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Joint production and maintenance strategy for economic production quantity model with imperfect production processes

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Abstract In this study, an economic production quantity (EPQ) model is generalized by considering maintenance and production programs for an imperfect process involving a deteriorating production system with increasing hazard rate. There are two types of preventive maintenance (PM), namely imperfect PM and perfect PM. The probability that perfect PM is performed depends on the number of imperfect maintenance operations performed since the last renewal cycle. Following a failure, the delayed repair performs some restorations and reduces production rate to restore the system into an operating state (in-control state), but leaves its lower production rate until perfect PM is performed. That is, the production run period not always starts in normal production rate. This study considers backorders, as well as loss of inventory due to the lower production rate. For the EPQ model, the optimum run time, which minimizes the total cost, is discussed. Various special cases are considered, including the maintenance learning effect. Finally, a numerical example is presented to illustrate the effects of PM ability, repair cost and production decreasing rate on total costs and production period.

Keywords Production · Imperfect maintenance · Learning effect · Optimum · Backorder

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List of symbols

T	Time of each production run
T_1	Period of production stoppage and inventory depletion; $T_1 = (\frac{p}{d} - 1)T$
$*$	Implies an optimum value
p	Normal production rate in units per year
Q	Production lot
d	Demand rate in units per year; $p > d$
α	Production decreasing rate after delayed repair
$\bar{P}_j$	Probability that the first j PM are imperfect PM; $\bar{P}_0 = 1$
p_j	Probability that PM is perfect following the $(j-1)$ imperfect PM; $p_j = \bar{P}_{j-1} - \bar{P}_j$
$\{\bar{P}_j\}$	Sequence of $\bar{P}_j$, $j = 0, 1, 2, \ldots$
q_j	Probability that the j-th PM is an imperfect PM; $q_j = \bar{P}_j / \bar{P}_{j-1}$
θ_j	Probability that the j-th PM is a perfect PM; $\theta_j = 1 - q_j$
M	Number of PM preceding the first perfect PM
R_m	Cost of each PM
R_s	Setup cost for each production run
R_{ms}	Sum of R_m and R_s; $R_{ms} = R_m + R_s$
R_r	Delayed repair cost of time lapse between failure and perfect PM per unit of time, including rework cost
R_b	Backorder cost per unit
R_h	Holding cost per unit per year of the product
$J(T; \{\bar{P}_j\})$	Expected total production cost for the EPQ model
X	Time to failure of a new unit
$F(t)$	Failure distribution function of X
$f(t)$	Failure density function associated with $F(t)$
$\bar{F}(t)$	Survival function associated with $F(t)$
r	The learning rate

Introduction

In the competitive business environment, managers of manufacturing industries encounter the challenge everyday to find ways of minimizing production costs by Anis and Daoud (2004), Hajji et al. (2010) and Ouyang et al. (2002). Recently considerable attention has been devoted to maintenance as an integral aspect of production. Maintenance is launched when equipment fails or as planned preventive maintenance (PM) (Chiu 2010; Sheu et al. 2006; Radhoui et al. 2010; Liao et al. 2010). Manufacturers seek a production policy that also manages inventory levels under uncertain production failure and demand to provide better services than before to customers. The economic production quantity (EPQ) model is a useful inventory control model that has been studied in detail (Chang and Ho 2010).

EPQ model can be considered as an extension to the well known economic order quantity (EOQ) model that was introduced by Harris (1913). Traditional EPQ models assume that a production process always produces parts with perfect quality (Biskup et al. 2003). Production of defective items is a feature of real production systems (Salameh and Jaber 2000). Such a production process is called imperfect production (Liu and Yang 1996). Furthermore, Rosenblatt and Lee (1986) investigated the influence of process deterioration on optimal EPQ.

As a result, random machine shifts from 'in-control' state to 'out-of-control' state frequently occur during production runs. Repair can restore a failed system to working order ('in-control' state). Different actions are generally taken during system repair, and affect the system differently following repair. Major repairs reset the system failure intensity and the system is restored to the "as good as new" state (Yang and Klutke 2001). Minimal repairs do not improve the hazard rate of the system, but simply restores the system to operational status (Barlow and Hunter 1960; Nakagawa and Kijima 1989; Ja et al. 2001). Biswas et al. (2003) showed that the breakdown policy (delayed repair) can be applied to certain situations following failures, such as 'difficulty of access', and 'cost constraints', preventing continuous system monitoring, and also restricting the restoration of repaired units until the next scheduled inspection period. In certain cases of standby redundancy, the smaller redundant elements (spares) begin to work only when the active element has failed, and the failures are repaired until perfect PM. Smaller redundancy facilities reduce the production rate. If the products are performed at lower production rate during pre-determined intervals, then stockout is unavoidable. In many practical situations, the occurrence of shortages in inventory is a natural phenomenon due to various uncertainties. Hu et al. (2010) considered the economic production run time problem with imperfect production processes and allowable shortages.

The product system can be produced more efficiently using a PM program that significantly increases production process reliability (Yang et al. 2008). PM is performed regularly at pre-determined intervals (Schutz et al. 2011). The major difference between sequential PM policy and periodic PM policy is that periodic PM involves performing PM at fixed time intervals (Chiang and Yuan 2001; Nakagawa and Yasui 1991), which do not exist for sequential PM (Lin et al. 2000; Nakagawa 1988). The perfect PM model is assumed to return to an "as good as new" state following each PM action. Tseng (1996) devised a perfect maintenance policy to improve the reliability of deteriorating systems. However, recognizing imperfect PM is more realistic. Nakagawa and Yasui (1987) and Nakagawa (1979) introduced an imperfect PM model, in which PM is "bad as old" with probability p, and "good as new" with probability $\bar{p} = 1 - p$. Numerous studies based on maintenance and production analyses have attempted to apply the imperfect production model to various real-world situations, including imperfect PM (Lin et al. 2003; Pham and Wang 1996). Ben-Daya (2002) assumed that, following each PM, the system age reduces proportional to the PM level for an integrated EPQ model with imperfect process.

This investigation assumes that the probability of perfect PM depends on the number of imperfect maintenance operations performed since the previous renewal cycle. The purpose of this paper is to extend the model of Liao et al. (2009) incorporating delayed repair, variable production rate and backorder to study the joint effects of PM cost, setup cost, backorder cost, holding cost and production rate.

Section "General model" describes the integrated maintenance and production programs in the EPQ model. Following a failure, the delayed repair performs some restorations and reduces production rate. The proposed model incorporates the occurrence of inventory shortages due to insufficient production following lower production rate. Restated, PM is performed after production run period. The integrated EPQ model is determined, and used to identify the optimal production policy.

The remainder of this study is organized as follows: Section "Special cases" details various special cases. Section "Example and analysis" then presents numerical results for special cases. Next, a study is conducted to examine the effects of these parameters on the solution. Finally, the last section presents "Concluding remarks".

General model

PM helps maintain a production system in top operating conditions. This section considers a generalized EPQ model with backorder, delayed repair and PM using the following

PM scheme. A system is classified as one of two PM types; that is, following periodic PM, the system may be either (1) unchanged, or (2) renewed. Type I PM is termed imperfect PM, while type II PM is labeled as perfect PM. This system enables the probability of type II PM to depend on the number of PM undertaken since the previous renewal cycle. Furthermore, M denotes the number of PM until the occurrence of the first type II PM. Additionally, let $\bar{P}_j = P(M > j)$. That is, $\bar{P}_j$ represents the probability that the first j outcomes are type I PM. Based on Sheu et al. (2006), we assume that the domain of $\bar{P}_j$ is $\{0, 1, 2, \ldots\}$, and that $1 = \bar{P}_0 > \bar{P}_1 \geq \bar{P}_2 \geq \cdots$. The probability $\bar{P}_j$ does not increase with the number of PM items j. The notation $\{\bar{P}_j\}$ is used as an abbreviation for a sequence of probabilities. Furthermore, let $p_j = P(M = j) = \bar{P}_{j-1} - \bar{P}_j = \bar{P}_{j-1}\left(1 - \bar{P}_j/\bar{P}_{j-1}\right)$, with domain $\{1, 2, 3, \ldots\}$. Consequently, if the j-th PM occurs, a PM is either type I with probability $q_j = \bar{P}_j/\bar{P}_{j-1}$, or type II with probability $\theta_j = 1 - q_j$.

To model the problem, the inventory cycle is divided into two major periods: inventory building period (production run period) and inventory depletion period (production stoppage period). T and T_1 represent the occurrence times of the inventory building and inventory depletion periods, respectively. Figures 1, 2 and 3 illustrate the different inventory cycle types of the EPQ model. Moreover, the following assumptions are made:

1. The demand rate, setup cost and holding cost are known constants.
2. Backorder is permitted in inventory depletion period.
3. The original system begins operating at time 0. The production process starts in an in-control state and perfect items are produced.
4. At the start of each inventory cycle, the setup cost R_s will be incurred and the state of the process is assumed not always to be restored to normal production rate. The cycle time for each production lot is T. PM is performed after production run period. The cost of each PM is R_m.
5. A system has two types of PM at cumulative production run time $j \cdot T (j = 1, 2, \ldots)$, based on outcome:
   - type-I PM (imperfect PM) results in the unit having the same failure rate as before PM, with probability $q_j = \bar{P}_j/\bar{P}_{j-1}$ $(0 \leq q_j < 1)$,
   - type-II PM (perfect PM) makes the unit as good as new, with probability $\theta_j = 1 - q_j$.
6. Following a perfect PM, the system returns to age 0.

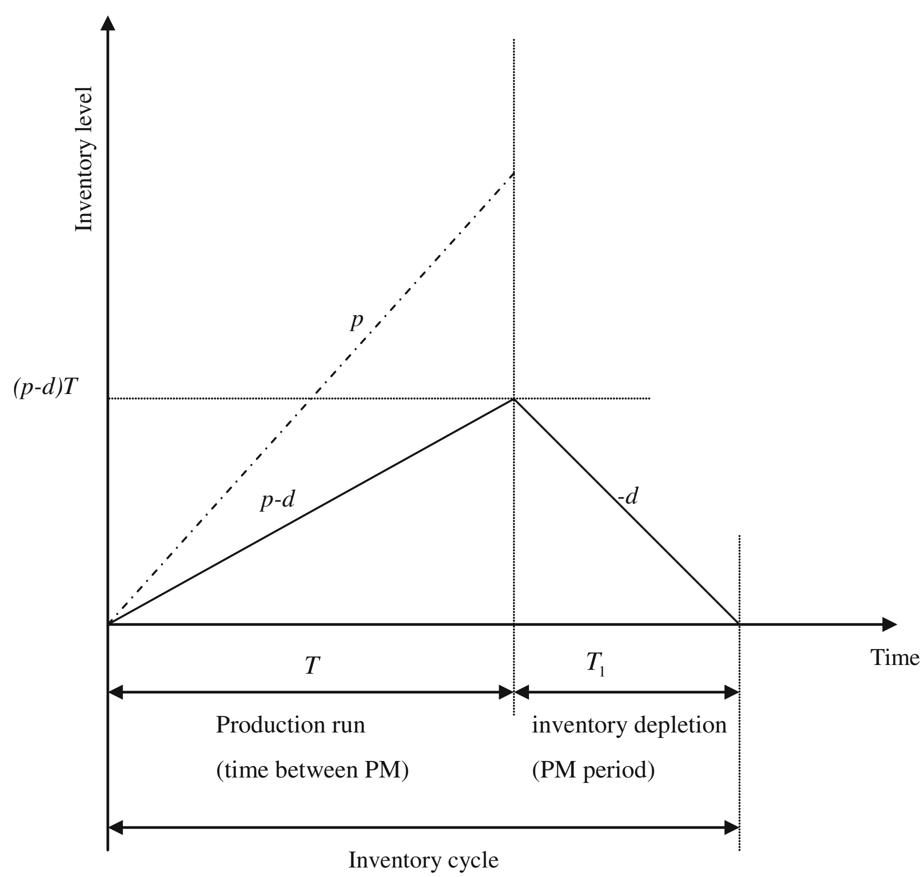


Fig. 1 The inventory cycle of the EPQ model with normal inventory building and depletion period

Fig. 2 The inventory cycle of the EPQ model with delayed repair and backorder

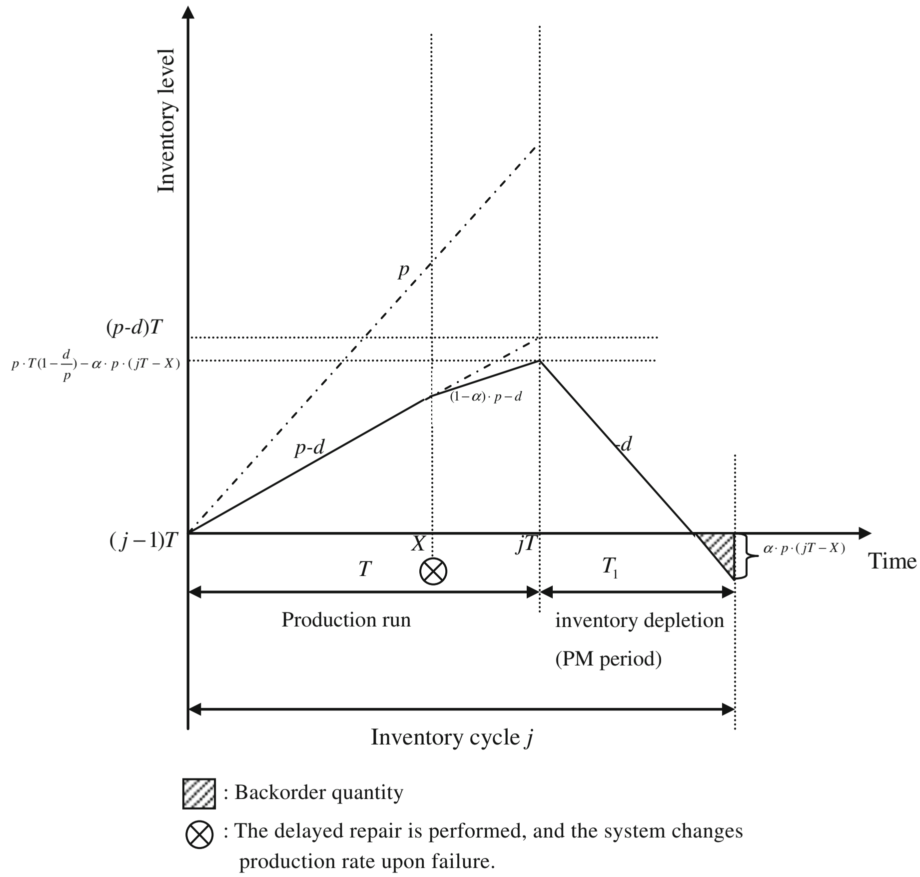
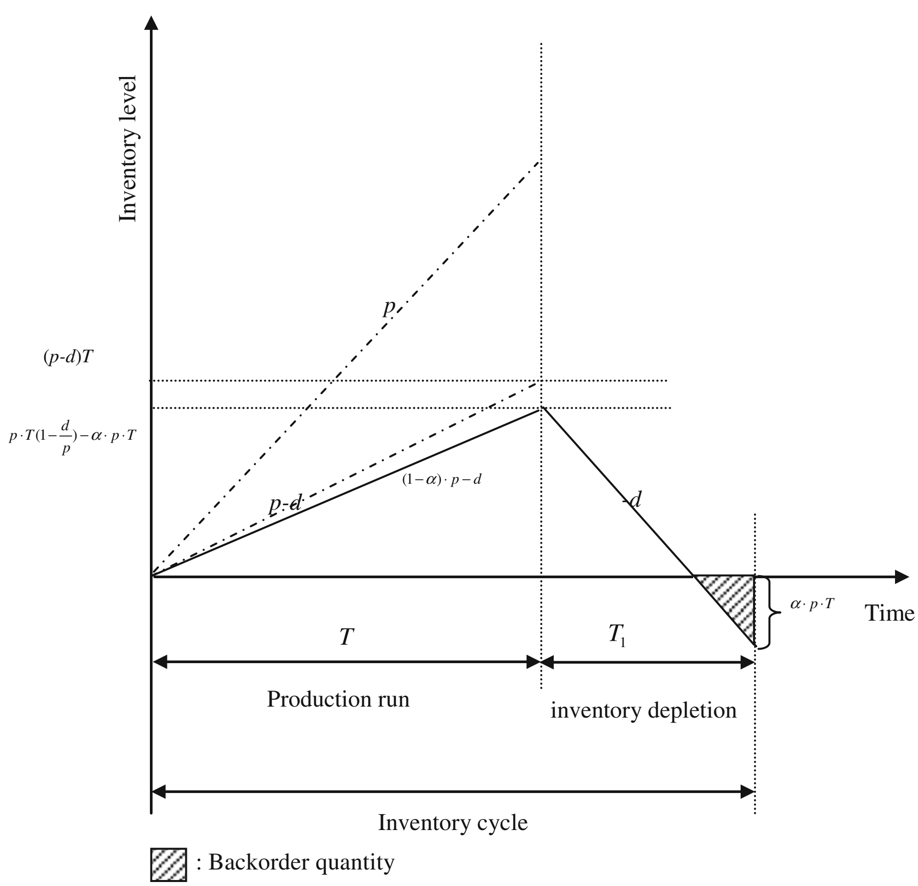


Fig. 3 The inventory cycle of the EPQ model starts in lower production rate



7. If failure occurs, the system shifts into the "out-of-control" state, and delayed repair is performed immediately. The delayed repair performs some restorations and production rate reduces to $(1-\alpha)p$, then returns the system to an operating state, but leaves its lower production rate until perfect PM. That is, the production process returns to the in-control condition, but leaves its hazard rate unchanged. The backorder occurs due to insufficient production after delayed repair. The delayed repair cost is R_r and backorder cost per unit is R_b.
8. The repair times are negligible.
9. The PM work must finish before next inventory cycle. Then, the PM time must be less than or equal to T_1.
10. $1-\frac{d}{p}-\alpha>0$. That is, backorder is not allowed in inventory building period.

Let DT_j be the failure time of the jth inventory cycle ($j=1,2,\ldots$). In addition, the expected failure time for one inventory cycle is $\sum_{j=1}^{\infty}E[DT_j]$.

$$\sum_{j=1}^{\infty}E[DT_j]=\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\left[\int_{(j-1)T}^{jT}(jT-t)f(t)\,dt\right]+\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}(T\cdot F((j-1)\cdot T)). \quad (1)$$

The first term on the right-hand side of equation (Eq. 1) is the mean failure time till production stoppage period when failure occurs at inventory cycle $j\,(j=1,2,\ldots)$; the system enters $(j-1)$ successive type I PM, so the delayed repair is performed and the production rate decreases immediately, as shown in Fig. 2. The second term is the situation that the production run period starts in lower production rate, as depicted in Fig. 3.

Figure 2 show that the breakdown occurs at times X during the time interval $((j-1)T, jT)$. The reduction in the inventory is $\alpha p(jT-X)$ due to the lower production rate after delayed repair. The inventory level is $pT(1-\frac{d}{p})-\alpha p(jT-X)$ at the end of the production run period. Then, the backorder quantity is $\alpha p(jT-X)$. This change in the lower production rate due to delayed repair will affect the next inventory cycle until perfect PM. The inventory level is $pT(1-\frac{d}{p})-\alpha pT$ at the end of the following production run period and the backorder quantity is αpT, as shown in Fig. 3.

From Eq. (1), we have

$$\sum_{j=1}^{\infty}E[DT_j]=\sum_{j=1}^{\infty}\frac{\bar{P}_{j-1}}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_{(j-1)T}^{jT}F(t)\,dt$$
$$=\sum_{j=1}^{\infty}\frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_{0}^{jT}F(t)\,dt. \quad (2)$$

Following a perfect PM without failure, as shown in Fig. 4 or a failure occurs at inventory cycle j and the system is renewed at perfect PM, as depicted in Fig. 5, the system resets to age 0.

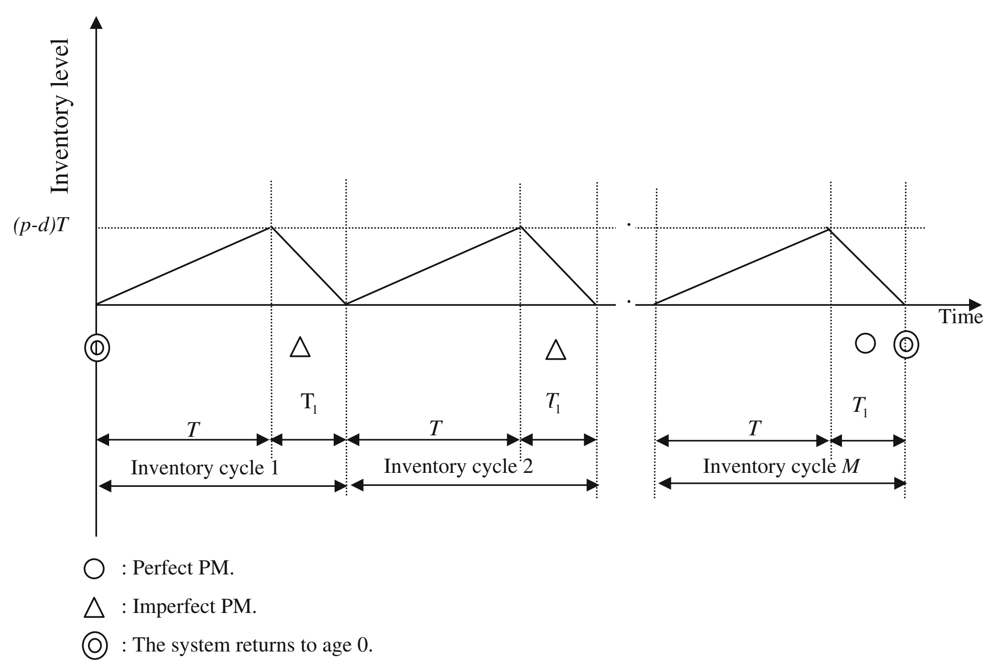


Fig. 4 The system is renewed at perfect PM without failure

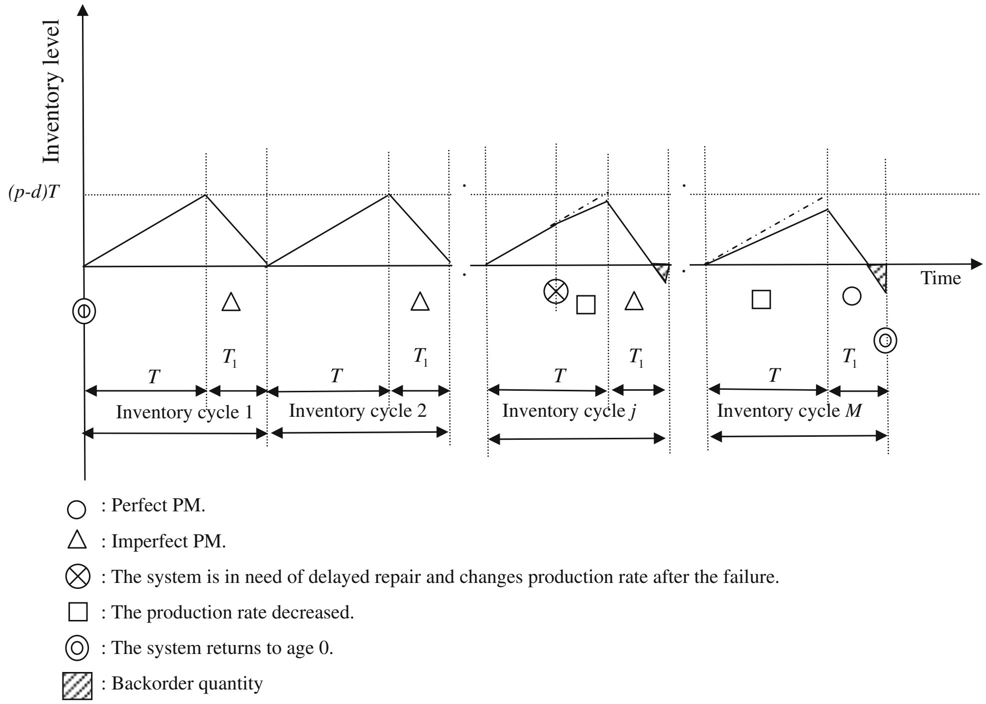


Fig. 5 The failure occurs at inventory cycle j and the system is renewed at perfect PM

The EPQ model with maintenance and production programs is

$$J(T;\{\bar{P}_j\}) = \text{Holding cost} + \text{Setup cost} + \text{PM cost} + \text{delayed repair, cost} + \text{backorder cost}. \quad (3)$$

The various costs of EPQ model are derived as follows:

1. Holding cost per year.

   The reduction in the inventory is $\alpha p \sum_{j=1}^{\infty} E[DT_j]$ due to the lower production rate after delayed repair. Inventory level must subtract loss of finished goods inventory, and the holding cost is

$$\frac{R_h}{2}\left[p\cdot T\left(1-\frac{d}{p}\right)-\alpha p\sum_{j=1}^{\infty}E[DT_j]\right]=\frac{R_h}{2}\times\left[p\cdot T\left(1-\frac{d}{p}\right)-\alpha p\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{jT}F(t)\,dt\right].$$

2. Setup cost is $\frac{d}{p\cdot T}R_s$ in a year, where $\frac{d}{p\cdot T}$ is the number of production cycles per year.
3. Cost of PM.
   For each inventory cycle, the PM process accrues once, and hence the expected PM cost is R_m. Cost of PM is $\frac{d}{p\cdot T}R_m$ in a year.
4. Delayed repair cost is
   $\frac{d}{p\cdot T}\left(R_r\times\sum_{j=1}^{\infty}E[DT_j]\right)=\frac{d}{p\cdot T}R_r\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\times\int_0^{jT}F(t)\,dt$ in a year.
5. Backorder cost is
   $\frac{d}{p\cdot T}\left(R_b\alpha p\times\sum_{j=1}^{\infty}E[DT_j]\right)=\frac{d}{p\cdot T}\alpha pR_b\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\times\int_0^{jT}F(t)\,dt$ in a year.

The expression for $J(T;\{\bar{P}_j\})$ can be obtained as follows:

$$J(T;\{\bar{P}_j\}) = \frac{1}{2}\left[p\cdot T\left(1-\frac{d}{p}\right)-\alpha p\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{jT}F(t)\,dt\right]\cdot R_h + \frac{d}{p\cdot T}\left[R_s+R_m+(R_r+\alpha pR_b)\times\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{jT}F(t)\,dt\right]$$

$$= \frac{1}{2}\left[p \cdot T\left(1-\frac{d}{p}\right) - \alpha p \sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \int_0^{\mathrm{jT}} F(t)\,dt\right] \cdot R_h + \frac{d}{p \cdot T}\left[R_{ms} + (R_r + \alpha p R_b) \times \sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \int_0^{\mathrm{jT}} F(t)\,dt\right]. \quad (4)$$

Find the optimal time T^*; Q^* can be easily determined by pT^*, which minimize $J(T;\{\bar{P}_j\})$. Differentiating $J(T;\{\bar{P}_j\})$ with respect to T and set the derivation to zero.

$$\frac{pR_hT^2}{2}\left[\left(1-\frac{d}{p}\right) - \alpha \sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} jF(jT)\right] + \frac{d}{p}\left[(R_r+\alpha pR_b)\sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} jTF(jT) - R_{ms} - (R_r+\alpha pR_b)\sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \int_0^{\mathrm{jT}} F(t)\,dt\right] = 0 \quad (5)$$

The following theorem demonstrates that a unique solution exists which satisfies Eq. (5) under certain reasonable conditions. Therefore, T^* can easily be determined by any numerical search procedure, for example bisection search.

Theorem 1 *If* $\sum_{j=1}^{\infty} \frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}}[(\frac{d}{p}R_r+\alpha dR_b)\int_0^{\mathrm{jT}} t\,dF(t) - \frac{\alpha pR_hT^2}{2} jF(jT)]$ *is strictly increasing in* T*; then there exists a finite and unique optimal solution* T^* *which minimizes* $J(T;\{\bar{P}_j\})$*, and*

$$J(T^*;\{\bar{P}_j\}) = pT^*\left(1-\frac{d}{p}\right)R_h - \frac{\alpha pR_h}{2}\sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} \times \left(\int_0^{\mathrm{jT}^*} F(t)\,dt + jT^*F(jT^*)\right) + \left(\frac{d}{p}R_r + \alpha dR_b\right)\sum_{j=1}^{\infty} \frac{\left(\bar{P}_{j-1}-\bar{P}_j\right)}{\sum_{j=1}^{\infty} \bar{P}_{j-1}} jF\left(jT^*\right). \quad (6)$$

Proof See Appendix 1

Special cases

Case 1: $\bar{P}_0 = 1;\ \bar{P}_j = 0 (j = 1, 2, \ldots)$

This case involves an operating system that must be as good as new following PM. From Eqs. (4) and (6),

$$J(T;\{1,0,0,\ldots,0\}) = \frac{1}{2}\left[pT\left(1-\frac{d}{p}\right) - \alpha p \int_0^{T} F(t)\,dt\right] \cdot R_h + \frac{d}{pT}\left(R_{ms} + (R_r+\alpha pR_b)\int_0^{T} F(t)\,dt\right), \quad (7)$$

$$J(T^*;\{1,0,0,\ldots,0\}) = pT^*\left(1-\frac{d}{p}\right)R_h - \frac{\alpha pR_h}{2}\left(\int_0^{T*} F(t)\,dt + T^* \cdot F(T^*)\right) + \left(\frac{d}{p}R_r + \alpha dR_b\right)F(T^*). \quad (8)$$

Case 2: $\bar{P}_0 = 1;\ \bar{P}_j = q^j (j = 1, 2, \ldots), 0 \le q < 1, \bar{q} = 1 - q$

Here, a random number M, of instances of PM until a type II PM is performed, is geometrically distributed. PM is "bad as old" with probability q, and "good as new" with probability $\bar{q} = 1 - q$. Using Eqs. (4) and (6),

$$J(T;\{q^j\}) = \frac{1}{2}\left[p \cdot T\left(1-\frac{d}{p}\right) - \alpha p\bar{q}^2\sum_{j=1}^{\infty} q^{j-1}\int_0^{\mathrm{jT}} F(t)\,dt\right] \cdot R_h + \frac{d}{p \cdot T}\left[R_{ms} + (R_r+\alpha pR_b)\bar{q}^2\sum_{j=1}^{\infty} q^{j-1}\int_0^{\mathrm{jT}} F(t)\,dt\right], \quad (9)$$

$$J(T^*;\{q^j\}) = pT^*\left(1-\frac{d}{p}\right)R_h - \frac{\alpha pR_h}{2}\bar{q}^2\sum_{j=1}^{\infty} q^{j-1} \times \left(\int_0^{\mathrm{jT}*} F(t)\,dt + \mathrm{jT}^*F(jT^*)\right) + \left(\frac{d}{p}R_r + \alpha dR_b\right)\bar{q}^2\sum_{j=1}^{\infty} q^{j-1} jF(jT^*). \quad (10)$$

Case 3: $\bar{P}_0 = 1; \bar{P}_1 \neq 0; \bar{P}_j = \bar{P}_1 \cdot j^b (j = 1, 2, \ldots)$, $0 < r < 1$

Given instruction and through repetition, PM workers learn to perform tasks more effectively. This study applies the traditional learning curve model to the PM model, substituting $\bar{P}_j$ for total direct labor hours per unit. The probability $\bar{P}_j$ resembles bile duct injury rate as in Moore ane Bennett (1995) and quality by Li and Rajagopalan (1997). Following the discussion, a learning curve is developed, and the following assumptions are made;

1. $\bar{P}_{j-1} > \bar{P}_j (j = 1, 2, \ldots)$.
2. If each doubling of the number of PM reduces imperfect PM probability by $(1-r)$, then $\bar{P}_j = \bar{P}_1 j^b (j = 1, 2, \ldots)$.
3. $b = \frac{\log r}{\log 2}$.

In this case, learning curves provide their greatest advantage in performing the early PM in response to new causes of failure. As the number of times PM has been performed becomes large, the learning effect is less noticeable. That is, $\bar{P}_j - \bar{P}_{j+1} > \bar{P}_{j+1} - \bar{P}_{j+2} (j = 1, 2, \ldots)$. Using Eqs. (4) and (6),

$$\begin{aligned} J(T; \{1, \bar{P}_1, \bar{P}_1 \cdot 2^b, \ldots\}) &= \frac{R_h}{2}\left[p \cdot T\left(1 - \frac{d}{p}\right)\right] - \alpha p \left[\frac{(1-\bar{P}_1)}{1 + \bar{P}\sum_{j=2}^{\infty}(j-1)^b} \right. \\ &\times \int_0^{\mathrm{T}} F(t)\,dt + \bar{P}_1 \sum_{j=2}^{\infty} \frac{(j-1)^b - j^b}{1 + \bar{P}_1 \sum_{j=2}^{\infty}(j-1)^b} \\ &\left. \times \int_0^{\mathrm{jT}} F(t)\,dt \right] + \frac{d}{p \cdot T}\left\{R_{ms} + (R_r + \alpha p R_b)\right. \\ &\times \left[\frac{(1-\bar{P}_1)}{1 + \bar{P}\sum_{j=2}^{\infty}(j-1)^b} \int_0^{\mathrm{T}} F(t)\,dt \right. \\ &\left.\left. + \bar{P}_1 \sum_{j=2}^{\infty} \frac{(j-1)^b - j^b}{1 + \bar{P}_1 \sum_{j=2}^{\infty}(j-1)^b} \int_0^{\mathrm{jT}} F(t)\,dt \right]\right\}. \end{aligned} \tag{11}$$

and

$$\begin{aligned} J(T^*; \{1, \bar{P}_1, \bar{P}_1 \cdot 2^b, \ldots\}) &= pT^*\left(1 - \frac{d}{p}\right) R_h - \frac{\alpha p R_h}{2} \\ &\times \left[\frac{(1-\bar{P}_1)}{1 + \bar{P}_1\sum_{j=2}^{\infty}(j-1)^b} \left(\int_0^{\mathrm{T}^*} F(t)\,dt + \mathrm{T}^* F\left(T^*\right)\right) \right. \\ &+ \bar{P}_1 \sum_{j=2}^{\infty} \frac{(j-1)^b - j^b}{1 + \bar{P}_1 \sum_{j=2}^{\infty}(j-1)^b} \\ &\left. \times \int_0^{\mathrm{jT}^*} F(t)\,dt + j\mathrm{T}^* F(jT^*) \right] \\ &+ \left(\frac{d}{p} R_r + \alpha d R_b\right)\left[\frac{(1-\bar{P}_1)}{1 + \bar{P}_1 \sum_{j=2}^{\infty}(j-1)^b} F(T^*) \right. \\ &\left. + \bar{P}_1 \sum_{j=2}^{\infty} \frac{(j-1)^b - j^b}{1 + \bar{P}_1 \sum_{j=2}^{\infty}(j-1)^b} jF(jT^*) \right]. \end{aligned} \tag{12}$$

Case 4: $\bar{P}_0 = 1; \bar{P}_j = q^{j^\beta} (j = 1, 2, \ldots), 0 \leq q < 1, \beta > 0$

Here, a random number M, of instances of PM until a perfect PM is performed, follows a discrete Weibull distribution. From Eqs. (4) and (6),

$$\begin{aligned} J(T; \{q^{j^\beta}\}) &= \frac{1}{2}\left[p \cdot T\left(1 - \frac{d}{p}\right) \right. \\ &\left. - \alpha p \sum_{j=1}^{\infty} \frac{\left(q^{(j-1)^\beta} - q^{j^\beta}\right)}{\sum_{j=1}^{\infty} q^{(j-1)^\beta}} \int_0^{\mathrm{jT}} F(t)\,dt \right] \cdot R_h \\ &+ \frac{d}{p \cdot T}\left[R_{ms} + (R_r + \alpha p R_b) \right. \\ &\left. \times \sum_{j=1}^{\infty} \frac{\left(q^{(j-1)^\beta} - q^{j^\beta}\right)}{\sum_{j=1}^{\infty} q^{(j-1)^\beta}} \int_0^{\mathrm{jT}} F(t)\,dt \right]. \end{aligned} \tag{13}$$

and

$$\begin{aligned} J(T^*; \{q^{j^\beta}\}) &= pT^*\left(1 - \frac{d}{p}\right) R_h - \frac{\alpha p R_h}{2} \sum_{j=1}^{\infty} \frac{\left(q^{(j-1)^\beta} - q^{j^\beta}\right)}{\sum_{j=1}^{\infty} q^{(j-1)^\beta}} \\ &\times \left(\int_0^{\mathrm{jT}^*} F(t)\,dt + j\mathrm{T}^* F(jT^*)\right) + \left(\frac{d}{p} R_r + \alpha d R_b\right) \\ &\times \sum_{j=1}^{\infty} \frac{\left(q^{(j-1)^\beta} - q^{j^\beta}\right)}{\sum_{j=1}^{\infty} q^{(j-1)^\beta}} jF(jT^*). \end{aligned} \tag{14}$$

Example and analysis

Two tables list optimal times T^* and the expected cost for the special cases. Table 1 lists the special case 1. Table 2

Table 1 Optimal policy and optimal expected cost given the parameters in special case 1

α	R_r/R_{ms}	Special case 1	
		$\bar{P}_0 = 1$; $\bar{P}_j = 0$ $(j = 1, 2, \ldots)$	
		T^*	$J(T^*; \{1, 0, 0, \ldots, 0\})$
0.05	0.5	0.3409	206.0201
	2	0.3383	206.7559
	3	0.3366	207.2322
0.1	0.5	0.3587	206.6960
	2	0.3555	207.3806
	3	0.3535	207.8715
0.15	0.5	0.3798	208.0528
	2	0.3759	208.7212
	3	0.3734	209.1665

presents special cases 2, 3 and 4. The following cost and distribution parameters of the special cases are considered to illustrate the policy; $p = 1550, d = 650, R_{ms} = 80, R_r =$ 0.05–3.0 R_{ms}, $R_h = 0.7, \alpha = 0.05$–0.1, r or q = 0.01–0.4, $R_b = 0.2$, $F(t) = gamf(t; 2)$ (gamma Cdf with shape parameter 2).

The parameters q, r, α, β or R_r/R_{ms} were varied to clarify their influence on the optimal solution. Tables 1, and 2, and Figs. 6 and 7 summarize the results, and illustrate the following.

1. The total cost $J(T; \{\overline{P}_j\})$ and optimal time T^* increase with increasing α. Decreasing total cost requires focusing on reducing α with each failure.
2. As R_r/R_{ms} increases, the optimal time T^* falls and the total cost $J(T; \{\overline{P}_j\})$ become large. Restated, it is best to perform inventory building frequently when R_r/R_{ms} is high.

Table 2 Optimal policy and optimal expected cost with the parameters in special case 2, 3 and 4

α	R_r/R_{ms}	r or q	Special case 2		Special case 3		Special case 4a		Special case 4b	
			$\bar{P}_j = q^j$ *or* $\bar{P}_j = q^{j^\beta}$, $\beta = 1$		$\bar{P}_j = \bar{P}_1 \cdot j^b$, $\bar{P}_1 = r$		$\bar{P}_j = q^{j^\beta}$, $\beta = 0.5$		$\bar{P}_j = q^{j^\beta}$, $\beta = 1.5$	
			T^*	$J(T^*; \{q^j\})$	T^*	$J(T^*; \{1, \bar{P}_1, \bar{P}_1 \cdot 2^b, \ldots\})$	T^*	$J(T^*; \{q^{j^\beta}\})$	T^*	$J(T^*; \{q^{j^\beta}\})$
0.05	0.5	0.01	0.3408	205.9816	0.3408	205.9817	0.3408	205.9920	0.3408	205.9922
		0.10	0.3407	206.1448	0.3406	206.1116	0.3405	206.4374	0.3406	206.0520
		0.20	0.3404	206.2789	0.3404	206.4822	0.3402	207.3943	0.3404	206.1281
		0.30	0.3401	206.5042	0.3407	207.5316	0.3406	208.9211	0.3403	206.2829
		0.40	0.3398	206.8489	0.3427	209.9561	0.3419	210.8154	0.3401	206.4164
	2	0.01	0.3382	206.8201	0.3382	206.8204	0.3381	206.8205	0.3381	206.6296
		0.10	0.3369	207.3036	0.3367	207.3550	0.3348	208.6013	0.3370	206.9994
		0.20	0.3351	208.0103	0.3344	209.0131	0.3298	212.9549	0.3359	207.4583
		0.30	0.3329	209.0089	0.3314	213.5013	0.3253	219.5724	0.3347	207.9568
		0.40	0.3302	210.4250	0.3306	224.4800	0.3232	228.0677	0.3333	208.5549
	3	0.01	0.3364	207.3010	0.3364	207.3015	0.3363	207.3345	0.3363	207.0168
		0.10	0.3344	208.0254	0.3342	208.1723	0.3313	210.0983	0.3347	207.6289
		0.20	0.3318	209.1798	0.3305	210.6054	0.3233	216.5688	0.3330	208.3074
		0.30	0.3285	210.7025	0.3257	217.4931	0.3159	226.5296	0.3312	209.0710
		0.40	0.3244	212.8078	0.3230	234.0421	0.3115	239.2480	0.3291	209.9777
0.1	0.5	0.01	0.3798	208.0366	0.3798	208.0365	0.3799	208.0810	0.3795	208.0246
		0.10	0.3803	208.1333	0.3804	208.1506	0.3818	208.4234	0.3799	208.0936
		0.20	0.3811	208.2959	0.3821	208.4472	0.3866	209.1695	0.3804	208.198
		0.30	0.3822	208.4986	0.3872	209.2479	0.3949	210.3845	0.3809	208.2795
		0.40	0.3838	208.7774	0.4006	211.0845	0.4064	211.8278	0.3815	208.3689
	2	0.01	0.3758	208.7720	0.3758	208.7722	0.3758	208.8206	0.3755	208.6141
		0.10	0.3747	209.2388	0.3746	209.3061	0.3735	210.4633	0.3745	208.9365
		0.20	0.3732	209.9067	0.3732	210.8260	0.3715	214.5774	0.3736	209.3884
		0.30	0.3714	210.8184	0.3736	215.1009	0.3723	220.9047	0.3725	209.8094
		0.40	0.3694	212.1529	0.3823	225.5412	0.3781	229.0427	0.3714	210.4083
	3	0.01	0.3732	209.2396	0.3732	209.2400	0.3731	209.2700	0.3729	208.9846
		0.10	0.3711	209.9390	0.3709	210.0531	0.3684	211.8648	0.3712	209.5645
		0.20	0.3683	210.9806	0.3676	212.3581	0.3622	218.0826	0.3693	210.1532
		0.30	0.3649	212.4013	0.3653	218.9761	0.3586	227.7438	0.3674	210.8768
		0.40	0.3608	214.3736	0.3710	235.0474	0.3608	240.2389	0.3651	211.6768

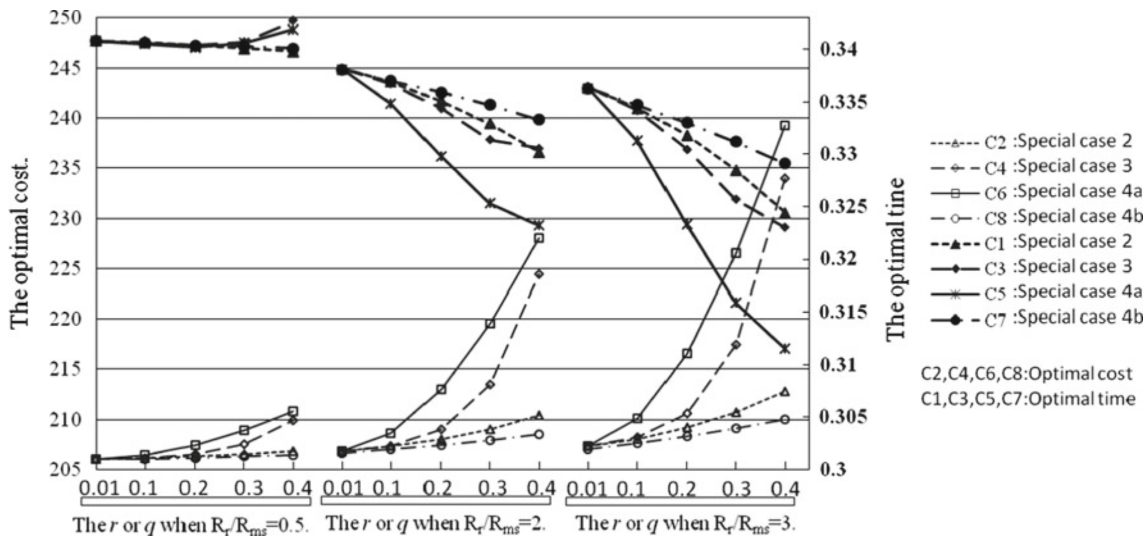


Fig. 6 Optimal policy and optimal expected cost with $\alpha = 0.05$ in special cases 2, 3 and 4

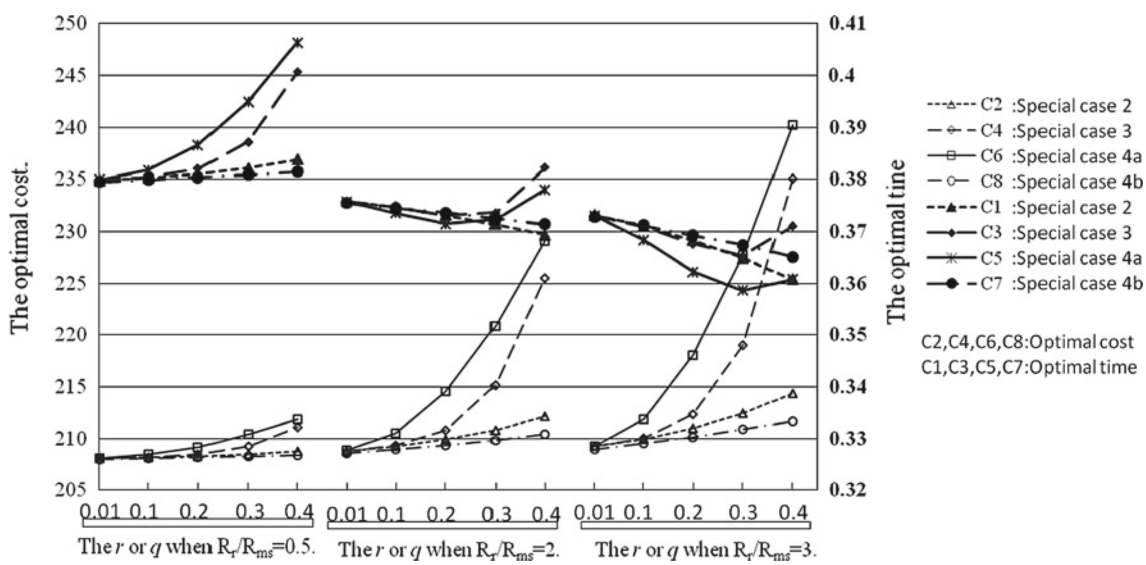


Fig. 7 Optimal policy and optimal expected cost with $\alpha = 0.1$ in special cases 2, 3 and 4

3. When q or r is low, the total cost $J(T; \{\overline{P}_j\})$ decreases. It needs to focus on developing maintenance ability to reduce total cost.
4. Figure 6 show that the optimal time $T^*$is decreased as q or r increases with $\alpha = 0.05$ and $R_r/R_{ms} = 2 \sim 3$. The optimal time $T^*$is increased as q or r increases with $\alpha = 0.15$ and $R_r/R_{ms} = 0.5$, as depicted in Fig. 7. The savings in holding cost from reduced production rate may affect the production policy.
5. Increasing $\beta(\beta > 0)$ reduces the expected cost $J(T; \{\overline{P}_j\})$. Moreover, $q_j < q_{j+1} < 1 (j = 1, 2, \ldots)$ when $0 < \beta < 1$, and $1 > q_j > q_{j+1} (j = 1, 2, \ldots)$ when $1 < \beta$. That is, it will be cost effective to improve q_j. Special case 2 can also be obtained for a special case 4 with $\beta = 1$, where the expected cost $J(T; \{\overline{P}_j\})$ with $\beta = 1.5$ is better than that in the special case 2.

Concluding remarks

This article presents an integrate EPQ model that incorporates production and maintenance programs has been analyzed. This model considers the impact of restoration action such as delayed repair and periodic PM on the damage of a deteriorating production system. At the beginning of the production cycle, the state of the process is assumed not always to be restored to normal production rate. The conditions are studied in the case of the EPQ model undergoing a backorder due to lower production rate after delayed repair. The model for optimizing the times T^* was examined. The nature of a PM process and policy produces the hypothesis that the probability of perfect PM being obtained depends on the number of imperfect PM performed since the previous renewal cycle. The results of investigating the optimal policy conditions demonstrate that such a policy is more general and flexible than policies already reported in the literature. Special cases were examined in detail. Using an example demonstrated that the optimal run times T^* were found in special cases. Analysis reveals the effect of the input parameters on the solution, and also obtains some further insights.

This study finds that developing maintenance ability reduces production related costs. If a maintenance technician training and accreditation program can be established, analysts can use the information to develop PM and production plans and the product system can be produced more efficiently using an integrated EPQ model that links maintenance programs.

Appendix

Proof of Theorem 1

$$\frac{d}{dT}J(T;\{\bar{P}_j\}) = 0$$

implies Eq. (5).

$$\frac{pR_hT^2}{2}\left[\left(1-\frac{d}{p}\right)-\alpha\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}jF(jT)\right] + \frac{d}{p}\left[(R_r+\alpha pR_b)\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{\mathrm{jT}}t\,dF(t)-R_{ms}\right]=0 \quad \text{(A1)}$$

Let $Q(T)$ be the left-hand side of Equation (A1).

$$Q(T) = \frac{pR_hT^2}{2}\left[\left(1-\frac{d}{p}\right)-\alpha\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}jF(jT)\right] + \frac{d}{p}\left[(R_r+\alpha pR_b)\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{\mathrm{jT}}t\,dF(t)-R_{ms}\right]. \quad \text{(A2)}$$

$Q(T)$ is strictly increasing when $\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\left[\left(\frac{d}{p}R_r+\alpha dR_b\right)\int_0^{\mathrm{jT}}t\,dF(t)-\frac{\alpha pR_hT^2}{2}jF(jT)\right]$ is also strictly increasing.

$$Q(0) = \frac{pR_h\cdot 0}{2}\left[\left(1-\frac{d}{p}\right)-0\right] + \frac{d}{p}\left[(R_r+\alpha pR_b)\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}\int_0^{0}t\,dF(t)-R_{ms}\right] = -\frac{d}{p}R_{ms}<0, \quad \text{(A3)}$$

$$Q(\infty)=\infty. \quad \text{(A4)}$$

If $\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}[(\frac{d}{p}R_r+\alpha dR_b)\int_0^{\mathrm{jT}}t\,dF(t)-\frac{\alpha pR_hT^2}{2}j \times F(jT)]$ is strictly increasing in T, then $Q(0)<0$ and $Q(\infty)>0$.

Thus from the strictly increasing property of $Q(T)$, there exists a unique and finite T^*, $(0<T^*<\infty)$ satisfying Eq. (5), which minimizes $J(T;\{\bar{P}_j\})$. If T^* is the solution, then from Eq. (5),

$$J(T^*;\{\bar{P}_j\}) = pT^*\left(1-\frac{d}{p}\right)R_h-\frac{\alpha pR_h}{2}\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}} \times\left(\int_0^{\mathrm{jT}*}F(t)\,dt+\mathrm{jT}^*F\left(jT^*\right)\right)+\left(\frac{d}{p}R_r+\alpha dR_b\right) \times\sum_{j=1}^{\infty}\frac{(\bar{P}_{j-1}-\bar{P}_j)}{\sum_{j=1}^{\infty}\bar{P}_{j-1}}jF\left(jT^*\right). \quad \text{(A5)}$$

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