A^+ = \left\{v_1^+, \ldots, v_n^+\right\} \tag{3}$$
$$A^- = \left\{v_1^-, \ldots, v_n^-\right\} \tag{4}$$

where $A^+$ denotes the positive ideal solution and $A^-$ denotes the negative ideal solution. If the $j$th criterion is beneficial criterion, $v_j^+ = \max\{v_{ij}, i = 1, \ldots, m\}$ and $v_j^- = \min\{v_{ij}, i = 1, \ldots, m\}$. On the contrary, if the $j$th criterion is cost criterion, $v_j^+ = \min\{v_{ij}, i = 1, \ldots, m\}$ and $v_j^- = \max\{v_{ij}, i = 1, \ldots, m\}$.

Step 4: Calculate the distances from each alternative to positive ideal solution and negative ideal solution:

$$D_i^+ = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^+\right)^2}, \quad i = 1, \ldots, m \tag{5}$$
$$D_i^- = \sqrt{\sum_{j=1}^{n} \left(v_{ij} - v_j^-\right)^2}, \quad i = 1, \ldots, m \tag{6}$$

where $D_i^+$ denotes the distance between the $i$th alternative and the positive ideal solution, and $D_i^-$ denotes the distance between the $i$th alternative and the negative ideal solution.

Step 5: Calculate the relative closeness to the ideal solution:

$$y_i = \frac{D_i^-}{D_i^+ + D_i^-} \tag{7}$$

Step 6: Rank the alternatives sorting by the value $y_i$ in decreasing order.

## Experimental design

Experimental design (also called design of experiment, DoE for short) is a statistical method used to determine simultaneously the individual and interactive effects of many factors on the system response (Kirk 1982). The main advantage of experimental design is that multiple variables, or multiple levels of a variable, can be tested simultaneously, which saves a great deal of time considering some pressing and time-sensitive issue. In DoE, factors are the input parameters or the variables need to be studied. Levels are the discrete possible values or different states of each factor. Response is the system output or result obtained by evaluating a certain experimental sample set. The combinations of all factors at different levels construct the design matrix. Table 1 presents a list of some common DoE techniques and their brief descriptions involving parameter study, full factorial design, fractional factorial technique, orthogonal array technique, central composite design, Latin hypercube design and optimal Latin hypercube design. For a detailed explanation of these DoE methods, the reader is referred to Dean and Voss (1999).

In the experimental design application, full factorial design, orthogonal array technique, Latin hypercube design and optimal Latin hypercube design are four commonly used DoE methods. Full factorial design provides extensive information for accurate estimation of factors and interaction effects. However, this method is costly to execute due to the need of large quantity of experiment analysis. The orthogonal array technique utilizes properties of fractional factorial experiment to efficiently determine the best combination of factor levels (Ross 1995), which is very cost effective. Optimal Latin hypercube design generates more evenly distributed samples compared with normal Latin hypercube design (Jin et al. 2005). In our paper, the DoE method is combined with TOPSIS method to create a data set that can be used to generate approximation models of the multiple criteria. To reduce the number of experiments, optimal Latin hypercube design and orthogonal array technique are carried out for the weight experiment and criterion experiment, respectively. The double DoE tries to decrease the effects of criterion weights and helps the DM to identify critical attributes.

## Chebyshev orthogonal polynomial

Orthogonal polynomial approximation is a regression technique that uses orthogonal basis functions to model a functional relationship between the multiple factors and the response. An advantage of using orthogonal functions as a basis for fitting is that the autocorrelation between the response values can be greatly reduced (Gautschi et al. 2004). In the classical orthogonal polynomials, notably Chebyshev polynomials of the first two kinds are of considerable importance for purposes of approximation (Mason and Handscomb 2010).

Chebyshev polynomials are a sequence of orthogonal polynomials that are solutions of a special kind of differential equation called a Chebyshev differential equation. The Chebyshev differential equation is written as:

$$\left(1 - x^2\right) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + n^2 y = 0 \tag{8}$$

Chebyshev polynomials can be of two kinds. The Chebyshev polynomials of the first kind in one dimension are defined by the recurrence relation:

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_{n+1}(x) &= 2x T_n(x) - T_{n-1}(x) \end{aligned} \tag{9}$$

**Table 1** Brief description of some common DoE techniques

| DoE method | Descriptions |
|---|---|
| Parameter study | One factor at a time |
| Full factorial design | All combinations of all factors at all levels are evaluated |
| Fractional factorial technique | A certain fractional subset (1/2, 1/4, etc. for two-level factors) of the full factorial experiment is selected |
| Orthogonal arrays | Maintain orthogonality among the various factors and certain interactions |
| Central composite design | 2-Level full factorial experiment enhanced with a center point and two additional points for each factor |
| Latin hypercube design | Design space for each factor is divided uniformly and factor levels are randomly combined |
| Optimal latin hypercube design | The combination of factor levels is optimized, rather than randomly combined |

The Chebyshev polynomials of the second kind can be defined as:

$$
\begin{aligned}
T_0(x) &= 1 \\
T_1(x) &= 2x \\
T_{n+1}(x) &= 2xT_n(x) - T_{n-1}(x)
\end{aligned}
\tag{10}
$$

The roots of these polynomials are not equally spaced. Taguchi (1987) describes a set of one-dimensional polynomials that have equally spaced roots. When these equally spaced roots are assumed to be the factor levels in an orthogonal array, a quadrature procedure is available for approximating a response using Chebyshev polynomials as individual terms. Isight implements Taguchi's method of fitting Chebyshev polynomials from an orthogonal array.

The following equations show the Chebyshev polynomials with equally spaced roots in one dimension:

$$
\begin{aligned}
T_1(x) &= (x - \bar{x}) \\
T_2(x) &= (x - \bar{x})^2 - b_2 \\
T_3(x) &= (x - \bar{x})^3 - b_3(x - \bar{x})
\end{aligned}
\tag{11}
$$

where $\bar{x}$ is the average value of the levels. Taguchi generates multivariate polynomials by taking products of Chebyshev polynomials in each variable as listed above. Taguchi also provides tables for computing the coefficients of these terms for an orthogonal array.

Suppose we have three variables $x_1$, $x_2$, and $x_3$ to which we want to fit a response $f$ using quadratic Chebyshev polynomials with cross terms. We can generate the following multivariate polynomial basis:

$$
\begin{aligned}
&\text{linear term: } \{T_1(x_1), T_1(x_2), T_1(x_3)\} \\
&\text{quadratic term: } \{T_2(x_1), T_2(x_2), T_2(x_3)\} \\
&\text{cross term: } \{T_1(x_1)T_1(x_2), T_1(x_1)T_1(x_3), T_1(x_2)T_1(x_3)\}
\end{aligned}
\tag{12}
$$

Therefore, the function $f$ is approximated as:

$$
\begin{aligned}
f(x_1, x_2, x_3) = {} & a_{11}T_1(x_1) + a_{12}T_1(x_2) + a_{13}T_1(x_3) \\
& + a_{21}T_2(x_1) + a_{22}T_2(x_2) \\
& + a_{23}T_2(x_3) + a_{24}T_1(x_1)T_1(x_2) \\
& + a_{25}T_1(x_1)T_1(x_3) + a_{26}T_1(x_2)T_1(x_3)
\end{aligned}
\tag{13}
$$

The coefficients $a_{ij}$ are calculated using least squares regression method.

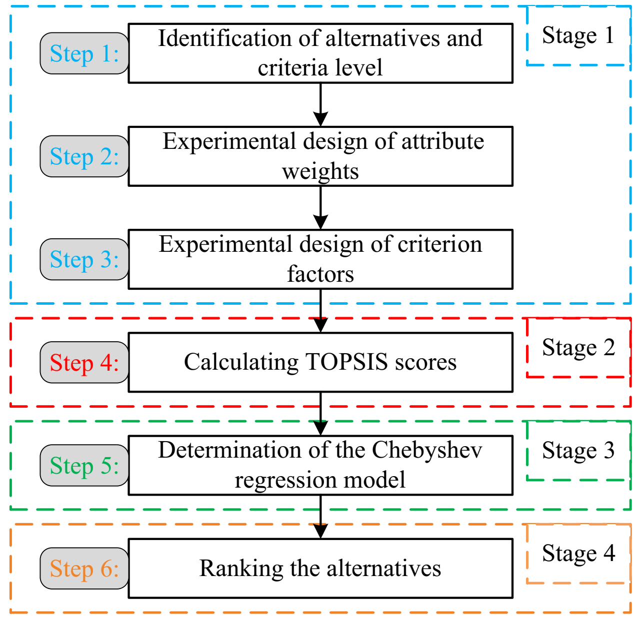


**Fig. 1** Steps of the improved TOPSIS method

## The proposed methodology

### Framework of the improved TOPSIS method

The integrated TOPSIS approach, composed of DoE and Chebyshev orthogonal polynomial regression technique, for MCDM problems consists of 4 basic stages: (1) DoE, (2) TOPSIS evaluation, (3) approximation, (4) decision making. In the first stage, input attributes and their levels are determined and the DoE procedure is conducted. After that, a data set of the entire selected criterion factors at all levels under different attribute weights is created. These determined attributes are used as input values to the TOPSIS model. The second stage of the proposed methodology is computing the ranking scores using aforementioned TOPSIS method. In the third stage, Chebyshev regression model is constructed to determine how the selected criteria affect ranking scores of a MCDM problem. In the last stage, a DM is able to use the regression model for evaluating alternatives and determining the final rank. Flow diagram of the proposed approach is illustrated in Fig. 1.

In the following section, a detailed applying process of the improved TOPSIS method is demonstrated. The whole selection process is finished using commercially available software Isight. Isight provides a suite of tools to execute simulation-based design processes in a visual and flexible way. We mainly use the DoE and approximation components to construct our model. The DoE component provides an easy access to intelligently sampling the design space. The approximation component is used to generate a regression model, which we can use for quick and efficient design

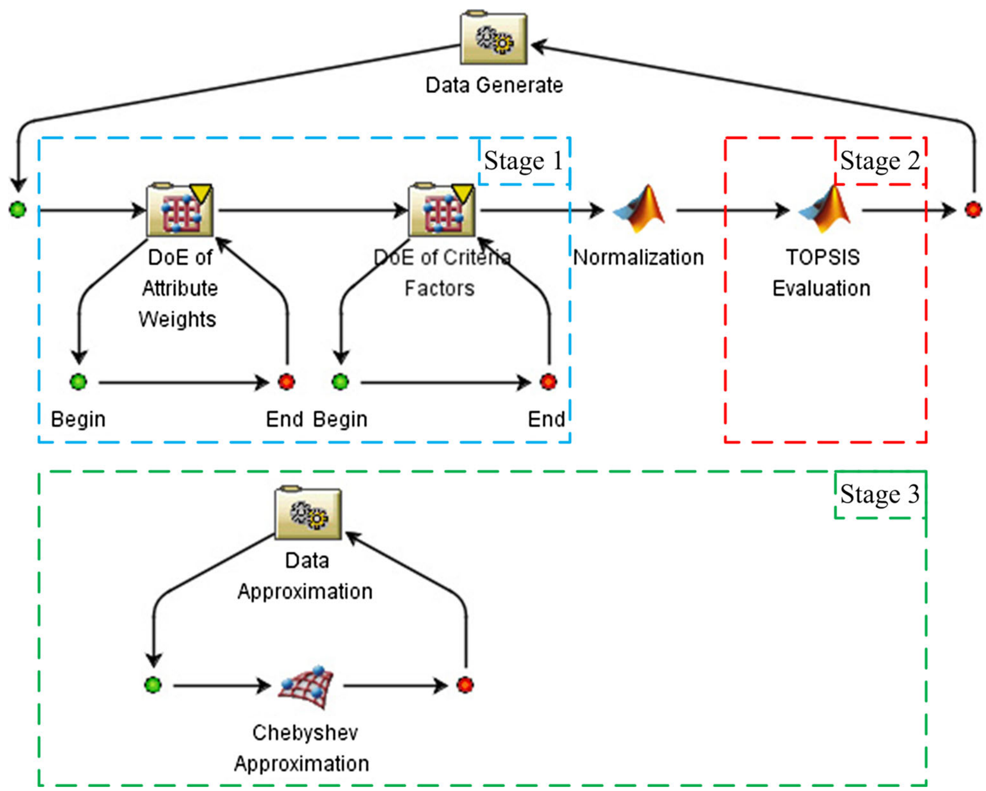


**Fig. 2** Workflow in Isight

studies. The workflow of the improved TOPSIS method built in Isight is shown in Fig. 2.

## Illustrative case study

### *Problem description*

In this section, an equipment selection problem adapted from Dağdeviren (2008) is conducted to particularize the application process and demonstrate the capabilities of the proposed MCDM method. This application is realized in a manufacturing company which is located in Ankara, Turkey. The company wants to purchase a few milling machines (MM) to reduce the work-in-process inventory and to replace its old equipment. The high technology equipments make significant improvements in the manufacturing processes of the firms and the correct decision made at this stage brings the companies competitive advantage. Therefore, selecting the most proper milling machines is of great importance for the company. But it is hard to choose the most suitable one among the milling machines which dominate each other in different characteristics (Dağdeviren 2008).

In this implementation, emphasis is put on explaining the detail process of the proposed methodology. Thus, the step by step problem solving process is explained and discussed for this decision making problem.

### *Detailed steps*

The steps for building a regression model using the proposed hybrid TOPSIS technique and making a scientific decision for the equipment selection problem are summarized below.

Step 1: Identification of alternatives and criterion level

According to Rao (2007), five possible milling machines suitable for the needs of the company are determined. The six attributes, namely price, weight, power, spindle, diameter and stroke will be taken into consideration in the selection process. The available values of different attributes are presented in Table 2. Out of the six attributes, the price and weights are considered as non-beneficial attributes as their lower values are desired and the other four attributes are considered as beneficial attributes where higher values are preferable. Decision hierarchy structured with the determined alternative equipments and criteria is provided in Fig. 3.

As shown in Table 2, price with minimum level 580 and maximum level 1,265, weight with minimum level 3.5 and maximum level 6, power with minimum level 900 and maximum level 2,000, spindle with minimum level 21,000 and maximum level 25,000, diameter with minimum level 8 and maximum level 12.7, and stroke with minimum level 50 and maximum level 65 are determined to be the criterion levels affecting the equipment selection decision.

**Table 2** Attributes for equipment selection problem

| Alternatives | Price ($x_1$) | Weight ($x_2$) | Power ($x_3$) | Spindle ($x_4$) | Diameter ($x_5$) | Stroke ($x_6$) |
|---|---|---|---|---|---|---|
| MM-1 | 936 | 4.8 | 1,300 | 24,000 | 12.7 | 58 |
| MM-2 | 1,265 | 6 | 2,000 | 21,000 | 12.7 | 65 |
| MM-3 | 680 | 3.5 | 900 | 24,000 | 8 | 50 |
| MM-4 | 650 | 5.2 | 1,600 | 22,000 | 12 | 62 |
| MM-5 | 580 | 3.5 | 1,050 | 25,000 | 12 | 62 |
| Min/max | Min | Min | Max | Max | Max | Max |
| Low level | 580 | 3.5 | 900 | 21,000 | 8 | 50 |
| High level | 1,265 | 6 | 2,000 | 25,000 | 12.7 | 65 |

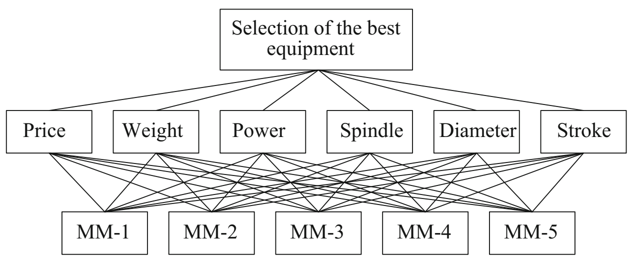


**Fig. 3** The decision hierarchy of equipment selection problem

**Table 3** Nine-point intensity of importance scale

| Definition | Intensity of importance |
|---|---|
| Equally important | 1 |
| Moderately more important | 3 |
| Strongly more important | 5 |
| Very strongly more important | 7 |
| Extremely more important | 9 |
| Intermediate values | 2, 4, 6 and 8 |

Step 2: Experimental design of attribute weights

The value of attribute weight indicates how many times more important one factor is over another in terms of a given criterion. In the MCDM applications, there exist two different weight assignment techniques namely the expert assignment and the eigenvector method (AHP method). Since human judgments including preference are often vague and cannot estimate his preference with an exact numerical value. A more realistic way may be to use linguistic terms to describe the desired value or the intensity of importance. The 9 point scale (Saaty 1994), a simple, logical and useful assignment criterion, has established the relationship between the linguistic terms and the intensity of criterion importance. Table 3 shows a typical nine-point scale proposed by Saaty.

Since we are not the member of the equipment decision making team and we have little knowledge about the milling machines, we cannot directly evaluate the relative importance of one attribute over others. Therefore, the experimental design method is used to determine the attribute weights in the equipment selection problem. To reduce the number of experiments, optimal Latin hypercube design is selected as the DoE technique. This design method allows a total freedom in selecting the number of designs to run as long as it is greater than the number of factors. Thus, we set the experiment number to $n + 1$, where $n$ is the number of criterion factors. For this selection problem, $n = 6$. The design matrix of attribute weights is shown in Table 4.

**Table 4** Attribute weights for equipment selection problem

| Weight no. | Price | Weight | Power | Spindle | Diameter | Stroke |
|---|---|---|---|---|---|---|
| W1 | 1 | 4 | 4 | 2 | 1 | 4 |
| W2 | 8 | 8 | 2 | 8 | 2 | 2 |
| W3 | 2 | 9 | 9 | 6 | 6 | 5 |
| W4 | 6 | 1 | 6 | 9 | 5 | 8 |
| W5 | 4 | 6 | 1 | 5 | 8 | 9 |
| W6 | 9 | 5 | 8 | 1 | 4 | 6 |
| W7 | 5 | 2 | 5 | 4 | 9 | 1 |

Step 3: Experimental design of criterion factors

The experimental design of criterion factors is utilized to assess the impact of criterion factors on ranking results. In this case study, the two-level (low-high) orthogonal array technique is used to create a data set for approximation. The selected DoE method can efficiently determine the best combination of factor levels. The orthogonal arrays with two-level are expressed by:

$$L_A\left(2^B\right) \tag{14}$$

where $A = 2^N$ is the number of experimental designs to run. $N$ is a positive integer which is greater than 1. 2 is the number of levels for each factor, and $B$ denotes the number of columns in the orthogonal array. The letter '$L$' comes from 'Latin', the idea of using orthogonal arrays for experimental design having been associated with Latin square designs from the outset. The two-level standard orthogonal arrays

**Table 5** Experimental design results of criterion factors

| Alternative no. | Factor levels | | | | | |
|---|---|---|---|---|---|---|
| | Price | Weight | Power | Spindle | Diameter | Stroke |
| A1 | 580 | 3.5 | 900 | 21,000 | 8 | 50 |
| A2 | 580 | 3.5 | 900 | 25,000 | 12.7 | 65 |
| A3 | 580 | 3.5 | 2,000 | 21,000 | 12.7 | 65 |
| A4 | 580 | 3.5 | 2,000 | 25,000 | 8 | 50 |
| A5 | 580 | 6 | 900 | 21,000 | 12.7 | 65 |
| A6 | 580 | 6 | 900 | 25,000 | 8 | 50 |
| A7 | 580 | 6 | 2,000 | 21,000 | 8 | 50 |
| A8 | 580 | 6 | 2,000 | 25,000 | 12.7 | 65 |
| A9 | 1,265 | 3.5 | 900 | 21,000 | 12.7 | 50 |
| A10 | 1,265 | 3.5 | 900 | 25,000 | 8 | 65 |
| A11 | 1,265 | 3.5 | 2,000 | 21,000 | 8 | 65 |
| A12 | 1,265 | 3.5 | 2,000 | 25,000 | 12.7 | 50 |
| A13 | 1,265 | 6 | 900 | 21,000 | 8 | 65 |
| A14 | 1,265 | 6 | 900 | 25,000 | 12.7 | 50 |
| A15 | 1,265 | 6 | 2,000 | 21,000 | 12.7 | 50 |
| A16 | 1,265 | 6 | 2,000 | 25,000 | 8 | 65 |

most often used in practice are $L_4(2^3)$, $L_8(2^7)$, $L_{16}(2^{15})$, and $L_{32}(2^{31})$ (Tsai et al. 2004). In this paper, two-level standard orthogonal arrays $L_{16}(2^6)$ is used. The orthogonal table based on two levels, six factors is given in Table 5.

Step 4: Calculating TOPSIS scores

Before calculating TOPSIS scores, the experimental data of criterion factors shown in Table 5 should be normalized. The normalization strategy is expressed in Eq. (1). After that, the DoE results of attribute weights and the normalized criterion factors have to be combined. Concretely speaking, for W1 = (1, 4, 4, 2, 1, 4) in Table 4, each row of the normalized criterion factors shall be multiplied by W1. This procedure is repeated until each subset of the attribute weights is incorporated into the decision matrix. In the application process of equipment selection problem, the experimental combination runs 7 times as there exit 7 different weight sets (Table 4). After the combination, a 112 × 6 decision matrix is generated. The decision matrix is used as input values to the TOPSIS model. Then we can calculate the evaluation scores using TOPSIS analysis method described in section "TOPSIS method". Table 6 shows the calculation results (TOPSIS scores) under 7 different attribute weights. The decision matrix with its TOPSIS scores forms a data set that can be used to generate approximation models of the multiple criteria.

Step 5: Determination of the Chebyshev regression model

Chebyshev regression method is widely used for its convenience and a nice property regarding its error. We use this method to model a functional relationship between the multiple criteria and the evaluation scores. The orthogonality means the inner product of any two Chebyshev polynomial terms is zero. In this case study, we use the approximation component in Isight to generate a regression model of our data. Detailed calculation process is described in "Chebyshev orthogonal polynomial" section. The Chebyshev orthogonal polynomial terms are determined and the coefficients of the terms are estimated. The results calculated by Isight are tabulated in Table 7. There are six input variables and the degree

**Table 6** TOPSIS scores under different attribute weights

| Alternative no. | TOPSIS scores under different attribute weights | | | | | | |
|---|---|---|---|---|---|---|---|
| | W1 | W2 | W3 | W4 | W5 | W6 | W7 |
| A1 | 0.4119 | 0.7555 | 0.4024 | 0.4356 | 0.4852 | 0.5239 | 0.4024 |
| A2 | 0.4637 | 0.8337 | 0.4726 | 0.5599 | 0.8940 | 0.5580 | 0.6053 |
| A3 | 0.9151 | 0.8370 | 0.8911 | 0.8114 | 0.8731 | 0.9814 | 0.9031 |
| A4 | 0.7607 | 0.8750 | 0.7314 | 0.6711 | 0.5025 | 0.7894 | 0.5618 |
| A5 | 0.2738 | 0.5556 | 0.2919 | 0.5308 | 0.6093 | 0.5183 | 0.5853 |
| A6 | 0.1749 | 0.5642 | 0.1715 | 0.4593 | 0.3561 | 0.4852 | 0.3949 |
| A7 | 0.5540 | 0.5651 | 0.5407 | 0.6337 | 0.3517 | 0.7089 | 0.5432 |
| A8 | 0.6074 | 0.5957 | 0.6127 | 0.9310 | 0.6297 | 0.7752 | 0.8630 |
| A9 | 0.4005 | 0.4125 | 0.4400 | 0.2590 | 0.5519 | 0.2651 | 0.4502 |
| A10 | 0.4388 | 0.4270 | 0.4085 | 0.2934 | 0.4632 | 0.2569 | 0.1650 |
| A11 | 0.8028 | 0.4279 | 0.7150 | 0.4888 | 0.4598 | 0.4960 | 0.4094 |
| A12 | 0.7393 | 0.4521 | 0.8148 | 0.5210 | 0.5699 | 0.5006 | 0.6107 |
| A13 | 0.2230 | 0.0668 | 0.1324 | 0.2396 | 0.2952 | 0.1470 | 0.0383 |
| A14 | 0.1317 | 0.1889 | 0.2609 | 0.2996 | 0.4320 | 0.1657 | 0.4434 |
| A15 | 0.5434 | 0.1916 | 0.5771 | 0.4926 | 0.4283 | 0.4614 | 0.5904 |
| A16 | 0.5805 | 0.2276 | 0.5464 | 0.5117 | 0.3218 | 0.4571 | 0.4020 |

**Table 7** Chebyshev regression results for equipment selection problem

| Coefficients | Chebyshev orthogonal terms | Coefficients | Chebyshev orthogonal terms |
|---|---|---|---|
| $a_0 = 0.5693$ | $T_0 = 1$ | $a_{14} = -0.0305$ | $T_{14} = (x_1 - 1.1718)(x_3 - 1.1687)$ |
| $a_1 = -0.1968$ | $T_1 = (x_1 - 1.1718)$ | $a_{15} = -0.0611$ | $T_{15} = (x_1 - 1.1718)(x_4 - 1.2453)$ |
| $a_2 = -0.1540$ | $T_2 = (x_2 - 1.2088)$ | $a_{16} = -0.0146$ | $T_{16} = (x_1 - 1.1718)(x_5 - 1.2190)$ |
| $a_3 = 0.1935$ | $T_3 = (x_3 - 1.1687)$ | $a_{17} = 0.0075$ | $T_{17} = (x_1 - 1.1718)(x_6 - 1.2395)$ |
| $a_4 = 0.0106$ | $T_4 = (x_4 - 1.2453)$ | $a_{18} = -0.0010$ | $T_{18} = (x_2 - 1.2088)(x_3 - 1.1687)$ |
| $a_5 = 0.0910$ | $T_5 = (x_5 - 1.2190)$ | $a_{19} = 0.0604$ | $T_{19} = (x_2 - 1.2088)(x_4 - 1.2453)$ |
| $a_6 = 0.0824$ | $T_6 = (x_6 - 1.2395)$ | $a_{20} = 0.0197$ | $T_{20} = (x_2 - 1.2088)(x_5 - 1.2190)$ |
| $a_7 = -0.0602$ | $T_7 = ((x_1 - 1.1718)\hat{}2 - 1.2559)$ | $a_{21} = -0.0387$ | $T_{21} = (x_2 - 1.2088)(x_6 - 1.2395)$ |
| $a_8 = -0.0726$ | $T_8 = ((x_2 - 1.2088)\hat{}2 - 1.1011)$ | $a_{22} = -0.0668$ | $T_{22} = (x_3 - 1.1687)(x_4 - 1.2453)$ |
| $a_9 = -0.0144$ | $T_9 = ((x_3 - 1.1687)\hat{}2 - 1.2665)$ | $a_{23} = 0.0166$ | $T_{23} = (x_3 - 1.1687)(x_5 - 1.2190)$ |
| $a_{10} = 0.0888$ | $T_{10} = ((x_4 - 1.2453)\hat{}2 - 0.8133)$ | $a_{24} = 0.0244$ | $T_{24} = (x_3 - 1.1687)(x_6 - 1.2395)$ |
| $a_{11} = 0.1589$ | $T_{11} = ((x_5 - 1.2190)\hat{}2 - 1.0449)$ | $a_{25} = -0.0663$ | $T_{25} = (x_4 - 1.2453)(x_5 - 1.2190)$ |
| $a_{12} = 0.0967$ | $T_{12} = ((x_6 - 1.2395)\hat{}2 - 0.8867)$ | $a_{26} = -0.2067$ | $T_{26} = (x_4 - 1.2453)(x_6 - 1.2395)$ |
| $a_{13} = 0.0209$ | $T_{13} = (x_1 - 1.1718)(x_2 - 1.2088)$ | $a_{27} = 0.1203$ | $T_{27} = (x_5 - 1.2190)(x_6 - 1.2395)$ |

of fit-polynomial is 2. In addition to the main effects of the six factors, interactions among the factors are also included. The polynomial regression model is expressed as follows:

$$y = a_0 T_0 + a_1 T_1 + \cdots + a_n T_n \tag{15}$$

where $y$ is the TOPSIS score, $T_n$ is the Chebyshev orthogonal polynomial term and $a_n$ is the coefficient.

Step 6: Ranking the alternatives

With the above regression model [Eq. (15)], the DM is able to evaluate the performance of different equipment alternatives and make the final decision. As this regression model is based on the normalized data set, the original attribute values shown in Table 2 should be normalized before substituting. The normalization procedure is the same as that in TOPSIS method. After that, the normalized attribute values shall be multiplied by attribute weights. In step 2, the experimental design of attribute weights has been conducted and the obtained regression model is based on different attribute weights. So in the last step, the importance of all the performance attributes is assumed to be equal. That is to say, the weights of the six attributes are all the same. To better fit the regression model and generate more reliable ranking scores, the attribute weight is not simply set as 1. It is the mean value of the whole DoE weight matrix (Table 4). The mean attribute weight is calculated as Eq. (16).

$$\bar{w} = \frac{1}{s \times n} \sum_{i=1}^{s} \sum_{j=1}^{n} w_{ij} \tag{16}$$

where $s$ is the number of weight experiments being conducted, $n$ is the number of criterion factors. In this case, the mean attribute weight is 5. By substituting the weighted normalized attribute values, the DM can obtain the ranking scores of alternative equipment. Results are shown in Table 8. The calculated ranking scores represent the equipment's performance. A higher score corresponds to a better performance. In this way, MM-5 is selected as the best alternative for this manufacturing company.

**Table 8** Ranking results of equipment selection problem

| Equipment alternatives | Ranking scores | Ranking results | | | |
|---|---|---|---|---|---|
| | | Improved TOPSIS | Original TOPSIS | AHP | ELECTRE |
| MM-1 | 0.4181 | 3 | 3 | 2 | 2 |
| MM-2 | 0.1321 | 5 | 5 | 5 | 5 |
| MM-3 | 0.3821 | 4 | 2 | 4 | 4 |
| MM-4 | 0.6622 | 2 | 4 | 3 | 3 |
| MM-5 | 0.7231 | 1 | 1 | 1 | 1 |

*Case analysis*

As discussed in the introduction, currently there are various MCDM methods that have been developed. To assure the validity of the proposed methodology, we make some comparisons between the improved TOPSIS method and other MCDM methods in the literature. In Dağdeviren (2008) paper, the same problem has been analyzed with AHP, TOPSIS and ELECTRE method. The ranking results tabulated in column 4 to 6 of Table 8 come from Dağdeviren (2008) paper. We also made a line chart in Fig. 4 to display the comparison results in a visual way. According to Table 8, the top-ranked alternatives are all the same. MM-5 is the most

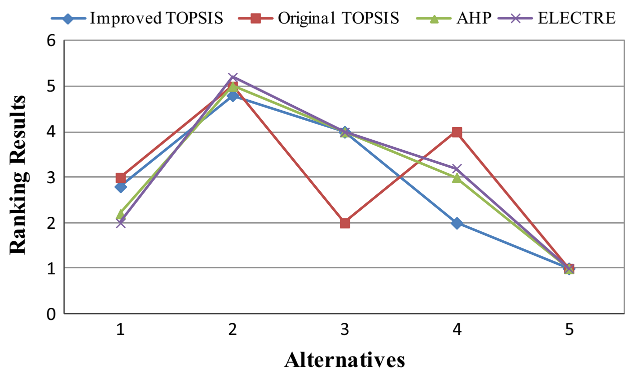


**Fig. 4** Rankings of the alternatives for equipment selection problem

suitable equipment to choose. There are slight discrepancies between the intermediate rankings of the alternatives. In our paper, a ranking of the alternatives is obtained as 3-5-4-2-1. Using original TOPSIS method, a ranking of the equipment is obtained as 3-5-2-4-1. As for the AHP and ELECTRE method, the ranking result is 2-5-4-3-1. The differences may be caused by the DM's subjective preferences. For MCDM problems, a decision maker lays more emphasis on the top-ranked alternatives. In this case, the first ranked equipment alternatives of the four methods exactly match. This implies the potential applicability of the proposed methodology.

## Applications and discussion

It is not easy to say which MCDM approach is more reasonable and reliable for a given decision making problem. A more reasonable and reliable way to prioritize alternatives is to apply several MCDM approaches to the same problem, compare their results, and then make the final decision (Kuo et al. 2008). In order to demonstrate the applicability and validity of the improved TOPSIS method in solving real-life MCDM problems, the following two illustrative examples are considered in this section.

### Layout selection

The layout design problem is a strategic issue and has a significant impact on the efficiency of a manufacturing system. In this case study, a practical layout design problem adapted from Yang and Kuo (2003) is presented. It is an IC (integrated circuit) packaging plant. There are 6 performance attributes and 18 alternative layouts. The company has to choose the most suitable layout alternative to assure the efficiency of production activities. The 6 performance attributes are flow distance, adjacency score, shape ratio, flexibility, accessibility and maintenance. For a layout design problem, we would like to minimize both the flow distance and shape ratio, while maximizing adjacency score, flexibility, accessi-

**Table 9** Attributes for layout selection problem

| Alternatives | Flow distance | Adjacency score | Shape ratio | Flexibility | Accessibility | Maintenance |
|---|---|---|---|---|---|---|
| 1 | 185.95 | 8 | 8.28 | 0.0494 | 0.0294 | 0.0130 |
| 2 | 207.37 | 9 | 3.75 | 0.0494 | 0.0147 | 0.0519 |
| 3 | 206.38 | 8 | 7.85 | 0.0370 | 0.0147 | 0.0519 |
| 4 | 189.66 | 8 | 8.28 | 0.0370 | 0.0147 | 0.0519 |
| 5 | 211.46 | 8 | 7.71 | 0.0617 | 0.0147 | 0.0390 |
| 6 | 264.07 | 5 | 2.07 | 0.0494 | 0.0147 | 0.0519 |
| 7 | 228.00 | 8 | 14.00 | 0.0247 | 0.0735 | 0.0649 |
| 8 | 185.59 | 9 | 6.25 | 0.0370 | 0.0441 | 0.0390 |
| 9 | 185.85 | 9 | 7.85 | 0.0741 | 0.0441 | 0.0519 |
| 10 | 236.15 | 8 | 7.85 | 0.0741 | 0.0588 | 0.0649 |
| 11 | 183.18 | 8 | 2.00 | 0.0864 | 0.1029 | 0.0909 |
| 12 | 204.18 | 8 | 13.3 | 0.0370 | 0.0588 | 0.0260 |
| 13 | 225.26 | 8 | 8.14 | 0.0247 | 0.0735 | 0.0519 |
| 14 | 202.82 | 8 | 8.00 | 0.0247 | 0.0588 | 0.0519 |
| 15 | 170.14 | 9 | 8.28 | 0.0864 | 0.1176 | 0.1169 |
| 16 | 216.38 | 9 | 7.71 | 0.0741 | 0.0735 | 0.0519 |
| 17 | 179.80 | 8 | 10.30 | 0.0988 | 0.1324 | 0.0909 |
| 18 | 185.75 | 10 | 10.16 | 0.0741 | 0.0588 | 0.0390 |
| Min/max | Min | Max | Min | Max | Max | Max |
| Low level | 170.14 | 5 | 2 | 0.0247 | 0.0147 | 0.0130 |
| High level | 264.07 | 10 | 14 | 0.0988 | 0.1324 | 0.1169 |

**Table 10** Chebyshev regression results for layout selection problem

| Coefficients | Chebyshev orthogonal terms | Coefficients | Chebyshev orthogonal terms |
|---|---|---|---|
| $a_0 = 0.5655$ | $T_0 = 1$ | $a_{14} = -0.0238$ | $T_{14} = (x_1 - 1.2217)(x_3 - 1.0000)$ |
| $a_1 = -0.1350$ | $T_1 = (x_1 - 1.2217)$ | $a_{15} = -0.0286$ | $T_{15} = (x_1 - 1.2217)(x_4 - 1.0719)$ |
| $a_2 = -0.0227$ | $T_2 = (x_2 - 1.1859)$ | $a_{16} = -0.0100$ | $T_{16} = (x_1 - 1.2217)(x_5 - 0.9760)$ |
| $a_3 = -0.0903$ | $T_3 = (x_3 - 1.0000)$ | $a_{17} = -0.0045$ | $T_{17} = (x_1 - 1.2217)(x_6 - 0.9762)$ |
| $a_4 = 0.0625$ | $T_4 = (x_4 - 1.0719)$ | $a_{18} = 0.0106$ | $T_{18} = (x_2 - 1.1859)(x_3 - 1.0000)$ |
| $a_5 = 0.0532$ | $T_5 = (x_5 - 0.9760)$ | $a_{19} = -0.0223$ | $T_{19} = (x_2 - 1.1859)(x_4 - 1.0719)$ |
| $a_6 = 0.0836$ | $T_6 = (x_6 - 0.9762)$ | $a_{20} = -0.0208$ | $T_{20} = (x_2 - 1.1859)(x_5 - 0.9760)$ |
| $a_7 = -0.0551$ | $T_7 = ((x_1 - 1.2217)\hat{}2 - 1.0279)$ | $a_{21} = -0.0081$ | $T_{21} = (x_2 - 1.1859)(x_6 - 0.9762)$ |
| $a_8 = 0.0513$ | $T_8 = ((x_2 - 1.1859)\hat{}2 - 1.2042)$ | $a_{22} = 0.0052$ | $T_{22} = (x_3 - 1.0000)(x_4 - 1.0719)$ |
| $a_9 = -0.0085$ | $T_9 = ((x_3 - 1.0000)\hat{}2 - 1.6017)$ | $a_{23} = -0.0005$ | $T_{23} = (x_3 - 1.0000)(x_5 - 0.9760)$ |
| $a_{10} = 0.0010$ | $T_{10} = ((x_4 - 1.0719)\hat{}2 - 1.5012)$ | $a_{24} = -0.0043$ | $T_{24} = (x_3 - 1.0000)(x_6 - 0.9762)$ |
| $a_{11} = -0.0203$ | $T_{11} = ((x_5 - 0.9760)\hat{}2 - 1.6261)$ | $a_{25} = -0.0129$ | $T_{25} = (x_4 - 1.0719)(x_5 - 0.9760)$ |
| $a_{12} = 0.0280$ | $T_{12} = ((x_6 - 0.9762)\hat{}2 - 1.6259)$ | $a_{26} = -0.0160$ | $T_{26} = (x_4 - 1.0719)(x_6 - 0.9762)$ |
| $a_{13} = -0.1181$ | $T_{13} = (x_1 - 1.2217)(x_2 - 1.1859)$ | $a_{27} = -0.0124$ | $T_{27} = (x_5 - 0.9760)(x_6 - 0.9762)$ |

**Table 11** Ranking results of layout selection problem

| Alternatives | Ranking scores | Ranking results | | | |
|---|---|---|---|---|---|
| | | Improved TOPSIS | Original TOPSIS | SAW | GRA |
| 1 | 0.4779 | 16 | 17 | 14 | 10 |
| 2 | 0.5524 | 9 | 8 | 8 | 8 |
| 3 | 0.4860 | 15 | 15 | 15 | 15 |
| 4 | 0.4907 | 14 | 16 | 12 | 11 |
| 5 | 0.4982 | 13 | 14 | 11 | 13 |
| 6 | 0.5658 | 7 | 9 | 18 | 16 |
| 7 | 0.4598 | 17 | 13 | 16 | 17 |
| 8 | 0.5386 | 10 | 12 | 9 | 7 |
| 9 | 0.5897 | 4 | 6 | 5 | 5 |
| 10 | 0.5848 | 6 | 5 | 7 | 9 |
| 11 | 0.7783 | 1 | 1 | 3 | 3 |
| 12 | 0.4312 | 18 | 18 | 17 | 18 |
| 13 | 0.5244 | 12 | 10 | 13 | 14 |
| 14 | 0.5277 | 11 | 11 | 10 | 12 |
| 15 | 0.7712 | 2 | 2 | 1 | 1 |
| 16 | 0.5874 | 5 | 4 | 6 | 6 |
| 17 | 0.6887 | 3 | 3 | 2 | 2 |
| 18 | 0.5581 | 8 | 7 | 4 | 4 |

bility and maintenance. Table 9 represents the performance characteristics of the considered layout selection with respect to all the criteria.

Using the proposed methodology in our paper, a six factor regression model is developed. The detailed calculation process is omitted here. Table 10 shows the calculated Chebyshev polynomial terms and their coefficients. The regression formula is expressed in Eq. (15). By substituting the attribute values to the regression model, a ranking of the layout design is obtained as 16-9-15-14-13-7-17-10-4-6-1-18-12-11-2-5-3-8. In the literature, Yang and Kuo (2003) adopted TOPSIS and SAW methods in solving the same case study problem. The results showed that alternatives 11, 15, and 17 were the performance frontiers. Kuo et al. (2008) used GRA method to

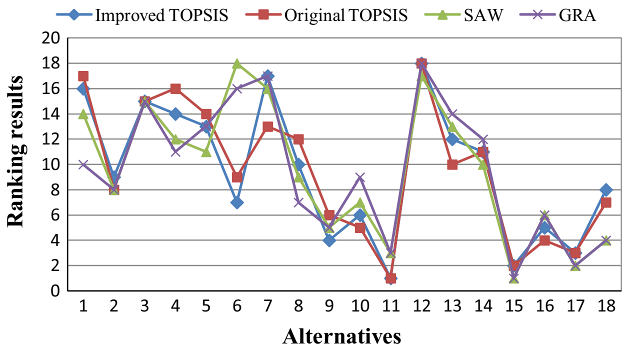


**Fig. 5** Rankings of the alternatives for layout selection problem

solve the layout design problem and drew the conclusion that alternative 15 was the best decision. The detailed comparison results are shown in Table 11 and Fig. 5. Considering that the selection problem has 18 alternatives, the ranking results of the 4 methods are roughly consistent. The top three ranked alternatives 11, 15, and 17 may be considered by the DMs. This case study shows that the improved TOPSIS method provides comparable ranking results with other approach.

## Pipe material selection

The material plays an important role in an engineering design process. The suitable material selection for a particular product is one of the vital tasks for the designers. In this section, we use the proposed methodology to solve a real problem in the sugar industry located at southern part of the India (Anojkumar et al. 2014). The DMs of the sugar industry are concerned with the issue of choosing the optimum material for the pipes to minimize the corrosive wear. Interested

**Table 12** Attributes for pipe material selection problem

| Alternatives | Yield strength | Tensile strength | % of elongation | Hardness | Cost | Corrosion rate | Wear rate |
|---|---|---|---|---|---|---|---|
| J4 | 382 | 728 | 48 | 98 | 112 | 0.16 | 2.75 |
| JSLAUS | 420 | 790 | 58 | 97 | 210 | 0.31 | 2.63 |
| 204Cu | 415 | 795 | 55 | 96 | 120 | 0.05 | 2.5 |
| 409M | 270 | 455 | 32 | 78 | 184 | 0.4 | 4 |
| 304 | 256 | 610 | 60 | 86 | 89 | 0.01 | 2.59 |
| Min/max | Max | Max | Max | Max | Min | Min | Min |
| Low level | 256 | 455 | 32 | 78 | 89 | 0.01 | 2.5 |
| High level | 420 | 795 | 60 | 98 | 210 | 0.4 | 4 |

**Table 13** Chebyshev regression results for pipe material selection problem

| Coefficients | Chebyshev orthogonal terms | Coefficients | Chebyshev orthogonal terms |
|---|---|---|---|
| $a_0 = 0.4847$ | $T_0 = 1$ | $a_{18} = 0.0139$ | $T_{18} = (x_1 - 1.2148)(x_5 - 1.1587)$ |
| $a_1 = 0.0627$ | $T_1 = (x_1 - 1.2148)$ | $a_{19} = 0.0030$ | $T_{19} = (x_1 - 1.2148)(x_6 - 0.9057)$ |
| $a_2 = 0.0431$ | $T_2 = (x_2 - 1.2062)$ | $a_{20} = 0.0194$ | $T_{20} = (x_1 - 1.2148)(x_7 - 1.2180)$ |
| $a_3 = 0.0987$ | $T_3 = (x_3 - 1.1958)$ | $a_{21} = -0.0564$ | $T_{21} = (x_2 - 1.2062)(x_3 - 1.1958)$ |
| $a_4 = 0.0320$ | $T_4 = (x_4 - 1.2420)$ | $a_{22} = -0.0249$ | $T_{22} = (x_2 - 1.2062)(x_4 - 1.2420)$ |
| $a_5 = -0.0769$ | $T_5 = (x_5 - 1.1587)$ | $a_{23} = 0.0344$ | $T_{23} = (x_2 - 1.2062)(x_5 - 1.1587)$ |
| $a_6 = -0.1177$ | $T_6 = (x_6 - 0.9057)$ | $a_{24} = 0.0110$ | $T_{24} = (x_2 - 1.2062)(x_6 - 0.9057)$ |
| $a_7 = -0.0215$ | $T_7 = (x_7 - 1.2180)$ | $a_{25} = -0.0152$ | $T_{25} = (x_2 - 1.2062)(x_7 - 1.2180)$ |
| $a_8 = 0.0089$ | $T_8 = ((x_1 - 1.2148)\hat{}2 - 1.0694)$ | $a_{26} = 0.0191$ | $T_{26} = (x_3 - 1.1958)(x_4 - 1.2420)$ |
| $a_9 = 0.0248$ | $T_9 = ((x_2 - 1.2062)\hat{}2 - 1.1146)$ | $a_{27} = 0.0125$ | $T_{27} = (x_3 - 1.1958)(x_5 - 1.1587)$ |
| $a_{10} = 0.0579$ | $T_{10} = ((x_3 - 1.1958)\hat{}2 - 1.1627)$ | $a_{28} = 0.0284$ | $T_{28} = (x_3 - 1.1958)(x_6 - 0.9057)$ |
| $a_{11} = 0.0888$ | $T_{11} = ((x_4 - 1.2420)\hat{}2 - 0.8584)$ | $a_{29} = 0.0197$ | $T_{29} = (x_3 - 1.1958)(x_7 - 1.2180)$ |
| $a_{12} = -0.0515$ | $T_{12} = ((x_5 - 1.1587)\hat{}2 - 1.2990)$ | $a_{30} = -0.0209$ | $T_{30} = (x_4 - 1.2420)(x_5 - 1.1587)$ |
| $a_{13} = 0.0621$ | $T_{13} = ((x_6 - 0.9057)\hat{}2 - 1.6771)$ | $a_{31} = -0.0095$ | $T_{31} = (x_4 - 1.2420)(x_6 - 0.9057)$ |
| $a_{14} = -0.0025$ | $T_{14} = ((x_7 - 1.2180)\hat{}2 - 1.0508)$ | $a_{32} = -0.0997$ | $T_{32} = (x_4 - 1.2420)(x_7 - 1.2180)$ |
| $a_{15} = -0.0261$ | $T_{15} = (x_1 - 1.2148)(x_2 - 1.2062)$ | $a_{33} = 0.0109$ | $T_{33} = (x_5 - 1.1587)(x_6 - 0.9057)$ |
| $a_{16} = -0.0529$ | $T_{16} = (x_1 - 1.2148)(x_3 - 1.1958)$ | $a_{34} = 0.0445$ | $T_{34} = (x_5 - 1.1587)(x_7 - 1.2180)$ |
| $a_{17} = 0.0223$ | $T_{17} = (x_1 - 1.2148)(x_4 - 1.2420)$ | $a_{35} = -0.0018$ | $T_{35} = (x_6 - 0.9057)(x_7 - 1.2180)$ |

**Table 14** Ranking results of pipe material selection problem

| Equipment alternatives | Ranking scores | Ranking results | | | |
|---|---|---|---|---|---|
| | | Improved TOPSIS | Original TOPSIS | ELECTRE | PROMETHEE |
| J4 | 0.4565 | 4 | 3 | 3 | 2 |
| JSLAUS | 0.5365 | 3 | 4 | 4 | 4 |
| 204Cu | 0.5743 | 2 | 2 | 2 | 1 |
| 409M | 0.3417 | 5 | 5 | 5 | 5 |
| 304 | 0.6406 | 1 | 1 | 1 | 3 |

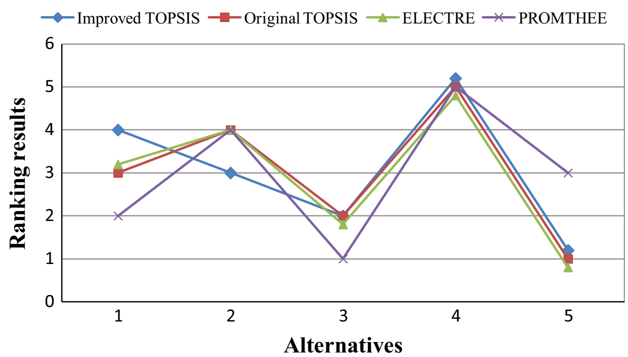


**Fig. 6** Rankings of the alternatives for pipe material selection problem

readers are referred to Anojkumar et al. (2014) for a detailed description of the selection problem. First of all, we should identify the alternatives and criterion level. Through literature (Prado et al. 2010; Wesley et al. 2012) and experts in the industry, five stainless steel grades such as J4, JSLAUS, J204Cu, 409M, 304 and seven evaluation criteria such as yield strength, ultimate tensile strength, percentage of elongation, hardness, cost, corrosion rate and wear rate are considered to choose the suitable material. Table 12 shows the attribute values for 5 alternative pipe materials, where yield strength, ultimate tensile strength, percentage of elongation, hardness are the beneficial criteria and cost, corrosion rate, wear rate are the non-beneficial criteria.

After conducting steps 1 to 6 in “Detailed steps” section, we are able to generate a regression model for evaluating alternatives. The calculated Chebyshev polynomial terms and their coefficients are tabulated in Table 13. Using the regression model expressed in Eq. (15), we obtain the ranking of the alternatives as 4-3-2-5-1. The detailed results are shown in Table 14 and Fig. 6. For comparison, the ranking results of TOPSIS, ELECTRE and PROMETHEE methods are shown in column 4, 5, and 6 of Table 14, respectively. These results come from Anojkumar et al. (2014) paper. A close examination of Table 14 reveals that the four MCDM techniques deliver the very similar results. Although there does not exist an optimal solution for MCDM problems, three of the four methods lead to the choice of material 304 as a possible final decision. Therefore, material 304 would be the optimum material for the pipes in the sugar industry. From above analysis, we can see that the improved TOPSIS method can generate reliable solutions efficiently when they are benchmarked with the results from the existing methodologies.

## Conclusion

This paper presents an improved TOPSIS method based on experimental design and Chebyshev orthogonal polynomial regression in solving MCMD problems. Three examples are considered to illustrate the application capability of this method. The ranking results are compared with some commonly used MCDM methods involving original TOPSIS, AHP, GRA, SAW, ELECTRE and PROMETHEE. In all the cases, it is observed that the top-ranked alternatives exactly match with those derived by the past researchers. This indicates that the proposed methodology can be used for solving real-life MCDM problems. It can provide an accurate evaluation of the alternatives and offer a more reasonable selection.

Building a regression model normally involves two steps: (1) employing design of experiments to sample the computer simulation, and (2) selecting an approximation model to represent the data and fit the model with the sample data. In this paper, optimal Latin hypercube design and orthogonal array technique are carried out for the weight experiment and criterion experiment, respectively. The experimental data is approximated using Chebyshev orthogonal polynomial regression method. The combination of these methods with TOPSIS technique has not ever been presented within the context of MCDM. Compared with other MCDM methods, the improved TOPSIS method mainly has two advantages. First, the weight assignment is conducted using DoE technique. This technique helps DM by quantifying the relative importance of each criterion statistically. In classical MCDM methods, the weights of experts’ opinions play an important role in the decision making. The DM’s evaluations on multiple criteria are always subjective and thus imprecise. Ranking results are very sensitive to the changes in the attribute weights. If the weighting procedure of a MCDM method is not made correctly, then the weights will be generated incorrectly, which directly affect the outcome of the MCDM approach. In our method, there is no need for an expert to assign exact numerical values to the comparison judgments. This avoids the subjectivity of human preference in making a decision and decreases the sensitivity to the weight change. Thus, the ranking results become more reasonable and reliable. Second, a regression model is generated to help the DMs make the decision. For a proper and effective evaluation, the DMs always need a large amount of data to be analyzed and many factors to be considered. The DM should be an expert or at least be very familiar with the selection problem. In our paper, a MCDM model is obtained by using the integrated DoE and Chebyshev regression approach. When the regression equation is obtained, the alternative evaluation process can be easily facilitated. The MCDM model ranks the alternatives and the highest ranked one is recommended as the best alternative. The DMs do not need to have technical knowledge in MCDM fields or a strong background in mathematics. They can use the obtained regression model to choose and analyze factors and attributes easily. Moreover, if a new alternative is added to or removed from the MCDM problem, all the DMs need to do is to use the regression model and the final results would be got. It is quite convenient and practicable.

Although the proposed methodology is successfully applied in solving some manufacturing MCDM problems, it should be noted that this approach also has some limitations. First, the generated regression model contains too many Chebyshev terms when the interaction effects of different criteria are considered. In the layout selection problem, there are 6 performance attributes and 18 alternative layouts. However, the obtained regression model contains 28 Chebyshev terms shown in Table 10. When the performance attributes increased to 7 (the pipe material selection problem), the number of Chebyshev terms shown in Table 13 increased to 36. In this paper we only use quadratic Chebyshev polynomials to do the approximation job. It's known that the approximation accuracy increases with the degree of fit-polynomial. But using higher order of Chebyshev polynomials means the regression equation will be more complicated. The contradiction should be balanced. Second, the calculation process becomes less efficient when the decision matrix contains a large number of attributes. As the regression model is constructed under different weight sets, the DoE results of attribute weights and the normalized criterion factors have to be combined. In the application process of equipment selection problem, the experimental combination runs 7 times and a $112 \times 6$ decision matrix is generated. A large number of attributes means a large number of MCDM evaluations have to be performed. But as the computer technology develops, the time consumed on the calculation process will be less and less. Future study will focus on improving the proposed methodology and overcoming the limitations mentioned above.

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