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# A multi-objective facility location model with batch arrivals: two parameter-tuned meta-heuristic algorithms

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**Abstract** Many research works in mathematical modeling of the facility location problem have been carried out in discrete and continuous optimization area to obtain the optimum number of required facilities along with the relevant allocation processes. This paper proposes a new multi-objective facility-location problem within the batch arrival queuing framework. Three objective functions are considered: (I) minimizing the weighted sum of the waiting and the traveling times, (II) minimizing the maximum idle time pertinent to each facility, and (III) minimizing the total cost associated with the opened facilities. In this way, the best combination of the facilities is determined in the sense of economical, equilibrium, and enhancing service quality viewpoints. As the model is shown strongly NP-hard, two meta-heuristic algorithms, namely genetic algorithm (GA) and simulated annealing (SA) are proposed to solve the model. Not only new coding is developed in these solution algorithms, but also a random search algorithm is proposed to justify the efficiency of both algorithms. Since the solution-quality of all meta-heuristic algorithms severely depends on their parameters, design of experiments and response surface methodologies have been utilized to calibrate the parameters of both algorithms. Finally, computational results obtained by implementing both algorithms on several problems of different sizes demonstrate the performances of the proposed methodology.

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## Introduction and literature review

The introduction and literature review section of the paper is organized into four subsections, in which research works on the facility location problem, is first reviewed. Then, some relevant research on queuing theory is surveyed. Next, the multi-criteria decision-making works are assessed. Finally, scope and purpose of the research is given.

### Facility location problem (FLP)

The facility location problem (FLP), formally introduced by Weber (1909), is still receiving significant attentions due to its wide applications. Balinski (1965) also addressed the problem of locating a new set of facilities based on different criteria. The FLP attractions are mostly due to both economical viewpoint and customer satisfaction. In fact, making a decision on a location of a facility is a critical element in strategic planning so that a wrong decision leads to long-term resource wastage and customer dissatisfaction. Decision makers need to determine the optimal number of facilities such that different constraints are met and the facilities are profitable during their lifetime when environmental factors change, populations shift, and market trends evolve. Love et al. (1988), Marianov and ReVelle (1995), and Hodgson and Berman (1997) have comprehensively proposed several models and different solutions methodologies for FLP. Moreover, in order to solve FLPs, McKendall and Hakobyan (2010) introduced a boundary search technique, where they tested heuristics on some instances of the dynamic FLP and static facility layout problem in the literature.

Different classifications of discrete location–allocation models include set covering, maximal covering, p-center, p-dispersion, p-median, fixed charge, and hub. Among these models, the p-median model is the concentration of this paper. The specification of an optimal p-median has two components: (1) selecting vertices to be median locations and (2) assigning vertices to medians that are always interpreted decision variables (Francis et al. 1992).

The simple facility location and p-median models are the two well-known deterministic un-capacitated FLP, which are based on some assumptions that make them unrealistic. As an example, in real environments the demand and the service rate are not constants but random variables (Boffey et al. 2007). Nowadays, combinations of FLP with other aspects of industrial and operational management such as supply chain, queuing theory, and pricing management have received considerable attentions. For example, Dong et al. (2009) presented a new type of dynamic multi-stage facility layout problem under dynamic business environment with shortest path. Chan and Kumar (2009) developed a leagile supply chain based model for manufacturing industries to emphasize the various aspects of leagile supply chain modeling and implementation. They proposed a new hybrid chaos-based fast genetic Tabu simulated annealing (CFGTSA) algorithm to solve the model.

In this research, in order to make the p-median model more realistic, we investigate FLP within the framework of a queuing approach for each facility. In fact, queuing theory is utilized to optimize decisions in decreasing waiting time of both customers and manufactures (Cooper 1980).

## Queuing theory

One of the oldest and the best-developed analytical techniques to model waiting lines is the queuing theory (Porter et al. 1991). As a main purpose of manufactures and service providers, customer satisfaction is mirrored as customer-desired characteristics (Berman and Krass 2001). Receiving goods or services as soon as possible is one of these characteristics. To optimize decisions and to decrease waiting time for customers, manufactures and service providers usually need to utilize queuing theory. Besides, the most important factors for improving queuing systems performance are made by appropriate resource allocation and getting to customer satisfaction especially in highly competitive environments.

Expanded applications that involve combinations of FLP and queuing theory are used in many services and industries. Besides, based on the service type, queuing facility-location problems (QFLPs) can be divided into two categories. The first category is so-called immobile servers where customers travel to a facility for a service. Automated teller machines (ATMs), internet mirror sites, vending machines (VM), intercity service centers (e.g. hotels and restaurants), and the like are examples of immobile servers. The second category is mobile servers where servers travel from facilities to the users to provide services. Ambulances, taxis that use wireless equipments, restaurants providing foods based on telephone orders and the like are examples of mobile servers. Berman and Krass (2001) provided some coverage of the mobile servers.

Wang et al. (2002) proposed a facility location model based on an M/M/1 queuing system, where customers visit the closest facility and a maximum expected waiting time considered a restriction. Visiting the nearest facility was also considered constraint in some location-allocation models such as Hakimi (1964), Ghosh and Rushton (1987), Current et al. (2002). Berman et al. (2006) proposed a similar model to minimize the number of facilities where the amount of lost demand was considered a constraint. Furthermore, the proposed model of Wang et al. (2002) was extended by Berman and Drezner (2007), in which more than one server could be allocated to each facility and the M/M/m queuing framework was employed. Pasandideh and Niaki (2010) proposed a bi-objective facility location problem within M/M/1 queuing framework on the p-median problem. They solved their model using a genetic algorithm (GA) in which the desirability function technique was utilized. Recently, Chambari et al. (2011) proposed a bi-objective model for the facility location problem with M/M/1/k queues under a congestion system. To solve their model, two Pareto-based meta-heuristic algorithms including non-dominated sorting genetic algorithms (NSGA-II) and non-dominated ranking genetic algorithms (NRGA) were provided.

## Multi criteria decision making

Multiple criteria decision-making (MCDM) can be defined as the body of methods and procedures by which the concern for multiple conflicting criteria can be formally incorporated into an analytical process (Ehrgott and Gandibleux 2000). MCDM are consisted of multi-attribute decision-making (MADM) and multi-objective decision-making (MODM) techniques.

MADM usually provides a limited number of predetermined alternatives to satisfy each objective in a specified level. Then, regarding to the priority of each objective and the interaction between them, the decision-maker (DM) selects the best solution among all alternatives. While there are many MADM techniques, the most popular ones are dominant, maximin, maximax, lexicographic, permutation, simple additive weighting (SAW), elimination, choice expressing reality (ELECTRE), technique for order preference by similarity to ideal solution (TOPSIS), and linear programming for multidimensional analysis of preference (LINMAP) (Hwang and Yoon 1981).

The applications of multi-objective optimization techniques in engineering sciences grew over the recent decades. Among these techniques, MODM tries to design the best alternative with various interactions that satisfies DM by attaining some acceptable levels of a set of objectives. Among many MODM techniques, global criterion method, utility function, metric L-P methods, bounded objective method, lexicographic method, goal programming (GP), and goal attainment methods are the most popular ones (Farahani et al. 2009).

Ohsawa (1999) investigated a single facility, quadratic Euclidean distance bi-criteria model defined in the continuous space, with convex combination of the minisum and minimax objectives including efficiency and equity. Costa et al. (2008) presented a bi-criteria approach to the single allocation hub location problem. The first objective was a minisum cost and the second one had two forms of minisum or minimax of the process time. For large-scale manufacturing facilities, Kerbache and Smith (2000) proposed a multi-objective routing model employing an open finite queuing network with a multi-objective set of performance measures. Harewood (2002) considered a queuing probabilistic location set covering problem with Euclidean distances and maxisum and minisum objectives including coverage and cost for locating ambulances. Singh and Singh (2010) proposed solving the issues of selecting the objective weights that makes the design process of multi-objective FLP completely designer independent.

## Scope and purpose

Although a considerable amount of research works has been devoted to model development and solution procedures of both the single-objective and the multi-criteria facility location problem in the past decade, less attention has been given to the multi-objective QFLP (a facility location problem within queuing framework). In other words, each facility is considered to serve customers in QFLP problem and not only is the traveling time important, but also utilization of the facilities is essential. In this research, a novel tri-objective QFLP model is proposed to FLP where the servers are immobile and the customer demand is stochastic. Moreover, in real-world FLPs, sometimes a batch of primary customers arrives into the system. In this case, none of the existing models of the FLP that utilize queuing approach is designed to deal with this situation. In this research, we also intend to develop a new multi-objective QFLP model within batch arrival queuing framework. Furthermore, while an unreal constraint on the available budget is considered in many research works, the minimization of the total cost is a more realistic goal of operation managers. Including this kind of objective to the model generates a desired combination of the facilities that are more economical. Consequently, to obtain more efficacious solutions, another objective has been considered to provide more appropriate combination of the decision variables. The other contributions of this paper that make it more applicable to real-world problems involve: (1) assigning weights for each part of the first objective function and (2) determining a coefficient to enhance the service quality level in the capacity constraints of the model. Finally, to determine the number of required facilities associated with the allocation of the customers to the facilities, two meta-heuristic algorithms namely GA and simulated annealing (SA) are used. While various discrete chromosome structures are considered to code the solution of the QFLP in the literature (Berman et al. 2006; Aytug and Saydam 2002; Topcuoglua et al. 2005), a new type of representation is proposed in this research to enhance the feasibility of the chromosomes in satisfying more constraints. The justification of the obtained solutions of both algorithms has been performed by comparing them with the results obtained by a random search method. Moreover, since the output quality of any meta-heuristic algorithm severely depends on its parameters, the response surface methodology (RSM) has been utilized to increase the accuracy and precision of model solutions. It should be mentioned that the current research concentrates on a discrete facility location problem where a finite number of possible locations is available.

The remainder of the paper is organized as follows: In the next section, the problem, the assumptions, the parameters, the decision variables are first defined and then the model is described. "The proposed multi-objective decision making technique" concentrates on combining the objectives into one. In "Solving methodologies", the characteristics of the proposed GA and SA are illustrated. In order to obtain more accurate solutions, a parameter tuning procedure is proposed in "Tuning the parameters". "Analysis of results and comparisons" demonstrates the performance of the proposed solving methodologies on different problems of various sizes. Finally, conclusions are made and possible future research works are provided in "Conclusion and directions for future researches".

## Model description

The number of required facilities and the allocation of the customers to the facilities are the two main questions involved in FLPs. While considerable research works have been devoted to a single-objective discrete FLP with immobile servers and stochastic demand, many real-world problems involve simultaneous optimization of several objectives. The objectives in these problems are usually conflicting such that there is usually no single optimal solution. Hence, finding a set of alternative solutions in a search space is desired so as in a broader sense no other solutions can be found superior. These solutions are known Pareto-optimal. Therefore, a general multi-objective problem can be defined to minimize a function $f(x)$ with $p$, $(p > 1)$, decision variables and $Q$ objectives, $(Q > 1)$, subject to several constraints (Deb 2001) in Eq. (1).

$$\text{Minimize } f(x) = \left[f_1(x)\,,\ f_2(x)\,, \ldots,\ f_Q(x)\right] \tag{1}$$

Subject to

$x \in X$

where $X \subseteq \mathbb{R}^Q$ is the feasible solution space and $x = \{x_{1,}, x_2, \ldots, x_p\}$ is set of $p$-dimensional decision variables.

To make model more realistic, the three objective functions of this research that require to be minimized simultaneously are:

(I) The aggregate travel time of customers plus the aggregate waiting time of customers per unit time
(II) Maximum idle time pertinent to each facility
(III) Fixed cost of establishing the facilities

Simultaneous aggregation of the above three objectives provides equilibrium between the customer's and the owner's goals. It should be mentioned that the second objective has been considered to provide more appropriate combination of the decision variables. In fact, objective equilibrium is constructed to obtain more efficacious solutions. Although available budget is usually considered a constraint, it is better to concentrates on the minimization of the total cost to obtain solutions that are more economical.

The next big step in developing the model concerns with the fact that each facility acts as a batch arrival queuing system. An interesting variant of the $M^{[x]}/M/1$ queuing model occurs when one assumes batch arrivals. That is, each arrival epoch now corresponds to the arrival of a batch of customers where the batch sizes are independent, identically distributed random variables (Cooper 1980). The customers within a batch are served one by one at a time, and, as before, the service times of the customers are independent, identically distributed random variables. Furthermore, a random process can model the number of customers in the batches. It should also be mentioned that customers are assumed to visit the closest open facility and each facility have finite capacity. To make the model more realistic, as other contributions of this paper, we investigate several conditions including (1) considering the weight for each part of the first objective function and (2) contemplating the coefficient to increase service quality level. For more clarification, the scheme of a queuing facility problem with batch arrival is shown in Fig. 1.

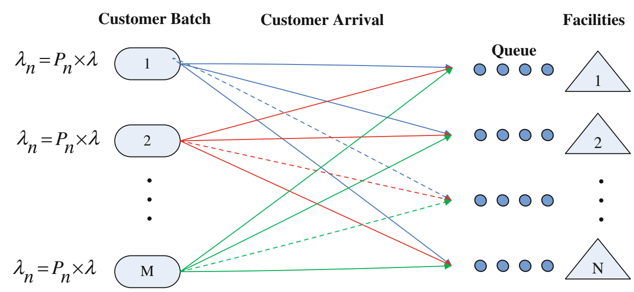



**Fig. 1** Queuing facility location problem (QFLP) scheme

In the following subsections, the assumptions, the parameters, and the variables of the model are first defined. Then, the Non-linear mixed- integer programming model is presented. Finally, the objective functions and the constraints are illustrated.

## Assumptions

The assumptions to formulate the problem at hand are:

- In order to receive the service batch of customers travel to each facility (immobile servers)
- An open facility behaves like an $M^{[x]}/M/1$ queue. It means that: (I) the service request of each batch follows an independent Poisson distribution; (II) each open facility has only one server with exponential service time.
- Each batch of customers can only be assigned to a facility.
- Batch size is considered a random variable.

## Notations

The notations to be used in this article are described as follow.

*Indices*

$i$ : Index for batch nodes (customers); $i = 1, 2, \ldots, M$
$j$ : Index for potential facility nodes; $j = 1, 2, \ldots, N$

*Parameters*

$V$ : Maximum number of on-duty servers; $(V \leq N)$
$t_{ij}$ : Travelling time from a customer in batch $i$ to facility node $j$
$\mathrm{w}_j$ : Expected waiting time of customer batches assigned to facility node $j$
$\lambda_i$ : Demand rate of service requests by customers in batch $i$
$\mu_j$ : Service rate of server $j$
$\tau_j$ : Demand rate at open facility node $j$
$\mathrm{c}_j$ : Fixed cost of establishing a facility at potential node $j$
$\alpha$ : Weight factor used in the first objective function

$\beta$ : Coefficient of service quality level
$U$ : A large positive number
$\pi_{Oj}$ : Probability of open facility $j$ being idle
$\rho_j$ : Utilization factor of facility $j$
$S$ : Batch size as a random variable
$E(S)$ : Mean of the batch size
$V(S)$ : Variance of the batch size

*Decision variables*

$$\boldsymbol{x}_{ij} = \begin{cases} 1; & \text{if } \textit{customer } i \text{ is assigned to facility } j \\ 0; & \text{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1; & \text{if facility } j \textit{ is} \text{ opened} \\ 0; & \text{otherwise} \end{cases}$$

The proposed mathematical model

The first objective function is divided into two parts. The first part represents aggregate travel time of customers per unit time and the second shows aggregate waiting time of customers per unit time. To derive both parts, the demand rate at each opened facility, $\tau_j$, is first obtained as

$$\tau_j = \sum_{\forall i} \mathrm{E(S)} \lambda_i \boldsymbol{x}_{ij}; \quad j = 1, 2, \ldots, n \tag{2}$$

Then, since each open facility behaves like an $\mathrm{M}^{[x]}$/M/1 queue, the expected waiting time at open facility $j$ is given as (Gross and Harris 1998)

$$\mathrm{w_j} = \frac{\frac{\rho_j}{1-\rho_j} + \frac{\rho_j\left(\frac{E(S^2)}{E(S)}-1\right)}{2(1-\rho_j)}}{\tau_j}; \quad j = 1, 2, \ldots, n \tag{3}$$

where $\rho_j = \tau_j / \mu_j$ is the utilization factor of facility $j$. As a result, the first objective that is the sum of aggregate traveling and waiting time can be obtained.

The second objective considers another aspect of the system provider's goal and involves probabilities of facilities being idle. While previous research works concentrated on minimizing the average idle probability, this research makes an important contribution and defines the idle probabilities using their maximum. Minimizing the average probabilities does not necessarily cause the idle probabilities of all facilities to be minimized. However, minimizing the maximum of the idle probabilities causes all idle probabilities to become less. In other words, we first define the idle probability of a facility $j$ by $\pi_{0j} = 1 - \frac{\tau_j}{\mu_j}$ (Costa et al. 2008). Then, the second objective function becomes minimizing the maximum of these probabilities. Furthermore, the idle probabilities are used for the facilities that are open. Hence, minimizing the maximum of a weighted sum of these probabilities, the weights being $y_j$, is the second objective function. Eventually, the extended model will be:

$$\mathrm{Min}\ T_1 = \alpha \sum_{\forall i} \sum_{\forall j} \lambda_i t_{ij} x_{ij} + (1-\alpha) \sum_{\forall i} \sum_{\forall j} \lambda_i \frac{\frac{\rho_j}{1-\rho_j} + \frac{\rho_j\left(\frac{E(S^2)}{E(S)}-1\right)}{2(1-\rho_j)}}{\tau_j} x_{ij} \tag{4}$$

$$\mathrm{Min}\ T_2 = \max_j \left\{ \left(1 - \frac{\tau_j}{\mu_j}\right) y_j \right\} \tag{5}$$

$$\mathrm{Min}\ T_3 = \sum_{\forall j} c_j y_j \tag{6}$$

Subject to:

$$\sum_{\forall j} y_j \le V \tag{7}$$

$$\sum_{\forall j} \boldsymbol{x}_{ij} = 1; \quad i = 1, 2, \ldots, m \tag{8}$$

$$\mathrm{E(S)} \sum_{\forall i} \lambda_i \boldsymbol{x}_{ij} < \beta \mu_j y_j; \quad j = 1, 2, \ldots, n \tag{9}$$

$$\sum_{\forall k \in N} t_{ik} x_{ik} \le \left(t_{ij} - U\right) y_j + U; \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n \tag{10}$$

$$x_{ij} \in \{0, 1\}, \quad y_j \in \{0, 1\}; \ i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \tag{11}$$

where $\tau_j$ is given in Eq. (2) and $\rho_j = \tau_j / \mu_j$.

As seen, the first objective function of the extended model in Eq. (4) includes two parts; the aggregate travel time of customers per unit time and the aggregate waiting time of customers per unit time. These two parts are multiplied by $\alpha$ and $(1-\alpha)$, respectively. Equation (5) is the second objective function that minimizes the maximum of ideal time pertinent to each facility. The third objective function given in Eq. (6) minimizes the fixed cost of establishing opened facilities. The first constraint (Eq. 7) ensures maximum number of open facilities. Equation (8) ensures that each customer must be assigned only to a facility. Equation (9) considers capacity constraint for each server. Actually, the input to each server should be less than its capacity. Moreover, to have a high service quality, the service rate is multiplied by a coefficient $\beta$. Equation (10) ensures that the assignment is carried out to the closest facility. Finally, Eq. (11) enforces the binary restrictions on the decision variables.

## The proposed multi-objective decision making technique

One of the most widely used methods for solving multi-objective optimization problems is transforming a multi-objective problem into a single one. When an appropriate collection of solutions is obtained by the single-objective optimization problem, the solutions approximate a Pareto front or Pareto surface in objectives space (Stadler 1984).

In order to transform the three objective functions given in Eqs. (4)–(6) into a single one, the $L_P$-metric ($P$ is an integer in $([1, \infty) \cup \{\infty\})$ is utilized as a weighting scheme in this research (Stadler 1984; Ehrgott and Gandibleux 2003). In this method that is given in Eq. (12), the differences between the objective functions and their optimum values are minimized. Besides, this method chooses a desired point $T_i^* \in \mathbb{R}^Q$ and looks for an optimal solution that is as close as possible to this point.

$$\text{Min } F(x) = \left[ \sum_{i=1}^{Q} \left[ \eta_i \left| \frac{T_i^* - T_i(x)}{T_i^*} \right| \right]^P \right]^{\frac{1}{P}} \qquad (12)$$

Subject to:

$x \in X \subset \mathbb{R}^Q$

where $\eta_i$ is the weight of objective function $T_i(x)$ determined by DM and $Q$ indicates the number of investigated objective functions. The $L_P$-metric for $P = 1$ is called Manhattan metric and $L_2$ is the Euclidean metric that is used in this research.

In the next section, two common meta-heuristic algorithms of a population-based (GA) and an individual-based (SA) are proposed to solve the tri-objective model at hand. Literature shows GA and SA are very useful in the area of location problems. Besides, since the verification of selected solving methodologies is essential in every optimization problem, a random search method is also chosen for verifications. Then, in order to select the appropriate meta-heuristic algorithm, the computational results in the selection process of the solving methodologies will be investigated.

## Solving methodologies

Since the proposed mathematical model of the problem at hand is of a constrained non-linear integer programming (NLIP) type that is NP-hard, an exact solution is hard (if not possible) to obtain (Ehrgott and Gandibleux 2000). Instead, the use of meta-heuristic algorithms, as a common and efficient way, is justified. Formerly, GA has been victorious in solving models similar to the proposed model of this research (Ghosh and Rushton 1987; Harewood 2002). To increase the performance of GA, a new chromosome representation is developed in each of which three constraints of the proposed model are satisfied. In other words, only feasible chromosomes are generated. Besides, another meta-heuristic search algorithm, namely simulated annealing (SA), is developed to validate the solution obtained by GA. To justify the results obtained and to evaluate the efficiencies and intelligence of both algorithms, a random search method is employed as well. The parameters of these algorithms are tuned using response surface methodology (RSM) to obtain better solutions.

In the following subsections, the steps involved in the proposed GA are illustrated.

### A GA for QFLP

Genetic algorithm (GA) is one of the most well known meta-heuristic optimization technique that was originally developed by Holland (1975). Vose (1991) provided the whole concept of a basic GA. Haupt and Haupt (2004) investigated a bit of updating, including some of the latest research results on GA.

Briefly, the GA mechanism is based on natural selection process that starts with an initial set of random solutions (population). Each individual in the population (chromosome) indicates a solution to the problem at hand. During a generation, the chromosomes are evaluated using a cost function. In order to produce next generation, two operators are used in GA. The first, called crossover, merges two chromosomes of a current generation to create offspring and the other, called mutation, modifies a chromosome. Then, based on the cost function values, some of the parents and offspring with better cost function values form a new generation. By this way, better chromosomes of successive generations have higher probabilities of being selected and the algorithm converges to the best chromosome that expectantly indicates the optimum or a near optimal solution to the problem after several generations. Since GA can find the global optimum solution with a high probability (Gen et al. 2008), in this research it is selected to be a meta-heuristic algorithm to solve the model.

In the next subsections, the steps involved in the GA are explained.

#### *Initialization*

In this step the parameters of the GA, i.e., the population size ($nPop$) that is the number of the chromosomes in each generation, the number of iterations ($nIt$), the crossover probability ($P_c$) and the mutation probability ($P_m$) are first initialized. Then, to generate initial population, a random generation policy is utilized in this step. Further, since the solutions obtained by a meta-heuristic algorithm are sensitive to their parameters values, a statistical procedure is used to tune the parameters.

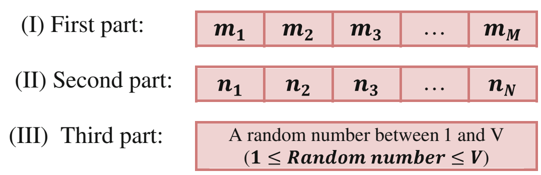



**Fig. 2** Chromosome structure

*The developed coding process*

In order to increase the feasibility of the chromosomes in satisfying more constraints, a new type of chromosome is proposed in this research to code the solution. The coding process takes place in two steps, encoding and decoding, described in the following two subsections.

*Encoding scheme*

The numbers of required facilities associated with the allocation of the customers to the facilities are decision variables that must be considered in a chromosome. In the following three steps, the three parts of a chromosome by which some constraints are satisfied based on the values of theses decision variables are described.

(I) The customer nodes are coded in the first part of the chromosome using a $1 \times M$ vector. Each member of this vector contains a random number between zero and one.
(II) The facility nodes are coded in the second part of the chromosome using a $1 \times N$ vector. Similarly, each member of this vector includes a random number between zero and one.
(III) The third part of a chromosome is consisted of a random number between one and the maximum number of servers that can be on-duty $(V)$.

The most important feature of the proposed chromosome structure is satisfying constraints (6), (7), and a part of constraint (8), in which each customer is assigned only to an open facility ($\boldsymbol{x}_{ij} \leq y_j$ ; $\forall i, j$). Figure 2 shows the general form of a chromosome and the upper section of the flowchart given in Fig. 3 describes the proposed coding process.

*Decoding scheme*

The decoding process that comes after chromosome representation is one of the most important steps in meta-heuristic algorithm. The decoding process of this research takes place in an order shown in Fig. 3. The parentheses containing numbers in different boxes of this flowchart correspond to step numbers.

In order to better illustrate the coding process consisting of encoding and decoding schemes, consider a numerical example in which $M = 7$, $N = 5$, and $V = 4$. Then, the steps (1)-(8) shown in the boxes of the flowchart in Fig. 3 are taken to both encode and decode the chromosomes. The descriptions of these steps follow:

(1) Regarding the third part of a chromosome, a random number $(v)$ is generated between one and four, say $v = 3$.
(2) Based on the second part of a chromosome, vector $\vec{n}$ with five genes is generated in which each gene contains a number between zero and one. Let Fig. 4 show the $\vec{n}$ vector.
(3) Sort the genes of vector $\vec{n}$ in ascending order while reserving the positions. Figure 5 show this step.
(4) As illustrated in Fig. 6, the first three $(v)$ genes of the sorted vector are chosen to be open facilities.
(5) This step reports the facilities that are selected for the assignment process. The position number of the potential facilities before the sorting process represents the selected facilities. Figure 7 illustrates this step.
(6) According to the second part of a chromosome, vector $\vec{m}$ with seven genes is generated in which each gene contains a number between zero and one (see Fig. 8).
(7) Obtain $\vec{H}$ vector where $\mathrm{H_i} = \lfloor \mathrm{v} \times \mathrm{m_i} \rfloor + 1$. Figure 9 illustrates this step.
(8) In the final step, each customer is allocated to a number (facility) in vector $\vec{H}$ shown in Fig. 10. In this figure, the selected facilities have been distinguished using different colors.

By taking the above eight steps, chromosomes representing solutions of the proposed model are generated.

*Cost function evaluation and constraint handling*

To evaluate chromosomes of each generation, the combined objective function given in Eq. (12) is utilized for $Q = 3$. Besides, to handle non-feasibilities of chromosomes in terms of constraint satisfaction the well-known penalty policy is employed, where the penalty is defined a big positive constant. The penalty is considered zero, when a chromosome is feasible and it takes a positive value, even if one of the constraints is not satisfied. In other words, regarding to a general form of the constraints as $g(x) \leq b$, the penalty value of a chromosome is defined as (Yeniay and Ankare 2005):

$$P(x) = U \times Max\left\{\left(\frac{g(x)}{b} - 1\right), 0\right\} \tag{13}$$

where $U$, $g(x)$, and $P(x)$ indicate a big positive number, the constraint, and the assigned penalty for chromosome $x$, respectively. For infeasible chromosomes, the penalty is then multiplied by $F(x)$ to obtain the corresponding cost function. In other words

$$F(x) = \begin{cases} F(x); & x \in \text{feasible region} \\ F(x) \times P(x); & x \notin \text{feasible region} \end{cases} \tag{14}$$

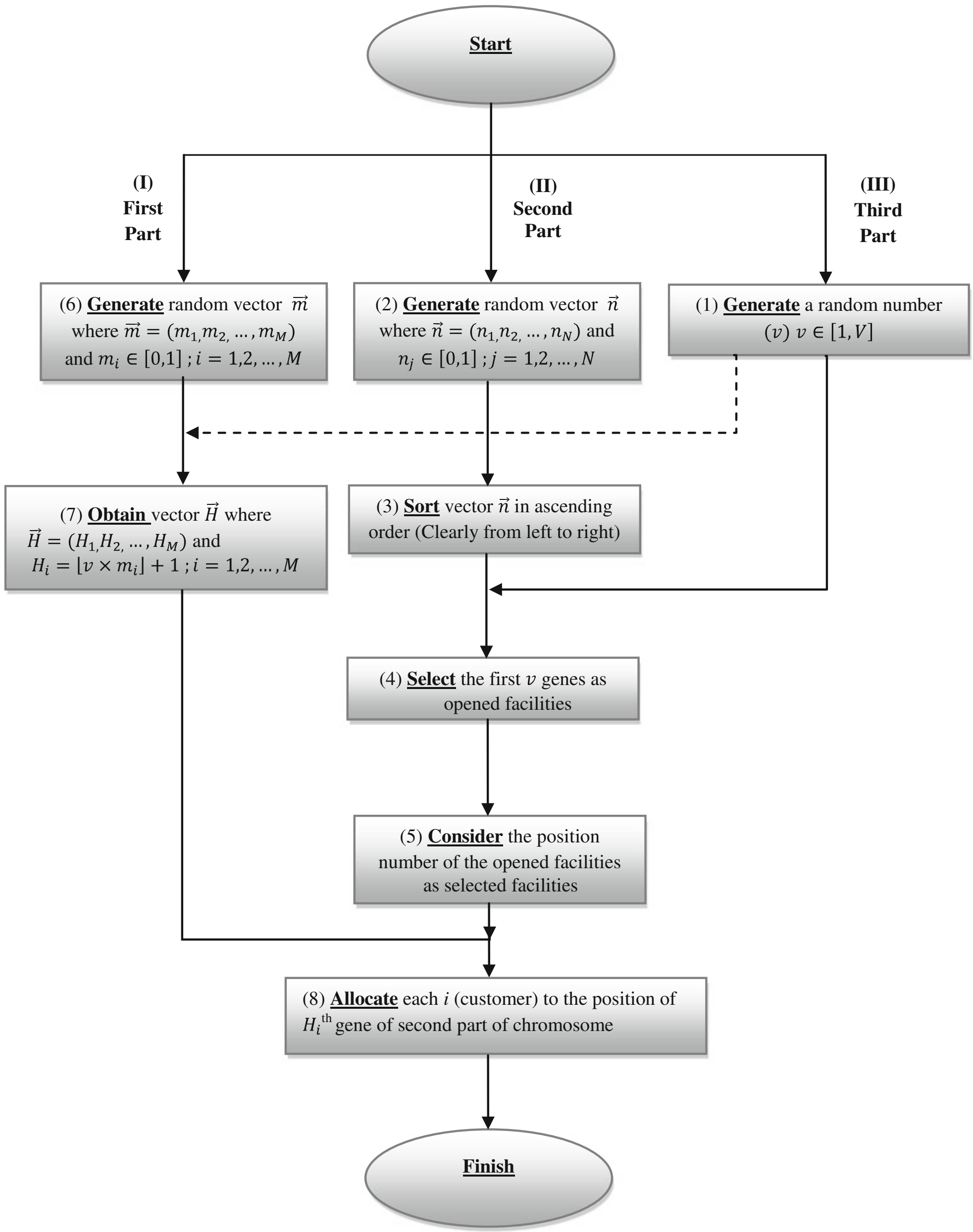



**Fig. 3** The proposed coding process

| | | | | |
|---|---|---|---|---|
| **0.43** | **0.12** | **0.58** | **0.81** | **0.29** |

**Fig. 4** Generated vector *n*

| | | | | |
|---|---|---|---|---|
| **0.12 (2)** | **0.29 (5)** | **0.43 (1)** | **0.58 (3)** | **0.81 (4)** |

**Fig. 5** Sorted vector *n*

Implicitly, two points need to be mentioned: (1) when the constant is selected larger, the penalty is considered larger; (2) for each type of constraint, the average of violations is considered. Moreover, in order to normalize all constraints, a normalization policy within the penalty function framework is considered in this research.

*Parent selection*

In each generation, a collection of offspring chromosomes is generated through a recombination process of parents using the roulette wheel procedure. The steps involved in this procedure follow:

| | | | | |
|---|---|---|---|---|
| 0.12 (2) | 0.29 (5) | 0.43 (1) | 0.58 (3) | 0.81 (4) |

*Open* $v = 3$ *facilities*

**Fig. 6** Opened facilities

| | | | | |
|---|---|---|---|---|
| 0.12 (2) | 0.29 (5) | 0.43 (1) | 0.58 (3) | 0.81 (4) |

*Selected (active) facilities* *2, 5, 1*

**Fig. 7** Number of selected facilities

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.65 | 0.32 | 0.19 | 0.09 | 0.73 | 0.97 | 0.66 |

**Fig. 8** Generated vector $m$

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 1 | 1 | 1 | 3 | 3 | 2 |

**Fig. 9** Vector $\vec{H}$

(I) Sort the chromosomes in ascending order based on their objective function values (fitness).

(II) Select a particular population member to be a parent with a probability equal to its fitness divided by the total fitness of the population (Al Jadaan et al. 2006). In other words,

$$P_i = F_i / \sum_{\forall i} F_i; \quad i = 1, 2, \ldots, nPop \tag{15}$$

where $i$, $F_i$, and $P_i$ denote an individual, its objective function value, and its probability of being selected, respectively.

(III) To choose the other parents a random number between zero and one is generated and multiplied by the total sum of the fitness values.

(IV) The obtained value belongs to the interval formed by two successive values of the cumulative function.

(V) The number associated with the upper value is chosen a parent.

The above selection process is based on spinning the roulette wheel $nPop$ times. The individuals selected from the selecting process are then stored in a mating pool (Shavandi and Mahlooji 2006). It should be noted that an elitist strategy is also applied to achieve faster convergence, i.e., the lowest objective function value is reserved in the next generation during the search process without any alteration.

*The crossover operator*

The following steps demonstrates the crossover operation of this research:

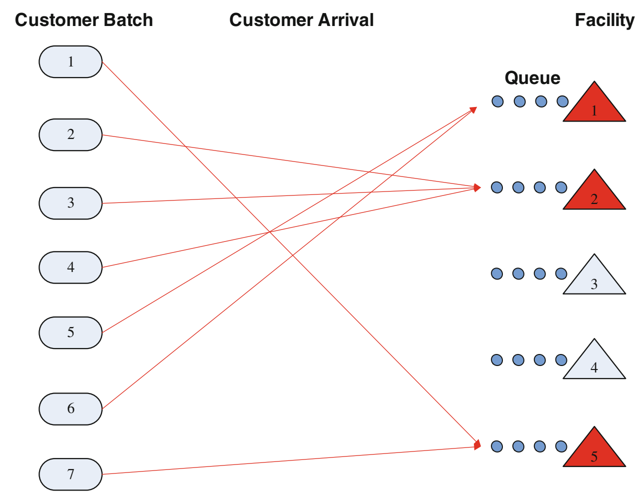



**Fig. 10** The allocation process

(I) At least, one of the three parts of a chromosome is considered.

(II) Regarding to the crossover probability ($P_c$), a number of chromosomes are randomly selected to generate offspring. In fact, the number of chromosomes for carrying out the crossover operator is obtained by $P_c \times nPop$.

(III) A continuous crossover operator is implemented in which a random vector ($\theta$) is first generated and then the offspring is generated based on Eq. (16) (Gross and Harris 1998).

$$\begin{aligned} Offspring_{(I)} &= \theta \times Parent_{(I)} + (1-\theta)\ Parent_{(II)} \\ Offspring_{(II)} &= \theta \times Parent_{(II)} + (1-\theta)\ Parent_{(I)} \end{aligned} \tag{16}$$

To illustrate this operation, suppose a random vector $\theta$ with a dimension equal to the size of the selected part (say the second part) of the selected chromosome (parent) is generated. Then, offspring is obtained using Eq. (16) as shown in Fig. 11. We note that in order to avoid generating infeasible offspring, each exchange is examined upon constraint (9) and (10) to assure feasibility of the generated offspring.

*The mutation operator*

Mutation operator alters a certain percentage of the bits in the list of chromosomes and keeps GA from converging too fast before sampling the entire cost surface (Deb 2001). The solution spaces that are not discovered by the crossover operator are found using the mutation operator. The steps involved in the mutation operation of this research at each iteration are:

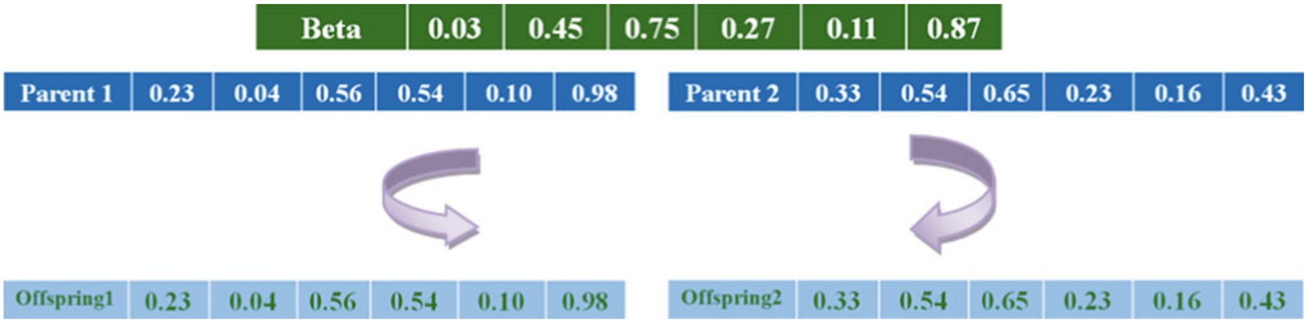



**Fig. 11** An example of the crossover operator

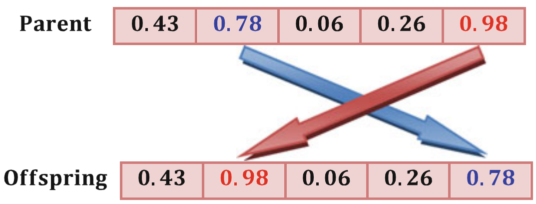



**Fig. 12** An example of the mutation operation

(I) At least, one of the three parts of a chromosome is considered.
(II) Regarding to the mutation probability ($P_m$), a number of chromosomes are randomly selected to generate offspring. This number is obtained by $P_m \times nPop$.
(III) The swap mutation is considered for mutation implementation (Gross and Harris 1998). In the swap mutation, two positions are randomly selected to swap with each other. Figure 12 illustrates this operation.

Once more, to avoid infeasible offspring, each exchange is examined upon constraints (9) and (10) to assure generating feasible offspring.

*Stopping criteria*

Stopping criteria is a set of conditions such that when satisfied a good solution is obtained. While different policies are taken to stop GA, in this research, when an improvement in the fitness function values for several successive generations is not achieved, GA stops.

In the next subsections, another meta-heuristic algorithm, simulated annealing (SA), is developed to solution verification purposes.

## A SA for QFLP

Simulated annealing was first introduced by Kirkpatrick et al. (1983) to obtain near optimum solutions of optimization models that are hard to solve using conventional procedures. Since then several authors employed SA in various optimization problems. SA is a general random search algorithm based on stochastic mechanism of physical annealing process in metallurgy. Generally, the objective value of a solution is equivalent to the internal energy state.

The steps involved in the developed SA of this research are explained in the following subsections.

*Initialization*

In this step, the input parameters of SA are initialized. The parameters are: (1) The initial temperature $T_0$ that is the starting point of temperature computation at each iteration, (2) The population size ***nPop*** *p* that is the number of the sustaining solutions at each generation, (3) The number of iteration in each temperature $nIt$, and (4) The temperature reduction rate $\beta$. Then, the temperature at iteration $h$, $T_h$, is obtained based on Eq. (17) (Kirkpatrick et al. 1983).

$$T_h = \beta \times T_{h-1}; \;\; h > 2\,, \;\; 0 < \beta < 1 \qquad (17)$$

Further, to generate initial population, the random generation policy is utilized in this research.

*The coding process*

As mentioned in "The developed coding process", to enhance feasibility of solutions and to satisfy more constraints, a new type of coding process that include encoding and decoding schemes is proposed. These schemes for SA are similar to the ones described for GA.

*Main loop of the SA*

SA starts with a high temperature and randomly chooses initial solution $\omega_0$. The initial value of $T_0$ acts as a controller parameter of the temperature. Then, a new solution $\omega_n$ within the neighborhood of the current solution $\omega$ is calculated at each iteration. In the minimization problem at hand, if the value of the combined objective function in Eq. (12), ($f(\omega_n)$), is less than the previous value $f(\omega)$, the new solution is accepted. Otherwise, in order to escape from the local optimal solution, the new solution is accepted with a probability ($Probabilty_{SA}$) given in Eq. (18) (Kirkpatrick et al. 1983).

$$Probability_{SA} = e^{-\Delta/T};\ \ \Delta = \frac{\mathrm{f}(\omega_n) - \mathrm{f}(\omega)}{\mathrm{f}(\omega_n)} \times 100 \quad (18)$$

This process is repeated until the desired state of the algorithm is reached.

*Neighborhood representation*

To represent the neighborhood structure, the proposed mutation operator of GA, described in "The mutation operator", is utilized to avoid fast convergence of SA.

*Stopping criteria*

This algorithm is also stopped after a predetermined number of iterations without improving the current best solution.

In the next section, the solutions obtained by GA and SA are verified by a comparative study using a random search method.

## Solution verification

Since the model at hand is of a minimization type, employing a random search method to solve it enables one to obtain an upper bound on the solution. This bound acts as a measure to assess the quality of the solutions obtained by GA and SA.

Let $\boldsymbol{f} : \mathbb{R}^Q \to \mathbb{R}$be the combined cost function that must be minimized and $\boldsymbol{x} \in \mathbb{R}^Q$ designate a position or candidate solution in the search space. The pseudo-code of the random search algorithm is shown in Fig. 13 (Rastrigin 1963).

As mentioned previously, since the solutions obtained by meta-heuristic algorithms are sensitive to their parameters, in the next section a statistical approach is taken to tune the parameters of GA and SA.

Initialize $\boldsymbol{x}$ with a random position in the search space.
Until a termination criterion (number of iterations) is met, repeat the following:

> Sample a new position $\boldsymbol{y}$ from the hyper sphere of a given radius surrounding the current position $\boldsymbol{x}$
> If $(f(\boldsymbol{y}) < f(\boldsymbol{x})$ then move to the new position by setting $\boldsymbol{x} = \boldsymbol{y}$

Now $\boldsymbol{x}$ holds the best-found position.

**Fig. 13** The random search procedure

## Tuning the parameters

In order to calibrate the parameters of the developed solving methodologies, design of experiments (DOE) approach is first employed in this Section to investigate the effect of the parameters (factors) on the solution (response) obtained. Then, the response function is estimated and finally using response surface methodology (RSM) a combination of the influential factor levels is obtained so that the response function is optimized.

Since the responses may have curvatures over the search ranges of the factors, the central composite design (CCD) of a $3^{k-p}$ fractional factorial with four central points is selected to run the experiments (Montgomery 2004), where there are $k = 4$ factors, each factor has three levels of low, medium, and high coed by $(-1)$, $(0)$, and $(+1)$, respectively and $p = 1$. The search ranges and the levels of the parameters are shown in Table 1.

The developed algorithms are coded in MATLAB 15 (R2010a) software environment (MATLAB 2010) and the experiments are performed on a laptop with a Pentium 1860 processor and one GB RAM, to estimate the response functions. The QFLP with 16 customers and 7 facilities is considered for experiments.

There are two parameters in the CCD that must be considered: (1) the distance $\alpha$ of the axial points from the design center and (2) the number of center runs (Montgomery 2004). The value of $\alpha$ depends on spherical property of the design. Since the region of interest is cubodial, the central composite face-centered design is utilized in which $\alpha = 1$ (Pasandideh and Niaki 2010; Najafi et al. 2009). It should be mentioned that the corresponding response of this paper is a combination of three objectives using the $L_P$-metric technique. The value of $P$ is first assumed two; resulting in Euclidean distance. However, it can be changed conveniently regarding the DM viewpoint.

In short, this research concentrates on $2^{4-1}$ fractional factorial central composite face-centered design with 10 axial points and 4 center runs. The design points along with the results of the experiments are shown in Tables 2 and 3 for GA and SA, respectively. In these tables, the values of all three objectives together with the fitness values that is obtained by the combined objective function are reported.

The results in Tables 2 and 3 are used to estimate the responses, $R_{GA}$ and $R_{SA}$, given in Eqs. (19) and (20) for GA and SA, respectively.

$$\begin{aligned} R_{GA} = 0.324922 &- 0.0451820\, P_c - 0.810931\, P_m \\ &- 0.806992\, nPop_{GA} - 0.0188820\, nIt_{GA} \\ &+ 0.310387\, P_c^2 + 0.286402\, P_m^2 \end{aligned}$$

**Table 1** Search range and the levels of the factors

| Solving methodologies | Parameter | Range | Low (−1) | Medium (0) | High (+1) |
|---|---|---|---|---|---|
| GA | $nPop_{GA}$ | 25–200 | 25 | 100 | 200 |
| | $P_c$ | 0.6–0.99 | 0.6 | 0.8 | 0.99 |
| | $P_m$ | 0.01–0.4 | 0.01 | 0.2 | 0.4 |
| | $nIt_{GA}$ | 100–500 | 100 | 300 | 500 |
| SA | $T_0$ | 500–1,000 | 500 | 750 | 1,000 |
| | $nPop_{SA}$ | 5–15 | 5 | 10 | 15 |
| | $nIt_{SA}$ | 100–500 | 100 | 300 | 500 |
| | $\beta$ | 0.9–0.99 | 0.9 | 0.95 | 0.99 |

**Table 2** Experimental results obtained by GA implementation

| Run order | GA parameters | | | | GA implementing | | | Combined objective function amount ($R_{GA}$) with $p = 2$ |
|---|---|---|---|---|---|---|---|---|
| | $nPop_{GA}$ | $P_c$ | $P_m$ | $nIt_{GA}$ | First objective | Second objective | Third objective | |
| 1 | 0 | 0 | 0 | 0 | 0.282050 | 0.082353 | 0.527780 | 0.357960 |
| 2 | −1 | 1 | 1 | −1 | 0.288060 | 0.153850 | 0.284480 | 0.480740 |
| 3 | 1 | −1 | 1 | −1 | 0.285460 | 0.129410 | 0.283630 | 0.492150 |
| 4 | 0 | 0 | 0 | 0 | 0.285010 | 0.102560 | 0.527780 | 0.330140 |
| 5 | 0 | 0 | 0 | 0 | 0.281980 | 0.082353 | 0.361110 | 0.443100 |
| 6 | 0 | 0 | 0 | 0 | 0.282050 | 0.102560 | 0.527780 | 0.315700 |
| 7 | −1 | −1 | 1 | 1 | 0.284820 | 0.063291 | 0.284620 | 0.803330 |
| 8 | 1 | −1 | −1 | 1 | 0.281980 | 0.128210 | 0.283810 | 0.524040 |
| 9 | 1 | 1 | 1 | 1 | 0.282050 | 0.102560 | 0.284140 | 0.536540 |
| 10 | −1 | −1 | −1 | −1 | 1.234600 | 15.484500 | 2.978700 | 4.328200 |
| 11 | −1 | 1 | −1 | 1 | 1.170400 | 5.257100 | 1.524900 | 4.337300 |
| 12 | 1 | 1 | −1 | −1 | 0.301100 | 0.223530 | 0.282900 | 0.837400 |
| 13 | 0 | 1 | 0 | 0 | 0.276350 | 0.112736 | 0.492352 | 0.453220 |
| 14 | 0 | −1 | 0 | 0 | 0.282050 | 0.063291 | 0.376235 | 0.768330 |
| 15 | 0 | 0 | 1 | 0 | 0.275250 | 0.102580 | 0.364525 | 0.298740 |
| 16 | 0 | 0 | −1 | 0 | 0.376530 | 0.217490 | 0.282900 | 0.874840 |
| 17 | 1 | 0 | 0 | 0 | 0.285120 | 0.081320 | 0.364525 | 0.346330 |
| 18 | −1 | 0 | 0 | 0 | 0.285010 | 0.124317 | 0.597634 | 0.675840 |
| 19 | 0 | 0 | 0 | 1 | 0.282050 | 0.082353 | 0.527780 | 0.465430 |
| 20 | 0 | 0 | 0 | −1 | 0.432620 | 0.217490 | 0.226527 | 0.897940 |

$$+\,0.210698\,nPop_{GA}^2 + 0.381298\,nIt_{GA}^2 - 0.0524613\,P_c\,P_m + 0.0612837\,P_c\,nPop_{GA} + 0.903706\,P_c\,nIt_{GA} \tag{19}$$

$$R_{SA} = 0.408939 + 0.0115532\,T_0 - 0.0685529\,\beta - 0.0577630\,nPop_{SA} - 0.0326809\,nIt_{SA} - 0.0427505\,T_0^2 + 0.0221682\,\beta^2 + 0.0310227\,nPop_{SA}^2 + 0.0108662\,nIt_{SA}^2 - 0.0524613\,T_0\,\beta + 0.0612837\,T_0 nPop_{SA} + 0.903706\,T_0 nIt_{SA} \tag{20}$$

Moreover, the analysis of variance results that are given in Tables 4 and 5 for GA and SA, show that both regression functions are appropriate and can be used in RSM. Then, the models in (19) and (20) are solved by LINGO software within the range of the parameters and the optimum combinations of the parameters are shown in Table 6 for each algorithm.

**Table 3** Experimental results obtained by SA implementation

| Run order | SA parameters | | | | SA implementing | | | Combined objective function amount ($R_{SA}$) with $p = 2$ |
|---|---|---|---|---|---|---|---|---|
| | $T_0$ | $nPop_{SA}$ | $\beta$ | $nIt_{SA}$ | First objective | Second objective | Third objective | |
| 1 | 0 | 0 | 0 | 0 | 0.283431 | 0.058828 | 0.527780 | 0.366630 |
| 2 | 1 | 1 | −1 | −1 | 0.288060 | 0.075380 | 0.296320 | 0.365324 |
| 3 | −1 | 1 | 1 | −1 | 0.285329 | 0.056433 | 0.287590 | 0.346257 |
| 4 | 0 | 0 | 0 | 0 | 0.283431 | 0.063335 | 0.527780 | 0.362521 |
| 5 | 0 | 0 | 0 | 0 | 0.282432 | 0.052666 | 0.487653 | 0.292874 |
| 6 | 0 | 0 | 0 | 0 | 0.285430 | 0.102560 | 0.525232 | 0.373635 |
| 7 | −1 | 1 | −1 | 1 | 0.324565 | 0.145115 | 0.527820 | 0.526222 |
| 8 | −1 | −1 | 1 | 1 | 0.283645 | 0.058330 | 0.283210 | 0.291373 |
| 9 | 1 | 1 | 1 | 1 | 0.280284 | 0.503922 | 0.283748 | 0.282738 |
| 10 | −1 | −1 | −1 | −1 | 0.381262 | 0.172363 | 0.537321 | 0.398367 |
| 11 | 1 | −1 | −1 | 1 | 0.320921 | 0.632910 | 0.763210 | 0.524525 |
| 12 | 1 | −1 | 1 | −1 | 0.313081 | 0.223530 | 0.282900 | 0.627128 |
| 13 | 1 | 0 | 0 | 0 | 0.293822 | 0.052421 | 0.514550 | 0.345223 |
| 14 | −1 | 0 | 0 | 0 | 0.392742 | 0.151345 | 0.481475 | 0.467187 |
| 15 | 0 | 1 | 0 | 0 | 0.286545 | 0.053220 | 0.543636 | 0.288785 |
| 16 | 0 | −1 | 0 | 0 | 0.387424 | 0.198173 | 0.288724 | 0.653462 |
| 17 | 0 | 0 | 1 | 0 | 0.284633 | 0.042425 | 0.493400 | 0.324634 |
| 18 | 0 | 0 | −1 | 0 | 0.324150 | 0.117163 | 0.523720 | 0.635322 |
| 19 | 0 | 0 | 0 | 1 | 0.283763 | 0.052622 | 0.502502 | 0.352526 |
| 20 | 0 | 0 | 0 | −1 | 0.416252 | 0.082633 | 0.291873 | 0.567117 |

**Table 4** Analysis of variance for the performance of the response ($R_{GA}$)

| Source | Degree of freedom ($df$) | Seq-SS | Adj-SS | Adj-MS | $F$ test | $P$ value |
|---|---|---|---|---|---|---|
| Regression | 11 | 24.6739 | 24.6739 | 2.24308 | 9.63 | 0.002 |
| Linear | 4 | 13.1124 | 13.1124 | 3.27811 | 14.07 | 0.001 |
| Square | 4 | 4.9759 | 4.9759 | 1.24398 | 5.34 | 0.022 |
| Interaction | 3 | 6.5855 | 6.5855 | 2.19518 | 9.42 | 0.005 |
| Residual error | 8 | 1.8639 | 1.8639 | 0.23299 | | |
| Lack-of-fit | 5 | 1.8542 | 1.8542 | 0.37084 | 114.08 | 0.001 |
| Pure error | 3 | 0.0098 | 0.0098 | 0.00325 | | |
| Total | 19 | 26.5378 | | | | |

**Table 5** Analysis of variance for the performance of the response ($R_{SA}$)

| Source | $df$ | Seq-SS | Adj-SS | Adj-MS | $F$ test | $P$ value |
|---|---|---|---|---|---|---|
| Regression | 11 | 0.179528 | 0.179528 | 0.016321 | 1.13 | 0.442 |
| Linear | 4 | 0.092376 | 0.092376 | 0.023094 | 1.60 | 0.265 |
| Square | 4 | 0.008161 | 0.008161 | 0.002040 | 0.14 | 0.962 |
| Interaction | 3 | 0.078991 | 0.078991 | 0.026330 | 1.82 | 0.221 |
| Residual error | 8 | 0.115518 | 0.115518 | 0.014440 | | |
| Lack-of-fit | 5 | 0.111267 | 0.111267 | 0.022253 | 15.71 | 0.023 |
| Pure error | 3 | 0.004251 | 0.004251 | 0.001417 | | |
| Total | 19 | 0.295046 | | | | |

**Table 6** Optimum parameter levels

| Solving methodologies | Parameter | Optimum amount |
|---|---|---|
| GA | $nPop_{GA}$ | 25 |
| | $P_c$ | 0.6 |
| | $P_m$ | 0.4 |
| | $nIt_{GA}$ | 100 |
| SA | $T_0$ | 500 |
| | $nPop_{SA}$ | 5 |
| | $nIt_{SA}$ | 500 |
| | $\beta$ | 0.99 |

In the next section, using a set of test problems, the performances of both algorithms are investigated, where for each problem, the parameters are tuned.

## Analysis of results and comparisons

The combined objective function value is considered a measure to evaluate and compare the performances of the solution methodologies under different environments using several test problems. The experiments are implemented on 20 problems, each having four different sizes. Then, these instance problems are solved not only by the proposed two GA and SA algorithms, but also for a random search (RS) algorithm as well. In other words, 240 problems are solved in this research using the three methods. Furthermore, to eliminate uncertainties of the solutions obtained, each problem is used three times under different random environments. Then, the averages of these three runs are treated as the ultimate responses.

For illustration, consider the following scenario for which the input data and the results are shown in Table 7 for 20 test problems.

- The number of customer zones ($M$), the number of potential facility sites ($N$), and the number of servers that are on-duty ($V$) are given in the second, the third, and the fourth column of Table 9, respectively.
- The demand rate of service requests from customer batch node $i$follows a uniform distribution, that is $\lambda_i \sim Uniform[2, 15]$.
- The service rate for server $j$ follows a uniform distribution, i.e., $\mu_i \sim Uniform[65, 95]$.
- The travelling time $t_{ij}$ is calculated as a proportion of the Euclidean distance between customer batch $i$ and potential facility $j$, that is $t_{ij} \sim Uniform[65, 95]$.
- The batch size follows a geometric distribution with parameter 0.5, i.e., $S \sim Geometric(0.5)$
- The fixed cost of locating is related to service rate for each size of problem.
- The fixed cost of establishing facility $j$ at potential node $j$ follows a uniform distribution, that is $C_j \sim Uniform[100, 500]$.
- The other input data are $\alpha = 0.5$, $\beta = 0.95$, and $\eta = [0.5 \quad 0.2 \quad 0.3]$

For the 20 test problems given in Table 7, the experimental results show appropriate performances of both GA and SA in comparison with RS to solve the proposed QFLP model. Similar patterns were observed for the other test problems. Figure 14 provides a pictorial proof of this statement. It should be mentioned that the values of all objectives along with the combined objective function values are reported in Table 7 to show the performances of the algorithms, the difference between the tri-objectives and a single objective optimization, and non-dominated solving methodologies. Moreover, to show the convergence of the proposed GA and SA algorithms, the diagram of the objective function values in terms of iteration number for problem number 3 are shown in Fig. 15 and 16, respectively.

In order to compare the efficiency of proposed GA and SA to the RS procedure, the one-way analysis of variance has been utilized. Analysis of variance was introduced in the context of the linear model as a schematic way of calculating the residual sum of squares as a basis for estimating residual variance and then as a device for testing a null hypothesis constraining the parameter vector of the linear model to a subspace (Montgomery 2005). This process was performed in Minitab 15 software environment. Table 8 presents the significant difference between the proposed GA, SA, and RS. Moreover, the significant difference of RS from GA and SA can be observed from Fig. 17.

To provide the similar efficiency of GA and SA for solving the proposed QFLP model, once again the analysis of variance is performed on the results obtained. Table 9 and Fig. 18 both show the output of the Minitab software, in which identical performance is observed.

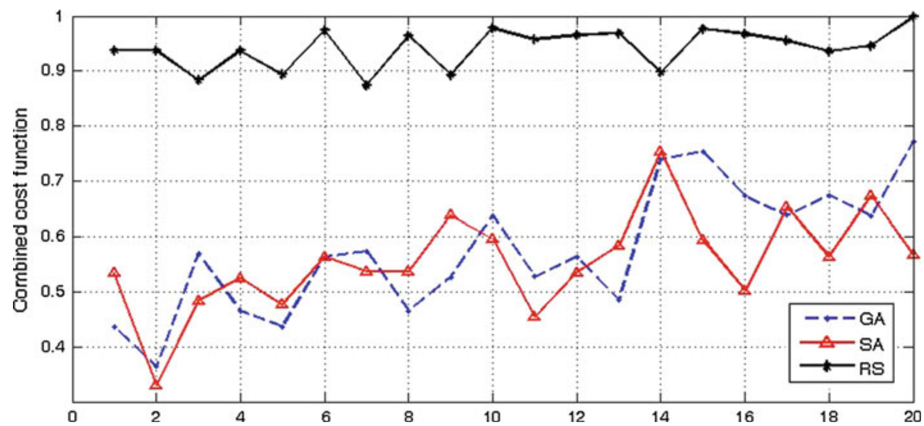
## Conclusion and directions for future researches

In this research, a new multi-objective facility location problem within $M^{[x]}/M/1$ queuing framework was proposed to determine the number of required facilities and the relevant allocation process. Three objective functions including (1) minimizing sum of the travel time and waiting, (2) minimizing maximum of ideal time pertinent to each facility, and (3) minimizing opened facilities total cost were involved. Moreover, two features (I) the weight for each part of the first objective function and (II) the

**Table 7** Computational results of solving methodologies

| Problem number | *M* | *N* | *V* | Objective function value (OFV) | | | | | | | | | | | | Non-dominated solution |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Proposed GA | | | | Proposed SA | | | | Proposed RS | | | | |
| | | | | OFV 1 | OFV 2 | OFV 3 | Integrated OFV | OFV 1 | OFV 2 | OFV 3 | Integrated OFV | OFV 1 | OFV 2 | OFV 3 | Integrated OFV | |
| 1 | 6 | 5 | 3 | 0.2821 | 0.0824 | 0.5171 | 0.4367 | 0.2834 | 0.0588 | 0.5278 | 0.5339 | 0.8675 | 0.4565 | 0.8723 | 0.9373 | GA and SA |
| 2 | 9 | 6 | 3 | 0.2881 | 0.1539 | 0.2845 | 0.3637 | 0.2881 | 0.0754 | 0.2963 | 0.3294 | 0.4587 | 0.3567 | 0.4939 | 0.9373 | GA and SA |
| 3 | 16 | 7 | 5 | 0.2855 | 0.1294 | 0.2836 | 0.5698 | 0.2853 | 0.0564 | 0.2876 | 0.4839 | 0.7757 | 0.2455 | 0.6188 | 0.8837 | GA and SA |
| 4 | 20 | 9 | 6 | 0.2850 | 0.1026 | 0.5171 | 0.4653 | 0.2834 | 0.0633 | 0.5278 | 0.5242 | 0.3586 | 0.1346 | 0.9282 | 0.9373 | GA and SA |
| 5 | 35 | 12 | 9 | 0.2820 | 0.0824 | 0.3611 | 0.4363 | 0.2824 | 0.0527 | 0.4877 | 0.4763 | 0.3667 | 0.0905 | 0.9262 | 0.8937 | GA and SA |
| 6 | 42 | 15 | 11 | 0.2821 | 0.1026 | 0.5278 | 0.5638 | 0.2854 | 0.1026 | 0.5252 | 0.5633 | 0.7964 | 0.3456 | 0.8622 | 0.9736 | GA and SA |
| 7 | 57 | 17 | 12 | 0.2848 | 0.0633 | 0.2846 | 0.5733 | 0.3246 | 0.1451 | 0.5278 | 0.5367 | 0.8124 | 0.4513 | 0.6363 | 0.8737 | GA |
| 8 | 62 | 21 | 14 | 0.2820 | 0.1282 | 0.2838 | 0.4654 | 0.2836 | 0.0583 | 0.2832 | 0.5367 | 0.9652 | 0.1455 | 0.6632 | 0.9644 | GA and SA |
| 9 | 77 | 25 | 18 | 0.2821 | 0.1026 | 0.2841 | 0.5267 | 0.2803 | 0.5039 | 0.2837 | 0.6378 | 0.7905 | 0.4567 | 0.3155 | 0.8927 | No domination |
| 10 | 81 | 30 | 20 | 0.8634 | 0.4845 | 0.8787 | 0.6373 | 0.3813 | 0.1724 | 0.5373 | 0.5957 | 0.8846 | 0.3478 | 0.8171 | 0.9776 | SA |
| 11 | 90 | 38 | 22 | 0.7704 | 0.7571 | 0.8524 | 0.5272 | 0.3209 | 0.6329 | 0.7632 | 0.4538 | 0.8345 | 0.8432 | 0.7992 | 0.9578 | SA |
| 12 | 105 | 42 | 25 | 0.3011 | 0.2873 | 0.2829 | 0.5633 | 0.3131 | 0.2235 | 0.2829 | 0.5353 | 0.5456 | 0.3444 | 0.6268 | 0.9659 | GA |
| 13 | 128 | 45 | 25 | 0.2764 | 0.1127 | 0.4924 | 0.4849 | 0.2938 | 0.0524 | 0.5146 | 0.5839 | 0.5839 | 0.1917 | 0.7293 | 0.9685 | GA and SA |
| 14 | 147 | 53 | 30 | 0.2821 | 0.0633 | 0.3762 | 0.7398 | 0.3927 | 0.1513 | 0.4815 | 0.7543 | 0.6889 | 0.4698 | 0.4931 | 0.8977 | GA |
| 15 | 175 | 68 | 38 | 0.2753 | 0.1026 | 0.3645 | 0.7534 | 0.2865 | 0.0532 | 0.5436 | 0.5938 | 0.8765 | 0.6844 | 0.6273 | 0.9765 | GA and SA |
| 16 | 196 | 70 | 45 | 0.3765 | 0.2175 | 0.2829 | 0.6733 | 0.3874 | 0.1982 | 0.2887 | 0.5020 | 0.8796 | 0.5511 | 0.3107 | 0.9675 | GA and SA |
| 17 | 220 | 88 | 52 | 0.2851 | 0.0813 | 0.3645 | 0.6383 | 0.2846 | 0.0424 | 0.4934 | 0.6537 | 0.6865 | 0.1294 | 0.5961 | 0.9554 | GA and SA |
| 18 | 250 | 95 | 70 | 0.2850 | 0.1243 | 0.5976 | 0.6738 | 0.3242 | 0.1172 | 0.5237 | 0.5638 | 0.8643 | 0.3471 | 0.6614 | 0.9356 | GA and SA |
| 19 | 350 | 105 | 75 | 0.2821 | 0.0824 | 0.5278 | 0.6373 | 0.2838 | 0.0526 | 0.5025 | 0.6732 | 0.8666 | 0.0932 | 0.9811 | 0.9465 | GA and SA |
| 20 | 500 | 110 | 86 | 0.4326 | 0.2175 | 0.2265 | 0.7717 | 0.4163 | 0.0826 | 0.2919 | 0.5671 | 0.9123 | 0.1955 | 0.3937 | 0.9987 | No domination |

Fig. 14 The intelligence of the proposed GA and SA in comparison with RS



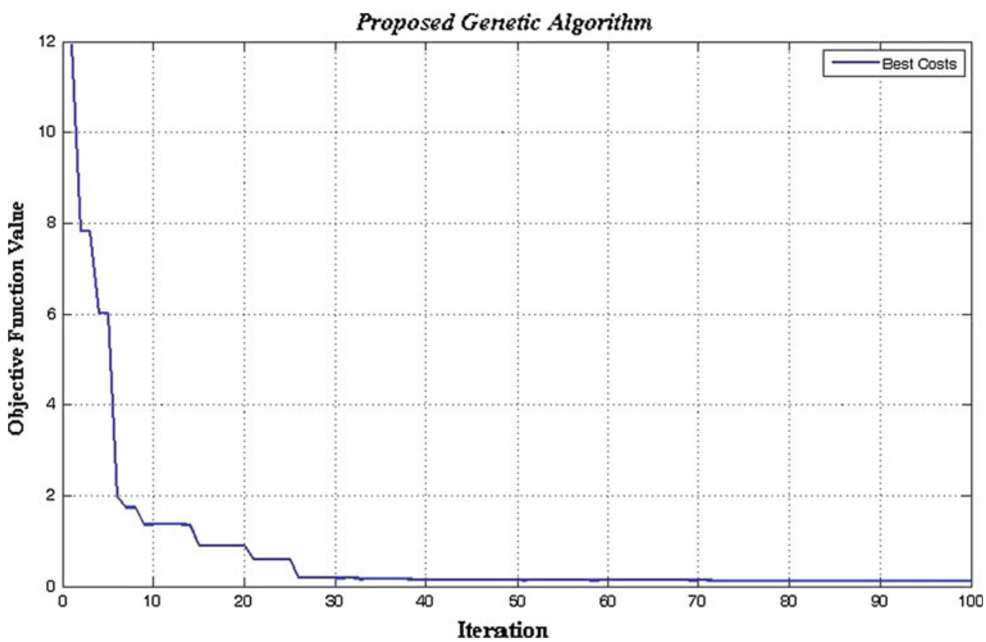




Fig. 15 Convergence diagram of GA for problem #3

coefficient to increase service quality level were considered. Since the problem was a NP-hard, two meta-heuristic algorithms, namely GA and SA, were developed with new coding schemes to solve the model. To verify the efficiency and intelligence of the both algorithms, a RS algorithm was used as well. Furthermore, RSM was utilized to tune the parameters of both algorithms. Finally, to demonstrate the efficiency of both algorithms, computational results on different problem sizes were reported. The results were in favor of both algorithms in solving different problems.

The following can be considered in future research works:

I. Other queuing disciplines can be considered to model QFLP.
II. A different QFLP model can be developed when customers encounter multi-echelon queuing networks.
III. Various service rates can be considered for facilities.
IV. A different all-feasible chromosome representation can be used.
V. Different multi-objective solution methodologies can be proposed.

The demand and the service rates can be considered fuzzy inputs to model a $\tilde{M}/\tilde{M}/1$ queuing system.

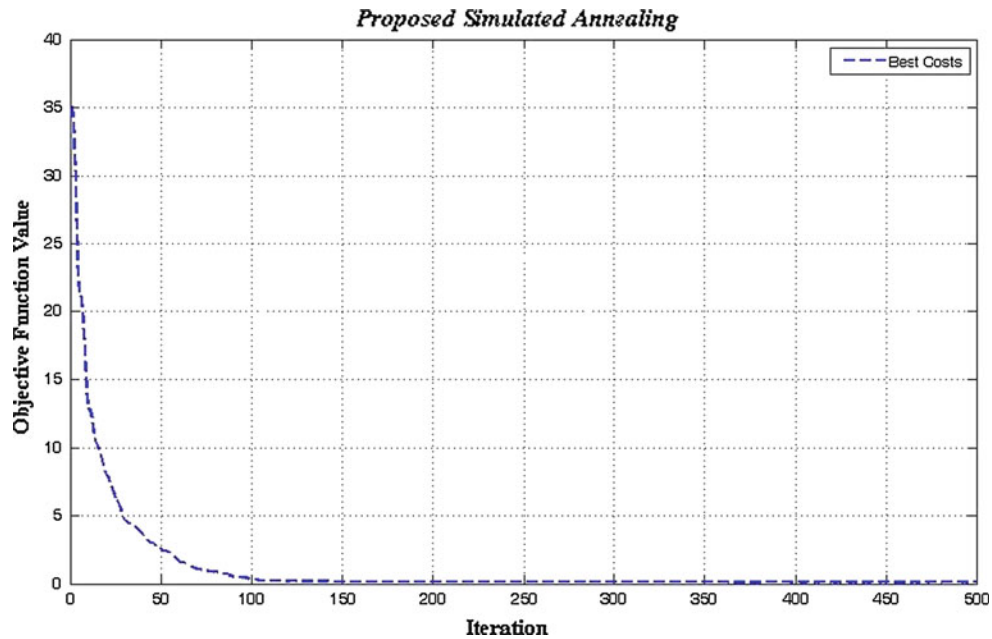



**Fig. 16** Convergence diagram of SA for problem #3

**Table 8** Analysis of variance for performance comparisons

| Source | *df* | SS | MS | *F* test | *P* value |
|---|---|---|---|---|---|
| Response | 2 | 1.89978 | 0.94989 | 127.21 | 0.000 |
| Error | 57 | 0.42562 | 0.00747 | | |
| Total | 59 | 2.32541 | | | |

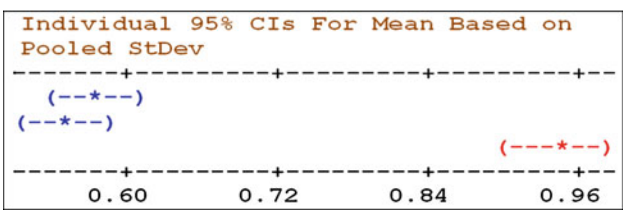



**Fig. 17** The significant difference of the RS method

**Table 9** Analysis of variance for the algorithms comparison

| Source | *df* | SS | MS | *F* test | *P* value |
|---|---|---|---|---|---|
| Response | 1 | 0.0041 | 0.0041 | 0.38 | 0.539 |
| Error | 38 | 0.4012 | 0.0106 | | |
| Total | 39 | 0.4052 | | | |

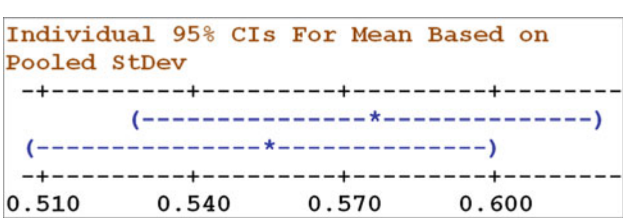



**Fig. 18** Identical performance of GA and SA

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