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Integrated inventory and transportation decision for ubiquitous supply chain management

Ki-sung Hong · Sang Soo Yeo · Hyoung Joong Kim · Ek Peng Chew · Chulung Lee

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Abstract We consider price-dependent demand and develop an integrated inventory and transportation policy with strategic pricing to maximize the total profit for a ubiquitous enterprise. The proposed policy provides the optimal ordering, shipment and pricing decision. We first assume that demand for a product is a linear function of the price. A mathematical model for the total profit under quantity based dispatch is developed in consideration of ordering, shipment and pricing variables. Optimality properties for the model are then obtained and an efficient algorithm is provided to compute the optimal parameters for ordering, shipment and pricing decision. Finally, we extend our results to a more general case where demand for the product is a convex or a concave function of the price.

Keywords Ubiquitous enterprise · Ubiquitous supply chain management · Pricing · Shipment consolidation · Linear demand function · General demand function

K. Hong
Graduate School of Information Management and Security, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea
e-mail: justlikewind@korea.ac.kr

S. S. Yeo
Division of Computer Engineering, Mokwon University, Daejeon 302-729, Republic of Korea
e-mail: sangsooyeo@gmail.com

H. J. Kim
Graduate School of Information Security, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea
e-mail: khj-@korea.ac.kr

E. P. Chew
Department of Industrial & Systems Engineering, National University of Singapore, 10 Kent Ridge Crescent, Singapore 119260, Singapore
e-mail: isecep@nus.edu.sg

C. Lee (✉)
Division of Industrial Management Engineering and Graduate School of Management of Technology, Korea University, Anamdong 5-ga, Seongbuk-gu, Seoul 136-713, Republic of Korea
e-mail: leecu@korea.ac.kr

Introduction

Recently, pervasive computing and ubiquitous networks have received significant attention from researchers (Weatherall and Jones 2002). The vision of ubiquitous technology is to create a smart space where users can enjoy ubiquitous services in an "anytime, anywhere, on any device" manner (Takahashi et al. 2005; Serrano and Fischer 2007; Su et al. 2008). The smart space realizing this vision is called a ubiquitous environment and the ubiquitous environment has motivated companies to create ubiquitous enterprise (Chen and Kotz 2000; Bohlen et al. 2005; Su et al. 2008).

The most successful application of ubiquitous technology has been run in the field of supply chain management (Bose and Pal 2005 and Yang 2008). Ubiquitous supply chain management (USCM) has been recognized as a new initiative for the effective supply chain management (SCM) and has been adapted to ubiquitous enterprise (Yang 2008). USCM, which differs from conventional SCM, makes a supply chain management decision in a real time based on information collected through ubiquitous technology. Thus, USCM needs an integrated model that computes the optimal values of supply chain control on a real time basis. This research addresses integrated supply inventory, transportation and pricing decision making for ubiquitous supply chain management (Fig. 1).

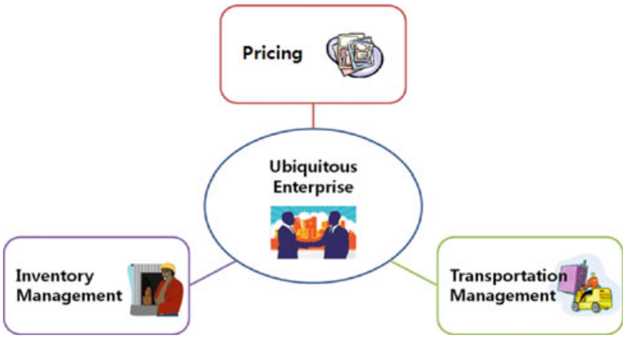

Fig. 1 Description of ubiquitous enterprise environment

Previous integrated supply chain optimization models are classified by two different approaches. One of the approaches is integrated inventory-transportation decision where the total relevant cost with the consideration of joint inventory and transportation decision is minimized. A stream of problems that deal with integrated inventory-transportation decisions is referred to as a shipment consolidation problem. Due to the advancement in information technology (e.g. electronic data interchange) and the cost reduction in information sharing, companies face a better opportunity to synchronize inventory and transportation decisions and began to use a shipment consolidation policy. Under a shipment consolidation policy, the company has the autonomy to consolidate small orders from customers until a larger dispatch quantity accumulates. Thus, fewer shipments of larger loads are dispatched (this practice is known as shipment consolidation), and the company improves the synchronization of the inventory and transportation decision (Chen et al. 2005). Using this method, companies can reduce the transportation cost.

However, shipment consolidation does not always reduce the supply chain cost. Delivery time and inventory holding time increase while several small orders are consolidated into a larger shipment. Thus, customer waiting cost and inventory holding cost increase in return. Hence, such trade-offs must be considered when making decisions about shipment consolidation.

On the other hand, recent operations management literature has started to focus on developing integrated models that can simultaneously optimize the relevant inventory (operations) and pricing (marketing) decisions (Sajadieh and Akbari Jokar 2009). The purpose of these literatures is to determine the operations and marketing decision variables that maximize the company's profit. The first model of this kind was investigated by Whitin (1955) who incorporated pricing into the traditional EOQ model through a linear price-sensitive relation for the customers.

The shipment consolidation literature and the integrated inventory-pricing literature deal with the ordering and shipment policies, and the ordering and pricing policies, respectively. In this paper, we integrate the two above mentioned literature branches in a model where the shipment, ordering and pricing policies are optimized all together and develop an integrated inventory-transportation-marketing model for ubiquitous enterprise.

This paper is organized as follows: "Literature review" provides an overview of USCM, shipment consolidation and integrated inventory-pricing literature. In "Integrated inventory-transportation-pricing decision", we have developed an integrated inventory-transportation-pricing model to maximize the total profits. A mathematical model with a linear demand function has been developed and an efficient algorithm is provided to obtain the optimal parameters for the proposed policy. We then extend our results in "Extensions" to a more general case where the demand for the product is a convex or a concave function of the price. Finally, "Conclusion" summarizes our conclusions.

Literature review

Most research relevant to USCM is carried out recently and tends to focus on technical perspective. Roussos (2006) addressed the SCM standard in ubiquitous commerce. He reviewed the history of unique identifier and product classification system and examined a global cataloguing schemes and standards for ubiquitous commerce. Hackenbroich et al. (2006) described enterprise software SAP's SCM and Auto-ID technology. They provide a good example of better understanding the relationship between ubiquitous technology and U-business application. In addition, Johnson (2006) examined tracking technologies focusing on the business case for investment and discussed the benefits of automated identification and tracking as compared with traditional legacy systems like bar codes. Thiesse et al. (2006) described the design and adaption of a real-time identification and localization system using RFID and ultrasound sensor technologies and Yang (2008) identified the critical success factors and examined their relationship with the benefits of USCM. Recently, Schoenemann et al. (2009) introduced a Peer-to-Peer-based architecture for exchanging distributed information, which are shared among participants of a supply chain facilitated with ubiquitous information technologies.

Shipment consolidation has received increasing academic attention in the last two decades (Chen et al. 2005). Research on shipment consolidation employs three different approaches in consolidating orders: Quantity-based dispatch policy, time-based dispatch policy and hybrid dispatch policy. The time-based dispatch policy and the quantity-based dispatch policy are the two most frequently used methods.

The quantity-based policy ships accumulated loads when a predetermined economical dispatch quantity, q is accumulated, whereas the time-based policy ships accumulated loads (all outstanding orders) every T period. Under the time-based policy, each order is dispatched on pre-specified shipment release dates, even if the dispatch quantity does not necessarily satisfy transportation scale economies. On the other hand, under the quantity-based policy, the dispatch quantity assures transportation scale economies, but a specific dispatch time cannot be guaranteed. An alternative to these two policies is a hybrid policy aimed at balancing the trade-offs between the timely delivery of the time-based policy and the transportation cost savings associated with the quantity-based policy. Under the hybrid policy, a dispatch decision is made either when the size of a consolidated load exceeds q_H (pre-specified dispatch quantity), or when the time since the last dispatch exceeds T_H (pre-specified dispatch time).

Cetinkaya and Bookbinder (2003); Chen et al. (2005); Centinkaya and Lee (2002); Moon et al. (2011); Ching and Tai (2005); Cetinkaya et al. (2006) have developed the optimization models for shipment consolidation. For the demand arrival following a Poisson process, Cetinkaya and Bookbinder (2003), and Chen et al. (2005) have developed the optimal quantity based policy. Centinkaya and Lee (2002) present an optimization model for coordinating inventory and transportation decisions at an outbound distribution warehouse that serves a group of customers located in a given market. Moon et al. (2011) developed joint replenishment and consolidated freight delivery policies for a TPW that handles multiple items. They extended the results of Centinkaya and Lee (2002) to consider the joint replenishment of multiple items and introduce two time-based policies for the warehouse (stationary policy and non-stationary policy, respectively). The optimal hybrid dispatch policy with stochastic demand is studied by Ching and Tai (2005) and Cetinkaya et al. (2006). They analyzed the advantages and the disadvantages of the quantity-based policy and the time-based policy, and have proposed hybrid policies. Recently, Günther and Seiler (2009) investigated an operational transportation planning problem based on a real industry case on shipment consolidation.

These traditional studies on shipment consolidation have assumed that the price of the product is constant. However, the price of the product, if allowed to be adjusted, would influence the demand and the optimal dispatch quantity, accordingly. Thus, the price of the product should also be considered a decision variable. Different from existing cost minimization research, we develop the optimization model to jointly optimize ordering, shipment and pricing decisions in order to maximize the total profit.

The integration of inventory and pricing decisions is first studied by Whitin (1955). Whitin (1955) incorporated pricing decision into the traditional EOQ model and this model was later explicitly solved by Portueus (1985). Other researchers such as Mills (1959); Karlin and Carr (1962); Hempenius (1970); Lau and Lau (1988) and Polatoglu (1991) build an integrated model consider a price-dependent demand. Abad (1996) then investigated a similar problem for a more general demand function.

Kunreuther and Schrage (1973) determine the optimal static price and the optimal production quantity under the assumption of deterministic demand. For stochastic demand, Federgruen and Heching (1999) address the problem of determining optimal pricing and inventory replenishment quantities. They build both finite and infinite horizon and obtain an optimal combined pricing and inventory policy.

Lau and Lau (2003) investigated a joint pricing-inventory model and they found that the nature of the price-demand relationship may have a considerable effect on the results of inventory-pricing decision. Ray et al. (2005) introduced an integrated marketing-inventory model for two pricing policies, price as a decision variable and mark-up pricing. Another recent paper in this area is by Bakal et al. (2008) who presented two inventory models with a price-dependent demand. They introduced two different pricing strategies (1) the firm chooses to offer a single price in all markets, and (2) a different price is set for each market. Recently, Abad (2008) investigated the pricing and lot-sizing problem for a product subject to general rate of deterioration and partial backordering. Sajadieh and Akbari Jokar (2009) developed an integrated production-inventory-marketing model to determine the relevant profit-maximizing decision variable values. Kannegiesser et al. (2009) considered on integrated sales and supply decision for commodities in a value chain and developed a mathematical model to determine the optimal sales quantity, production quantity and price to maximize the profit. Recently, Kannegiesser and Günther (2010) proposed coordinated decision making for production, distribution, sales and procurement of a global supply chain network and introduced a linear optimization model for tactical value chain planning.

These literatures on integrated inventory-pricing problem only deal with the ordering and pricing decisions and did not consider transportation decision. Explained ahead, company can reduce the transportation cost using shipment consolidation. Different from existing studies, we integrate these two streams of literature and develop a model where the shipment, ordering and pricing policies are optimized all together.

Integrated inventory-transportation-pricing decision

In this section, we develop a mathematical model for integrated inventory-transportation-pricing decision of USCM. The price of the product affects the total revenue as well as the total cost. For example, if the price is reduced, the demand increases, which will shorten the replenishment cycle. While the total cost may be reduced, the revenue may also be reduced. Thus, the price must be determined considering this trade-off between the revenue and the cost, and we employ the objective function of maximizing the profit, which is computed by the difference between revenue and cost.

In this study, we consider a quantity-based dispatch policy where the company releases a shipment as soon as the size of an outbound load waiting to be released reaches a critical dispatch quantity denoted by q. In this context, the time between two successive outbound dispatch decisions is called a dispatch cycle, and all orders arriving during a dispatch cycle are combined to form a large outbound load. Let Q denote the replenishment quantity. Observe that one can safely substitute $Q = nq$, where n is an integer denoting the number of dispatch cycles within an inventory replenishment cycle. In this paper, we assume that each customer requests one unit of the product, and the demand arrives according to a Poisson process with mean rate λ. Customers can wait, however, keeping customers waiting has negative impact on firm's goodwill. Such loss of goodwill associated with delayed receipt of goods is represented by a customer waiting cost. We also assume that the shipment cost is irrespective of the customer location (transportation distance). In this paper, we assume that the delivery lead time is negligible, i.e., customers are located in a relatively close proximity. Under this assumption, the shipment cost consists of a fixed cost of hiring trucks (or other transportation means) and a variable cost that is determined by volume, not by distance.

The followings are the additional assumptions of the model:

- The inventory level is under continuous review.
- The lead time for inventory replenishment is assumed to be negligible so that there is no inventory during the last dispatch cycle of an inventory replenishment cycle, as in Fig. 2.
- The demand arrival rate λ is a non-increasing and linear function of the price, as shown in Fig. 3 (Lau and Lau 1988; Polatoglu 1991; Abad 1996; Jung and Hwang 2009) and (Yassine 2010).

As a result, the problem is to compute the optimal price, p, the optimal number of dispatch cycles within a replenishment cycle, n, and the optimal dispatch quantity, q, in order to maximize the total profit.

Mathematical model

In this section, we present a mathematical model for the quantity-based dispatch policy to consider pricing.

The following notation is employed in this study:

p: Unit product price (decision variable)
F_R: Fixed cost of replenish inventory

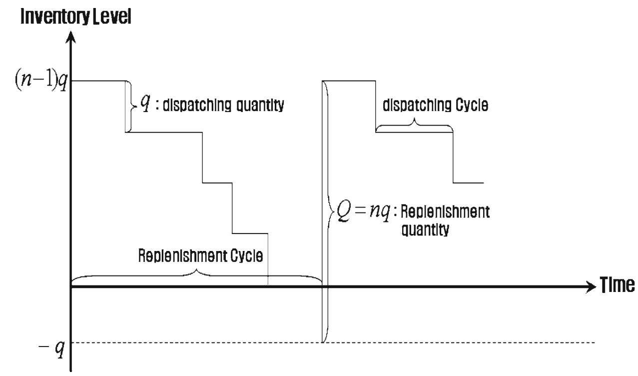

Fig. 2 Firm's inventory level

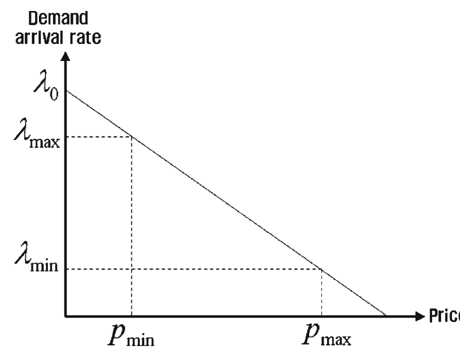

Fig. 3 Linear demand function

C_R: Unit replenish cost
F_D: Fixed cost of dispatching shipment to customer
C_D: Unit dispatch cost
h: Holding cost per unit per unit time
w: Waiting cost per unit per unit time
λ: Poisson demand rate, a linear function of product price ($\lambda = \lambda_0 - \alpha p$)
α: Price sensitivity of demand ($\alpha > 0$)
q: Dispatch quantity (integer, decision variable)
n: Number of dispatch cycles within an inventory replenishment cycle (integer, decision variable)
Q: Replenishment quantity ($Q = nq$)

As Fig. 2 has shown, the process under consideration is a renewal process (for the classical treatment of this kind of system). Thus, using the Renewal Reward Theorem, the expression for the long-run average profit, $TP(p, n, q)$, is determined by dividing E[Replenishment Cycle Profit] by E[Replenishment Cycle Length].

As the demand process is a Poisson process, the expectation of the dispatch cycle length will be q/λ. Since the number of dispatch cycles within an inventory replenishment cycle is n, the expectation of the replenishment cycle length is:

$$\text{E[Replenishment Cycle Length]} = nq/\lambda \quad (1)$$

We now compute revenue and four different cost elements (replenishment, dispatch, holding and waiting costs) during a replenishment cycle:

- **Revenue**: Since the quantity of product dispatched during a replenishment cycle is nq, the corresponding revenue is:

$$\text{Revenue } = npq \quad (2)$$

- **Replenishment Cost**: Since the replenishment quantity, Q, is equal to nq, the replenishment cost is:

$$\text{Replenishment Cost } = F_R + C_R \cdot nq \quad (3)$$

- **Dispatch Cost**: Since the dispatch quantity is q, the dispatch cost in a dispatch cycle is $F_D + C_D \cdot q$. There are n dispatch cycles during a replenishment cycle, and thus, the dispatch cost during the cycle is:

$$\text{Dispatch Cost} = n\,(F_D + C_D \cdot q) \quad (4)$$

- **Inventory Holding Cost**: At the beginning of a replenishment cycle, the inventory level is $(n-1)q$. This implies that the inventory level is kept at $(n-1)q$ throughout the first dispatch cycle, and hence incurs an expected holding cost of $h \cdot (n-1)q \cdot \frac{q}{\lambda}$. For the ith dispatch cycle, the expected holding cost is $h \cdot (n-i)g \cdot \frac{q}{\lambda}$. Hence, the total expected inventory holding cost is given by:

$$\text{Inventory Holding Cost} = \sum_{i=1}^{n}\left[h \cdot (n-i)q \cdot \frac{q}{\lambda}\right] = \frac{h \cdot n(n-1) \cdot q^2}{2\lambda} \quad (5)$$

- **Waiting Cost**: Since the company will not dispatch their products until q units of demand accumulate, the waiting time for the jth demand is $\frac{(q-j)}{\lambda}$. Thus, the customer waiting cost per dispatch cycle is $\sum_{j=1}^{q}\left[w \cdot \frac{(q-j)}{\lambda}\right] = w \cdot \frac{q(q-1)}{2\lambda}$. Since there are n dispatch cycles in a replenishment cycle, the customer waiting cost per replenishment cycle is:

$$\text{Waiting Cost } = n\frac{w \cdot q(q-1)}{2\lambda} \quad (6)$$

Using the above results, the expected profit during a replenishment cycle is computed by

$$\begin{aligned}&\text{E[Replenishment Cycle Profit]}\\ &\quad = nqp - (F_R + C_R \cdot nq)\\ &\quad -n(F_D + C_D \cdot q) - \frac{h \cdot n(n-1) \cdot q^2}{2\lambda}\\ &\quad -\frac{n \cdot w \cdot (q-1)q}{2\lambda}\end{aligned} \quad (7)$$

Conversely, the expression for the long-run average profit, $TP(n, q, p)$, is given by

$$\begin{aligned}&TP(n,q,p)\\ &= \frac{nqp-(F_R+C_R \cdot nq)-n(F_D+C_D \cdot q)-\frac{h \cdot n(n-1)\cdot q^2}{2\lambda}-\frac{n \cdot w \cdot (q-1)q}{2\lambda}}{\frac{nq}{\lambda}}\\ &= p\lambda - \frac{F_R\lambda}{nq} - C_R\lambda - \frac{F_D\lambda}{q} - C_D\lambda - \frac{h(n-1)q}{2}\\ &-\frac{w(q-1)}{2}\end{aligned} \quad (8)$$

Since $\lambda = \lambda_0 - \alpha p$, $TP(p, n, q)$ is

$$\begin{aligned}TP(n,q,p) &= p(\lambda_0 - \alpha p) - \frac{F_R(\lambda_0-\alpha p)}{nq}\\ &-C_R(\lambda_0-\alpha p) - \frac{F_D(\lambda_0-\alpha p)}{q}\\ &-C_D(\lambda_0-\alpha p) - \frac{h(n-1)q}{2} - \frac{w(q-1)}{2}\end{aligned} \quad (9)$$

The value of p, n and q that maximize the total profit per unit time follow the optimality conditions below.

Proposition 1 *For given values of n and q, the total profit function is a concave function of p. Thus, the optimal price p is obtained by taking the first order derivative of the total profit function, as given by Eq. (10):*

$$p^* = \frac{1}{2\alpha}\left[\lambda_0 + \frac{\alpha F_R}{nq} + \alpha C_R + \frac{\alpha F_D}{q} + \alpha C_D\right] \quad (10)$$

Proof Taking the first order and second order partial derivatives of (9) with respect to p, we have

$$\frac{dTP(n,q,p)}{dp} = \lambda_0 - 2\alpha p + \frac{\alpha F_R}{nq} + \alpha C_R + \frac{\alpha F_D}{q} + \alpha C_D \quad (11)$$

and

$$\frac{d^2TP(n,q,p)}{dp^2} = -2\alpha \quad (12)$$

respectively. Since the second order derivative is always less than zero, $TP(n, q, p)$ is concave with respect to p for given values of n and q. □

Proposition 2 *For given values of p and q, the optimal value of n always satisfies the following condition:*

$$n^*(n^*-1) \le \frac{2F_R(\lambda_0-\alpha p)}{hq^2} \le n^*(n^*+1) \quad (13)$$

Proof For given values of p and q, the optimal value of n always satisfies the following:

$TP(n^*-1) \leq TP(n^*)$ and $TP(n^*+1) \leq TP(n^*)$

Using Eq. (1), an optimality condition for n is:

$$n^*(n^*-1) \leq \frac{2F_R(\lambda_0-\alpha p)}{hq^2} \leq n^*(n^*+1)$$

□

Proposition 3 *For given values of p and n, the optimal value of q satisfies the following condition:*

$$q^*(q^*-1) \leq \frac{2\left[F_R(\lambda_0-\alpha p)+nF_D(\lambda_0-\alpha p)\right]}{n\left[h(n-1)+w\right]} \leq q^*(q^*+1) \quad (14)$$

Proof For given values of p and n, the optimal value for q follows:

$TP(q^*-1) \leq TP(q^*)$ and $TP(q^*+1) \leq TP(q^*)$

Similarly, using Eq. (1), an optimality condition of q is:

$$q^*(q^*-1) \leq \frac{2\left[F_R(\lambda_0-\alpha p)+nF_D(\lambda_0-\alpha p)\right]}{n\left[h(n-1)+w\right]} \leq q^*(q^*+1)$$

□

Proposition 4 *The upper bound of n satisfies the following condition:*

$$n_{\max}(n_{\max}-1) \leq \frac{2F_R\lambda_0}{h} \leq n_{\max}(n_{\max}+1) \quad (15)$$

where $n_{\max}$ denote the upper bound of n.

Proof The value, $\frac{2F_R(\lambda_0-\alpha p)}{hq^2}$ in Eq. (13) is a non-increasing function of p and n. So, the maximum value of possible n is determined when $p=0$ and $q=1$. □

Proposition 5 *The upper bound of q satisfies the following condition:*

$$q_{\max}(q_{\max}-1) \leq \frac{2\left[F_R\lambda_0+F_D\lambda_0\right]}{w} \leq q_{\max}(q_{\max}+1) \quad (16)$$

where $q_{\max}$ denote the upper bound of q.

Proof The value, $\frac{2[F_R(\lambda_0-\alpha p)+nF_D(\lambda_0-\alpha p)]}{n[h(n-1)+w]}$ in Eq. (14) is non-increasing function of p and q. So, the maximum value of possible n is determined when $p=0$ and $n=1$. □

Using the above optimality conditions, we develop a simple enumeration algorithm to obtain the optimal parameters for the proposed policy. The simple enumeration algorithm always guarantees the optimal solution. The procedure is as follows:

Table 1 The parameter values for our example

Parameter	Value
F_R	40
F_D	5
λ_0	2
h	1
w	2
α	0.01

Table 2 Summary of the solution procedure

Parameter	Value
$n_{\max}$	13
$q_{\max}$	9
Optimal p	103.0556
Optimal n	3
Optimal q	3
Total profit	88.98225

The simple enumeration algorithm (SEA)

(Step 1) Compute the upper bound of n and q using Eqs. (15) and (16), respectively.
(Step 2) For all combination of $n(n=1,\ldots,n_{\max})$ and $q(q=1,\ldots,q_{\max})$, compute the optimal p using Eq. (10).
(Step 3) For given combination of n and q, p, compute the total profit $TP(n,q,p)$ using Eq. (9).
(Step 4) Select the (n,q,p) with the maximum $TP(n,q,p)$

A numerical example is employed to illustrate how the algorithm proceeds. The parameter values for this example are provided in Table 1.

The upper bound of n and q, as well as the resulting optimal values, are provided in Table 2.

Computational complexity of the proposed algorithm

The computational complexity of the simple enumeration algorithm is determined by the number of dispatch cycles and the number of dispatch quantity. The proposed algorithm requires computing p^* by Eq. (10) at most $q_{\max}\cdot n_{\max}$ times, where $q_{\max}$ and $n_{\max}$ are tight upper bounds obtained by Propositions 4 and 5.

Computational experiments

In this subsection, we compare the performance of the two policies (Quantity-based dispatch policy with pricing (QWP) and without pricing (QWOP)). For this comparison, we will use the data from (2006), a set of 1,024 problem instances,

Table 3 Comparison of the profits under QWP and QWOP

F_R	F_D	λ_0	h	w	
40	5	2	1	2	
p	n	q	Total profit		
Quantity-based dispatch policy with pricing					
103.0556	3	3	88.98225		
p_j	n	q	Total profit	Profit gap	$\Delta P\%$
Quantity-based dispatch policy without pricing					
12.25	2	6	7.176459	81.80579	91.94
24.5	3	4	27.98375	61.0285	68.59
36.75	3	4	45.51209	43.47017	48.85
49	3	3	59.76222	29.22003	32.84
61.25	2	5	71.54688	17.43538	19.59
73.5	2	5	80.1525	8.829758	9.92
85.75	3	3	85.95743	2.99482	3.37
98	3	3	88.72666	0.255592	0.29
110.25	2	4	88.34	0.64225	0.72
122.5	2	4	85.0376	3.888496	4.37

generated using a full factorial design and considering $C_R = 0 : C_D = 0 : F_R = 40, 80, 160, 320$; $F_D = 5, 10, 20, 40$; $h = 1, 2, 4, 8$; $w = 2, 4, 8, 16$; and $\lambda_0 = 2, 4, 8, 16$.

To compare the performance of QWP for different values of p, we compute maximum value of p, $p_{\max}$, and divide the range $[0, p_{\max}]$ into ten equally spaced values of $p_j : (p_1, p_2, \ldots, p_{10})$. Thus, ten different values of p are considered. For each value of p, 1,024 problems are generated and solved using both QWP and QWOP for a total of 10,240 problems. The particular scenario satisfying $F_R = 40$, $F_D = 5$, $\lambda_0 = 2$, $h = 1$, $w = 2$ is shown in Table 3. Table 3 demonstrates the profit difference among a QWP and QWOP. The optimal price of QWP is calculated by Proposition 1, and the price of QWOP is given in column 1. For each $p_j = 12.25, 24.5, \ldots, 122.5$, the percentage profit increase that result from optimal pricing is reported under the column heading $\Delta P\%$. These values are computed by

$$\Delta P\% = 100\frac{\text{Profit of QWP} - \text{Profit of QWOP}}{\text{Profit of QWP}}.$$

As shown in Table 3, the values of n and q are adjusted according to the price, and the total profit under QWP consistently increases. The profit difference between QWP and QWOP is at least 0.29% and can be up to 91.94% in a particular scenario.

Tables 4 and 5 analyze the performance of the QWP with different the fixed cost of replenishing inventory and dispatch shipment. Table 4 shows that the replenishment quantity ($Q = nq$) increase and the profit decreases as F_R increases from 40 to 320. This agrees with the intuition that if fixed cost of replenishing inventory is high, we order more products to reduce the replenishment cost.

Table 4 Performance of QWP with different F_R ($F_D = 5, \lambda_0 = 2, h = 1, w = 2$)

F_R	n	q	p	Profit
40	3	3	103.06	88.98
60	4	3	103.13	86.94
80	4	3	103.96	85.34
100	5	3	103.83	83.84
120	5	3	104.50	82.57
140	5	3	105.17	81.30
160	6	3	104.86	80.22
180	6	3	105.42	79.17
200	6	3	105.97	78.13
220	7	3	105.60	77.22
240	7	3	106.07	76.33
260	7	3	106.55	75.44
280	8	3	106.15	74.61
300	8	3	106.56	73.83
320	8	3	106.98	73.06

Table 5 shows that the shipment quantity increases and profit decrease as F_D increases from 5 to 40. This agrees with the intuition that if the fixed cost of dispatch shipment is high, we dispatch more orders to reduce the transportation cost.

Table 6 shows that the replenishment quantity and the profit decrease as the unit holding cost increases from 1

Table 5 Performance of QWP with different F_D ($F_R = 40$, $\lambda_0 = 2, h = 1, w = 2$)

F_D	n	q	p	Profit
5	3	3	103.06	88.98
7.5	2	4	104.38	88.23
10	2	4	105.00	87.63
12.5	2	5	105.13	87.07
15	2	5	105.75	86.57
17.5	2	5	106.38	86.07
20	2	5	107.00	85.57
22.5	2	5	107.63	85.07
25	2	5	108.25	84.56
27.5	2	5	108.88	84.06
30	2	6	109.17	83.59
32.5	2	6	109.79	83.15
35	2	6	110.42	82.70
37.5	2	6	111.04	82.26
40	2	6	111.67	81.81

Table 6 Performance of QWP with different h ($F_R = 40$, $F_D = 5$, $\lambda_0 = 16, w = 16$)

h	n	q	p	Profit
1	13	2	800.96	6,347.72
1.5	10	2	801.25	6,342.54
2	9	2	801.39	6,338.27
2.5	8	2	801.56	6,334.55
3	5	3	801.83	6,331.38
3.5	5	3	801.83	6,328.38
4	4	3	802.29	6,326.06
4.5	4	3	802.29	6,323.81
5	4	3	802.29	6,321.56
5.5	4	3	802.29	6,319.31
6	3	3	803.06	6,317.20
6.5	3	3	803.06	6,315.70
7	3	3	803.06	6,314.20
7.5	3	3	803.06	6,312.70
8	3	3	803.06	6,311.20

to 8. This agrees with the intuition that if the unit holding cost is high, we keep fewer inventories to reduce the holding cost.

Table 7 shows that the shipment quantity and the total profit decrease as the unit waiting cost increases from 2 to 16. This agrees with the intuition that if the unit waiting cost is high, we dispatch fewer orders to reduce the total waiting cost because the waiting cost increases as shipment quantity increases.

Table 7 Performance of the QWP with different w($F_R = 40$, $F_D = 5$, $\lambda_0 = 16, h = 4$)

w	n	q	p	Profit
2	1	19	803.55	6,363.01
3	1	15	803.83	6,354.97
4	1	13	804.04	6,348.29
5	1	12	804.17	6,342.48
6	2	6	802.92	6,339.70
7	2	6	802.92	6,337.20
8	3	4	802.50	6,335.39
9	3	4	802.50	6,333.89
10	3	4	802.50	6,332.39
11	4	3	802.29	6,331.06
12	4	3	802.29	6,330.06
13	4	3	802.29	6,329.06
14	4	3	802.29	6,328.06
15	4	3	802.29	6,327.06
16	4	3	802.29	6,326.06

Extensions

In this section, we extend our results to a more general case where the demand for the product is a convex or a concave function of the product price. In previous sections, we assumed that the demand for the product is a linear function of the price. But the relation between price and demand is not accurately represented by an inverse proportion in real life. Thus, we consider a more general case where the demand for the product is a convex or a concave function of the price, as shown in Fig. 4.

Concave demand function

When the demand for the product is a general concave function ($\lambda = f(p)$ where $\frac{df(p)}{dp} < 0$ and $\frac{d^2 f(p)}{dp^2} < 0$), $TP(n, q, p)$ is

$$TP(n, q, p) = pf(p) - \frac{F_R f(p)}{nq} - C_R f(p) - \frac{F_D f(p)}{q} - C_D f(p) - \frac{h(n-1)q}{2} - \frac{w(q-1)}{2} \quad (17)$$

The values of p, n and q that maximize the total profit per unit time follow the optimality conditions below.

Proposition 6 *For given values of n and q, the total profit function is a concave function of p. Thus, the optimal price p is obtained by taking the first order derivative of the total profit function.*

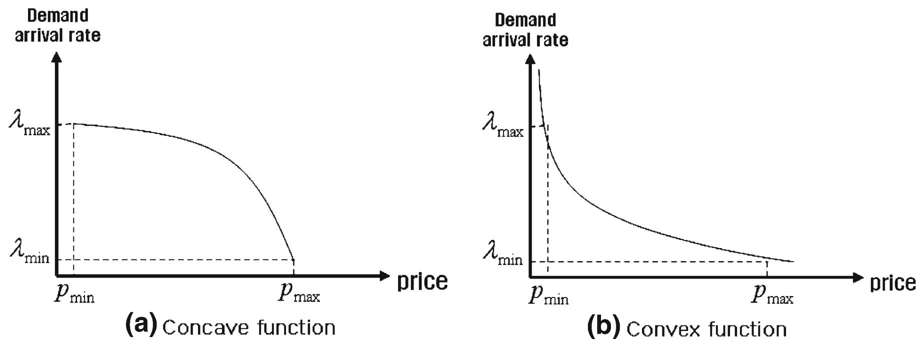

Fig. 4 Concave and convex demand functions

Proof Taking the first order and second order partial derivatives of (17) with respect to p, we have

$$\frac{dTP(n,q,p)}{dp} = f(p) + p \cdot f'(p) - \frac{F_R \cdot f(p)}{nq} - C_R \cdot f'(p) - \frac{F_D \cdot f(p)}{q} - C_D \cdot f'(p) \quad (18)$$

and

$$\frac{d^2TP(n,q,p)}{dp^2} = 2f'(p) + f''(p)\left[p - \frac{F_R}{nq} - C_R - \frac{F_D}{q} - C_D\right] < 0 \quad (19)$$

respectively. Since the second order derivative is always less than zero, $TP(n, q, p)$ is concave with respect to p for given values of n and q. □

Proposition 7 *For given values of p and q, the optimal value of n always satisfies the following condition:*

$$n^*(n^*-1) \leq \frac{2F_R \cdot f(p)}{hq^2} \leq n^*(n^*+1) \quad (20)$$

Proof For given values of p and q, the optimal value of n always satisfies the following:

$TP(n^*-1) \leq TP(n^*)$ and $TP(n^*+1) \leq TP(n^*)$

Using Eq. (15), an optimality condition for n is:

$$n^*(n^*-1) \leq \frac{2F_R \cdot f(p)}{hq^2} \leq n^*(n^*+1)$$

□

Proposition 8 *For given values of p and n, the optimal value of q always satisfies the following condition:*

$$q^*(q^*-1) \leq \frac{2\left[F_R \cdot f(p) + nF_D \cdot f(p)\right]}{n\left[h(n-1)+w\right]} \leq q^*(q^*+1) \quad (21)$$

Proof For given values of p and n, the optimal value of q follows:

$TP(q^*-1) \leq TP(q^*)$ and $TP(q^*+1) \leq TP(q^*)$

Similarly, using Eq. (15), an optimal condition of q is:

$$q^*(q^*-1) \leq \frac{2\left[F_R \cdot f(p) + nF_D \cdot f(p)\right]}{n\left[h(n-1)+w\right]} \leq q^*(q^*+1)$$

□

Proposition 9 *The upper bound of n satisfies the following condition:*

$$n_{max}(n_{max}-1) \leq \frac{2F_R f(0)}{h} \leq n_{max}(n_{max}+1) \quad (22)$$

Proof The value, $\frac{2F_R f(p)}{hq^2}$ in Eq. (20) is a non-increasing function of p and q. So, the maximum value of possible n is determined when $p = 0$ and $q = 1$. □

Proposition 10 *The upper bound of q satisfies the following condition:*

$$q_{max}(q_{max}-1) \leq \frac{2\left[F_R f(0) + F_D f(0)\right]}{w} \leq q_{max}(q_{max}+1) \quad (23)$$

Proof The value, $\frac{2[F_R f(p)+nF_D f(p)]}{n[h(n-1)+w]}$ in Eq. (21) is a non-increasing function of p and n. So, the maximum value of possible n is determined when $p = 0$ and $n = 1$. □

Using the above optimality conditions, we can develop an enumeration algorithm (like SEA in "Integrated inventory-transportation-pricing decision") to obtain the optimal parameters for the proposed policy.

Convex demand function

When the demand for the product is a general convex function ($\lambda = f(p)$ where $\frac{df(p)}{dp} < 0$ and $\frac{d^2 f(p)}{dp^2} > 0$), the optimality conditions for upper bound of n and q are provided by Propositions 9 and 10. However, it is difficult to obtain an optimality condition for p. Thus, we consider the two most widely-employed convex demand functions, $f(p) = \frac{k}{p^\beta}$ and $f(p) = \gamma \exp(-\delta p)$, where k, β, γ and δ are positive constants. These functions are introduced in numerous standard Economics references (Lau and Lau 2003).

Iso-elastic demand function

When the demand for the product is an iso-elastic demand function, $f(p) = \frac{k}{p^\beta}$ where $\beta > 1$, $TP(n, q, p)$ is

$$TP(n,q,p) = p\left(\frac{k}{p^\beta}\right) - \frac{F_R\left(\frac{k}{p^\beta}\right)}{nq} - C_R\left(\frac{k}{p^\beta}\right) - \frac{F_D\left(\frac{k}{p^\beta}\right)}{q} - C_D\left(\frac{k}{p^\beta}\right) - \frac{h(n-1)q}{2} - \frac{w(q-1)}{2} \quad (24)$$

The value of p that maximizes the total profit per unit time follows the optimality conditions below.

Proposition 11 *For given values of n and q, the total profit function is a concave function of p. Thus, the optimal price p is obtained by taking the first order derivative of the total profit function, as given by Eq. (25).*

$$p^* = \frac{\beta\left(\frac{F_R}{nq} + C_R + \frac{F_D}{q} + C_D\right)}{(\beta-1)} \quad (25)$$

Proof Taking the first order and second order partial derivatives of (24) with respect to p, we have

$$\frac{dTP(n,q,p)}{dp} = \frac{k}{p^{\beta+1}}\left[p - \beta p + \beta\left(\frac{F_R}{nq} + C_R + \frac{F_D}{q} + C_D\right)\right] \quad (26)$$

and

$$\frac{d^2TP(n,q,p)}{dp^2} = -\frac{2\beta k}{p^{\beta+1}} + \frac{\beta(\beta+1)k}{p^{\beta+2}}\left[p - \frac{F_R}{nq} - C_R - \frac{F_D}{q} - C_D\right] = -\frac{2\beta k}{p^{\beta+1}} + \frac{\beta(\beta+1)k}{p^{\beta+1}} - \frac{\beta(\beta+1)k}{p^{\beta+2}} \times \left[\frac{F_R}{nq} + C_R + \frac{F_D}{q} + C_D\right] \quad (27)$$

respectively. The unique value of p that satisfies $\frac{dTP(n,q,p)}{dp} = 0$ exists, and this value is

$$p = \frac{\beta\left(\frac{F_R}{nq} + C_R + \frac{F_D}{q} + C_D\right)}{(\beta-1)} \quad (28)$$

Using Eq. (28), $\left[\frac{F_R}{nq} - C_R - \frac{F_D}{q} - C_D\right] = \frac{(\beta-1)}{p}$. Substituting this equation into Eq. (27), we obtain (after rearranging the terms):

$$\frac{d^2TP(n,q,p)}{dp^2} = -\frac{2\beta k}{p^{\beta+1}} + \frac{\beta(\beta+1)k}{p^{\beta+1}} - \frac{(\beta+1)(\beta-1)k}{p^{\beta+1}} = \frac{(1-\beta)k}{p^{\beta+1}} < 0 \quad (29)$$

Since the second order derivative is always less than zero, $TP(n, q, p)$ is concave with respect to p for given values of n and q. □

Using the above optimality conditions, we can develop an enumeration algorithm (like SEA in "Integrated inventory-transportation-pricing decision") to obtain the optimal parameters for the proposed policy.

Exponential demand function

When the demand for the product is an exponential demand function, $f(p) = \gamma \exp(-\delta p)$, $TP(n, q, p)$ is

$$TP(n,q,p) = p(\gamma\exp(-\delta p)) - \frac{F_R(\gamma\exp(-\delta p))}{nq} - C_R(\gamma\exp(-\delta p)) - \frac{F_D(\gamma\exp(-\delta p))}{q} - C_D\gamma\exp(-\delta p) - \frac{h(n-1)q}{2} - \frac{w(q-1)}{2} \quad (30)$$

The value of p that maximizes the total profit per unit time follows the optimality conditions below.

Proposition 12 *For given values of n and q, the total profit function is a concave function of p. Thus, the optimal price p is obtained by taking the first order derivative of the total profit function, as given by Eq. (31).*

$$p^* = \frac{1}{\delta} + \left(\frac{F_R}{nq} + C_R + \frac{F_D}{q} + C_D\right) \quad (31)$$

Proof Taking the first order and second order partial derivatives of (30) with respect to p, we have

$$\frac{dTP(n,q,p)}{dp} = \gamma\exp \times(-\delta p)\left[1 - \delta\left(p - \frac{F_R}{nq} - C_R - \frac{F_D}{q} - C_D\right)\right] \quad (32)$$

and

$$\frac{d^2TP(n,q,p)}{dp^2} = -2\delta\gamma\exp(-\delta p) + \delta^2\gamma\exp(-\delta p)\left[p - \frac{F_R}{nq} - C_R - \frac{F_D}{q} - C_D\right] \quad (33)$$

respectively. The unique value of p that satisfies $\frac{dTP(n,q,p)}{dp} = 0$ exists, and this value is

$$p = \frac{1}{\delta} + \left(\frac{F_R}{nq} + C_R + \frac{F_D}{q} + C_D\right) \quad (34)$$

Using Eq. (34), $\left[p - \frac{F_R}{nq} - C_R - \frac{F_D}{q} - C_D\right] = \frac{1}{\delta}$. Substituting this equation into Eq. (33), we obtain (after rearranging the terms)

$$\frac{d^2TP(n,q,p)}{dp^2} = -\delta\gamma\exp(-\delta p) < 0 \quad (35)$$

Since the second order derivative is always less than zero, $TP(n, q, p)$ is concave with respect to p for given values of n and q. □

Using the above optimality conditions, we can develop an enumeration algorithm (like SEA in "Integrated inventory-transportation-pricing decision") to obtain the optimal parameters for the proposed policy.

Conclusion

For USCM, suppliers must simultaneously optimize inventory, transportation and pricing on a real time basis. The proposed model jointly determines the optimal price, order quantity, and shipment quantity with polynomial complexity.

We consider the price-dependent demand and develop an integrated inventory and transportation policy with strategic pricing to maximize the total profit for a ubiquitous enterprise. We first assume that demand for the product is a linear function of the price and extend our results to a more general case where the demand for the product is a convex or a concave function of the price. An efficient algorithm is provided to obtain the optimal parameters for the proposed policy. Numerical results show that the Quantity-based dispatch policy with pricing (QWP) significantly improve the total profit. To the best of our knowledge, this is the first attempt to jointly optimize ordering, shipment and pricing decisions for USCM.

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