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A Lagrangean heuristic for a two-echelon storage capacitated lot-sizing problem

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Abstract In this paper, we study a structure issued from a real case. Raw materials (RMs) are sent by suppliers to a distribution center (DC) and then transported to a unique plant where they can be stored. The inventory capacity is limited in the plant as well as in the DC. The transportation capacity between the DC and the plant is also limited. The objective is to determine the flows between suppliers and the DC, and from the DC to the plant in order to satisfy the demand during the planning horizon while minimizing the global cost. A mixed-integer programming (MIP) formulation is presented and a Lagrangean relaxation solution procedure is proposed. Computational experiments are carried out.

Keywords Lot-sizing · Capacitated storage · Lagrangean heuristic

Introduction

Coordination in supply chains represents a crucial factor of competitiveness for companies. This paper deals with coordination in a two-echelon supply chain extracted from a real case. More precisely, Raw Materials (RMs) are sent by suppliers to a distribution center (DC) and then shipped to a unique plant where they can be stored. The inventory capacity is limited in the plant as well as in the DC and the transportation capacity is also limited. The objective is to minimize the global cost generated by the various activities in the supply process of RMs (procurement, transportation, storage).

Several papers in the literature deal with coordination in supply chains. In the paper of Samiento and Nagi (1999), one can find a state of the art of contributions in this field. The classical Lot-Sizing Problem (LSP), which consists of determining lot sizes to produce in order to minimize production and inventory costs, can be cited among the first integrated planning problems (Akbalik et al. 2008). Brahimi et al. (2006) give a literature review for different variants of one-echelon lot-sizing problem. For the multi-echelon LSP, (van Hoesel et al. 2005) consider a model in which production, multi-level storage, and transportation decisions are integrated under production capacities and concave cost functions. They present algorithms which are polynomial in the size of the planning horizon. Another relevant paper concerning a multi-echelon LSP is due to Kaminsky and Simchi-Levi (2003).

In this paper, we use Lagrangean relaxation (LR) in order to get a lower bound for our problem. Starting from the solution of the Lagrangean problem, we develop an efficient heuristic procedure which builds a good feasible solution for the initial problem. Lagrangean relaxation is a method which has been widely and successfully used in supply chain coordination. Jayaraman and Pirkul (2001) in the resolution of their model on "planning and coordination of production and distribution facilities for multiple commodities", dualize three constraints in their initial problem. They subdivide the lagrangean problem thus obtained, into three sub-problems which are easier to solve. More recently, Chen and Chu (2003) developed a heuristic procedure for a supply chain planning problem modeled as a multi-item

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multi-level capacitated lot-sizing problem. Their heuristic combines (LR) with local search. Other relevant references in this area can be found in the following papers Robinson and Lawrence (2004), Sambasivan and Yahya (2005).

Some industrial cases related to the supply chain coordination can also be cited. For example, Sharma (1990) optimizes a fertilizer distribution system in India, while integrating the production and inventory activities. Dhaenens-Flipo and Finke (2001) study the coordination between the production and distribution activities in a multi-plant, multi-product and multi-period environment for a steel industry. In a study carried out in "Digital Equipment Corporation", Arntzen et al. (1995) show a profit more than 100 M$ after a global optimization of the chain. Haq et al. (1991) study a real case (manufacturer of urea fertilizer), where they coordinate production-inventory-distribution activities in one model to find the quantity to produce, to store and to distribute each period. Matta and Miller (2004) integrate production, storage and transportation decisions between two plants. They study the influence of different parameters on the integrated decisions and on the total profit in the chain. They use data obtained from a pharmaceutical firm. Gnoni et al. (2003) study the scheduling and lot sizing problems in multi-site and multi-product environment, under the assumptions of capacity constrained production and stochastic demands. They propose a hybrid model to solve the integrated problem issued from the automotive industry.

The rest of the paper is organized as follows. Section "Description of the problem" presents the problem description and the structure studied in this paper. In section "Problem formulation", the mathematical formulation is given. The Lagrangean problem and the solution approach are presented in section "Lagrangean relaxation". In the section "The lagrangean heuristic solution procedure", the different steps of the Lagrangean heuristic procedure are described. The computational experiments are given in section "Computational experiments". Finally, some concluding remarks and on-going work are presented in section "Concluding remarks and on-going work".

Description of the problem

The problem that we consider here originates from a real case previously described in Akbalik et al. (2008). It deals with a plant of an international firm that is located in Morocco. Several RMs are assembled in order to produce multiple finished goods. These RMs come from various suppliers located throughout the world (Asia, Europe, USA). Some of them are firstly stored in a distribution center DC located in Europe (because of foreign currency exchange considerations), before being sent to the plant by trucks. Each supplier delivers only one RM and a RM comes from only one supplier. The transportation of RMs from a supplier to the DC generates a fixed cost depending on the supplier localization. In Akbalik et al. (2008), we assumed that the storage of RMs was not allowed at the plant. This enabled us to reduce the model to the supply of RMs between suppliers and the DC. Here, we take into account the possibility of storing RMs in the plant. The storage capacity in the plant is limited, as well as in the DC. We also assume, in accordance with the practice, that the number of trucks transporting RMs from the DC to the plant is constant over the planning horizon (see Fig. 1).

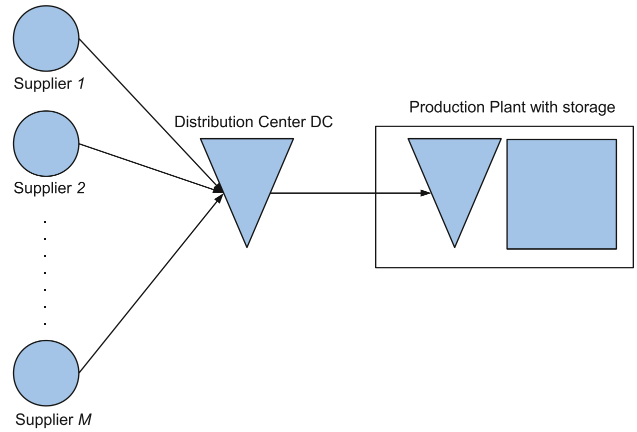

Fig. 1 The supply chain structure

The objective of our study is to optimize simultaneously the physical flows in this supply chain. The different coordinated activities are the supply of RMs from suppliers to the DC, their storage at DC, their transportation between the DC and the plant and their storage in the plant. The aim is to get a procurement planning, which can be used at the tactical and operational levels. This planning must take into account the suppliers capacities constraints, while minimizing transportation and inventory costs.

Problem formulation

We use the following notations for the parameters:

- M: the number of RMs (or equivalently suppliers),
- T: the number of periods in the planning horizon,
- $d_{m,t}$: the customer aggregated demand at period t for RM m (assumed to be integer),
- $P_{m,t}$: the capacity of the supplier m at period t (assumed to be integer),
- $pf_{m,t}$: the setup cost of RM m at period t,
- V_m^{DC}: the capacity of the vehicles supplying RM m between the supplier m and the DC,
- $tf_{m,t}^{DC}$: the fixed transportation cost per vehicle between the supplier m and the DC, at period t,

- IC_t^{DC}: the inventory capacity of the DC at period t,
- $iu_{m,t}^{DC}$: the unit inventory cost at the DC for RM m at period t,
- V^P: the constant transportation capacity between the DC and the plant,
- tf^P: the fixed transportation cost between the DC and the plant (which will be incurred at every period and therefore does not figure in the objective function of the MIP below),
- IC_t^P: the inventory capacity of RMs at the plant at period t,
- $iu_{m,t}^P$: the unit inventory cost at the plant for RM m at period t.

The decision variables are:

- $Y_{m,t}$: the quantity of RM m supplied at period t at the DC,
- $X_{m,t}$: the quantity of RM m shipped from the DC to the plant, at period t,
- $I_{m,t}^{DC}$: the inventory level of RM m at period t at the DC,
- $I_{m,t}^P$: the inventory level of RM m at period t at the plant,
- $z_{m,t}^{DC}$: the number of vehicles shipped from supplier m to the DC at period t,
- $q_{m,t}$: a binary variable set to 1 if RM m is supplied at period t, 0 otherwise.

A MIP formulation is given below:

$$(P) = \min \sum_{m=1}^{M} \sum_{t=1}^{T} \Big[pf_{m,t}.q_{m,t} + iu_{m,t}^{DC}.I_{m,t}^{DC} + iu_{m,t}^{P}.I_{m,t}^{P} + tf_{m,t}^{DC}.z_{m,t}^{DC} \Big]$$

$$I_{m,t}^{DC} = I_{m,t-1}^{DC} + Y_{m,t} - X_{m,t}; \quad \forall t, \quad \forall m \tag{1}$$

$$I_{m,t}^{P} = I_{m,t-1}^{P} + X_{m,t} - d_{m,t}; \quad \forall t, \quad \forall m \tag{2}$$

$$Y_{m,t} \leq P_{m,t}.q_{m,t}; \quad \forall m, \quad \forall t \tag{3}$$

$$Y_{m,t} \leq z_{m,t}^{DC}.V_m^{DC}; \quad \forall m, \quad \forall t \tag{4}$$

$$\sum_{m=1}^{M} I_{m,t}^{P} \leq IC_t^P; \quad \forall t \tag{5}$$

$$\sum_{m=1}^{M} I_{m,t}^{DC} \leq IC_t^{DC}; \quad \forall t \tag{6}$$

$$\sum_{m=1}^{M} X_{m,t} \leq V^P; \quad \forall t \tag{7}$$

$$Y_{m,t}, I_{m,t}^{DC} \geq 0; \quad \forall t, \quad \forall m \tag{8}$$

$$q_{m,t} \in \{0, 1\}; \quad z_{m,t}^{DC} \geq 0 \text{ and integer}; \quad \forall t, \quad \forall m \tag{9}$$

$$X_{m,t}, I_{m,t}^{P} \geq 0; \quad \forall m, \quad \forall t \tag{10}$$

The objective function minimizes the total cost which consists of the fixed procurement costs, the inventory costs at the DC and at the plant and the fixed transportation cost. Constraints (1) and (2) represent respectively the material balance for each period and for each RM at the DC and at the plant. The supplier capacity restriction per period for each RM is expressed by constraint (3). In constraint (4), the number of vehicles sent from each supplier to the DC is computed as a function of the quantity supplied for each RM at each period. Constraints (5) and (6) impose the limitation of inventory capacity in the DC and in the plant respectively. Constraint (7) expresses the limitation of shipment capacity between the DC and the plant. Constraint (9) ensures that the variable relating to the decision "to supply or not" in each period is binary and imposes the integrality restriction on the decision variable $z_{m,t}^{DC}$. Constraints (8) and (10) impose the non-negativity restriction on the decision variables $X_{m,t}$, $Y_{m,t}$, $I_{m,t}^{DC}$, $I_{m,t}^{P}$.

Lagrangean relaxation

The major difficulty in the resolution of our model lies in the coupling constraints (5), (6) and (7) (for details on (LR), the reader is referred to Beasley (1993). Let (LRP) be the Lagrangean problem resulting from the dualization of these constraints using the Lagrange multipliers α, β, γ respectively. Observe that each RM in (LRP) can now be treated independently. Our approach consists of solving each of the resulting problems in a sequential way, i.e by considering two sub-problems. The first sub-problem (LRP1) is related to the procurement of RMs from the DC to the plant. The optimal quantities ($X_{m,t}$) thus obtained are considered as "the demands" for the second sub-problem (LRP2) which relates to the procurement of RMs from the suppliers to the DC. The two sub-problems are as follows:

Sub-problem (LRP1): $(LRP1) = \min \sum_{m=1}^{M} \sum_{t=1}^{T} \big[(iu_{m,t}^{P} + \alpha_t) \cdot I_{m,t}^{P} + \gamma_t \cdot X_{m,t} \big] - \sum_{t=1}^{T} \big[\alpha_t \cdot IC_t^P + \gamma_t \cdot V^P \big]$ Subject to constraints (2) and (10).

Sub-problem (LRP2): $(LRP2) = \min \sum_{m=1}^{M} \sum_{t=1}^{T} \big[pf_{m,t} \cdot q_{m,t} + (iu_{m,t}^{DC} + \beta_t) \cdot I_{m,t}^{DC} + tf_{m,t}^{DC} \cdot z_{m,t}^{DC} \big] - \sum_{t=1}^{T} \beta_t \cdot IC_t^{DC}$ Subject to constraints (1), (3), (4), (8), (9).

In order to solve (LRP1), we use the dynamic programming algorithm developed by Shaw and Wagelmans (1998). (LRP2) is solved using the dynamic programming formula presented in Akbalik et al. (2008).

In Akbalik et al. (2008), these two solution approaches to solve the disaggregated model were proved to perform better than CPLEX for many instances.

The objective function of (LRP) is the sum of the objective functions of (LRP1) and (LRP2). The value that we get for the objective function of (LRP) is not necessarily a lower bound of the objective function of the initial problem (P). However, the corresponding solution of (LRP) yields through the Lagrangean heuristic (LH) described below a good feasible solution of (P), and for the determination of the Lagrange multipliers, a subgradient optimization method is used based on that value in place of the usual optimum of the current Lagrangean problem. By generating a solution starting from the data of the specific problem we are facing, we obtain an initial upper bound for (P) that is updated after each iteration of the Lagrangean heuristic solution procedure.

The Lagrangean heuristic solution procedure

Step 1: Satisfaction of the storage constraint at the plant (5).

The aim is to check if at each period t, constraint (5) is satisfied. If it is not true, then we must reduce some values of $I^P_{m,t}$. Note that the expression of inventory balance (2) in period $t+1$ is:

$$I^P_{m,t} = I^P_{m,t+1} - X_{m,t+1} + d_{m,t+1}; \quad \forall t, \quad \forall m$$

If constraint (5) is violated at period t then compute the extra-quantity to store $E^P_t = \left[\sum_{m=1}^{M} I^P_{m,t} - IC^P_t\right]$. Choose among the M RMs one that has the highest inventory cost (say RM k). Set $e = \min(E^P_t, I^P_{k,t}, X_{k,t})$. Substract e from $I^P_{k,t}$ and in order to satisfy the inventory balance (2), substract e from $X_{k,t}$, that is:

$$I^P_{k,t} := I^P_{k,t} - e$$
$$X_{k,t} := X_{k,t} - e$$

At period $t+1$, add e to the amount of RM k to be supplied $X_{k,t+1}$, that is:

$$X_{k,t+1} := X_{k,t+1} + e$$

Finally, substract e from the extra-quantity to store E^P_t, that is:

$$E^P_t := E^P_t - e$$

Continue the same process until the extra-quantity to store becomes equal to 0. Update the inventories $I^{DC}_{m,t}$ accordingly.

Step 2: Satisfaction of the storage constraint at the DC (6).

We start by updating the inventories $I^{DC}_{m,t}$ because in step 1, some of the values $X_{m,t}$ might have changed.

Set $e'_{m,t}$ the reductions to be carried out on the inventory of RM m at period t. Inventory balance constraints in periods t and $t+1$ at the DC are:

$$\left[I^{DC}_{m,t} - e'_{m,t}\right] = I^{DC}_{m,t-1} + \left[Y_{m,t} - e'_{m,t}\right] - X_{m,t};$$
$$\left[I^{DC}_{m,t+1} - e'_{m,t+1}\right] = \left[I^{DC}_{m,t} - e'_{m,t}\right] + \left[Y_{m,t+1} + e'_{m,t} - e'_{m,t+1}\right] - X_{m,t+1};$$

The following linear program (LP) allows us to determine such reductions (we set $E^{DC}_t = \sum_{m=1}^{M} I^{DC}_{m,t} - IC^{DC}_t$).

$$\min \sum_{m=1}^{M}\sum_{t=1}^{T}\left[\left(e'_{m,t-1} - e'_{m,t}\right)\cdot\frac{tf^{DC}_{m,t}}{V^{DC}_m} - e'_{m,t}.iu^{DC}_{m,t}\right]$$

$s.t.$

$$0 \le e'_{m,t} \le I^{DC}_{m,t}; \quad \forall t, \quad \forall m \tag{11}$$
$$0 \le Y_{m,t} + e'_{m,t-1} - e'_{m,t} \le P_{m,t}; \quad \forall t, \quad \forall m \tag{12}$$
$$\sum_{m=1}^{M} e'_{m,t} = E^{DC}_t; \quad \forall t, \left|\sum_{m=1}^{M} I^{DC}_{m,t} \ge IC^{DC}_t\right. \tag{13}$$
$$\sum_{t=1}^{T}\sum_{m=1}^{M} e'_{m,t} \le \sum_{t,|\sum_{m=1}^{M} I^{DC}_{m,t} \ge IC^{DC}_t} E^{DC}_t; \tag{14}$$
$$e'_{m,t} \ge 0 \quad \text{and integer} \tag{15}$$

The objective is to minimize the impact of inventory reductions at the DC on the transportation cost. Constraint (11) ensures the non-negativity of the inventory levels at the DC. Constraint (12) imposes the respect of suppliers capacities. According to constraint (13), after reduction of the inventory levels, a violated inventory capacity constraint at the DC becomes satisfied with equality. Constraint (14) says that the sum of all reductions is less than or equal to the sum of over-stocked quantities at the DC in the planning horizon. Having obtained the reductions, we update the values of decision variables ($I^{DC}_{m,t}$, $Y_{m,t}$, $z^{DC}_{m,t}$).

Step 3: Satisfaction of the transportation constraint between the DC and the plant (7).

If constraint (7) is violated at period t then compute the gap $E^{Tr}_t = \left[\sum_{m=1}^{M} X_{m,t} - V^P\right]$ and increase the volume of transportation between the DC and the plant. Recall that this volume V^P was initially assumed to be constant with a fixed transportation cost tf^P. The extra transportation volume corresponding to E^{Tr}_t is $\lceil \frac{E^{Tr}_t}{V^P} \rceil$ and the extra-transportation cost is $tf^P \times \lceil \frac{E^{Tr}_t}{V^P} \rceil$.

Computational experiments

The objective of the computational experiments is to compare the performances of this Lagrangean heuristic with an exact method (CPLEX 10.1 applied to the MIP with a gap parameter fixed to 0.0% to the optimal solution) on a Pentium Dual Core E4600 (2.40 Ghz, 0.99 Gb RAM) with respect to total cost and CPU time. The MIP and the Lagrangean heuristic were programmed in Java+Concert Technology.

Suppliers capacities are uniformly distributed in the interval [10, 20] and [100, 200] (notice that they may depend on the period). Tests have been conducted with demands varying with respect to the supplier capacity and inventory costs for RM m varying with respect to the fixed transportation cost between supplier m and the DC. For each test value of the parameters (M, T and the distribution of production capacities), ten different instances were generated. Each time, two gaps are computed, one is $(LH - MIP)/MIP$ in order to compare the cost obtained through the heuristic with the optimum cost, the other is $(LH - LB)/LB$ in order to evaluate the gap between the lower bound and the upper bound of the cost given by the heuristic. The average CPU time and the average gaps of these instances are reported in the tables.

The notations used are: t_{LH} average heuristic CPU time, t_{MIP} average MIP CPU time where the presolving time is included, UB "upper bound" for the Lagrangean heuristic, LB "lower bound" provided by (LRP); u represents the interval in which $P_{m,t}/d_{m,t}$ vary; u' and u'' are defined similarly.

For the subgradient optimization method, the stop condition is defined by a maximum number of iterations equals to 50.

In the tests, the unit inventory cost in the DC is lower than the inventory cost in the plan. The storage capacities both in the DC and the plant as well as the volume of vehicles between the DC and the plant have been generated in function of the demand distribution and the supplier capacity.

The results of the computational experiments show that, in the case of small suppliers capacities, the heuristic performs better than CPLEX as the demands get bigger compared with supplier capacities, and unit inventory costs (in the DC and the plant) get bigger compared with transportation fixed cost between suppliers and the DC (Tables 1, 2).

In Table 1, we observe that both gaps decrease when the number of RMs and the number of periods increase. At the same time, CPLEX running time increases compared to the heuristic running time. These gaps become very small in Table 2.

In the case of large suppliers capacities, the heuristic seems to behave badly compared with CPLEX. The instances reported in Table 3 have been selected so that they yield small inventory levels (this is why the gaps are equal to zero). However in some large instances ((M=15, R = 24) and (M = 20, R = 18)), CPLEX is much more time consuming compared with the Lagrangean heuristic.

In general, we noticed that when (LRP) solution violates many inventory capacity constraints because of low unit inventory costs or low demands, then CPLEX is better than the Lagrangean heuristic and that in the opposite case, the Lagrangean heuristic is better than CPLEX.

Concluding remarks and on-going work

In this paper, a model of an inbound supply chain planning problem issued from an industrial case is presented. It contains three different families of coupling constraints. One

Table 1 Comparison of MIP and LH for small capacities: part I

$d_{m,t}$ $u.P_{m,t}$	$I^{DC}_{m,t}$ $u'.tf^{DC}_{m,t}$	$I^{P}_{m,t}$ $u''.tf^{DC}_{m,t}$	M	T	t_{LH} (s)	t_{MIP} (s)	$\frac{LH-MIP}{MIP}$ (%)	$\frac{LH-LB}{LB}$ (%)
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	5	6	0.47	0.067	6.71	9.04
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	5	12	1.09	0.38	2.64	3.91
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	5	18	1.94	12.73	2.20	3.12
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	10	6	0.78	0.32	5.28	6.69
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	10	12	1.93	10 min 1 s	2.64	3.27
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	15	6	1.38	2.11	5.55	6.78
[0.3, 0.5]	[0.2, 0.4]	[0.6, 0.8]	20	6	4.41	4.34	5.12	6.14
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	5	6	0.37	0.05	2.99	4.20
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	5	12	0.87	0.25	1.85	2.70
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	5	18	1.45	4.76	1.98	2.46
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	5	24	11.24	7 min 17 s	0.69	1.02
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	10	6	0.63	0.17	3.45	4.31
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	10	12	1.33	13 min 31 s	1.91	2.34
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	15	6	1.05	0.56	2.84	3.21
[0.6, 0.8]	[0.2, 0.4]	[0.6, 0.8]	20	6	4.96	1.54	3.08	3.40

Table 2 Comparison of MIP and LH for small capacities: part II

$d_{m,t}$ $u.P_{m,t}$	$I^{DC}_{m,t}$ $u'.tf^{DC}_{m,t}$	$I^{P}_{m,t}$ $u''.tf^{DC}_{m,t}$	M	T	t_{LH} (s)	t_{MIP} (s)	$\frac{LH-MIP}{MIP}$ (%)	$\frac{LH-LB}{LB}$ (%)
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	5	6	0.14	0.09	0.34	0.45
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	5	12	0.33	0.051	0.2	0.20
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	5	18	0.78	0.12	0.27	0.35
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	5	24	0.93	0.19	0.08	0.12
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	10	6	0.22	0.054	0.45	0.51
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	10	12	0.41	0.26	0.19	0.20
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	10	18	1.21	3.85	0.15	0.19
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	10	24	1.94	31.77	0.11	0.13
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	15	6	0.13	0.068	0.08	0.08
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	12	1.08	1.04	0.23	0.25
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	18	1.75	29.15	0.12	0.13
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	24	2.69	11 min 41 s	0.079	0.082
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	20	6	0.39	0.16	0.27	0.30
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	20	12	0.074	4.01	0	0
[0.3, 0.5]	[0.6, 0.7]	[0.8, 0.9]	20	18	1.91	7 min 37 s	0.04	0.06
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	5	6	0.086	0.06	0.21	0.29
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	5	12	0.01	0.042	0.06	0.06
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	5	18	0.015	0.071	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	5	24	0.24	0.12	0.006	0.012
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	10	6	0.057	0.037	0.036	0.060
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	10	12	0.16	0.12	0.006	0.012
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	10	18	0.30	0.43	0.020	0.020
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	10	24	0.065	1.86	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	6	0.003	0.067	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	12	0.31	0.29	0.04	0.04
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	18	0.073	9.02	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	15	24	0.15	2 min 30 s	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	20	6	0.012	0.062	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	20	12	0.054	1.2	0	0
[0.6, 0.8]	[0.6, 0.7]	[0.8, 0.9]	20	18	0.13	3 min 33 s	0	0

Table 3 Comparison of MIP and LH for large capacities

$d_{m,t}$ $u.P_{m,t}$	$I^{DC}_{m,t}$ $u'.tf^{DC}_{m,t}$	$I^{P}_{m,t}$ $u''.tf^{DC}_{m,t}$	M	T	t_{LH} (s)	t_{MIP} (s)	$\frac{LH-MIP}{MIP}$ (%)	$\frac{LH-LB}{LB}$ (%)
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	5	6	0.17	0.04	0	0
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	5	12	7.94	0.042	0.144	0.146
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	5	18	10.02	0.079	0.078	0.09
[0.6, 0.8]	[0.05, 0.1]	[0.1, 0.3]	5	24	3.07	0.16	0	0
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	10	6	0.37	0.04	0	0
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	10	12	1.71	0.11	0	0
[0.6, 0.8]	[0.05, 0.1]	[0.1, 0.3]	10	18	23.58	0.9	0.014	0.018
[0.6, 0.8]	[0.05, 0.1]	[0.1, 0.3]	10	24	7.86	2.62	0	0
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	15	6	0.94	0.043	0	0
[0.6, 0.8]	[0.05, 0.1]	[0.1, 0.3]	15	12	4.37	0.79	0	0

Table 3 continued

$d_{m,t}$ $u.P_{m,t}$	$I_{m,t}^{DC}$ $u^{'}.tf_{m,t}^{DC}$	$I_{m,t}^{P}$ $u^{''}.tf_{m,t}^{DC}$	M	T	t_{LH} (s)	t_{MIP} (s)	$\frac{LH-MIP}{MIP}$ (%)	$\frac{LH-LB}{LB}$ (%)
[0.6, 0.8]	[0.05, 0.1]	[0.1, 0.3]	15	18	10.24	2.42	0	0
[0.6, 0.8]	[0.05, 0.1]	[0.1, 0.3]	15	24	18.52	4 min 35 s	0	0
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	20	6	8.94	0.048	0.067	0.068
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	20	12	7.12	1.06	0	0
[0.3, 0.5]	[0.05, 0.1]	[0.1, 0.3]	20	18	16.48	1 min 30 s	0	0

family is related to the limitation of the transport capacity between the DC and the plant, and the two others are related to storage capacity of RMs in the DC and in the plant. Using Lagrangean relaxation, these three different families are dualized into the objective function. The Lagrangean problem thus obtained is separated into two easier sub-problems which are solved by dynamic programming. Finally, an efficient heuristic solution procedure that uses the solution generated from Lagrangean relaxation is presented and tested computationaly.

Some improvements or extensions can be made. First of all, by adding valid inequalities to the MIP, the computation time could be reduced using CPLEX and therefore larger instances could be solved by this way. A comparison between MIP and LH on larger instances could then be interesting. Concerning the model, it could be more realistic to consider explicitely the transportation between the DC and the plant. We would then have to take into account the number of trucks and their volumes between the DC and the plant. This extension of our model would increase the difficuly of the problem, but the Lagrangean heuristic could be a good way to solve the problem.

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