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Tool path planning for 5-axis flank milling of ruled surfaces considering CNC linear interpolation

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Abstract This paper investigates tool path planning for 5-axis flank milling of ruled surfaces in consideration of CNC linear interpolation. Simulation analyses for machining error show insights into the tool motion that generates a precision machined surface. Contradicting to previous thoughts, the resultant tool path does not necessarily produce minimal machining error when the cutter contacts the rulings of a developable surface. This effect becomes more significant as the distance between two cutter locations is increased. An optimizing approach that adjusts the tool position locally may not produce minimal error as far as the entire surface is concerned. The optimal tool path computed by a global search scheme based on dynamic programming supports this argument. A flank milling experiment and CMM measurement further validate the findings of this work.

Keywords Five-axis machining · Flank milling · Ruled surface · Linear interpolation · Tool path planning · Developability

Introduction

5-axis CNC machining has been commonly used in aerospace, energy, automobile, and mold industries since the late 90's. It offers two additional degrees of freedom in tool motion, which increase not only the complexity of the machined geometry, but also the machining productivity as compared to conventional 3-axis machining. Tool path planning is considered a crucial task in any 5-axis machining operation, and the most time-consuming one in many cases. Poor tool path produces lower surface quality and lengthy machining time. It also induces tool collision, which seriously damage the machine tool, work part, and jig and fixture.

5-axis CNC milling can be categorized into two different operations: end milling and flank milling. In the former, material removal occurs at the cutting edges near the end of a tool. Moving at a title angle, namely Sturz milling, the tool can produce good surface quality with fewer tool paths. In contrast, the peripheral of a cutting tool does the main cutting in flank milling. Larger contact area of the tool leads to a faster material removal rate, but the machining geometry is limited. Flank milling is normally applied to ruled geometries. Serious tool interference occurs when a non-ruled surface is machined with the tool flank, or when excessive twist is present on the surface. To completely eliminate tool interference is difficult except for simple geometries such as cylindrical and conical surfaces. In practice, the machining quality is considered acceptable as long as the amount of tool interference can be maintained within a given tolerance.

Numerous previous literatures (Bedi et al. 2003; Liu 1995; Bohez et al. 1997; Tsay and Her 2001; Tsay et al. 2002; Chu and Chen 2006) have studied tool path planning in 5-axis machining of ruled surfaces. Bedi et al. (2003) analyzed the relationship between the tool position and the resultant machining errors like the amount of undercut and overcut. They place a cylindrical cutter along a boundary curve of ruled surface with the tool axis as the surface ruling at each point along the curve. Liu (1995) proposed a heuristic method that determines the tool axis by offsetting the points at the parameter values 0.25 and 0.75 on

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a ruling with a distance of the tool radius. However, the tool path generated in this manner may not guarantee minimal tool interferences in the machined surface. Bohez et al. (1997) developed a heuristic approach that generates the tool contact points. The tool axis is computed by offsetting the average vector of the surface normal at two end points of a ruling. They claim that the approach can significantly reduce the amount of tool interference. Tsay and Her (2001), Tsay et al. (2002) analyzed the influence of the yaw and tilt angles on the amount of tool interference at a cutter location. The result serves as look-up tables in a tool path planning algorithm. Chu and Chen (2006) computed a series of G1 developable patches approximating a ruled surface along which the tool moves without tool interference in 5-axis flank milling of the surface. This work demonstrates a good potential of the developable surface theory for multi-axis flank machining.

All those studies indicate that twist of a ruled surface along its rulings is the main reason causing tool interference in 5-axis flank milling. As a logical consequence, the cutter does not interfere with the machined surface if it remains tangent to both boundary curves, namely no twist. In other words, interference will not occur at least locally when the contact area of the cutter becomes developable. This statement is true in general; however, to derive path planning schemes for reducing the machining error based on it is tricky. Previous research may have overlooked some premises of the conclusion. Perhaps the most important factor that needs to be considered but often neglected is CNC interpolation of machine tools. The tool positions linearly interpolated from cutter locations may deviate from the ideally continuous path to a degree that influences the application of the developable condition. In addition, any approach that minimizes the machining error by adjusting individual cutter locations ought to analyze the resultant tool path regarding the machined surface as a whole. A combination of locally best tool positions may not guarantee a minimal machining error as far as the entire surface is concerned, especially when the distance between cutter locations is large. Consequently, moving along the surface rulings is not the best strategy for flank milling of a developable surface. This work attempts to elaborate these issues from simulation analyses of the machining error. We also propose a global optimization scheme based on dynamic programming that computes the optimal tool path in consideration of CNC interpolation. Finally, a 5-axis flank milling experiment and CMM measurement validate the above findings.

5-Axis flank milling of ruled surface

A ruled surface ***S*** is constructed with two boundary curves in 3D space as:

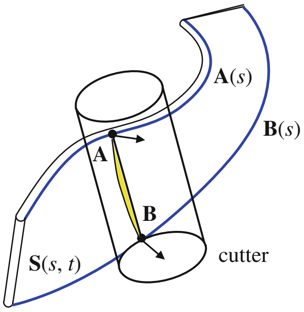


Fig. 1 Cutter contacts a ruled surface along the ruling

$$\mathbf{S}(s,t) = (1-t)\mathbf{A}(s) + t\mathbf{B}(s),\quad 0 \le t \le 1 \quad \text{and} \quad 0 \le s \le 1 \tag{1}$$

where **A**(s) and **B**(s) are the boundary curves. The ruling refers to the line segment **AB** that connects the corresponding points with equal s. A simple way of tool path generation in 5-axis flank milling of a ruled surface is to make the cutter contact with the surface rulings. The resultant tool position is likely to induce tool interference, as shown in Fig. 1. Figure 2a shows the tool engagement from the end view of a ruling. Tool interference occurs around the ruling, as the cutter does not contact tangentially with both boundary curves at the same time. The tangents **A'**(s) and **B'**(s) to the curves at the corresponding end points and the ruling **AB** are not coplanar. As a result, the cutter becomes tangent only to one boundary curve but not both, refers to as "twist". This normally causes overcut in the surface region near the boundary not tangent to the cutter.

The tool interference induced by the twist is eliminated when the cutter remains tangent to both boundary curves, i.e. **A'**(s), **B'**(s), and **AB** lie in the same plane (see Fig. 2b). The surface region near the ruling is developable at this circumstance (Chu and Séquin 2002). A ruled surface becomes a developable surface when the above property holds for all surface rulings. A logical conclusion is that the cutter can move along the rulings of a developable surface without inducing tool interference. This statement is true under the condition that the actual tool motion is continuous. G01 is still the description of CNC tool path most commonly used in industry. In this case, continuous tool motion will be transformed into discrete cutter locations at some point along the CAM/NC pipeline. A CNC controller generates the tool motion between cutter locations by linear interpolation of both the cutter center point and the cutter axis (Lartigue et al. 2003). Figure 3 shows an example for such interpolated tool positions. More sophisticated interpolation schemes like circular and spline interpolation are also available in high-end CNC controllers, but they are not widely used in practice due to higher computational load and insufficient support from tool path planning software.

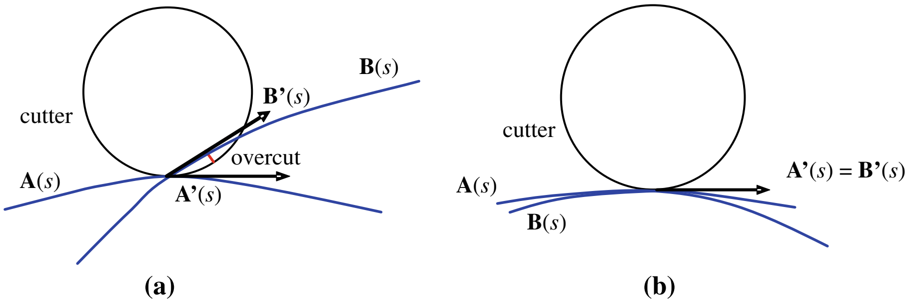


Fig. 2 Tool engagement for **a** non-developable and **b** developable surfaces

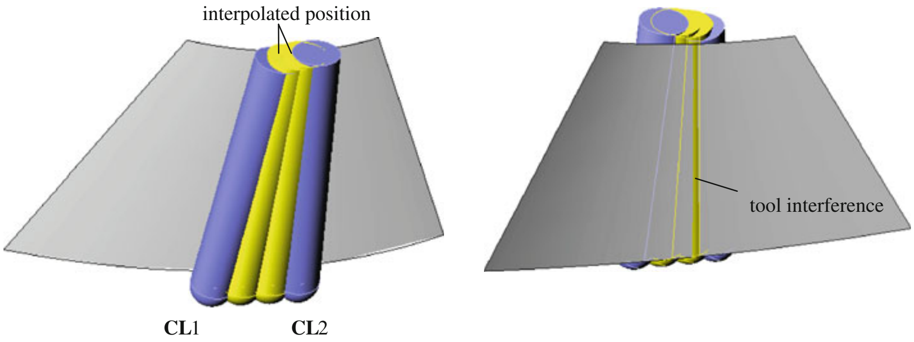


Fig. 3 Tool interference induced by interpolated positions

Previous studies have investigated how to discretize the tool path in 3-axis CNC machining while satisfying the maximal deviations specified by the user. They also discuss its implications on the tool path planning. However, little work has addressed the similar problem for 5-axis milling. In fact, the discrete nature of the final tool motion has profound impacts on multi-axis flank milling. The deviation induced by the interpolation not only influences the machining accuracy, but it also changes the way that precision tool path is generated. Misconception may occur in the path planning when overlooking this factor. The next section will discuss these issues in detail.

Influence of CNC linear interpolation

Machining error estimation

To characterize how the interpolation affects tool path planning in 5-axis flank milling requires effective estimation of the machining error. In theory, one can generate the shape of the machined stock by subtracting the tool swept volume determined by the tool motion from the stock. To obtain the exact shape involves time-consuming computation such as envelop surface construction (Lartigue et al. 2003) and Boolean operations. We propose a simplified method that represents the stock shape approximately to balance between precision and computation load.

Figure 4 shows the schematic of the error estimation method. It transforms the stock shape from volumetric representation into discrete straight lines. Users are allowed to control the line density, i.e. the number of points sampled from the surface. Two line segments which length is equal to the cutter radius are created from each point, one along a user-given direction and the other along its reversed direction. These two groups of lines represent the excess and gouge regions, respectively. For each pair of cutter locations, n intermediate tool positions are generated by linear interpolation of the tool center point and the tool axis at each location. The cutter intersects those line segments while it is moving. The length (or height) of the intersected lines is updated, thus generating the stock shape. The material left in the excess region after machining indicates undercut while the material removed in the gouge region suggests overcut.

Li et al. (2005) explored different error estimation methods for multi-axis flank machining. They classify the methods into three categories: radial, parametric, and closet-point approaches, and compare the advantages/disadvantages of each category. Our method is similar to the radial approach except that the deviation is estimated along a different direction from their work at sample points. The radial approach computes the error as the projection distance from a sample point onto the cutter. The projection direction varies with different points. We allow the user to specify a direction along which the error is calculated. Such error estimation mimics the metrology operation in practice, e.g. CMM measurement, in which the coordinates of a point in 3D space is measured by moving the probe along a given direction until contacting the point. The previous work (Wu and Chu 2008) has verified the effectiveness of the method in tool path optimization for 5-axis flank milling.

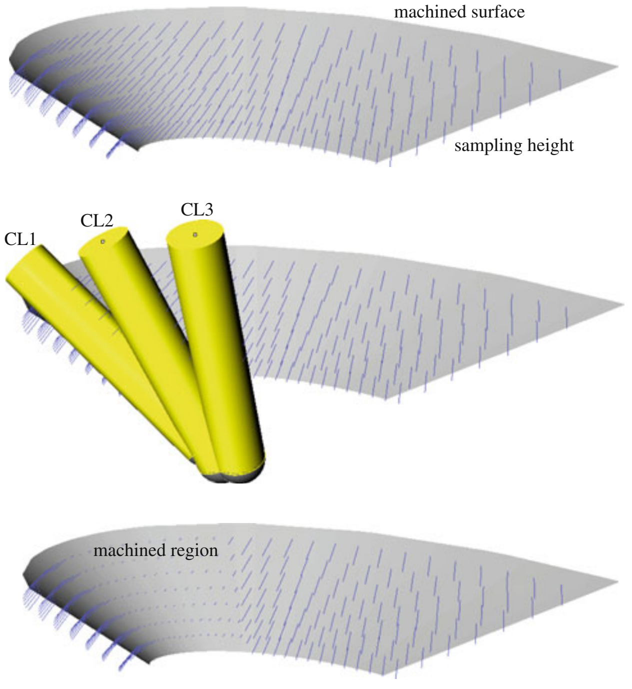
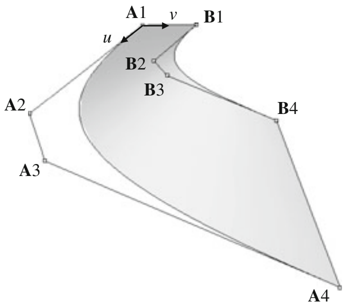


Fig. 4 Schematic of machining error estimation



A1	-15, 0, 5	**B1**	-6, 0, 2
A2	-5, 10, 5	**B2**	-2, 4, 2
A3	5, 12, 3	**B3**	2, 4.8, 1.2
A4	15, 2, 7	**B4**	6, 0.8, 2.8

Fig. 5 Test developable surface and its control points

Influence of linear interpolation

Linear interpolation of continuous tool motion inevitably induces deviation from the ideal values. The amount of the deviation depends on the number of the interpolated positions and their distribution along the motion. Here we use a developable surface as an example to illustrate how the interpolation number affects the machining error. As mentioned above, the cutter does not produce interference in the local region of the surface around the ruling that it contacts. The machining error, however, occurs between two such cutter locations. Figure 5 show the machined surface constructed with two cubic Bézier curves. The cutter makes contact with the surface rulings at discrete positions during its motion. The machining error is calculated based on the method described in the next section.

Table 1 lists three test conditions, in which 50, 100, 200 cutter locations are generated from the surface rulings of the same numbers. The numbers of interpolation between successive cutter locations are 39, 19, and 9, respectively, producing totally 4,000 interpolated positions in all three cases. 200 rows of sampling heights are generated with equal parameter increment in the u direction (see Fig. 5) for approximating the stock material. Each row contains 10 sampling heights along the v direction.

Figure 6 shows the simulation result of the surface portion ranged from the 200th to 300th tool positions. The error is defined as the maximal deviation among the sampling heights swept by the cutter. It is calculated for every interpolated tool position. The result indicates that the machining error is neglectible at the cutter locations while it is substantial at the interpolated positions. The error is larger in the intermediate positions as compared to that near the cutter locations. In addition, the amount of error increases with the distance between cutter locations. This tendency is observed for the entire surface. Notice that the number of interpolations is extremely high in a real CNC controller. However, increase of the interpolation density does not affect the error profile described above, i.e. the maximal deviation occurs in the intermediate position and its value does not have a significant change. Figure 6b shows the similar simulation result with double interpolated positions of that in Fig. 6a.

The simulation result reveals an important insight to tool path planning of 5-axis flank milling. That is, the deviation caused by CNC interpolation should be taken into account in the path planning. It cannot be ignored even though no interference occurs at discrete cutter locations. The tool path optimization thus requires sophisticated schemes, which previous studies failed to address.

Global optimization of tool path

Previous studies (Bedi et al. 2003; Liu 1995; Tsay and Her 2001; Menzel et al. 2004) concerned generation of optimal tool path for 5-axis flank milling in terms of minimal machining error. They have one common characteristic despite that different optimization methods were proposed, i.e. optimization was applied locally at every cutter location. For example, Bedi et al. (2003) developed a nonlinear optimization scheme for minimizing the tool interference. The scheme first adjusts the tool center point to a position with a smaller error; then it rotates the tool orientation around the contact ruling to further reduce the error. They (Menzel et al. 2004) proposed a modified method, a triple tangent positioning strategy, for improving the undercut produced by the previous method. Tool adjustment based on the method was conducted at individual cutter locations independently. Tsay and Her (2001) analyzed the error in planes perpendicular to the ruling line

Table 1 Test conditions of machining error simulation

Cutter radius	1 mm				
Cutter length	30 mm				
# Of sampling height	2,000				
# Of CL	I	50	# Of interpolation between CL	I	39
	II	100		II	19
	III	200		III	9

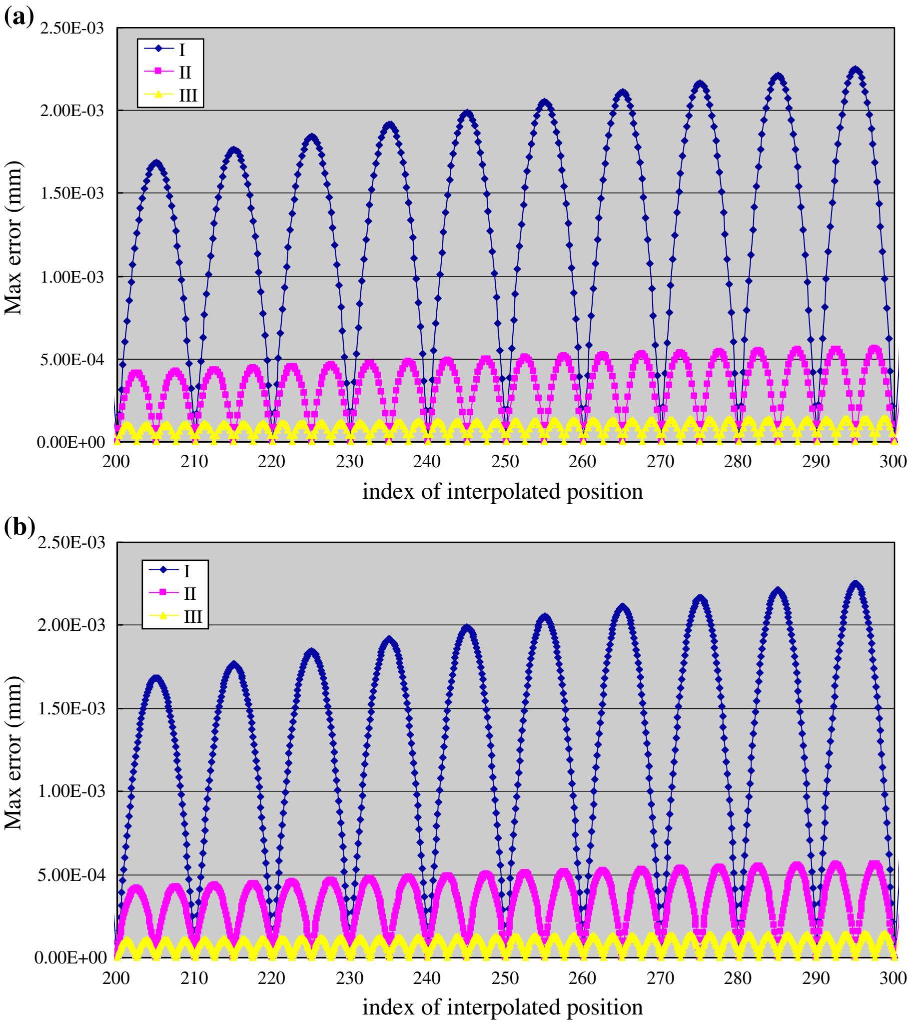


Fig. 6 Simulated machining error of the test developable surface

in 5-axis flank milling of ruled surfaces, and then used statistical analysis to derive equations to minimize this error. Likewise, they performed the minimization at each tool position separately.

Our previous research (Wu and Chu 2008) has demonstrated that optimizing the tool path in 5-axis flank milling with a global search scheme performs better than local optimization. The latter produces a larger machining error regarding the entire machined surface. This is analogous to the optimization theory: the sum of local optima is not equal to the global optimum. CNC linear interpolation aggravates the difference between them in optimizing tool path for 5-axis flank milling. The tool path should be computed by an optimization approach that does monolithic planning for the entire machined surface, rather than by local search conducted at discrete locations independently.

A good illustrative example is the flank milling of developable surfaces using a cylindrical cutter. Without considering the interpolation error, the optimal and perhaps the most straightforward way of the tool path planning is to make the cutter move along the surface rulings. No interference occurs at the contact lines due to the developability condition

(Chu and Chen 2006). Such a planning method is a local optimization approach, since the machining error is null at the cutter locations where the cutter contacts the surface. However, the approach may not correspond to minimal machining error when we account for the deviation induced by the interpolated positions between the cutter locations. There exist better strategies for positioning the cutter on the surface than moving along the surface rulings. The following example will validate this statement.

This work adopts the optimization scheme proposed by our previous work (Wu et al. 2008). The scheme searches for global optima based on discrete dynamic programming techniques. It consists of five steps as shown in Fig. 7. First, the boundary curves of a machined surface are discretized into a set of sampling points. Second, the user chooses one boundary as the focal curve for creating stages in dynamic programming. Each sample point on the focal curve represents a stage. The third step is to create all possible states at each stage. Each state contains a number of connection lines which connect a point on the focal curve with some points on the other boundary. The number of the states and the connection lines are determined by two user-defined parameters. The next step is to make links between states in adjacent stages. This step must satisfy two conditions. The first one is to avoid intersection between the connection lines within adjacent states, since we do not allow the cutter to move backwards. The second condition prevents the number of skipped sample points on the non-focal boundary from violating the value specified by the user. Each link denotes the machining error induced by the cutter moving from the position corresponding to the previous node to that of the next node.

We can thus construct a network that records all possible tool movements from the start ruling to the end ruling of the machined surface. Dynamic programming is applied to

Fig. 7 Steps in the global optimization of tool PATH planning

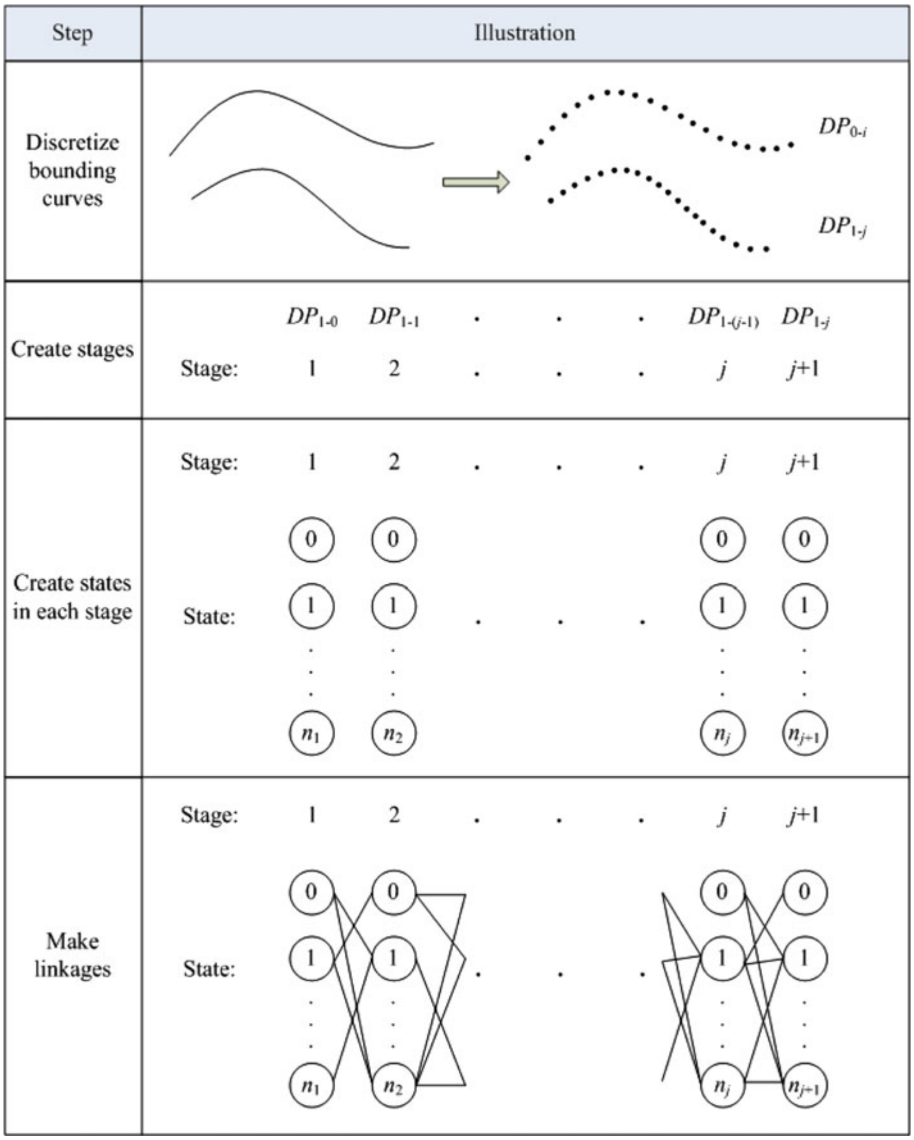


Table 2 Test conditions in the global optimization of tool path

Cutter radius	1 mm				
Cutter length	30 mm				
# Of interpolation between CL	9				
# Of CL	I	60	# Of sampling height	I	6,000
	II	120		II	12,000
	III	180		III	18,000

Table 3 Comparison between optimal and non-optimal machining errors

	# Of CL	Error (optimized)	Error (along rulings)
I	60	0.775389	0.815854
II	120	0.384153	0.41114
III	180	0.255797	0.274845

obtain an optimal route that corresponds to minimal machining error. The optimal route comprises a series of connection lines that link the two boundary curves. Each line represents a contact line and a cutter location on the machined surface. The optimization scheme provides a systematic method for achieving globally optimal tool path in terms of minimal machining error.

The subsequent optimization will employ the cubic Bézier developable surface shown in Fig. 5 as a test surface. Table 2 lists the parameter setting in the optimization process. Nine tool positions are linearly interpolated between consecutive cutter locations. 60, 120, and 180 rows of sampling heights are generated with equal parameter increment in the *u* direction for approximating the stock material in the three optimizations, respectively. Each row contains 10 sampling heights along the *v* direction. The global optimization scheme is conducted with objective function as the sum of the maximal error at each tool position. The number of skipped point is one. Table 3 shows the optimized machining errors in comparison with the result of the cutter moving along the surface rulings. Moving along the rulings is obviously not the better strategy of tool motion for reducing machining error. This validates the argument that the tool path needs to be optimized in a global manner in consideration of CNC linear interpolation. This is true for all three cases, regardless of the number of cutter locations.

Figure 8 shows the cutter contact lines for the resultant tool path in each optimization. The surface region in which the cutter does not follow the rulings is highlighted in a different color. Figure 9 is an enlarged view of the cutter contact lines in the case of 60 CL's. The dotted lines indicate the surface rulings. Apparently the optimized tool path contains many non-ruling contact lines. Smaller machining error is produced when the cutter is skew to the rulings. The result shows that following the rulings does not give the minimal machining error in consideration of CNC linear interpolation when a developable surface is milled with a cylindrical cutter.

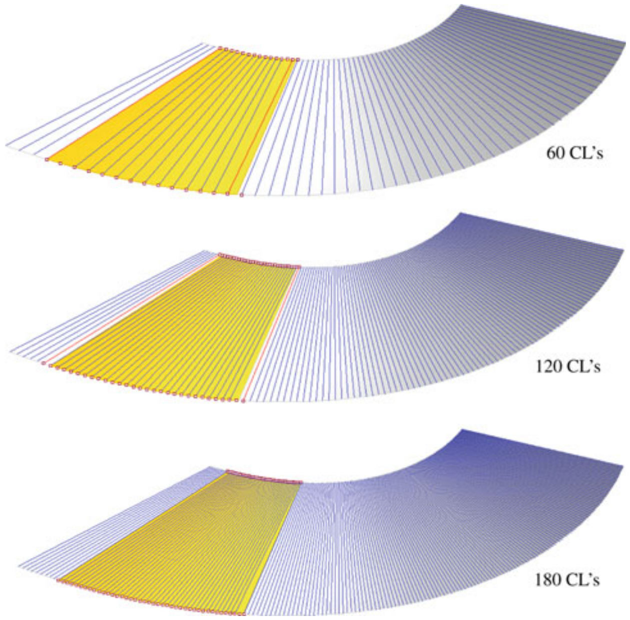


Fig. 8 The cutter contact lines in three optimizations

Experiment

A machining experiment is conducted to verify the issues we have discussed in pervious sections. The developable surface shown in Fig. 5 is flank-milled twice using a Deckel-Maho^{TM} DMG60T 5-axis CNC machine tool. The cutter moves along the surface rulings in the first cut. The second tool path is generated by the global optimized scheme with the parameter setting shown in Table 4. Table 5 lists the machining parameters in the experiment. They remain the same in the two cuts. Each cut utilizes a fresh milling cutter for eliminating influence of tool wear. A rough cut operation has been conducted prior to the experiment, leaving a layer of material for the finish cut. The stock is made of soft material Epoxy.

Figure 10 shows the machined part containing two surfaces. Performing two cuts in one part helps reduce the

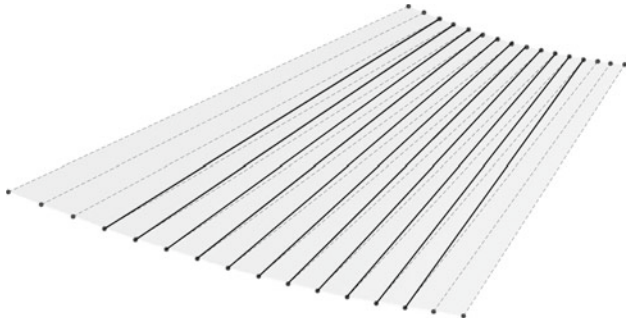


Fig. 9 An enlarged view of the cutter contact lines in the case of 60 CL's

Table 4 Parameter setting in the generation of optimal tool path

Cutter radius	2 mm
# Of sample points on each boundary	100
# Of interpolations between CL	9
# Of allowable skipped point	1
# Of sampling heights ($u \times v$)	500 × 8 = 4,000

Table 5 Machining parameters in the milling experiment

Cutter type	2-Flute straight ball-end milling cutter
Cutter radius	2 mm
Cutter length	30 mm
Rotational speed	14,000 rpm
Feedrate	224 mm/min

calibration variation during the subsequent measurement. The machined surfaces are inspected using a Zeiss^{TM} UMC 850 3D coordinate measuring machine (CMM). The measurement origin is chosen at one vertex of the part shown in the figure, determined by intersecting three reference planes. They can be readily determined by sampling points on the finished part. A touch probe makes contact with the surfaces at a set of pre-defined positions. There are 210 rows of measure points along the *u* direction and each row contains 4 points along the *v* direction. The machining error is computed as the distance between the actual position reported from the CMM and the ideal value in CAD model.

Table 6 shows the measured errors for the two cuts. Figure 11 illustrates the error distribution on the machined surface. The optimized tool path produces a smaller machining error. The result indicates that the cutter should not follow the surface rulings in the existence of the linear interpolation error, when minimizing the machining error is concerned. In addition, choosing locally optimal tool positions does not necessarily produce the best machining quality. Generation of the optimal tool path requires a global scheme that conducts search based on the machining error of the entire surface during the optimization process.

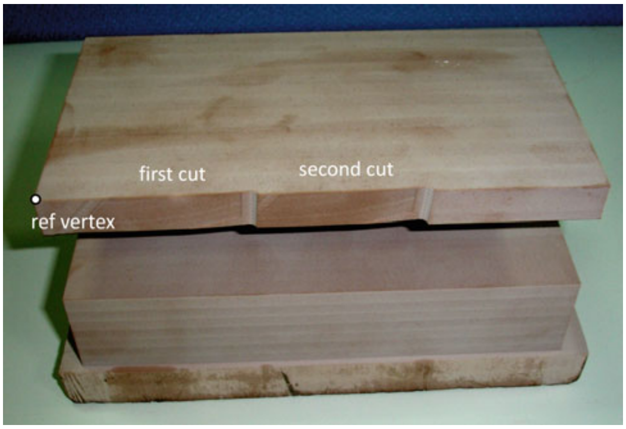


Fig. 10 Machined part with two experimental cuts

Table 6 CMM measurement of the two cuts

Optimal tool path (1st cut)	4.228 mm
Moving along the rulings (2nd cut)	9.494 mm

Conclusions

5-axis flank milling offers unique advantages over conventional 3-axis milling in machining productivity and capability. It has been widely used in industry since the late 90s. Tool path planning is considered a critical factor in the flank milling operation. A cylindrical cutter cannot produce the exact shape of a non-developable ruled surface in theory. As a result, high precision machining result requires sophisticated schemes for the tool path planning. Many previous studies have proposed different methods for reducing the machining error in 5-axis flank milling. They either rely on heuristics for positioning the cutter, or employ complicated optimization algorithms for adjusting the cutter locations locally. A common glitch of both is that they did not consider the deviation of the tool positions between cutter locations induced by CNC interpolation. The negligence may significantly influence the result of optimal tool path. Most related work overlooks this effect, leading to misconceptions in the path planning.

To overcome these problems, this work investigates tool path planning for 5-axis flank milling of ruled surfaces in consideration of CNC interpolation error. Based on the simulation result, we demonstrate that the machining error at the interpolated positions can be substantial when the distance between cutter locations is large. The interpolation density does not change the error notably. Thus, it is still present in actual CNC machining. In addition, the simulation result shows that moving along the surface rulings may not be the best strategy for flank milling of a developable surface. This reveals an important insight that past research failed to

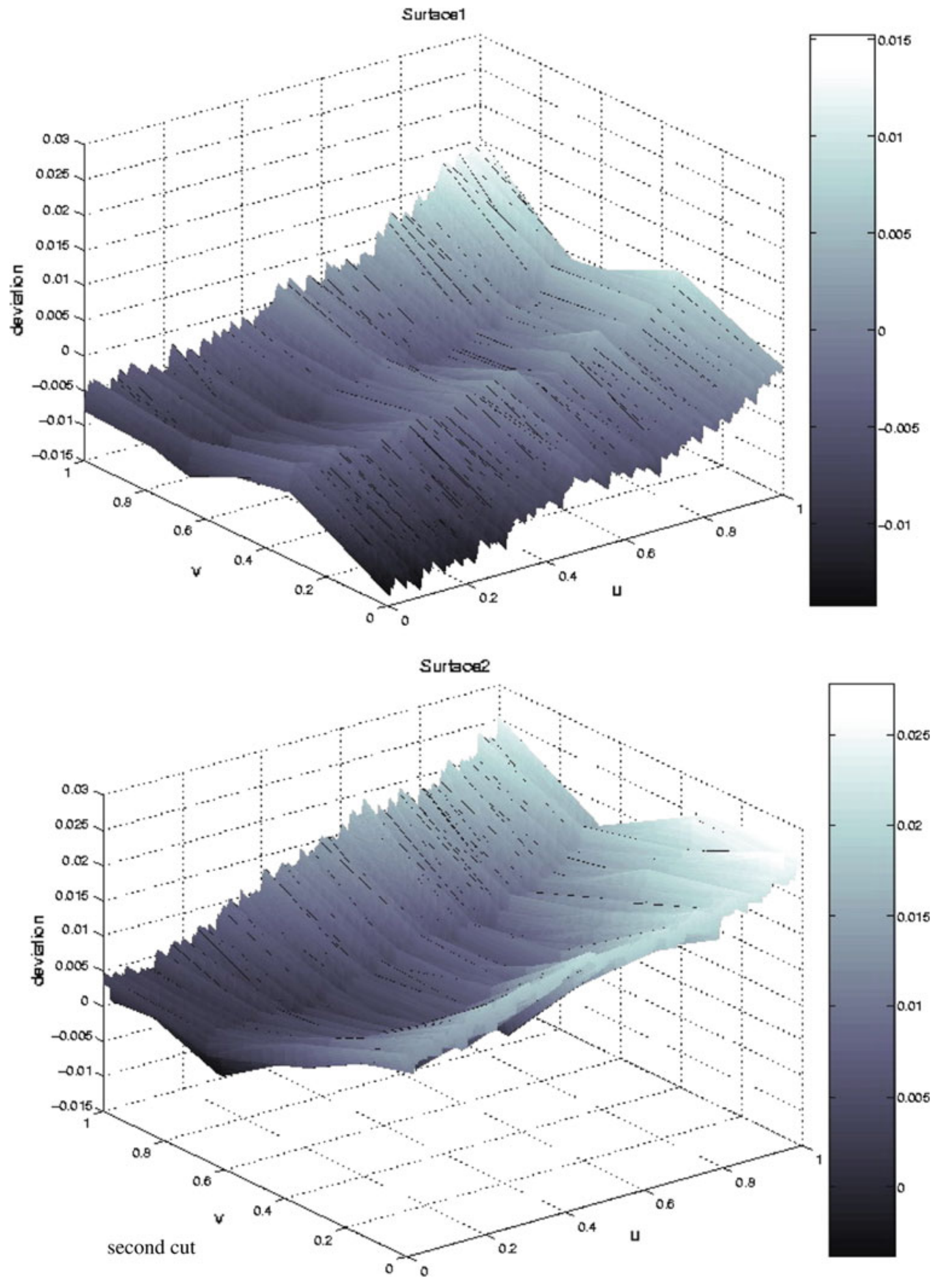


Fig. 11 Machining error distributions for the two cuts

identify. That is, any approach that locally adjusts the tool position for minimizing the machining error may not produce the best result for the entire surface. It is necessary to optimize the tool path planning in a global manner, i.e. determination of the cutter locations depend on the error of the entire machined surface, not just the error induced at each location. Following this idea, a global optimization scheme based on dynamic programming was introduced to obtain the optimal tool path. A flank milling experiment of a developable surface was conducted. The CMM measurement validates the above statements. This work highlights the significance of CNC linear interpolation on 5-axis flank milling. It provides a systematic method for optimizing the tool path in consideration of the deviation caused by the interpolation.

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