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Intelligent design of a dynamic machine layout in uncertain environment of flexible manufacturing systems

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Abstract Since Facility Layout Problem (FLP) affects the total manufacturing cost significantly, it can be considered as a critical issue in the early stages of designing Flexible Manufacturing Systems (FMSs), particularly in volatile environments where uncertainty in product demands is inevitable. This paper proposes a new mathematical model by using the Quadratic Assignment Problem formulation for designing an optimal machine layout for each period of a dynamic machine layout problem in FMSs. The product demands are considered as independent normally distributed random variables with known Probability Density Function (PDF), which changes from period to period at random. In this model, the decision maker's defined *confidence level* is also considered. The confidence level represents the decision maker's attitude about uncertainty in product demands in such a way that it affects the results of the problem significantly. To validate the proposed model, two different size test problems are generated at random. Since the FLP, especially in multi-period case is a hard Combinatorial Optimization Problem (COP), Simulated Annealing (SA) meta-heuristic resolution approach programmed in Matlab is used to solve the mathematical model in a reasonable computational time. Finally, the computational results are evaluated statistically.

Keywords Dynamic facility layout problem · Uncertain environment · Flexible manufacturing system · Simulated annealing

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Introduction

A Flexible Manufacturing System (FMS) consists of at least four automated and multifunctional machine centres such as Computer Numerical Control (CNC) machine tools, which are linked together mechanically by an automated material handling system and electronically by a distributed computer control system. The problems in the FMS can be categorized into designing, programming, scheduling and controlling. One of the most important steps in the design of the FMS is the arrangement of facilities (machines) called FLP. The Material Handling Cost (MHC) is one of the most appropriate measures to evaluate the efficiency of a facility layout so that an efficient layout has the minimum MHC. According to Tompkins et al. (2003), the MHC forms 20–50% of the total manufacturing costs and it can be decreased at least 10–30% by an efficient layout design. The MHC is calculated as the product of the flow of materials between facilities and travel distance between locations. Considering the known facility locations leads to the known and fixed travel distance. In this case, the MHC can be regarded as a function of the flow of materials. According to the nature of the flow of materials, the FLP can be *static* or *dynamic*.

In the Static Facility Layout Problem (SFLP), the flow of materials is deterministic and constant over the entire time planning horizon. In this problem, the optimum relative location of each facility is determined so that the total MHC is minimized. It is very difficult to forecast the product demands in a long period of time. Therefore, in the SFLP, the single time planning horizon is divided into several time periods so that each period has different and fixed product demand requirements. By doing so, the SFLP is become a multi-period layout problem named Dynamic Facility Layout Problem (DFLP). Actually, in the DFLP, the demand for products is deterministic and constant for each period, but it changes from period to period. Changes in the product demands lead to changes in the flow of materials. Increase in the flow of materials in the transition from the current period to the next period increases the MHC, which in turn leads to an inefficient layout. Therefore, it is necessary to rearrange the facilities in the next period to obtain the optimal layout. The rearrangement of facilities is a costly process. This cost is named the rearrangement cost. The objective of the DFLP is to design an optimum layout for each period so that the total material handling and rearrangement costs are minimized. The rearrangement cost consists of the rewiring cost of both material-handling devices and workstations, labour cost, dismantling and reconstruction costs, setup costs, and the cost due to some loss in production capacity throughout the rearrangement process (Afentakis et al. 1990; Benjaafar et al. 2002). According to Kochhar and Heragu (1999), the rearrangement cost can be classified into fixed and variable costs. The fixed cost is because of some loss in production capacity throughout the rearrangement process. This cost is also independent of present and future layouts. On the other hand, the variable cost is due to moving the facilities from current locations to the next locations. The variable cost depends on the travelling distance of facilities and also the dimensions of facilities. However, in this paper, the fixed relocating cost of facilities is considered.

The product demands are usually obtained by using inaccurate techniques such as forecasting methods or historical trends. Therefore, it would be more realistic if we consider the product demands as random variables. Considering uncertainty in the product demands in both of the aforementioned static (single period) and dynamic (multi-period) FLPs leads to two stochastic FLPs called Stochastic Static Facility Layout Problem (SSFLP) and Stochastic Dynamic Facility Layout Problem (SDFLP) respectively. Similar to the SFLP, in the SSFLP an optimal facility layout is designed in a single time planning horizon by minimizing the MHC. Like the DFLP, in the SDFLP, the objective is to find an optimum layout for each period so that the total material handling and rearrangement costs are minimized. However, this paper proposes a new mathematical model to design an optimal machine layout for each period of the SDFLP in FMSs.

Literature review

The research on FLP has been formally started since the early 1950s. Webster and Tyberghein (1980) regarded a layout with the lowest MHC over several product demand scenarios as the most flexible layout. Gupta (1986) used the Monte Carlo simulation approach to generate the materials flow between all pairs of facilities in the FLP with facilities, which are square in shape and equal in size. They also considered the independent product demands as random variables with known normal distribution functions. Kouvelis and Kiran (1991) solved the SSFLP and the SDFLP by using the QAP and Dynamic Programming (DP) respectively. They also considered changes in product mix, part routings, and process plans. Palekar et al. (1992) designed the SDFLP using quadratic integer programming model. They considered three degrees of uncertainties named *optimistic*, *most likely*, and *pessimistic* for product demands by assigning a probability of happening to these degrees. Rosenblatt and Kropp (1992) designed the SSFLP with multiple demand scenarios assigned to randomly generated probability of happening. Montreuil and Laforge (1992) addressed the SDFLP by a scenario tree of probable futures. Virtual layout (McLean et al. 1982; Drolet 1989), hybrid layout (Irani et al. 1993), and fractal layout (Venkatadri et al. 1997; Montreuil et al. 1999) were developed as dynamic layouts to cope with uncertainty in product demands. Yang and Peters (1998) suggested a design method named Expected Flow Density (EFD) with regard to multiple demand scenarios for each period of an unequal area SDFLP. Enea et al. (2005) proposed a fuzzy model for the stochastic FLP to design a robust layout that has an effectual performance over the possible demand scenarios. Tavakkoli-Moghaddam et al. (2007) proposed a new mathematical model to concurrent design of the optimal machine and cell layouts in a single time planning horizon of a cellular manufacturing system by considering the stochastic product demands with known normal PDF. Krishnan et al. (2008) proposed three mathematical models for designing both of the SSFLP and SDFLP by considering multiple product demand scenarios. Kulturel-Konak (2007) reviewed different approaches in the DFLP, SSFLP, and SDFLP. The above-mentioned review of the literature on the stochastic FLP is summarized in Table 1.

Quadratic Assignment Problem (QAP)

The QAP is a nonlinear COP. In general, the FLP is usually formulated as the QAP by considering the following conditions: (i) The facilities are equal in size, (ii) A number of facilities are assigned to the same number of locations, (iii) Discrete representation is considered. In discrete representation the shop floor is divided into a number of squares, which are equal in size. The squares are considered as the locations of the facilities. The first mathematical model of the QAP for the FLP was introduced by Koopmans and Beckman (1957) as follows:

$$\text{Minimize} \sum_{l=1}^{M} \sum_{q=1}^{M} f_{\pi(l)\pi(q)} d_{lq} \ \text{ over all permutations} \pi \in S_M$$

where, S_M is the set of all permutations of the set of positive integer numbers $N = \{1, 2, \ldots, M\}$, M is the number of facilities (or locations), $\pi(l)$ denotes the facility assigned to the location l, $f_{\pi(l)\pi(q)}$ represents the flow of materials between

Table 1 Stochastic FLP review

Authors (year)	SSFLP	SDFLP	Approach
Webster and Tyberghein (1980)	√		Minimizing MHC
Gupta (1986)	√		Monte Carlo simulation
Kouvelis and Kiran (1991)	√	√	QAP/DP
Palekar et al. (1992)		√	QAP
Rosenblatt and Kropp (1992)	√		QAP
Montreuil and Laforge (1992)		√	Scenario tree
McLean et al. (1982); Drolet (1989)	√		Virtual layout
Irani et al. (1993)	√		Hybrid layout
Venkatadri et al. (1997), Montreuil et al. (1999)	√		Fractal layout
Yang and Peters (1998)		√	EFD
Enea et al. (2005)	√		Fuzzy model
Tavakkoli-Moghaddam et al. (2007)	√		Bi-QAP
Krishnan et al. (2008)	√	√	Mathematical models
Kulturel-Konak (2007)	√	√	Survey

the facilities $\pi(l)$ and $\pi(q)$, and d_{lq} is the distance between the location l and the location q. The optimal solution of the problem is the permutation $\pi^* \in S_M$ so that the objective function is minimized.

The more suitable form of the QAP formulation for modelling of the SFLP is the following 0–1 integer programming:

$$\text{Minimize} \sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M} f_{ij} d_{lq} x_{il} x_{jq} \tag{1}$$

Subject to:

$$\sum_{i=1}^{M} x_{il} = 1; \quad \forall l \tag{2}$$

$$\sum_{l=1}^{M} x_{il} = 1; \quad \forall i \tag{3}$$

$$x_{il} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to location } l \\ 0 & \text{otherwise} \end{cases}$$

The QAP mathematical model includes three components, *decision variables*, *objective function*, and *constraints*. The decision variables x_{il} are the solution of the problem so that they determine the location of each facility. The objective function Eq. (1) is a second-degree (quadratic) function of the decision variables. In this equation, f_{ij} denotes the flow of materials between facilities i and j. The distance between locations l and q is denoted by d_{lq}. In fact, the objective function represents the total MHC, which is calculated as the summation of the product of materials flow between facilities and distance between the locations of these facilities. The constraints Eqs. (2) and (3) ensure that each facility must be assigned to exactly one location and each location must have only one facility.

According to Balakrishnan et al. (1992) the QAP mathematical model for the DFLP can be written as follows:

$$\text{Minimize} \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M} f_{tij} d_{lq} x_{til} x_{tjq} + \sum_{t=2}^{T}\sum_{i=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M} a_{tilq} y_{tilq} \tag{4}$$

Subject to:

$$\sum_{i=1}^{M} x_{til} = 1; \quad \forall t,\ l \tag{5}$$

$$\sum_{l=1}^{M} x_{til} = 1; \quad \forall t,\ i \tag{6}$$

$$x_{til} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to location } l \text{ in period } t \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$y_{tilq} = x_{(t-1)il} \times x_{tiq} \tag{8}$$

where,

- i, j — index for machines ($i, j = 1, 2, \ldots, M$); $i \neq j$
- l, q — index for machine locations ($l, q = 1, 2, \ldots, M$); $l \neq q$
- t — index for period ($t = 1, 2, \ldots, T$)
- f_{tij} — total material flow between machines i and j in period t
- a_{tilq} — fixed cost of shifting machine i from location l to location q in period t
- d_{lq} — distance between machine locations l and q
- x_{til} — decision variable
- T — number of periods under consideration
- M — number of machines/machine locations

The objective function Eq. (4) is the sum of two terms. The first term is the total MHC and the second term is the total rearrangement cost of the layout during the planning horizon. Equation (5) ensures that each location in each period contains only one facility. Equation (6) ensures that each facility in each period is located in only one location. Equation (8) indicates that $y_{tilq} = 1$ if the facility i is shifted from location l in period $t-1$(i.e. $x_{(t-1)il} = 1$) to location q in period t(i.e. $x_{tiq} = 1$).

Simulated Annealing (SA)

It can be proved that to solve the DFLP with M facilities and T periods a very large number of possible solutions (i.e. $(M!)^T$) must be checked. For example, for the DFLP with six facilities and five periods, a very large number of possible solutions (1.93×10^{14}) must be evaluated. The QAP is an NP-complete (Nondeterministic Polynomial) problem (Sahni and Gonzalez 1976). The computational time, which is needed for solving the QAP, is exponentially proportional to the size of the problem (Foulds 1983). Therefore, it is very difficult to be solved by the exact solution approaches. SA is one of the promising tools for solving the COPs such as the FLP (Alvarenga et al. 2000). In this paper, due to the above-mentioned reasons and the complexity of the proposed model, the SA meta-heuristic is used to solve the problem. The SA algorithm is a simulation of physical annealing process of solids in statistical mechanics. It has been used as a good method to solve hard COPs in a reasonable computational time since the early 1980s. In the physical annealing process in thermodynamics, the perfect structure of crystals can be obtained by melting a solid and then reducing the temperature very slowly so that the crystal can reach this minimum energy level named ground state. Consider the COP (S, f), where S is the solution space, including all of the possible solutions and f is the objective (cost) function. The SA algorithm finds the best solution $s = s^* \in S$ so that $f(s^*)$ is the minimum value of the objective function f. A general SA algorithm is illustrated in Fig. 1.

```
procedure Simulated Annealing
   A known or randomly generated
   initial solution  s^0, s = s^0, initial value
   for temperature T_0 , el = 0
   while outer-loop criterion not satisfied
         (i.e. el ≤ el_max ) % el denotes the
         outer loop counter
      while inner-loop criterion not satisfied
            (i.e. il ≤ il_max ) % il denotes the
            inner loop counter
         s′ = A randomly generated
              neighbouring solution(s)
         if f(s) ≤ f(s′)
            s = s′
         end
         if f(s) > f(s′)
            if randomly generated
 x ∈ (0,1) ≤ exp( (f(s) − f(s′)) / T_el )
               s = s′
            end
         end
       il = il + 1
      end
    T_el = T_0 α^el     % Update Temp T_el ,
          % α ∈ (0.80, 0.99) , cooling ratio
    el = el + 1
  end  Simulated Annealing
```

Fig. 1 A general simulated annealing algorithm

Proposed model

In this section, the new mathematical model is formulated with the following assumptions:

1. According to the definition of the FMS, at least four equal-sized facilities (machines) are considered.
2. The SDFLP is considered.
3. The product demands whose parts are made in the FMS are independent normally distributed random variables with known expected value and variance, changing from period to period at random.
4. The parts are flowed in batches between machines.
5. Both of the material handling and machine rearrangement costs are known.
6. Machines are laid out in a u-shaped configuration as shown in Fig. 2. In this figure, $L1, \ldots, L12$ are the known machine locations.
7. There isn't any constraint for dimensions and shapes of the shop floor.
8. The candidate machine locations are known in advance, which in turn leads to the known distance between locations.
9. The same number of machines and locations are considered.

Due to the assumption of using the same number of equal-sized machines and the known machine locations, the QAP model for the DFLP (Eqs. 4–8 in the preceding section) is used to obtain the new mathematical model for the SDFLP. The data on machine sequence, transfer batch size, part

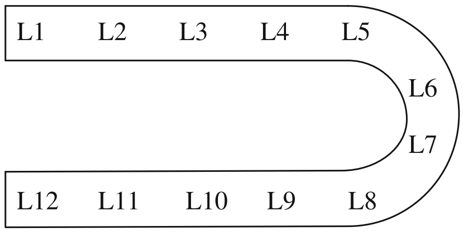

Fig. 2 Layout configuration for each period

movement cost, distance between machine locations, rearrangement costs, and normally distributed product demands with known expected value and variance, are the *inputs* of the model. The *output* of the model is the best layout for each period in the planning horizon. Actually, considering each period as a stage, the multi-period problem can be considered as a multi-stage dynamic system with optimal behaviour from stage to stage. The following indexes and parameters are used in this model besides the ones introduced in the previous section:

k index for parts ($k = 1, 2, \ldots, K$)
N_{ki} operation number for the operation done on part k by machine i
f_{tijk} materials flow for part k between machines i and j in period t
f_{tij} materials flow for all parts between machines i and j in period t
D_{tk} demand for part k in period t
B_k transfer batch size for part k
C_{tk} cost of movements for part k in period t
$C(\pi)$ total MHC for layout π
R_c rearrangement cost
Z_p standard normal Z value for percentile (confidence level) p
$E()$ expected value of a parameter
$Var()$ variance of a parameter
$Pr()$ probability of a parameter

The flow of materials for part k between machines i and j in period t can be calculated as follows:

$$f_{tijk} = \begin{cases} \frac{D_{tk}}{B_k} C_{tk} & \text{if } \left|N_{ki} - N_{kj}\right| = 1 \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where, the condition $|N_{ki} - N_{kj}| = 1$ refers to two consecutive operations, which are done on part k by machines i and j.

As mentioned in the assumptions of the problem, the demand for part k in period $t (D_{tk})$ is a random variable with normal distribution. Therefore, according to Eq. (9), the materials flow for part k between machines i and j in period $t (f_{tijk})$ is also a normally distributed random variable with the following expected value and variance:

$$E\left(f_{tijk}\right) = \begin{cases} \frac{E(D_{tk})}{B_k} C_{tk} & \text{if } \left|N_{ki} - N_{kj}\right| = 1 \\ 0 & \text{otherwise} \end{cases} \tag{10}$$

$$Var\left(f_{tijk}\right) = \begin{cases} \frac{Var(D_{tk})}{B_k^2} C_{tk}^2 & \text{if } \left|N_{ki} - N_{kj}\right| = 1 \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

The total flow between machines i and j in period t resulting from all parts (i.e. f_{tij}) can be written as follows:

$$f_{tij} = \sum_{k=1}^{K} f_{tijk} \tag{12}$$

Since f_{tijk} is a random variable with normal distribution, f_{tij} is also a normally distributed random variable with the following expected value and variance:

$$E\left(f_{tij}\right) = \sum_{k=1}^{K} E\left(f_{tijk}\right) \tag{13}$$

If we insert Eq. (10) into Eq. (13), then

$$E\left(f_{tij}\right) = \sum_{k=1}^{K} \frac{E\left(D_{tk}\right)}{B_k} C_{tk} \tag{14}$$

Since we assumed the part demands as independent variables, the variance of f_{tij} can be calculated as follows:

$$Var\left(f_{tij}\right) = \sum_{k=1}^{K} Var\left(f_{tijk}\right) \tag{15}$$

In a similar way, if we combine Eq. (11) with Eq. (15), then

$$Var\left(f_{tij}\right) = \sum_{k=1}^{K} \frac{Var\left(D_{tk}\right)}{B_k^2} C_{tk}^2 \tag{16}$$

According to Eq. (4), the MHC for the layout (permutation) π, (i.e. $C(\pi)$) and the rearrangement cost (i.e. R_c) are defined as follows:

$$C(\pi) = \sum_{t=1}^{T} \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{l=1}^{M} \sum_{q=1}^{M} f_{tij} d_{lq} x_{til} x_{tjq} \tag{17}$$

$$R_c = \sum_{t=2}^{T} \sum_{i=1}^{M} \sum_{l=1}^{M} \sum_{q=1}^{M} a_{tilq} y_{tilq} \tag{18}$$

If we insert Eq. (8) in Eq. (18), the rearrangement cost R_c can be rewritten as Eq. (19):

$$R_c = \sum_{t=2}^{T} \sum_{i=1}^{M} \sum_{l=1}^{M} \sum_{q=1}^{M} a_{tilq} x_{(t-1)il} x_{tiq} \tag{19}$$

The total cost T_c is the summation of the MHC $(C(\pi))$ and the rearrangement cost R_c as given in Eq. (20):

$$T_c = C(\pi) + R_c \tag{20}$$

Since f_{tij} is a random variable with normal distribution, then according to Eq. (17), $C(\pi)$ is also a normally distributed random variable with the following expected value and variance :

$$E\left(C(\pi)\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} E\left(f_{tij}\right)\sum_{l=1}^{M}\sum_{q=1}^{M} d_{lq}x_{til}x_{tjq} \quad (21)$$

Inserting the Eq. (14) into the Eq. (21) leads to the following equation:

$$E\left(C(\pi)\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K} \frac{E\left(D_{tk}\right)}{B_k} C_{tk}\sum_{l=1}^{M}\sum_{q=1}^{M} d_{lq}x_{til}x_{tjq} \quad (22)$$

Similarly, the variance can be calculated as follows:

$$Var\left(C(\pi)\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M} Var\left(f_{tij}\right)\left(\sum_{l=1}^{M}\sum_{q=1}^{M} d_{lq}x_{til}x_{tjq}\right)^2 \quad (23)$$

Inserting the Eq. (16) into the Eq. (23) results in the following equation:

$$Var\left(C(\pi)\right) = \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K} \frac{Var\left(D_{tk}\right)}{B_k^2} C_{tk}^2\left(\sum_{l=1}^{M}\sum_{q=1}^{M} d_{lq}x_{til}x_{tjq}\right)^2 \quad (24)$$

If the decision maker considers $U(\pi, p)$ as the maximum value (upper bound) of $C(\pi)$ with the confidence level p, then $U(\pi, p)$ can be minimized instead of minimizing $C(\pi)$. According to this assumption:

$$\Pr\left(C(\pi) \le U(\pi, p)\right) = p \quad (25)$$

This equation can be standardized as follows:

$$\Pr\left(\frac{C(\pi) - E\left(C(\pi)\right)}{\sqrt{Var\left(C(\pi)\right)}} \le \frac{U(\pi, p) - E\left(C(\pi)\right)}{\sqrt{Var\left(C(\pi)\right)}}\right) = p \quad (26)$$

If we assume that $Z = \frac{C(\pi)-E(C(\pi))}{\sqrt{Var(C(\pi))}}$, then the Eq. (26) can be rewritten in the following form:

$$\Pr\left(Z \le \frac{U(\pi, p) - E\left(C(\pi)\right)}{\sqrt{Var\left(C(\pi)\right)}}\right) = p \quad (27)$$

Thus, $Z \sim N(0, 1)$, i.e. Z is a variable with standard normal distribution. We assume that $F(z)$ is the Cumulative Distribution Function (CDF) of the random variable Z. Therefore:

$$F\left(\frac{U(\pi, p) - E\left(C(\pi)\right)}{\sqrt{Var\left(C(\pi)\right)}}\right) = p \quad (28)$$

If we consider F^{-1} as the inverse function for F, then the Eq. (28) can be rewritten as follows:

$$F^{-1}(p) = \frac{U(\pi, p) - E\left(C(\pi)\right)}{\sqrt{Var\left(C(\pi)\right)}} \quad (29)$$

Since Z_p is a standard normal Z value for percentile p, therefore, we can write the following equation:

$$F\left(Z_p\right) = p \quad (30)$$

The Eq. (30) can be rewritten in the following form by using the inverse function for F:

$$F^{-1}(p) = Z_p \quad (31)$$

If we compare Eq. (29) with Eq. (31), then we can have the following equation:

$$\frac{U(\pi, p) - E\left(C(\pi)\right)}{\sqrt{Var\left(C(\pi)\right)}} = Z_p \quad (32)$$

The Eq. (32) can be rearranged as follows:

$$U(\pi, p) = E\left(C(\pi)\right) + Z_p\sqrt{Var\left(C(\pi)\right)} \quad (33)$$

To obtain the optimal layout for each period, the Eq. (20) (objective function) must be minimized. In this equation, instead of minimizing $C(\pi)$, its upper bound i.e. $U(\pi, p)$ can be minimized. Therefore, the objective function Eq. (20) can be rewritten as follows:

$$\text{Minimize } U(\pi, p) + R_c \quad (34)$$

Using Eqs. (19, 22, 24, 33, 34), the mathematical model can be written as follows:

$$\text{Minimize}\left\{\begin{array}{l} \sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K} \frac{E(D_{tk})}{B_k} C_{tk}\sum_{l=1}^{M}\sum_{q=1}^{M} d_{lq}x_{til}x_{tjq} \\ +Z_p\sqrt{\sum_{t=1}^{T}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K} \frac{Var(D_{tk})}{B_k^2} C_{tk}^2\left(\sum_{l=1}^{M}\sum_{q=1}^{M} d_{lq}x_{til}x_{tjq}\right)^2} \\ +\sum_{t=2}^{T}\sum_{i=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M} a_{tilq}x_{(t-1)il}x_{tiq} \end{array}\right\} \quad (35)$$

Subject to:

$$\sum_{i=1}^{M} x_{til} = 1; \quad \forall t,\ l \quad (36)$$

$$\sum_{l=1}^{M} x_{til} = 1; \quad \forall t,\ i \quad (37)$$

$$x_{til} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to location } l \text{ in period } t \\ 0 & \text{otherwise} \end{cases} \quad (38)$$

$$\left|N_{ki} - N_{kj}\right| = 1 \quad (39)$$

Notice that the constraints (35), (36), (37), and (38) are the same as the Eqs. (5), (6), (7), and (8) respectively. They have been written again because of their importance. As mentioned, the Eq. (39) refers to two consecutive operations, which are done on part k by machines i and j.

Simulation results and discussion

To validate the proposed model, two randomly generated test problems are solved by the SA algorithm programmed in Matlab. These problems are different in size. Both of them have the same number of facilituies ($M = 12$), but they are different in the number of time periods ($T = 5$ for problem I and $T = 10$ for problem II). A personal computer with Intel 2.10 GHZ CPU and 3 GB RAM is used to run the SA algorithm. The example of input data for parts, including the data on machine sequence, batch size, and cost of movements are given in Table 2. In this table, for example the machine sequence for part 3 is $1 \rightarrow 12 \rightarrow 8$. It means that the first, second, and third operations on part 3 are done by machines 1, 12, and 8 respectively. In other words, according to the parameter N_{ki}, which denotes the operation number for the operation done on part k by machine i (please see the previous section), in this example N_{31}, $N_{3(12)}$, and N_{38} are equal to 1, 2, and 3 respectively.

The distance between machine locations and shifting cost for machines are given in Tables 3 and 4 respectively. According to the assumption (3) in the preceeding section, the product demands are assumed to be normally distributed random variables with randomly generated means and variances for each time period in the multi-period planning horizon. Each time period is considered as a year. The randomly generated mean and variance of part demands for each period is given in Table 5 (for problems I and II) and Table 6 (for problem II). Actually, in spite of the stochastic nature of the product demands, the proposed mathematical model has deteministic and known parameters. The solution of the problem is given as a matrix where each row represents a period, each column represents a location, and each element represents a machine number. For example, in Table 9 the element 1 placed at intersection of the row 2 and the column 6 indicates that the machine 1 is placed at location 6 in period 2. The initial solutions required by the SA algorithm to solve the problems I and II are given in Tables 7 and 8 respectively.

Table 2 Input data

Parts	Machine sequence	Batch size	Cost of movements for all periods
1	$5 \rightarrow 3 \rightarrow 10 \rightarrow 9 \rightarrow 11$	50	5
2	$11 \rightarrow 10 \rightarrow 3 \rightarrow 9 \rightarrow 5$	50	5
3	$1 \rightarrow 12 \rightarrow 8$	50	5
4	$12 \rightarrow 8 \rightarrow 1$	50	5
5	$8 \rightarrow 1 \rightarrow 12$	50	5
6	$7 \rightarrow 2 \rightarrow 6$	50	5
7	$2 \rightarrow 4 \rightarrow 7 \rightarrow 6$	50	5
8	$6 \rightarrow 7 \rightarrow 4 \rightarrow 2$	50	5
9	$2 \rightarrow 6$	50	5
10	$5 \rightarrow 10 \rightarrow 3$	50	5

Table 3 Distance between machine locations

To	1	2	3	4	5	6	7	8	9	10	11	12
From												
1	0	10	20	30	40	50	70	60	50	40	30	20
2	10	0	10	20	30	40	60	50	40	30	20	30
3	20	10	0	10	20	30	50	40	30	20	30	40
4	30	20	10	0	10	20	40	30	20	30	40	50
5	40	30	20	10	0	10	30	20	30	40	50	60
6	50	40	30	20	10	0	20	30	40	50	60	70
7	70	60	50	40	30	20	0	10	20	30	40	50
8	60	50	40	30	20	30	10	0	10	20	30	40
9	50	40	30	20	30	40	20	10	0	10	20	30
10	40	30	20	30	40	50	30	20	10	0	10	20
11	30	20	30	40	50	60	40	30	20	10	0	10
12	20	30	40	50	60	70	50	40	30	20	10	0

Table 4 Shifting cost for machines

Machine number	1	2	3	4	5	6	7	8	9	10	11	12
Shifting cost	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Table 5 Input data for part demands for periods 1–5

Part	Period 1		Period 2		Period 3		Period 4		Period 5	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
1	6,220	1,073	5,656	1,118	3,764	2,584	6,503	2,629	6,503	1,553
2	2,565	2,824	8,863	2,442	6,636	1,609	9,101	1,344	7,589	2,940
3	7,623	1,893	9,120	2,318	3,543	1,372	4,554	1,668	5,948	1,388
4	2,067	1,573	4,347	2,578	2,646	2,986	9,746	2,262	8,496	1,812
5	8,965	1,283	2,358	2,251	2,720	1,909	7,540	1,898	8,085	1,663
6	8,736	2,892	9,998	1,190	7,804	1,045	3,677	1,417	2,066	2,998
7	6,823	1,373	8,104	2,493	6,861	2,062	4,910	2,499	8,772	1,856
8	6,088	1,030	9,696	2,751	3,116	1,195	9,253	1,052	8,257	2,355
9	6,907	1,641	7,493	2,087	4,458	1,854	5,141	1,384	6,664	2,676
10	4,093	2,316	5,496	1,447	1,606	1,177	7,172	2,648	5,258	1,236

Table 6 Input data for part demands for periods 6–10

Part	Period 6		Period 7		Period 8		Period 9		Period 10	
	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance	Mean	Variance
1	5,468	1,713	6,510	1,464	1,060	2,562	7,409	2,923	2,132	2,190
2	7,614	1,907	2,045	2,356	1,231	2,389	2,802	1,073	4,250	2,045
3	7,543	2,469	8,514	2,561	6,809	1,738	3,857	1,720	1,380	2,752
4	3,220	2,587	1,847	1,439	6,784	2,841	4,097	1,464	1,386	2,403
5	3,502	1,871	7,706	2,098	6,153	1,587	9,746	1,673	5,346	2,680
6	1,784	2,296	3,538	1,882	8,842	2,395	2,321	2,437	4,873	2,881
7	2,627	1,332	2,278	2,642	2,833	2,193	1,311	2,632	6,757	2,135
8	1,487	2,122	6,682	1,391	1,104	2,158	3,767	1,999	6,488	2,999
9	8,362	2,277	9,602	2,681	5,478	1,933	7,343	1,850	6,680	2,676
10	4,417	2,322	2,105	2,501	2,384	1,881	8,405	2,703	4,007	2,499

Table 7 Initial solution for problem I

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	1	2	3	4	5	6	7	8	9	10	11	12
2	1	2	3	4	5	6	7	8	9	10	11	12
3	1	2	3	4	5	6	7	8	9	10	11	12
4	1	2	3	4	5	6	7	8	9	10	11	12
5	1	2	3	4	5	6	7	8	9	10	11	12

Table 8 Initial solution for problem II

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	1	2	3	4	5	6	7	8	9	10	11	12
2	1	2	3	4	5	6	7	8	9	10	11	12
3	1	2	3	4	5	6	7	8	9	10	11	12
4	1	2	3	4	5	6	7	8	9	10	11	12
5	1	2	3	4	5	6	7	8	9	10	11	12
6	1	2	3	4	5	6	7	8	9	10	11	12
7	1	2	3	4	5	6	7	8	9	10	11	12
8	1	2	3	4	5	6	7	8	9	10	11	12
9	1	2	3	4	5	6	7	8	9	10	11	12
10	1	2	3	4	5	6	7	8	9	10	11	12

Table 9 The best solution for problem I with confidence level $P = 0.75$ (10 trials)

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	2	6	5	3	10	9	11	12	1	8	7	4
2	2	6	5	8	12	1	11	10	3	9	7	4
3	1	5	9	3	10	11	6	2	7	4	8	12
4	1	8	12	11	10	3	5	9	2	4	7	6
5	1	8	12	11	10	3	5	9	2	4	7	6
OFV = 1,265,500 (Objective Function Value)				$P = 0.75$ (Confidence level)					Elapsed time = 6.245991 (Second)			

The following results are obtained by running the SA algorithm 10 times: Tables 9, 10, 11 (problem I) and Tables 13, 14 and 15 (problem II) display the results, including the best solution, Objective Function Value (OFV), and elapsed computational time for three different confidence levels. These results show that the larger problem (problem II) needs to more computational time than the smaller one (problem I). The computational time for problems I and II are about 6 and 17 seconds, respectively. The results obtained from statistical evaluation, including the worst, best, and Standard Deviation (Std. Dev.) of the Objective Function Values (OFVs) for three different values of the confidence level (p) for problems I and II are given in Tables 12 and 16 respectively. These results indicate that the objective function is directly proportional to the confidence level (p). These results also show that the OFVs are pretty nearly to each other.

According to the results shown in Tables 9, 10, 11, 13, 14 and 15, different optimal layouts are achieved by different confidence levels. In sensitivity analysis point of view, the optimal layout as the output of the model is changed considerably by little changes in the confidence level as the sensitive input parameter. In addition, the confidence level affects the stability of the output layout. The stability of a layout is definded as "the property of a layout to exhibit little sensitivity to demand variability" Braglia et al. (2005). In other words, a layout with minimum variance of product demands is the most stable layout. According to the Eq. (35), decrease in Z_p decreases the effect of the variance of the product demands, which in turn increases the stability of the output layout. Since the confidence level (p) is directly proportional to Z_p, therefore, the stability of the output layout is increased by decreasing the confidence level.

Conclusion

This paper proposed a new nonlinear mathematical model for designing a dynamic layout in uncertain environment of the FMS where the independent product demands are normally distributed random variables with known PDF, which changes from period to period. Two randomly generated test problems with 10 parts, 12 machines, but with two different time periods ($T = 5$ and $T = 10$) were solved by SA approach with three different confidence levels in a reasonable computational time. Finally, this work can be continued in the future researches as follows:

1. Design of a robust layout as the best layout over the entire multi-period time planning horizon.

Table 10 The best solution for problem I with confidence level $P = 0.85$ (10 trials)

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	11	6	2	7	4	8	1	12	5	3	10	9
2	11	10	3	9	5	1	12	8	4	7	2	6
3	11	10	3	9	5	1	12	8	4	7	2	6
4	11	9	6	7	4	2	1	8	12	5	3	10
5	11	9	10	3	5	12	1	8	2	4	7	6
OFV = 1,274,300 (Objective Function Value)				$P = 0.85$ (Confidence level)					Elapsed time = 6.161677 (Second)			

Table 11 The best solution for problem I with confidence level $P = 0.95$ (10 trials)

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	8	5	3	10	9	11	6	2	7	4	12	1
2	2	6	1	12	8	11	10	3	9	5	7	4
3	1	8	12	5	9	11	10	3	6	2	7	4
4	1	8	12	5	3	10	11	9	4	2	7	6
5	3	10	11	1	8	12	6	7	4	2	9	5
OFV = 1,301,100 (Objective Function Value)				$P = 0.95$ (Confidence level)					Elapsed time = 6.498778 (Second)			

Table 12 Statistical evaluation for problem I (10 trials)

Confidence level (P)	Objective Function Value (OFV)			
	Worst	Mean	Best	Std. Dev.
0.75	1,328,800	1,293,950	1,265,500	26,050.8584
0.85	1,382,300	1,308,990	1,274,300	28,707.0046
0.95	1,362,600	1,325,380	1,301,100	18,900.6055

Table 13 The best solution for problem II with confidence level $P = 0.75$ (10 trials)

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	8	12	1	5	10	3	11	9	6	2	7	4
2	8	12	1	11	10	3	5	9	4	2	7	6
3	6	2	7	4	11	10	3	9	5	8	12	1
4	6	5	9	3	10	11	1	8	12	4	2	7
5	5	9	3	10	11	12	8	1	2	4	7	6
6	4	7	2	6	1	12	8	5	3	10	9	11
7	11	9	10	3	5	4	7	2	6	8	12	1
8	11	9	4	7	2	6	1	8	12	3	10	5
9	11	8	1	12	2	6	7	4	5	3	10	9
10	2	4	10	3	9	5	8	1	12	11	7	6
OFV = 2,155,600 (Objective Function Value)				$P = 0.75$ (Confidence level)					Elapsed time = 17.143283 (Second)			

Table 14 The best solution for problem II with confidence level $P = 0.85$ (10 trials)

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	12	1	8	5	3	10	11	9	4	7	2	6
2	3	10	11	1	12	8	4	2	7	6	9	5
3	11	10	3	9	5	4	7	2	6	1	12	8
4	7	6	11	1	8	12	5	9	3	10	2	4
5	7	6	10	3	9	5	12	1	8	11	2	4
6	6	2	7	4	9	11	10	3	5	1	12	8
7	6	2	7	4	1	12	8	11	9	10	3	5
8	11	9	6	2	7	4	8	12	1	3	10	5
9	7	2	6	12	1	8	11	9	10	3	5	4
10	5	9	3	10	11	8	1	12	4	2	7	6
OFV = 2,173,000 (Objective Function Value)				$P = 0.85$ (Confidence level)					Elapsed time = 16.570847 (Second)			

Table 15 The best solution for problem II with confidence level $P = 0.95$ (10 trials)

Location	1	2	3	4	5	6	7	8	9	10	11	12
Period												
1	4	7	2	6	9	11	10	3	5	8	1	12
2	4	7	2	6	3	10	11	9	5	8	12	1
3	4	7	2	6	1	12	8	11	10	3	9	5
4	12	11	10	3	9	5	2	4	7	6	1	8
5	4	2	5	9	3	10	11	1	8	12	6	7
6	1	5	9	3	10	11	4	7	6	2	8	12
7	12	1	8	11	9	10	3	5	6	2	7	4
8	9	11	8	12	1	4	7	2	6	3	10	5
9	7	2	6	12	1	8	5	3	10	9	11	4
10	11	8	1	12	2	4	7	6	5	3	10	9
OFV = 2,241,200 (Objective Function Value)				$P = 0.95$ (Confidence level)					Elapsed time = 17.912468 (Second)			

Table 16 Statistical evaluation for problem II (10 trials)

Confidence level (P)	Objective Function Value (OFV)			
	Worst	Mean	Best	Std. Dev.
0.75	2,231,800	2,192,690	2,155,600	23,805.4359
0.85	2,236,000	2,205,950	2,173,000	21,140.7584
0.95	2,397,000	2,295,900	2,241,200	47,104.1871

2. Further investigation of the stability of the output layout by considering the confidence level as a fuzzy variable.
3. A new hybirid meta-heuristic approach can be developed by combining the SA with other intelligent approaches to solve this problem.
4. The proposed model in this paper can be used for concurrently design of inter- cell and intra-cell layout design in the FMS.
5. Moreover, the constraints such as unequal-sized machines, closeness ratio, aisles, routing flexibility, budget

constraint for total cost, and dependent product demands can also be considered in the future researches.

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