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Multi-criteria analysis for a maintenance management problem in an engine factory: rational choice

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Abstract The industrial organization needs to develop better methods for evaluating the performance of its projects. We are interested in the problems related to pieces with differing degrees of dirt. In this direction, we propose and evaluate a maintenance decision problem of maintenance in an engine factory that is specialized in the production, sale and maintenance of medium and slow speed four stroke engines. The main purpose of this paper is to study the problem by means of the analytic hierarchy process to obtain the weights of criteria, and the TOPSIS method as multicriteria decision making to obtain the ranking of alternatives, when the information was given in linguistic terms.

Keywords Maintenance management · Multicriteria decision making · Analytic hierarchy process · TOPSIS method · Fuzzy numbers · Cleaning system

Introduction

In multiple criteria decision analysis (MCDA) (Lootsma 1999; Luce and Raiffa 1967; Triantaphyllou 2000) a number of alternatives have to be evaluated and compared using several criteria. The aim of MCDA is to provide support to the decision-maker in the process of making the choice between alternatives. In other words, MCDA techniques help the decision-maker to articulate his/her preferences in a complex decision making environment. A pre-requisite in most MCDA methods is that the decision-maker is able to provide the necessary information.

Most of the times, the decision-maker is not able to define the importance of the criteria or the goodness of the alternatives with respect to each criterion in a strict way. In general for the decision-maker it is easier when he/she evaluates judgements by means of linguistic terms.

As is well known, the analytic hierarchy process (AHP) is a simple MCDA to deal with unstructured and multi criteria problems, which was developed by Saaty (1980). It consists of decomposing a complex problem into its components, organizing the components into levels to generate a hierarchical structure. The aim of constructing this hierarchy is to determine the impact of the lower level on an upper level, which is achieved by paired comparisons provided by the decision-maker. In this case, the AHP was only used in order to obtain the weight of criteria in the decision problem.

Practical problems are often characterized by several no commensurable and conflicting criteria, and there may be no solution satisfying all criteria simultaneously. Thus, the solution is a compromise solution according to the decision-maker's preferences. The information is located in a set of labels, and in a later step the decision-maker expresses his/her intuition about the meaning of these linguistic terms by means of fuzzy numbers.

The technique for order performance by similarity to ideal solution (TOPSIS), one of the known classical MCDM methods, was first developed by Hwang and Yoon (1981). It is based upon the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the farthest from the negative ideal solution (NIS). The final ranking is obtained by means of the closeness index.

The purpose of this article is to contribute in a maintenance problem, by means of a modification in the TOPSIS

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method (step of normalization). The new norm avoids the possibility that two similar valuations can provide different results, as occurs with the norm within the TOPSIS method proper.

The rest of the paper is organized as follows: "The statement of decision problem" is related with the problem in question. In the following sections, we describe the suggested method in detail. The linguistic variable and the fuzzy sets are described, as well as the AHP and TOPSIS methods, which will be used later. In "A decision problem in maintenance" we present the application of the methods. Finally, "Conclusions" details the most important conclusions.

The statement of decision problem

Any multi-criteria decision problem (MCDP) may be expressed by means of the following five elements:

$$\{C, D, r, I, \prec\}$$

where

1. $C = \{C_1, C_2, \ldots, C_m\}$ It is the set of criteria that represents the tools which enable alternatives to be compared from a specific point of view.
2. $D = \{D_1, D_2, \ldots, D_n\}$ It is the set of feasible alternatives to the decision-maker, and from which the decision-maker must choose one. In this case, the sets C and D as finite sets. This allows us to avoid convergence, integrability and measurability problems.
3. $r : D \times C \rightarrow$ R is a function where a real interval corresponds to every decision d_i and to every criterion C_j.

   $$(D_i, C_j) \rightarrow r(D_i, C_j) = r_{ij}$$

   Once that set of criteria and alternatives have been selected, then we need a measure of the effect produced by each alternative with respect to each criterion.
   By means of linguistic terms, the decision-maker represents the goodness of an alternative with respect to a criterion, the different values of r can be represented by means of matrix called the "Matrix of decision making".
4. A relation of preferences $\prec$ by the decision maker. We shall suppose a coherent decision-maker, therefore we shall try to maximize his profits or else minimize his losses. In this case the decision-maker needs to obtain the best alternative in function of the considered criteria.
5. Certain information about the criteria, in this case, the information is also linguistic. The decision-maker gives us linguistic information, and by means of the AHP, we obtain the importance for criteria.

Linguistic variable and fuzzy sets

Since Zadeh (1975) introduced the concept of fuzzy set and sets has been extraordinary. We are particularly interested in the role of linguistic variables, and their associated terms, in this case fuzzy numbers, that will be used in the multi-criteria decision making.

By a linguistic variable (Zadeh and Kacprzyt 1999a,b) we mean a variable whose values are words or sentences in a natural or artificial language. For example "Age" is a linguistic variable if its values are linguistic rather than numerical, i.e., young, not young, very young, quite young, old, not very old and not very young, etc., rather than numbers as 20, 21, 22, 23,…

Definition 1 A linguistic variable is characterized by a quintuple

$$\{X; T(X); U; G; M\}$$

in which

1. X is the name of the variable,
2. $T(X)$ is the term set of X, that is, the collection of its linguistic values
3. U is a universe of discourse,
4. G is a syntactic rule for generating the elements of $T(X)$ and
5. M is a semantic rule for associating meaning with the linguistic values of X.

In our case, we identify the linguistic variable with a fuzzy set (Bellman and Zadeh 1970; Kacprzyt and Yager 2001; Kerre 1982). In this paper, we only make reference at the operations on fuzzy sets that we will use in the application, as well as, the defuzzification process used.

Definition 2 A real fuzzy number A is described as any fuzzy subset of the real line $\Re$ with membership function f_A which possesses the following properties:

(1) $f_A(x)$ is a continuous mapping from $\Re$ to the closed interval $[0, w]$, $0 < w < 1$;
(2) $f_A(x) = 0$, for all $x \in (-\infty, a]$;
(3) $f_A(x)$ is strictly increasing on $[a, b]$;
(4) $f_A(x) = 1$, for all $x \in [b, c]$;
(5) $f_A(x)$ is strictly decreasing on $[c, d]$;
(6) $f_A(x) = 0$, for all $x \in (d, \infty]$,

where a, b, c, d are real numbers.

Unless elsewhere specified, it is assumed that A is convex and bounded, (i.e., $-\infty < a, d < \infty$).

Definition 3 The fuzzy number A it will be triangular (TFN) if its membership function is given by:

$$f_A(x) = \begin{cases} \dfrac{x-a}{b-a}, & a \le x \le b, \\ \dfrac{x-c}{b-c}, & b \le x \le c, \\ 0, & \text{otherwise}, \end{cases} \quad (1)$$

where a, b and c are real numbers. The value of b corresponds with the mode or core and $[a, c]$ with the support.

Definition 4 If T_1 and T_2 are two TFN defined by the triplets (a_1, b_1, c_1) and (a_2, b_2, c_2), respectively. For this case, the necessary arithmetic operations with positive fuzzy numbers are:

(a) Addition

$$T_1 \oplus T_2 = [a_1 + a_2, b_1 + b_2, c_1 + c_2] \quad (2)$$

(b) Subtraction

$$T_1 \Theta T_2 = T_1 + (-T_2) \text{ when the opposite}$$
$$-T_2 = (-c_2, -b_2, -a_2) \text{ then}$$
$$T_1 \Theta T_2 = [a_1 - c_2, b_1 - b_2, c_1 - a_2] \quad (3)$$

(c) Multiplication

$$T_1 \otimes T_2 = [a_1 \times a_2, b_1 \times b_2, c_1 \times c_2] \quad (4)$$

(d) Division

$$T_1 \emptyset t T_2 = [[a_1, b_1, c_1] \cdot [1/c_2, 1/b_2, 1/a_2]], \quad 0 \ne [a_2, b_2, c_2] \quad (5)$$

(e) Scalar multiplication

$$k \circ T_1 = (k \circ a_1, k \circ b_1, k \circ c_1) \quad (6)$$

(f) Maximum and minimum

$$\mathrm{Max}(T_1, T_2) = [\mathrm{Max}(a_1, a_2), \mathrm{Max}(b_1, b_2), \mathrm{Max}(c_1, c_2)]$$
$$\mathrm{Min}(T_1, T_2) = [\mathrm{Min}(a_1, a_2), \mathrm{Min}(b_1, b_2), \mathrm{Min}(c_1, c_2)] \quad (7)$$

Defuzzification

Definition 5 Let $A = (a, b, c)$ be a fuzzy number, with membership function f_A, we define the area related to the left side as $S_L(A_i) = b - \int_a^b f_A^L(x)\mathrm{d}x = \int_0^1 g_A^L(y)\mathrm{d}y$, the area related to the right side as $S_R(A_i) = b + \int_b^d f_A^R(x)\mathrm{d}x = \int_0^1 g_A^R(y)\mathrm{d}y$, and the area related with the mode as $S_M(A_i) = b$. The meaning of $S_L(A_i)$, $S_M(A_i)$ and $S_R(A_i)$ are expressed in Fig. 1.

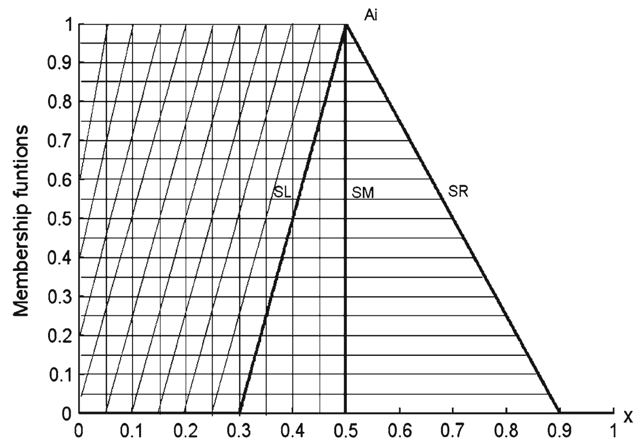


Fig. 1 Representation of $S_L(A_i)$, $S_M(A_i)$ and $S_R(A_i)$

In this way, we define an index that is a function of the three integrals previously defined.

Definition 6 The index associated with the ranking is a bi-convex combination:

$$\begin{aligned} I_{\beta,\lambda}(A_i) &= \beta S_M(A_i) + (1-\beta)[\lambda S_R(A_i) + (1-\lambda) S_L(A_i)] \\ &= \beta S_M(A_i) + (1-\beta)\lambda S_R(A_i) \\ &\quad + (1-\beta)(1-\lambda) S_L(A_i) \end{aligned} \quad (8)$$

$\beta \in [0, 1]$, is the index of modality that represents the importance of the central value against the extreme values and $\lambda \in [0, 1]$ is the degree of optimism of the decision maker. For more details, see Garcia-Cascales and Lamata (2007b).

Remark If we consider a TFN defined by the triplet (a, b, c), it is possible to consider different values β and λ in $I_{\beta,\lambda}(A_i)$. Thus, for example:

If $\lambda = 1/2$ and $\beta = 1/2 \Rightarrow I_{1/2,1/2}(A_i) = (\frac{a+6b+c}{8})$.

Analytic hierarchy process

The AHP methodology (Saaty 1980, 1989) has been accepted by the international scientific community as a robust and flexible multi-criteria decision making tool for dealing with complex decision problems. AHP has been applied to numerous decision problems such as energy policy (Kablan 2004), project selection (Cheng et al. 1999), measuring business performance (Al Harbi 2001), and evaluation of advanced manufacturing technology (Chan et al. 2000a,b). Basically, AHP has three underlying concepts: structuring the complex decision problem as a hierarchy of goal, criteria and alternatives, pair-wise comparison of elements al each level of the hierarchy with respect to each criterion on the preceding level, and finally vertically synthesizing the judgements over the different levels of the hierarchy. AHP attempts to estimate the impact of each one of the alternatives on the overall objective of the hierarchy. In this case, we only apply the method in order to obtain the criteria's weights.

We assume that the quantified judgements provided by the decision-maker on pairs of criteria (C_i, C_j) are represented in an $n \times n$ matrix as in the following:

$$C = \begin{array}{c} \\ C_1 \\ C_2 \\ \cdot \\ \cdot \\ C_n \end{array} \begin{bmatrix} c_1 & c_2 & \dots & c_n \\ c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \dots & c_{2n} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ c_{n1} & c_{n2} & \dots & c_{nn} \end{bmatrix} \tag{9}$$

The c_{12} value is supposed to be an approximation of the relative importance of C_1 to C_2, i.e., $c_{12} \approx (w_1/w_2)$. This can be generalized and the statements below can be concluded:

1. $c_{ij} \approx (w_i/w_j)\ i, j = 1, 2, \dots, n$
2. $c_{ii} = 1,\ i = 1, 2, \dots, n$
3. If $c_{ij} = \alpha, \alpha \neq 0$, then $a_{ji} = 1/\alpha,\ i = 1, 2, \dots, n$
4. If C_i is more important than C_j then $c_{ij} \cong (w_i/w_j) > 1$.

This implies that matrix A should be a positive and reciprocal matrix with 1's in the main diagonal and hence the decision maker needs only to provide value judgements in the upper triangle of the matrix. The values assigned to c_{ij} according to Saaty scale are usually in the interval of 1–9 or their reciprocals. In our case, Table 1 presents the linguistic decision-maker's preferences in the pair-wise comparison process.

It can be shown that the number of judgements (L) needed in the upper triangle of the matrix are:

$$L = n(n-1)/2 \tag{10}$$

where n is the size of the matrix C. In AHP problems, where the values are fuzzy, not crisp; instead of using lambda as an estimator to the weight, we will use the geometric normalized average, expressed by the following expression:

$$w_i = \frac{\prod_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij})}{\sum_{i=1}^{m} \prod_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij})} \tag{11}$$

where (a_{ij}, b_{ij}, c_{ij}) is a fuzzy number.

TOPSIS method

Technique for order performance by similarity to ideal solution is one of the known classical MCDM methods, that was developed by Hwang and Yoon (1981). It is based upon the concept that the chosen alternative should have the shortest distance from the PIS, and the farthest from the NIS.

This approach is employed for four reasons Wang and Chang (2007):

(a) TOPSIS logic is rational and understandable;
(b) the computation processes are straightforward;
(c) the concept permits the pursuit of best alternatives for each criterion depicted in a simple mathematical form, and
(d) the importance weights are incorporated into the comparison procedures.

In this study, the TOPSIS method, which is very simple and easy to implement, was used to select the preference order of the alternatives. The MCDM that includes both numeric and linguistic labels can be expressed in a matrix.

The computational steps of fuzzy TOPSIS

The fuzzy TOPSIS methods are derived from the generic TOPSIS method with minor differences, with the pertinent adaptation of the operations associated to the fuzzy numbers.

Step 1: Identify the evaluation criteria and the appropriate linguistic variables for the importance weight of the criteria and determine the set of feasible alternatives with the linguistic score for alternatives in terms of each criterion. Once the decision matrix is formed, the normalized decision matrix $(n_{ij}; i = 1, 2, \dots, m$ (number of alternatives); $j = 1, 2, \dots, n$ (number of criteria)) is constructed using Eq. 1:

$$\bar{n}_{ij}^{1} = \frac{z_{ij}}{\sqrt{\sum_{j=1}^{m} (z_{ij})^2}}, \quad j = 1, \dots, n,\ i = 1, \dots, m. \tag{12}$$

where z_{ij} is the performance score of alternative i against criteria j. This norm has the disadvantage that it performs dependent on the information in the following sense.
Two criteria and two alternatives are supposed, with valuations (5,7) and (6,7). The corresponding normalized points are (0.5812, 0.7592) and (0.8137, 0.6508), with the normalized values corresponding to the value "7" different in one case or the other. However, if we use the norm $\bar{n}_{ij}^{2} = z_{ij}/\underset{j}{\mathrm{Max}} z_{ij}$, $j = 1, \dots, n, i = 1, \dots, m$, we obtain the values (0.7142, 1) and (1, 0.8571), respectively, which we consider to be more appropriate.

Step 2: The weighted normalized decision matrix $\bar{v}_{ij}$ is calculated using Eq. 12. The weight of the criteria j is represented by w_j in Eq. 2:

$$\bar{v}_{ij} = w_j \otimes \bar{n}_{ij}, \quad j = 1, \ldots, n, \; i = 1, \ldots, m, \tag{13}$$

where, w_j such that $1 = \sum_{j=1}^{n} w_j$ is the weight of the jth attribute or criterion. It is well known that the weights of criteria in decision-making problem do not have the same mean and not all of them have the same importance.

Step 3: The ideal solution, $\bar{A}^+ \left(\bar{A}_i^+; i = 1, 2, \ldots, m\right)$, is made of all the best performance scores

$$\begin{aligned}\bar{A}^+ &= \{\bar{v}_1^+, \ldots, \bar{v}_n^+\} \\ &= \left\{\left(\max_i \bar{v}_{ij}, j \in J\right)\left(\min_i \bar{v}_{ij}, j \in J'\right)\right\} \\ i &= 1, 2, \ldots, m\end{aligned} \tag{14}$$

and the NIS, $\bar{A}^- \left(\bar{A}_i^-; j = 1, 2, \ldots, n\right)$, is made of all the worst performance scores at the measures in the weighted normalized decision matrix.

$$\begin{aligned}\bar{A}^- &= \{\bar{v}_1^-, \ldots, \bar{v}_n^-\} \\ &= \left\{\left(\min_i \bar{v}_{ij}, j \in J\right)\left(\max_i \bar{v}_{ij}, j \in J'\right)\right\} \\ i &= 1, 2, \ldots, m\end{aligned} \tag{15}$$

They are calculated using Eqs. 14 and 15 and where J is associated with benefit criteria, and J' is associated with cost criteria.

Step 4: The distance of an alternative to the ideal solution $\bar{d}_i^+$,

$$\bar{d}_i^+ = \left\{\sum_{j=1}^{n}\left(\bar{v}_{ij} - \bar{v}_j^+\right)^2\right\}^{\frac{1}{2}}, \quad i = 1, \ldots, m \tag{16}$$

and from the NIS $\bar{d}_i^-$ are calculated using Eqs. 3 and 4

$$\bar{d}_i^- = \left\{\sum_{j=1}^{n}\left(\bar{v}_{ij} - \bar{v}_j^-\right)^2\right\}^{\frac{1}{2}}, \quad i = 1, \ldots, m \tag{17}$$

in this case we use the m-multidimensional Euclidean distance

Step 5: The ranking score $\bar{R}_i$ is calculated using Eq. 18. The obtained ranking scores represent the alternatives' performance achievement within their status. A higher score corresponds to a better performance.

$$\bar{R}_i = \frac{\bar{d}_i^-}{\bar{d}_i^+ + \bar{d}_i^-}, \quad i = 1, \ldots, m \tag{18}$$

If $\bar{R}_i = 1 \rightarrow A_i = \bar{A}^+$
If $\bar{R}_i = 0 \rightarrow A_i = \bar{A}^-$
where the $\bar{R}_i$ value lies between 0 and 1. The closer the $\bar{R}_i = 1$ value implies a higher priority of the ith alternative.

Step 6: Rank the preference order

A decision problem in maintenance

We are going to study a decision problem in maintenance in an engine factory that is specialized in production, sale and maintenance of medium and slow speed four stroke engines (Garcia-Cascales et al. 2007).

One of the most important steps that should be done in a maintenance process and in the engine reparation is the cleaning of every component. The testing process and reconditioning of every component requires that every piece has a high quality cleaning; if not the reparation process will not be appropriate.

This implies certain types of pieces with some defined dimensions and a concrete grade of dirt that it is necessary to eliminate. The piece characteristics that affect the cleaning system are: the type of dirt, the size of the piece, the quantity of pieces to clean and the material which these pieces are made of.

The grades of dirt that we have to face are:

- *Solid carbon powder*: The solid carbon powder is an agglomerate of a fine powder of coal and not burnt compounds coming from the fuel or the oil. This type of dirt is where there are exhaust gases. This type of dirt is always forming solid scabs stuck strongly to the walls of the pieces.
- *Oil and grease*: They come from the oil motor and the additives of the diverse fluids that develop inside the circuits of the motor. This type of dirt is usually less stuck to the piece and does not include solid products.
- *Other*: Inside this section mention can be made of the layers of painting of the pieces; that must not be considered as dirt in principle; and the calcareous incrustation of the refrigeration water.
  The problems concerned with are pieces with diverse degrees of dirt, with very different geometry, a work process that demands speed and flexibility. Depending on the type of dirt a procedure is applied. The processes for the cleaning of pieces are very diverse and each one has advantages and their drawbacks. Most of them could be classified as follows:
- *Conventional cleaning*: Inside this group they are considered the industrial washing machines that apply a pressurized cleaning mixture on the piece; basically the

applied product is water with a percent of detergent with a specific cleaning additive.

- *Chemical cleaning*: The chemical cleaning consists in the application of an aggressive chemical product to dissolve the dirt. The results observed with this type of machines are satisfactory for pieces of light and moderate dirt, for pieces of high dirt as those that have solid carbon powder satisfactory results are obtained applying medium/long times of process.
- *Thermal cleaning*: The thermal cleaning is an alternative to the chemical cleaning in the cases of pieces that have solid carbon powder. This type of cleaning is based in the fact that the solids included in the solid carbon powder can be disintegrated if all the components are completely burnt.
  This procedure is very used by the restorers of diesel motors and very recommended for its effectiveness. The problem of this method besides that the opposing machines are not adjusted to the piece size and that they have a very high cost of acquisition, it is that the high temperatures necessary in the standard processes of maintenance of the engines cause unacceptable structural defects in the pieces. Therefore it makes us discard this alternative a priori.
- *Mechanical cleaning*: Inside this group we can consider the drip with sand or another sent out material (silica, glass) this method is good to eliminate inlays when they are in accessible places, but in the pieces there are usually conduits and turns that are not accessible or they are susceptible of catching abrasive particles that later are loosened during the operation, producing failures. Therefore, this alternative will not be considered either a priori.
- *Ultrasonic cleaning*: The ultrasonic cleaning is in fact a mixed process, it uses the action of a conventional detergent and the mechanical action of some shock waves and cavitations that take place in the recipient that contains the piece.

Structuring the problem

The global objective of the problem is to decide which the best system for cleaning of pieces is. As we have commented above, the problems concerned with are pieces with diverse degrees of dirt, with very different geometry, a work process that demands speed and flexibility. The problem of the thermal method besides that the opposing machines are not adjusted to the piece size and that they have a very high cost of acquisition, it is that the high temperatures necessary in the standards processes of maintenance of the engines cause unacceptable structural defects in the pieces. Therefore, it makes us discard this alternative a priori. The case of the mechanical process is sometimes essential, it cannot be considered as a complete process of cleaning but as a support in certain very concrete cases. Therefore, this alternative will not be considered either a priori. Therefore, we only take into account three alternatives.

Criteria like the total cost of annual operation, the productivity of the used system, the load capacity of the system, the cleaning efficiency and the healthiness of the used products must be taken into account. Being quantifiable numerically the criteria total cost of annual operation and the productivity of the used system, while the rest of the criteria are qualitative described by means of linguistic labels and quantified by means of fuzzy numbers.

In this way, we are dealing with a problem characterized by the following components. We use a hierarchy structure with two levels as representation of this problem (Fig. 2).

1. *Objective*

   Choose the best cleaning system

2. *Alternatives*

   - A1: Conventional cleaning
   - A2: Chemical cleaning
   - A3: Ultrasonic cleaning

3. *Criteria*

   - C1: Total cost of annual operation
   - C2: Productivity volumetric of the system
   - C3: Capacity of load of the system
   - C4: Efficiency in the cleaning
   - C5: Healthiness

Data collection

Questionnaire

Here, we describe the procedure of acquisition and processing of linguistic labels used by the decision-maker, both for the labels which measure the importance of the criteria as well as for the labels that evaluate the goodness of the alter-

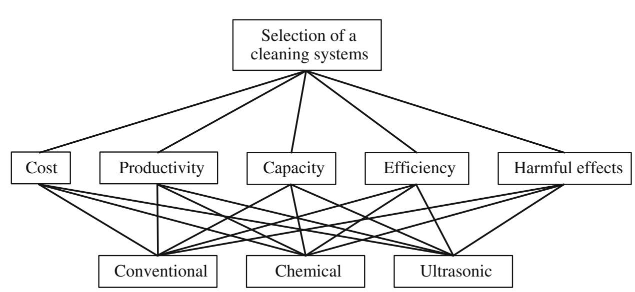


Fig. 2 Hierarchy structure of the problem

natives with respect to the criteria. In this case the decision-maker is the assembly workshop manager (Garcia-Cascales and Lamata 2007a) and the process to obtain these values is as follows:

- Step 1: The evaluator is asked to mark an interval in favour of the meaning of the linguistic term used.
- Step 2: The evaluator is asked to mark only one point that represents the meaning of the linguistic term used in.
- Step 3: Repeat the above steps for every linguistic term.
- Step 4: Obtain the fuzzy number corresponding to every linguistic label defined; where the support of the fuzzy number corresponds with the interval defined by the evaluator (a, c) in step 1 and the central value of the fuzzy number with the point value (b) obtained in step 2.

Weights

We consider the AHP approach, in which the decision-maker compares these five criteria (Badri 2001; Bolloju 2001; Deng 1999; Lipovetsky and Conklin 2002; Zhu et al. 1999) with the information given by the assembly workshop manager, and which was considered in linguistic terms is reduced at the function

$$f(x) = \begin{cases} [0.58, x, 1.71] & \text{for } x = 1 \\ [x - 1.3, x, x + 0.7] & \text{for } x = 2, 3, \ldots, 8 \\ [7.7, x, 9] & \text{for } x = 9 \end{cases} \quad (19)$$

Taking into account (19), the table of verbal judgements of preferences following Saaty, see Table 1, this stays for the fuzzy case, which is what concerns us, since: with this function (19) and the application of AHP method, together with the normalized geometric average, we obtain Table 2.

Ratings

Once that set of criteria and alternatives have been selected, then we need a measure of the effect produced by each alternative with respect to each criterion.

By means of linguistic terms, the decision-maker represents the goodness of the alternatives with respect to criteria C_1, C_2 and C_3. The different values of r can be represented by means of matrix called the "Matrix of decision making". The relative rating to C_1 represents the current cost, to which we have added a percentage as an estimate of a possible increase in the price. However, for C_2 the data relating to the three systems are facilitated by the manufacturer.

$$\begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{ccccc} C_1(\text{miles €}) & C_2(m^3/h) & C_3 & C_4 & C_5 \end{array}$$
$$\begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} \begin{pmatrix} 21{,}399{+}2\%{+}4\% & 0.768 \pm 3\% & \text{Fair} & \text{Fair} & \text{Medium Good} \\ 27{,}971{+}2\%{+}4\% & 0.468 \pm 3\% & \text{Very Good} & \text{Medium Good} & \text{Very Poor} \\ 45{,}479{+}2\%{+}4\% & 2.520 \pm 3\% & \text{Medium Good} & \text{Medium Good} & \text{Medium Good} \end{pmatrix}$$

Evaluation

In this case, as mentioned above, we use the TOPSIS multicriteria decision-making with the new norm, $\bar{n}_{ij}^2 = z_{ij}/\underset{j}{\text{Max}}\, z_{ij}$, $j = 1, \ldots, n, i = 1, \ldots, m$.

Table 1 Decision-maker's preferences in the pair-wise comparison process

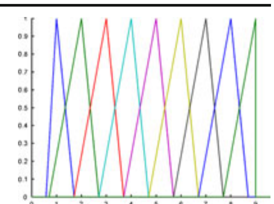
Verbal judgements of preferences between alternative i and alternative j	Decision maker fuzzy numbers	Linguistic representation
A_i y A_j is equally important to (EI)	[0.58, 1, 1.71]	
A_i is slightly more/less important than A_j(SmI)/(SlI)	[1.7, 3, 3.7]/[0.27, 0.33, 0.59]	
A_i is strongly more/less important than A_j(SMI)/(SLI)	[3.7, 5, 5.7]/[0.17, 0.2, 0.27]	
A_i is very strongly more/less important than A_j(VSMI)/(VSLI)	[5.7, 7, 7.7]/[0.13, 0.14, 0.17]	
A_i is extremely more/less important than A_j(EMI)/(ELI)	[7.7, 9, 9]/[0.11, 0.11, 0.13]	
And its intermediate values +(legend)		

Table 2 Pair-wise comparison of the criteria with respect to the goal

	C1	C2	C3	C4	C5	Average	Normalized
C1	(EI)	+(EI)	+(SmI)	(SmI)	(SmI)	1.2594, 2.3522, 3.1230	[0.1357, 0.3942, 0.9035]
C2	−(EI)	(EI)	(SmI)	+(EI)	(SmI)	0.8463, 1.5518, 2.4616	[0.0912, 0.2601, 0.7121]
C3	−(SlI)	(SlI)	(EI)	−(SlI)	−(EI)	0.3030, 0.3642, 0.7230	[0.0327, 0.0610, 0.2092]
C4	(SlI)	−(EI)	+(SmI)	(EI)	+(EI)	0.6425, 1.0571, 1.7887	[0.0693, 0.1772, 0.5175]
C5	(SlI)	(SlI)	+(EI)	−(EI)	(EI)	0.4054, 0.6418, 1.1811	[0.0437, 0.1076, 0.3417]
						3.4566, 5.9671, 9.2773	

Computing the local priority vector

Table 3 Computation weighted normalizad decision matrix and ideal solution $\bar{A}^{+}$ and the negative ideal solution $\bar{A}^{-}$

	C1 (0.14, 0.39, 0.90)	C2 (0.09, 0.26, 0.71)	C3 (0.03, 0.06, 0.21)	C4 (0.07, 0.18, 0.52)	C4 (0.04, 0.10, 0.34)
A_1	(21.349, 21.776, 22.203)	(0.745, 0.768, 0.791)	(4.2, 4.8, 5)	(4.2, 4.8, 5)	(5, 5.8, 6.8)
A_2	(27.971, 28.530, 29.090)	(0.454, 0.468, 0.482)	(7.9, 9, 10)	(5, 5.8, 6.8)	(0, 1, 1.9)
A_3	(45.479, 46.388, 47.298)	(2.446, 2.52, 2.595)	(5, 5.8, 6.8)	(5, 5.8, 6.8)	(5, 5.8, 6.8)
Dreadful	(45.479, 46.388, 47.298)	(0.454, 0.468, 0.482)	(0, 1, 1.9)	(0, 1, 1.9)	(0, 1, 1.9)
Excellent	(21.349, 21.776, 22.203)	(2.446, 2.520, 2.595)	(7.9, 9.10)	(7.9, 9.10)	(7.9, 9.10)
$\bar{n}_{ij}^{2} A_1$	(0.4514, 0.4694, 0.4882)	(0.2871, 0.3048, 0.3234)	(0.42, 0.53, 0.63)	(0.42, 0.53, 0.63)	(0.5, 0.64, 0.86)
$\bar{n}_{ij}^{2} A_2$	(0.5914, 0.6150, 0.6396)	(0.1750, 0.1857, 0.1971)	(0.79, 1.00, 1.26)	(0.50, 0.64, 0.86)	(0.00, 0.11, 0.24)
$\bar{n}_{ij}^{2} A_3$	(0.9615, 1.0000, 1.0400)	(0.9426, 1.0000, 1.0609)	(0.50, 0.64, 0.86)	(0.50, 0.64, 0.86)	(0.50, 0.64, 0.86)
Dreadful	(0.9615, 1.0000, 1.0400)	(0.1750, 0.1857, 0.1971)	(0.00, 0.11, 0.24)	(0.00, 0.11, 0.24)	(0.00, 0.11, 0.24)
Excellent	(0.4514, 0.4694, 0.4882)	(0.9426, 1.0000, 1.0609)	(0.79, 1.00, 1.26)	(0.79, 1.00, 1.26)	(0.79, 1.00, 1.26)
$\bar{v}_{ij}^{2} A_1$	(0.0613, 0.1850, 0.4411)	(0.0262, 0.0793, 0.2303)	(0.01, 0.03, 0.13)	(0.03, 0.09, 0.32)	(0.02, 0.07, 0.29)
$\bar{v}_{ij}^{2} A_2$	(0.0803, 0.2424, 0.5779)	(0.0160, 0.0483, 0.1403)	(0.02, 0.06, 0.26)	(0.03, 0.11, 0.44)	(0.00, 0.01, 0.08)
$\bar{v}_{ij}^{2} A_3$	(0.1305, 0.3942, 0.9396)	(0.0860, 0.2601, 0.7555)	(0.02, 0.04, 0.18)	(0.03, 0.11, 0.44)	(0.02, 0.07, 0.29)
Dreadful	(0.1305, 0.3942, 0.9396)	(0.0160, 0.0483, 0.1403)	(0.00, 0.01, 0.05)	(0.00, 0.02, 0.12)	(0.00, 0.01, 0.08)
Excellent	(0.0613, 0.1850, 0.4411)	(0.0860, 0.2601, 0.7555)	(0.02, 0.06, 0.26)	(0.05, 0.18, 0.65)	(0.03, 0.11, 0.43)
$\bar{A}^{+}$	(0.0613, 0.1850, 0.4411)	(0.0860, 0.2601, 0.7555)	(0.02, 0.06, 0.26)	(0.00, 0.02, 0.12)	(0.03, 0.11, 0.43)
$\bar{A}^{-}$	(0.1305, 0.3942, 0.9396)	(0.0160, 0.0483, 0.1403)	(0.00, 0.01, 0.05)	(0.05, 0.18, 0.65)	(0.00, 0.01, 0.08)

The awful and the excellent alternatives are fictitious

Table 4 Computation distance to ideal solution $\bar{d}_i^{+}$ and from the negative ideal solution $\bar{d}_i^{-}$ and the ranking score, by means of fuzzy numbers

A_1d+	(0.0674, 0.2045, 0.6479)
A_2d+	(0.0828, 0.2475, 0.7509)
A_3d+	(0.0738, 0.2228, 0.5646)
A_1d−	(0.0801, 0.2329, 0.5912)
A_2d−	(0.0663, 0.1868, 0.5290)
A_3d−	(0.0827, 0.2411, 0.7370)
R A_1	(0.0646, 0.5325, 4.0101)
R A_2	(0.0518, 0.4301, 3.5488)
R A_3	(0.0636, 0.5197, 4.7079)

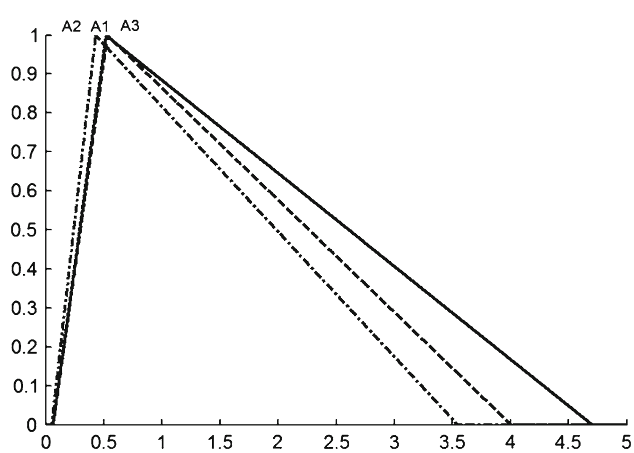


Fig. 3 Ranking score

At this moment, we have two possibilities. The first consists in choosing the best alternative from these three, although it may not be the best in absolute terms. This is the consequence of it being a compensatory method. In the second option, we would obtain the best alternative in absolute terms, to do so we introduce for assessment, in addition to the three alternatives to be considered, a further two fictitious alternatives. These two alternatives would correspond to the awful and excellent alternatives. The worst would be that which received a very poor valuation in all the attributes. On the other hand, the excellent alternative would have very good valuations for all the criteria.

It must be taken into account that criterion C_1 represents the cost, so for this, for the excellent alternative and the value of A^{+} it will be necessary to consider the one that has a

Table 5 Ranking with different values of λ y β

	$\lambda = 1/2, \quad \beta = 1/2$	$\lambda = 1/2, \quad \beta = 1/3$	$\lambda = 1/2, \quad \beta = 2/3$
I A_1	0.9087	1.0341	0.7833
I A_2	0.7727	0.8868	0.6585
I A_3	0.9862	1.1417	0.8307
Ranking	$\mathbf{A_3 > A_1 > A_2}$	$\mathbf{A_3 > A_1 > A_2}$	$\mathbf{A_3 > A_1 > A_2}$

value from among those possible, which is the minimum (Tables 3, 4).

Since the ranking according to the fuzzy numbers to which we have reached is not direct, as an alternative does not exist so that all their values may be greater than the corresponding values of the other two alternatives (Fig. 3), it is necessary to apply the process of defuzzification.

Therefore, the IA_i, is computed taking into account the formula (8), for several possible values of λ and β (Table 5).

Conclusions

The most common drawback of existing multicriteria methods, at least for some classes of problems, is the need to translate the decision makers' knowledge about a decision problem into numbers and functions. There are decision problems in which qualitative judgment prevails over more or less exact quantitative evaluation. For such problems, a natural choice is to use models that incorporate qualitative (descriptive, linguistic, ordinal) variables.

In this paper, we have studied a problem of maintenance management, the selection of a cleaning system for pieces of four stroke engines. So that, by surveys to the decision-maker, in this case the assembly workshop manager, we have developed the study by means of linguistic variables, which we have transformed into fuzzy numbers.

It is possible to see how the last alternative, "the ultrasonic cleaning" A_3, is the best alternative with this method, for different values of λ and β, in the defuzzification step.

Having presented the results to the decision-maker, he considers that his satisfaction is more in accordance with the results of the solution.

For the decision-maker the best option was A_3 the ultrasonic cleaning, because it is the best solution for all the criteria specially for the healthiness criterion and the decision-maker does not take into account another counterpart of this alternative, while the chemical cleaning A_2 is a good option for the decision-maker but it has the problem of healthiness; and finally the conventional cleaning A_1 cannot solve the problem.

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