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Condition based maintenance optimization considering multiple objectives

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Abstract In condition based maintenance (CBM) optimization, the main optimization objectives include maximizing reliability and minimizing maintenance costs, which are often times conflicting to each other. In this work, we develop a physical programming based approach to deal with the multi-objective condition based maintenance optimization problem. Physical programming presents two major advantages: (1) it is an efficient approach to capture the decision makers' preferences on the objectives by eliminating the iterative process of adjusting the weights of the objectives, and (2) it is easy to use in that decision makers just need to specify physically meaningful boundaries for the objectives. The maintenance cost and reliability objectives are calculated based on proportional hazards model and a control limit CBM replacement policy. With the proposed approach, the decision maker can systematically and efficiently make good tradeoff between the cost objective and reliability objective. An example is used to illustrate the proposed approach.

Keywords Condition based maintenance · Physical programming · Proportional hazards model · Reliability · Multi-objective optimization

Abbreviations
CBM Condition based maintenance
PHM Proportional hazards model

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Introduction

Condition based maintenance (CBM) aims to achieve reliable and cost-effective operation of engineering systems such as aircraft systems, manufacturing systems, power stations, etc. CBM is based on the understanding that a piece of equipment goes through multiple degraded states before failure. The health conditions can be monitored and predicted (Dong and He 2007a,b; Yang and Chen 2009), and optimal maintenance actions can be scheduled for preventing equipment breakdown and minimizing total operation costs (Jardine et al. 2006).

A CBM optimization approach based on proportional hazards model (PHM) has been developed, aiming at determining an optimal replacement policy, that is, an optimal risk threshold control limit in this approach, for minimizing the long-run replacement cost (Makis and Jardine 1992; Banjevic et al. 2001; Ghasemi et al. 2007). This approach was developed into the CBM optimization software EXAKT (Banjevic et al. 2001), and it has been successfully applied in many industries, including mining industry, food processing industry, utility industry, manufacturing industry, etc. In "Proportional hazards model and its applications", we will summarize the basics of PHM and the applications of the PHM based CBM methods.

In condition based maintenance optimization, there are multiple and typically conflicting design objectives, such as minimizing the maintenance costs, maximizing the reliability, minimizing equipment downtime, etc. The existing methods only consider single optimization objective (Banjevic et al. 2001). In some cases, minimizing cost is the only optimization objective. In some other cases, maximizing reliability, or minimizing failure probability, is the only optimization objective. When cost is the optimization objective, for example, reliability can be used as a constraint, and vice versa. However, the disadvantage of single-objective optimization is that we cannot systematically investigate the trade-off between the optimization objectives and find the optimal solution that best represents the decision maker's preference on the optimization objectives. Physical programming is an effective multi-objective optimization approach developed in 1996 (Messac 1996), and it has proved its effectiveness in addressing a wide array of multi-objective optimization problems in the field of structural optimization, product family design, control, robust design, reliability optimization, etc. (Huang et al. 2005b; Tian and Zuo 2006; Tian et al. 2008). In this work, we develop an approach based on physical programming to deal with the multiple optimization objectives involved in CBM optimization, that is, the cost objective and the reliability objective. Physical programming presents two major advantages: (1) it is an effective approach to capture the decision makers' preferences on the objectives by eliminating the iterative process of adjusting the weights of the objectives, and (2) it is easy to use in that decision makers just need to specify physically meaningful boundaries for the objectives. An example will be used to illustrate the approach.

Proportional hazards model and its applications

Proportional hazards model

Since the proportional hazards model (PHM) was introduced in 1972 by D.R. Cox, it has been utilized in many fields such as biomedicine, maintenance, transportation, politics, etc. (Xu and Gamst 2007; Kumar et al. 1997). It was especially widely applied in the field of biomedicine and thousands of papers related to this topic can be found. But the research and application in the field of maintenance reliability engineering has not yet come to maturity. From 1990s, interest in applications of the PHM in this field has greatly increased. PHM has begun to be adopted to deal with various components and systems, such as aircraft engines, machine tools, and power transmission cables. The most important reason that the PHM is a good approach to analyze the reliability data is that it can take into account the condition monitoring data and operating condition data to estimate the failure probability and for making maintenance decisions. The proportional hazards model is a valuable statistical procedure to estimate the risk of failure of equipment when it is under condition monitoring. In this model, the effects of different covariates influencing the time to failure of a system can be estimated, such as the parts per million (PPM) of iron or lead in the lubricant oil in the truck transmission system. The PHM can take various forms, but all of them are made up of two parts: a baseline hazard function and a function including all the covariates which affect the time to failure.

The basic model of PHM combines a Weibull baseline hazard function with a component including all the covariates which affects the time to failure, as follows (Jardine et al. 2006):

$$h(t, Z(t)) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \exp\left\{\sum_{i=1}^{m} \gamma_i z_i(t)\right\} \tag{1}$$

where $h(t, Z(t))$ is the hazard value, or failure rate value, at time t, given the values of $z_1(t), z_2(t), \ldots, z_m(t)$. The first part of this model is the baseline hazard function $\beta/\eta\,(t/\eta)^{\beta-1}$, which takes into account the age of the equipment at time of inspection, given the values of parameters β and η. The second part $\exp\{\gamma_1 z_1(t) + \gamma_2 z_2(t) + \cdots + \gamma_m z_m(t)\}$ takes into account the covariates which can be considered to be the key factors reflecting the health condition of equipment.

Applications of the proportional hazards model based methods

The applications of PHM based methods in maintenance optimization and reliability engineering are reviewed in this section.

Applications in maintenance optimization

Applications in maintenance optimization combine the age data with the condition monitoring data in PHM, so as to more accurately represent the equipment health condition and failure probability. Jardine et al. (2008a) described the development of an optimal predictive maintenance program for shear pump bearings in the food processing industry. Measurements are taken in three directions for the bearings under investigation: axial, horizontal and vertical. In each of these directions, the velocity spectrum is obtained in five frequency bands. In addition, overall velocity and acceleration are also measured in the three directions. Lin et al. (2006) proposed the application of a principal components proportional hazards regression model in condition based maintenance (CBM) optimization. The oil analysis data set was collected from transmissions on haul trucks in a mining company. A similar research using oil data from transmissions in 240-ton heavy hauler trucks can be found in the paper by Makis et al. (2006).

PHM was utilized by Vlok et al. (2002) to determine the optimal replacement policy for a vital item which is subject to vibration monitoring. In their study they chose circulating pumps in a coal wash plant as the research case. The lifetime data was collected based on a period of 2 years. Their study showed that, even with some problems in collected data, vibration measurements can be used in proportional hazards model and that a useful decision policy can be obtained. Kobbacy et al. (1997) proposed a heuristic approach for implementing the PHM to schedule future preventive maintenance actions on the basis of the equipment's full condition history, using data from four pumps operating in a continuous process industry.

Jardine et al. (2008b) used the EXAKT software to build an optimized condition based maintenance policy model for the interpretation of inspection data from hydro-dyne seals in a nuclear reactor station. The PHM based statistical decision methodology was applied to determine the optimal times at which to perform proactive maintenance. Ansell and Phillips (1997) used PHM to illustrate the repairable data from the hydrocarbon industry. The data set consists of failure data in a pipeline arising from a set of different causes and information supplied on a daily basis on average temperature and the stress the system was under. In the research by Rao & Prasad (2001), PHM was used to analyze failure data and plan maintenance intervals for dumpers in mining industry. In this paper PHM was applied to model the failure rate of a repairable equipment whose performance is affected by concomitant variables. Graphical methods were used to determine the maintenance intervals.

Applications in reliability analysis

Applications in reliability analysis are applying PHM in the measurement and prediction of the reliability of equipment by using covariates to describe different operating conditions. Kumar et al. (1992) used PHM to examine the effects of two different designs and maintenance on the reliability of a power transmission cable of an electric mine loader. The study by Prasad and Rao (2002) considered failure data from cables of an electrical loader hauler dumper in an underground coal mine. The failures due to electrical problems, compressed air and cable fault were found to be significant. They also applied PHM to study the reliability of repairable systems considering the effect of operating conditions with an example of thermal power unit. Elsayed and Chan (1990) developed a PHM method to estimate thin oxide dielectric reliability and time-dependent dielectric breakdown hazard rates, including two groups of models: group one doesn't consider interactions between temperature and electric field, while group two analyzes interactions between these two factors. JoWiak (1992) developed an approach to utilize PHM in reliability exploration of microcomputer systems. In this approach, he examined the effects of two concomitant variables, temperature and mean daily user's exploitation time of the system, on system reliability, and found that the PHM with Weibull baseline failure rate has considerable potential for estimating equipment failure rate in the presence of time-dependent and time-independent concomitant variables.

Campean et al. (2001) presented a general PHM based methodology for camshaft timing-belt life prediction modeling. The approach aimed to establish a correlation between the degradation mechanism, the real world customer usage profile and rig life testing. The proportional hazards model is also widely utilized in many other fields, such as aircraft engines (Jardine and Anderson 1987), locomotive diesel engines (Jardine et al. 1989), etc.

Multi-objective optimization techniques

Review of multi-objective optimization methods

Engineering optimization problems typically involve multiple conflicting objectives. A general multi-objective optimization problem is to find the design variables that optimize a set of different objectives over the feasible design space. A mathematical formulation of the multi-objective optimization problem is given as follows (Huang et al. 2005a):

$$\begin{aligned} &\text{minimize } f(x) = \{f_1(x),\ f_2(x), \ldots, f_m(x)\} \\ &\text{Subject to } x \in \mathrm{X} \end{aligned} \tag{2}$$

where x is an n-dimensional vector of design variables, X is the feasible design space, m is the number of design objectives, $f_i(x)$ is the objective function for the ith design objective, and $f(x)$ is the design objective vector.

Because of the conflicting nature among different design objectives, it is typically impossible to achieve the best values for all the objectives simultaneously. One of the most widely used methods for multi-objective optimization is the Weighted-sum Method. It converts a multi-objective optimization problem into a single-objective optimization problem by using a weighted sum of all the objective functions as the single objective. The mathematical model of the Weighted-sum Method takes the following form:

$$\begin{aligned} &\text{minimize } f = \sum_{i=1}^{m} w_i f_i(x) \\ &\text{Subject to } x \in \mathrm{X} \end{aligned} \tag{3}$$

where w_i is the weight of objective i, and $\sum_{i=1}^{m} w_i = 1$, $w_i \geq 0$, $i = 1, 2, \ldots, m$.

Augment weighted Tchebycheff programs (AWTPs) is another widely used method for multi-objective optimization. The mathematical model of AWTPs is presented as follows (Huang et al. 2005a):

$$\begin{aligned} &\min\ \alpha + \rho \sum_{i=1}^{m} (1 - z_i) \\ &\text{s.t. } \alpha \geq \lambda_i (1 - z_i),\ \forall i \\ &z_i = \frac{f_i(x) - f_i^{\text{nadir}}}{f_i^{\text{ideal}} - f_i^{\text{nadir}}},\ \forall i \\ &x \in \mathrm{X} \end{aligned} \tag{4}$$

where a is a variable satisfying the condition in the constraint, and ρ is a small positive scalar. f^{ideal} represents the utopian point, that is, f_i^{ideal} is the optimization result with the ith design objective as objective function and $x \in \mathrm{X}$ as constraints. λ_i is the weight of objective i, and $\sum_{i=1}^{m} \lambda_i = 1$, $\lambda_i \geq 0$, $i = 1, 2, \ldots, m$. f_i^{nadir} denotes the worst value of the ith objective function among all the Pareto points, or non-dominant point. A design variable vector is said to be Pareto solution if there exists no feasible design variable vector that would improve some objective functions without causing a simultaneous increase in at least one objective function. z_i^{nadir} can be estimated by the optimization result with the minus of the ith design objective as objective function and $x \in \mathrm{X}$ as constraints, or it can be estimated by simply being assigned a value based on experience.

The Weighted-sum method and the AWTPs method are hard to use, because we need to specify the weigh values, which do not have physical meanings, and there are no systematic approaches to find the best set of weight values for these models. Other more sophisticated multi-objective optimization methods have been reported in the literature. Huang et al. (2005a) proposed an interactive multi-objective optimization method, and applied it to reliability optimization. Baykasoglu et al. (2004) presented a tabu search method for solving multi-objective flexible job shop scheduling problems. Taboada et al. (2008) developed a multi-objective multi-state genetic algorithm for system reliability optimal design. Li et al. (2009) presented a two-stage approach for multi-objective decision making. Konak et al. (2006) gave a tutorial on multi-objective optimization using genetic algorithms. However, comparing to the multi-objective optimization methods presented above, the physical programming method is easier to use, because it does not include the interactive process of adjusting the weighs, and it is very effective in capturing the designers' preferences on different design objectives. The physical programming method will be discussed in more details in the next section.

The physical programming method

Physical Programming is an effective multi-objective optimization method that explicitly incorporates the designer's preferences on different design objectives. Physical programming captures the designer's preferences using class functions. A class function is a function of a design objective. The value of a class function represents the preference of the designer on the objective function value, and the smaller the class function value is, the better. Class functions are classified into four classes: smaller is better (i.e., minimization), larger is better (i.e., maximization), value is better, and range is better. Consider for example the case of class-1 soft class function (class 1-S), the qualitative meaning of the preference function is given in Fig. 1. The value of the objective function, g_i, is on the horizontal axis, and the corresponding class function, $\bar{g}_i$, is on the vertical axis.

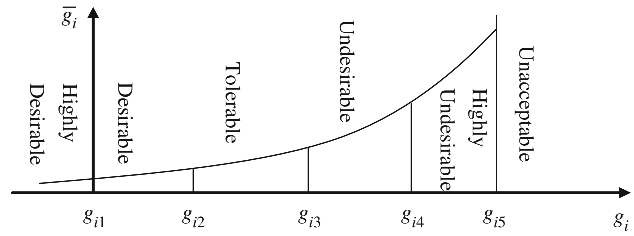

Fig. 1 The Class-1 soft class function

Physical programming allows the designers to express ranges of differing levels of preference with respect to each design objective with more flexibility. For Class 1-S, as shown in Fig. 1, there are six ranges: Highly desirable range ($g_i \leq g_{i1}$), Desirable range ($g_{i1} \leq g_i \leq g_{i2}$), Tolerable range ($g_{i2} \leq g_i \leq g_{i3}$), Undesirable range ($g_{i3} \leq g_i \leq g_{i4}$), Highly undesirable range ($g_{i4} \leq g_i \leq g_{i5}$), and Unacceptable range ($g_i \geq g_{i5}$). The parameters g_{i1} through g_{i5} are physically meaningful constants associated with design objective i. What the designer needs to do is just to specify ranges of different degrees of desirability (highly desirable, desirable, tolerable, undesirable, highly undesirable, and unacceptable) for the class function of each objective. Given the specified boundary values, the class function for a design objective can be constructed using the method developed by Messac (1996).

The range boundary values define the intra-objective preference, while the "One versus Others" criteria rule (OVO rule) describe the inter-objective preference. Suppose there are two options: (1) full reduction for one criterion across a given preference range, say, the tolerable range; (2) full reduction for all the other criteria across the next better range, say, the desirable range. The OVO rule decides that option (3) is preferred over option (4). The OVO rule is incorporated in the method of constructing the class functions.

The aggregate objective function is built by combining all the soft class functions, and thus the multi-objective optimization problem is converted into a single-objective optimization problem, and can be solved using optimization codes. Typically, only Class 1-S functions (to be minimized) and Class 2-S functions (to be maximized) are the soft class functions that we have, and the physical programming problem model can be formulated as follows (Messac 1996; Tian and Zuo 2006):

$$\begin{aligned} &\min_{x} g(\boldsymbol{x}) = \log_{10}\left\{\frac{1}{n_{sc}} \sum_{i=1}^{n_{sc}} \bar{g}_i[g_i(\boldsymbol{x})]\right\} \\ &\text{s.t.} \quad g_i(\boldsymbol{x}) \leq g_{i5} \qquad (\text{for class 1-S}) \\ &\qquad\ g_i(\boldsymbol{x}) \geq g_{i5} \qquad (\text{for class 2-S}) \\ &\qquad\ x_{jm} \leq x_j \leq x_{jM} \end{aligned} \tag{5}$$

where x_{jm} and x_{jM} are the corresponding minimum and maximum values for design variable j, n_{sc} is the number of the soft design objectives in the problem.

The physical programming method has the following advantages (Messac 1996; Tian and Zuo 2006). Equation (1) the iterative weight-adjusting process is eliminated, thus the computational burden is substantially reduced; (2) The designers only need to specify the physically meaningful boundary values for each design objective, not those meaningless weights, which makes this approach very easy to use; (3) The designers' preferences are specified on each design metric individually, therefore, physical programming is suitable to deal with a larger number of design objectives.

The approach for multi-objective CBM optimization using physical programming

In this section, we present the approach for multi-objective CBM optimization using physical programming. The methods for calculating the cost and reliability objective values will be presented first.

Calculation of the cost and reliability values

The CBM optimization approach based on proportional hazards model, and the method for calculating the cost and reliability objective function values, were developed in Makis and Jardine (1992) and Banjevic et al. (2001). A summary of method is given in this section. In the PHM based CBM policy, if the observed hazard rate $h(t, z(t))$ multiplied by K at the given inspection point of time is greater than a certain risk threshold value d, preventive replacement action should be taken; otherwise operation can continue. If a failure occurs, a failure replacement will be performed. Thus, the risk threshold value d determines the PHM based CBM policy. The objective of CBM optimization is to find the optimal risk threshold to optimize the cost and reliability objectives.

The cost objective C, that is, the total expected cost per unit of time, can be calculated based on the following formula (Banjevic et al. 2001):

$$C = \frac{C_p(1 - Q(d)) + (C_p + K)\, Q(d)}{W(d)}, \tag{6}$$

where C is the average cost per unit of time. C_p is the preventive replacement cost and $C_p + K$ the failure replacement cost. $Q(d)$ is the probability that failure replacement will occur, that is, $Q(d) = P(T_d \geq T)$. $T_d = \inf\{t \geq 0 : Kh(t, z(t)) \geq d\}$ is the preventive time at the risk level d. $W(d)$ is the expected time until replacement, regardless of whether it is a preventive action or failure, that is, $W(d) = E(\min\{T_d, T\})$, where T is the failure time. Once the optimal risk level, $d*$, is determined, the item is replaced at the first moment t when $\beta/\eta(t/\eta)^{\beta-1}\exp(\gamma Z(t)) \geq d*/K$.

The reliability under a CBM policy is defined as the probability of performing preventive replacements, that is, the probability of preventing failure from occurring. Thus, the reliability objective R can be calculated using the following equation:

$$R = 1 - Q(d). \tag{7}$$

The cost and reliability objective values can be calculated using the method developed by Banjevic et al. (2001).

The physical programming model

The physical programming model for the multi-objective CBM optimization is developed in this section. In this work, we have two optimization objectives, cost and reliability. The cost objective class function is an increasing function, as shown in Fig. 2. The lower the cost, the better it is. The values in the figure are just to qualitatively illustrate the cost class function. The reliability class function is a decreasing function of reliability value, as shown in Fig. 3. The higher the reliability, the better it is.

The physical programming approach transforms a multi-objective optimization problem into a single-objective optimization model. The soft class functions of design objectives are combined into the aggregate objective function f, which is to be minimized. The physical programming-based optimization model for CBM optimization problem is given as:

$$\begin{aligned} &\min f(d) = \log_{10}\left\{\tfrac{1}{2}\left[\bar{g}_R(R(d)) + \bar{g}_C(C(d))\right]\right\} \\ &\text{s.t.} \\ &R \geq R_0,\ C \leq C_0 \\ &d > 0 \end{aligned} \tag{8}$$

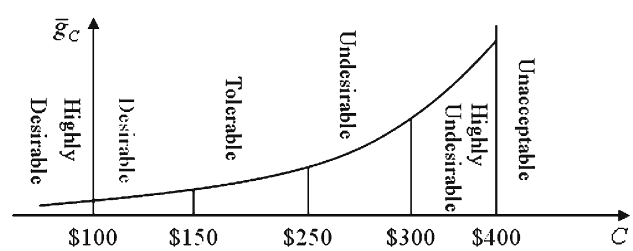

Fig. 2 The cost objective class function

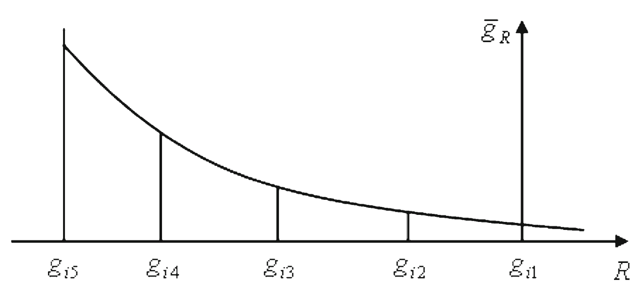

Fig. 3 The reliability objective class function

where $\bar{g}_R$ and $\bar{g}_C$ are the class functions for the reliability objective and the cost objective, respectively. R_0 and C_0 are the reliability and cost constraint values, respectively. Given a risk threshold value d, the corresponding cost and reliability under the CBM policy can be calculated. The objective function values are used to further calculate the corresponding class functions. The aggregate objective function can then be calculated and optimized to find the optimal risk threshold value. As can be seen from (7), the optimization problem can be formulated as a single-variable optimization problem. Many optimization methods can be used to solve such an optimization problem.

An example

To illustrate the proposed approach, we use the example of CBM of shear pump bearings in a food processing plant. The case was reported by Banjevic et al. (2001). The objective is to find an optimal condition based replacement policy to minimize total long-run expected replacement cost, and to improve reliability, given the condition monitoring data (vibration data) and replacement histories.

Totally 21 vibration measurements were collected using accelerometers, including vibration data in axial, horizontal and vertical directions for the overall velocity, velocities in 5 bands and acceleration. There are 25 histories in the recorded data, including 13 failure replacements (ended with failure) and 12 preventive replacements (ended with suspension). Using the software EXAKT (Banjevic et al. 2001), the significance analysis was performed, and three significant covariates were identified: VEL#1A (band 1 velocity in the axial direction), VEL#1V (band 1 velocity in the vertical direction), and VEL#2A (band 2 velocity in the axial direction). The PHM parameters can thus be estimated, and the resulting hazard function is given as follows:

$$\begin{aligned}h(t, Z(t)) &= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} \\ &\quad \cdot \exp\{\gamma_1 z_{1A}(t)+\gamma_2 z_{2A}(t)+\gamma_3 z_{1V}(t)\} \\ &= \frac{4.992}{1{,}584}\left(\frac{t}{1{,}584}\right)^{4.992-1} \\ &\quad \cdot \exp\{5.831 z_{1A}(t)+36.55 z_{2A}(t) \\ &\quad +24.05 z_{1V}(t)\}\end{aligned} \tag{9}$$

The transition probability matrix is required for calculating the cost and reliability measures. The transition probability matrix indicates the probabilities of a covariate in different ranges at the next inspection time given its current range. Assume the inspection interval is 20 days. The transition probability matrices for the three covariates can also be estimated using EXAKT. The transition probability matrix for covariate VEL#1A is shown in the Fig. 4. As can be seen, the covariate is divided into 5 ranges, and the transition probability values are shown in the figure. The matrices for the other two covariates will not be listed here.

In this application, the preventive replacement cost is estimated to be \$1,800, and the failure replacement cost is \$16,200. Thus we have K equal to \$14,400. Given a certain risk threshold value d, the corresponding cost and reliability value can be calculated. The cost versus risk threshold plot and the reliability versus risk threshold plot are shown in Figs. 5 and 6, respectively. The risk threshold d is in the logarithm scale, since the relationships can be better presented in the plots in this way.

To use the physical programming approach, we need to first indicate the preferences on the objectives by specifying the boundary values for each objective. Suppose the specified boundary values for cost and reliability are given as follows:

$$\begin{aligned}&[g_{C1}, g_{C2}, g_{C3}, g_{C4}, g_{C5},] = [8, 10, 12, 15, 20,], \\ &[g_{R1}, g_{R2}, g_{R3}, g_{R4}, g_{R5},] = [0.99, 0.98, 0.95, 0.90, 0.80,].\end{aligned} \tag{10}$$

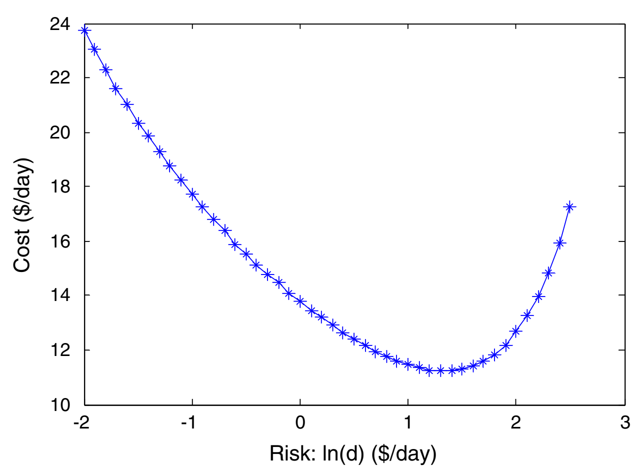

Fig. 5 The cost versus risk threshold plot

Fig. 4 The transition probability matrix for covariate VEL#1A

VEL_1A	0 to 0.035266	0.035266 to 0.2519	0.2519 to 1.08821	1.08821 to 2.51648	Above 2.51648
0 to 0.035266	0.765522	0.214501	0.0187137	0.00123314	3.01141e-005
0.035266 to 0.2519	0.0419512	0.809202	0.134907	0.0134952	0.000445182
0.2519 to 1.08821	0.00436408	0.160862	0.683157	0.144277	0.00734044
1.08821 to 2.51648	0.000138356	0.00774194	0.0694142	0.838071	0.0846349
Above 2.51648	0	0	0	0	1

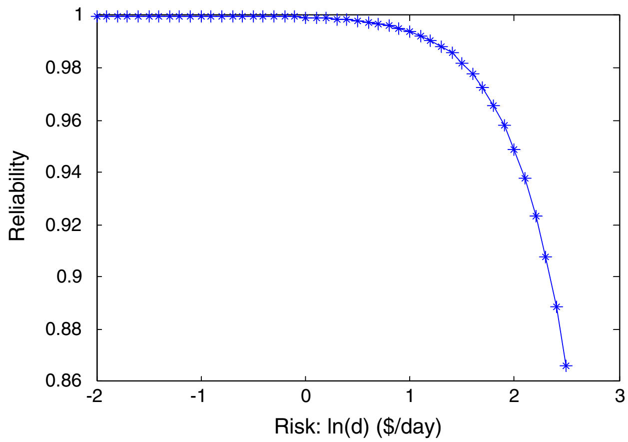

Fig. 6 The reliability versus risk threshold plot

Using the Malab Optimization Toolbox to perform the optimization, we can obtain the following optimal solution:

$$d^* = 10.79 \text{ /day}, \quad C^* = 10.48 \text{ /day}, \quad R^* = 0.9915 \qquad (11)$$

As can be seen, the optimal cost falls into the tolerable range, and the optimal reliability is in the highly desirable range. The optimization results can reflect the designer's preferences on the objectives, and the tradeoff between the two design objectives.

Now let's investigate another scenario, in which the decision maker has high requirement on the reliability objective. The decision maker can specify such preference on the reliability objective by specifying the boundary value settings for the reliability objective. Assume the cost objective boundary values are kept the same.

$$\begin{aligned} &[g_{C1}, g_{C2}, g_{C3}, g_{C4}, g_{C5},] = [8, 10, 12, 15, 20,], \\ &[g_{R1}, g_{R2}, g_{R3}, g_{R4}, g_{R5},] \\ &\quad = [0.9999, 0.999, 0.995, 0.99, 0.95,]. \end{aligned} \qquad (12)$$

Conducting the optimization, we can obtain the following optimal solution:

$$d^* = 3.4897 \text{ /day}, C^* = 11.22 \text{ /day}, R^* = 0.9973. \qquad (13)$$

As can be seen, when there is a higher requirement on reliability, the optimal risk threshold value decreases. Both of the optimal cost and optimal reliability fall into the tolerable ranges, in order to make the best tradeoff between these two objectives. The optimization results reflect the change in the designer's preferences.

Conclusions

In condition based maintenance optimization, main optimization objectives include maximizing reliability and minimizing maintenance costs, which are often times conflicting to each other. In this work, we develop a physical programming based approach to deal with the multi-objective condition based maintenance optimization problem. The physical programming method presents two major advantages: (1) it is an efficient approach to capture the decision makers' preferences on the objectives by eliminating the iterative process of adjusting the weights of the objectives, and (2) it is easy to use in that decision makers just need to specify physically meaningful boundaries for the objectives. With the proposed approach, the decision maker can systematically and efficiently make good tradeoff between the cost objective and reliability objective. The example illustrates the effectiveness of the proposed approach.

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