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An intelligent approach to machine tool selection through fuzzy analytic network process

Z. Ayağ · R. G. Özdemir

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Abstract In this study, we utilize analytic network process (ANP), a more general form of AHP, for justifying stand-alone machine tools out of available alternatives in market due to the fact that AHP cannot accommodate the variety of interactions, dependencies and feedback between higher and lower level elements. However, due to the vagueness and uncertainty on judgments of a decision-maker, the crisp pair wise comparison in the conventional ANP seems to be insufficient and imprecise to capture the right judgments of the decision-maker. That is why, also in this paper, fuzzy number logic is introduced in the pair wise comparison of ANP to make up for this deficiency in the ANP. In short, here, an intelligent approach to machine tool selection (MTS) problem through fuzzy ANP is proposed to improve the imprecise ranking of company's requirements which is based on the conventional ANP. In order to reach to final solution, a preference ratio (PR) analysis is done by using the results of the fuzzy ANP, and investment costs of alternatives. In addition, a numerical example is presented to illustrate the proposed approach.

Keywords Fuzzy logic · Analytic network process (ANP) · Multiple-criteria decision making (MCDM) · Machine tool selection

Z. Ayağ (✉)
Faculty of Engineering, Department of Industrial Engineering, Kadir Has University, Kadir Has Campus, 34083 Cibali, Istanbul, Turkey
e-mail: zekia@khas.edu.tr

R. G. Özdemir
Faculty of Engineering and Architecture, Department of Industrial Engineering, Istanbul Kültür University, Atakoy Campus, D-100 Yanyol, 34156 Bakirkoy, Istanbul, Turkey

Introduction

Selecting a proper stand-alone machine tool among various alternatives in market has been very important issue for manufacturing companies due to the fact that improperly selected one can negatively affect the overall performance of a manufacturing system. In addition, the outputs of manufacturing system (i.e. throughout, quality, cost) are mostly dependent on what kinds of properly selected and implemented machines tools are used. On the other hand, the selection of a new machine tool is a time-consuming and difficult process that requires advanced knowledge and experience and experience deeply. So, the process can be hard task for engineers and managers, and also for machine tool manufacturer or vendor, to carry out. For a proper and effective evaluation, the decision-maker may need a large amount of data to be analyzed and many factors to be considered. The decision-maker should be an expert or at least be very familiar with the specifications of machine tool to select the most suitable among the others. However, a survey conducted by Gerrard (1988a) reveals that the role of engineering staff in authorization for final selection is 6%, the rest belongs to middle and upper management (94%). Gerrard also indicated the need for a simplified and practical approach for the machine selection process.

Machine tool selection (MTS) problem is typical multiple-criteria decision making (MCDM) problem in the presence of various selection criteria and a set of possible alternatives. Among the available multi-attribute approaches, only the analytic hierarchy process (AHP) approach, first introduced by Saaty (1981) has the capabilities to combine different types of criteria in a multi-level decision structure to obtain a single score for each alternative to rank the alternatives (Yurdakul 2004). In AHP, a hierarchy considers the distribution of a goal amongst the elements being compared, and judges which element has a greater influence on that goal. In reality, a holistic approach like analytic network process (ANP), a more general form of AHP is needed if all attributes and alternatives involved are connected in a network system that accepts various dependencies. Several decision problems cannot be hierarchically structured because they involve the interactions and dependencies in higher or lower level elements. Not only does the importance of the attributes determine the importance of the alternatives as in AHP, but the importance of alternatives themselves also influences the importance of the attributes.

In conventional ANP developed by Saaty, the pair wise comparisons for each level with respect to the goal of the best alternative selection are conducted using a nine-point scale of Saaty (1989).

The application of Saaty's ANP has some shortcomings as follows: (1) The ANP method is mainly used in nearly crisp decision applications, (2) The ANP method creates and deals with a very unbalanced scale of judgment, (3) The ANP method does not take into account the uncertainty associated with the mapping of one's judgment to a number, (4) Ranking of the ANP method is rather imprecise, (5) The subjective judgment, selection and preference of decision-makers have great influence on the ANP results.

Furthermore, a decision-maker requirements for evaluating machine tool alternatives always contain ambiguity and multiplicity of meaning. Additionally, it is also recognized that human assesment on qualitative atrributes is always subjective and thus imprecise. Therefore, the conventional ANP seems to be inadequate to capture the decision-maker‘s requirements explicitly. In order to model this kind of uncertainty in human preference, fuzzy sets could be incorporated with the pair wise comparison as an extension of ANP, called *fuzzy ANP*.

Fuzzy set theory is a mathematical theory pioneered by Zadeh (Lootsma 1997), designed to model the vagueness or imprecision of human cognitive processes. This theory is basically a theory of classes with non-sharp boundaries. What is important to recognize is that any crisp theory can be made fuzzy by generalizing the concept of a set within that theory to the concept of a fuzzy set (Zadeh 1994).

In this paper, an intelligent approach to machine tool selection problem through fuzzy ANP is proposed to make up the vagueness and uncertainty existing in the importance attributed to judgment of the decision-maker. In order to reach to final solution, a preference ratio (PR) analysis is done by using the results of the fuzzy ANP, and investment costs of alternatives. In addition, to prove the applicability of the proposed approach, a numerical example is presented.

Literature survey

Procurement of a new machine tool requires that many alternatives have to be evaluated under several conflicting factors (table size, spindle speed, power, axis travel, positioning accuracy, repeatability, work-piece material and sizes, cutting tool requirements, etc). In the literature of machine tool selection problem, there are quite good numbers of studies proposing multi criteria decision making models. Some of them structured the analytic hierarchy process framework to solve the machine tool selection problem (Cimren et al. 2004; Yurdakul 2004). A CNC machine selection methodology using DEA studied by Sun (2002). Georgakellos (2005) proposed a scoring model in which technical and commercial criteria are involved. Layek and Lars (2000) and Gopalakrishnan et al. (2004) studied on machine center selection problem and modeled expert systems as a solution methodology. Fuzzy logic is incorporated in machine tool selection models when imprecise and/or vague data need to be processed. Chu and Lin (2003) proposed a fuzzy TOPSIS model for robot selection. A fuzzy multiple criteria decision making model that helps decision makers solving the machine selection problem was proposed by Wang et al. (2000). They particularly dealt with the machine selection problem in a flexible manufacturing cell. Jiang and Hsu (2003) used a fuzzy analytic hierarchy process for selecting advanced manufacturing technologies. Iç and Yurdakul (2009) developed a decision support system in which a pre-selection module with several questions determines a feasible set of machining centers. The developed model uses either fuzzy analytical hierarchy process or fuzzy TOPSIS according to accuracy required to rank the feasible machining centers. Yurdakul and Iç (2009) studied on the benefit generated by using fuzzy numbers in multi criteria decision making model for machine tool selection problems. They suggested employing fuzzy numbers when a high level of vagueness exits in data, otherwise crisp evaluation should be preferred. Dura'n and Aguilo (2008) developed an analytic hierarchical process based on fuzzy numbers, the proposed multi-attribute method for the evaluation and justification of an advanced manufacturing system is then illustrated by an example problem. Tabucanon et al. (1994) technique for developed a decision support system for multi-criteria MTS problem for FMS, and used the AHP the selection process. Wang et al. (2000) proposed a fuzzy MCDM model to assist the decision-maker to deal with the MTS problem for a FMS. MTS from fixed number of available machines is considered by Atmani and Lashkari (1998). They developed a model for MTS and operation allocation in FMS. The model assumes that there is a set of machines with known processing capabilities. The AHP is also proposed by Lin and Yang (1994) to evaluate what type of machine tool is the most appropriate for machining the certain parts. Goh et al. (1995) proposed a revised weighted sum decision model for robot selection by using weights assigned by a group of experts. Gerrard (1988b) also proposed a step-by-step methodology for the selection and introduction of new machine tools. Yurdakul (2004) defined a model the links between machine tool alternatives and manufacturing strategy for MTS. He presented such a strategic justification tool for machine tools by using AHP and ANP. Oeltjenbruns et al. (1995) proposed AHP for MTS problem. Arslan et al. (2004) also proposed a multi-criteria weighted average (MCWA) method for MTS. They classified all of machine tools in the market to create a database so that decision attributes can be easily determined to use in the related method. Almutawa et al. (2005) developed an approach for optimizing the number of machines acquired for batch processing in a multi-stage manufacturing system.

Fuzzy set theory and fuzzy logic have been applied in a great variety of applications, as reviewed by several authors (Klir and Yuan 1995; Zimmermann 1996). In literature, in the most of studies, triangular fuzzy numbers (TFNs) have been used to construct pair wise comparisons for the AHP by applying extent analysis (Chan 1996; Bozdag et al. 2003; Chan et al. 2003; Kahraman et al. 2004; Ayag 2005, 2006; Ayag and Ozdemir 2006b,c,e). In fuzzy ANP, the linguistic assessment is transformed to TFNs that are used to build a pair-wise comparison matrix for the ANP and, by applying extent analysis, one can obtain the weights for attributes on each level. In fuzzy ANP, the calculation of weights are more simple to calculate than for conventional ANP. Several authors have applied the fuzzy ANP-based approach to solve complex decision-making scenarios as follows: Buyukozkan et al. (2004) used ANP to prioritize design requirements by taking into account the degree of the interdependence between the customer needs and design requirements and the inner dependence among them. They also integrated fuzzy logic with ANP and used TFN to improve the quality of the responsiveness to customer needs and design requirements due to the fact that human judgment on the importance of requirements is always imprecise and vague. Mikhailov and Singh (2003) applied fuzzy ANP to the development of decision support systems. Ayag and Ozdemir (2006a) used the fuzzy ANP for ERP software package selection. More studies have been also realized by many researchers (Lee and Kim 2000; Karsak et al. 2002; Chung et al. 2005).

Procurement of a new machine tool requires that many alternatives have to be evaluated under several conflicting factors (table size, spindle speed, power, axis travel, positioning accuracy, repeatability, work-piece material and sizes, cutting tool requirements, etc). In literature, many methods have been used for the machine tool selection problem. The following methods have been generally proposed: TOPSIS, ELECTRE, AHP, ANP, weighted sum model (WSM) and weighted product model (WPM). These methods use numeric techniques to help DM(s) choose among a discrete set of machine tool alternatives. This is achieved on the basis of the impact of the alternatives on certain criteria, and thereby on the overall utility of the DM(s). Despite the criticism that multi-dimensional methods have received, some of them are widely used. The WSM is the earliest and probably the most widely used method. The weighted product model can be considered as a modification of the WSM, and has been proposed in order to overcome some of its weaknesses. Both methods: WSM and WPM are not used for this study because these methods use actual values which are not certain in the MTS problem.

The analytic hierarchy process (AHP), as proposed by Thomas L. Saaty is a later development and it has recently become increasingly popular. But, analytic network process (ANP), a more general form of AHP, is more powerful than the AHP, because AHP cannot accommodate the variety of interactions, dependencies and feedback between higher and lower level elements.

Some other widely used methods are the ELECTRE and TOPSIS. The TOPSIS (Techniques for Order Preference by Similarity to an Ideal Solution) method which is a multiple criteria method to identify solution from finite set of points. The basic principle is that the chosen points should have the "shortest" distance from the positive ideal and the "farthest" distance from the negative ideal solution. ELECTRE is used for ranking a set of alternatives based on evaluation criteria. This method takes the quantitative and qualitative criteria into consideration. In this method, the output is a set of ranks such that the necessary concordance will be provided in the most appropriate form. ELECTRE uses a new concept known as outranking. All the alternatives are assessed using the outranking comparisons and the non-effective alternatives are omitted. Pair comparisons performed based on agreement rank of weights and difference rank from weighting assessment values and are tested simultaneously for alternatives assessment. All these steps are planned according to a concordant and a discordant set that is known as concordance analysis.

In this work, we selected the ANP method integrated with fuzzy logic to solve the MTS problem. The fuzzy ANP is to make up the vagueness and uncertainty existing in the importance attributed to judgment of the DM. A fuzzy logic method providing more accuracy on judgments is applied. The resulting fuzzy ANP enhances the potential of the conventional ANP for dealing with imprecise and uncertain human comparison judgments.

Proposed approach

In this section, we firstly construct the ANP framework in which the critical determinants, dimensions and attribute-enablers are identified for the MTS problem (i.e. selection

Table 1 Determinant, dimensions and attribute-enablers in the ANP-based network for MTS problem

Determinants	Dimensions	Code	Definition	Code
Improved customer satisfaction (ICS)	Increased productivity	IPR	Spindle speed	SPS
			Main power	MPW
			Cutting feed	CFD
			Traverse speed	TSP
	Higher flexibility	HFL	Tool change time	TCT
			Capacity of rotary table	CRT
			Average set-up time for product change	AST
	Effective use of space	EUS	Machine dimensions	MDM
			Area for accessories	ARA
			Difficulty degree to locate in-site	DDL
Increased profitability (IPF)	Better adaptability	BAD	DNC integration	DNC
			CNC capability	CNC
			Upgradeability	UPG
	Better precision and accuracy	BPA	Repeatability	RPT
			Thermal deformation	TDF
			Checking probe installed	CPI
	Increased reliability	IRL	Bearing failure rate	BFR
			Reliability of drive system	RDS
			Reliability of computer-controlled system	RCC
	More safety and environment	MSE	Operator training for safety	OTS
			Proportion of recycling components	PRC
			Safety accessories (i.e. mist collector)	SAC
	Satisfied maintenance and service	SMS	Specialized training	STR
			On-time repair service	ORS
			Regular maintenance	RMN

of CNC vertical turning centers for general use). Then, the fuzzy logic and its steps are presented to form the fuzzy ANP.

Building of ANP framework

To build the ANP framework related to MTS problem, first we determined the elements (i.e. determinants, dimensions and attribute-enablers) based upon the needs and expectations of a typical manufacturing system in which the ultimate machine tool will be used (Table 1). We also utilized knowledge of experts, vendors in the field, together with a deep review of the literature. Then, we constructed the schematic representation of ANP-based framework and its decision environment as illustrated in Fig. 1. The overall objective is to find out the weights of machine tool alternatives for the preference ratio (PR) analysis. The investment cost of each alternative used in PR analysis is evaluated separate criterion rather than the elements in the ANP hierarchy, to compare the benefits and costs of alternatives in a good manner.

This framework represents relationships hierarchically but does not require as strict a hierarchical structure, and therefore allows for more complex interrelationships among the decision levels and attributes. After constructing this flexible hierarchy, the decision-maker is asked to compare the elements at a given level on a pair wise basis to estimate their relative importance in relation to the element at the immediate proceeding level. In conventional ANP, the pair wise comparison is made using a ratio scale. A frequently used scale is the nine-point scale (Saaty 1989) which shows the participants' judgments or preferences. Even though this nine-point scale has the advantages of simplicity and easiness for use, it does not take into account the uncertainty associated with the mapping of one's perception or judgment to a number. Therefore, the conventional ANP seems to be inadequate to capture decision maker‘s requirements explicitly. In order to model this kind of uncertainty in human preference, fuzzy sets could be incorporated with the pair wise comparison as an extension of ANP, called *fuzzy ANP*.

Fuzzy logic

The key idea of fuzzy set theory is that an element has a degree of membership in a fuzzy set (Negoita 1985 and

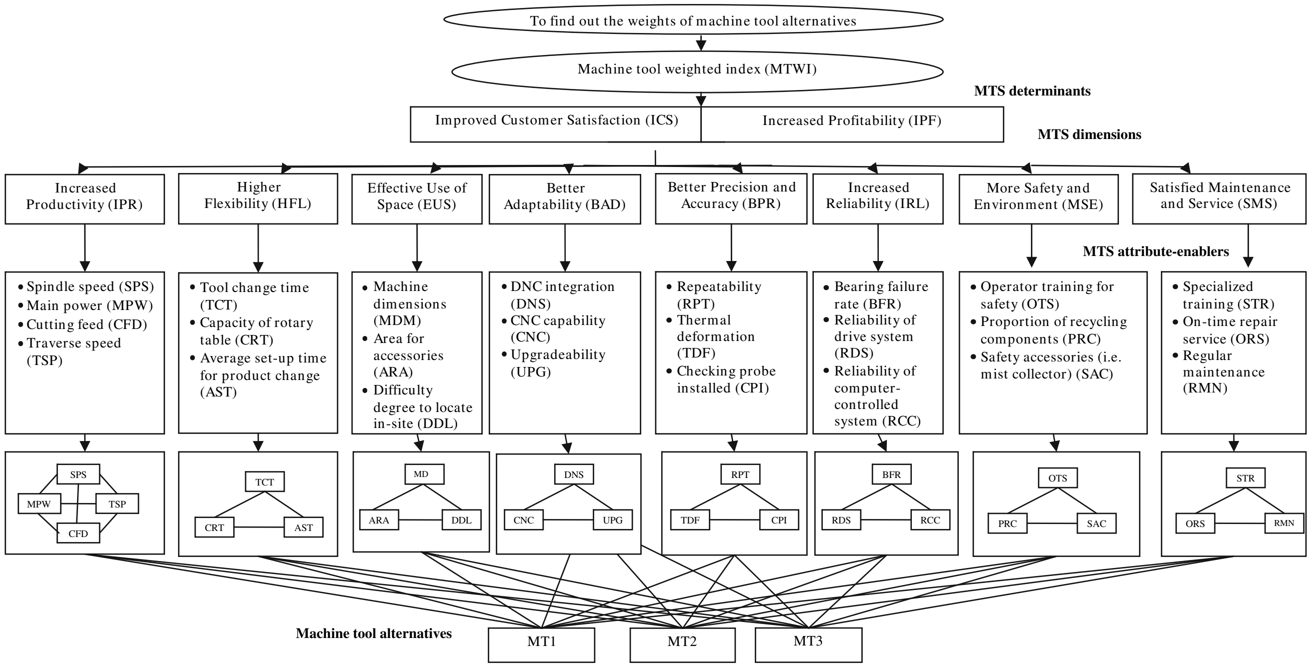

Fig. 1 Fuzzy ANP-based framework for MTS problem

Table 2 Definition and membership function of fuzzy number (Ayag 2005)

Intensity of importance [a]	Fuzzy number	Definition	Membership function
1	$\tilde{1}$	Equally important/preferred	(1,1,2)
3	$\tilde{3}$	Moderately more important/preferred	(2,3,4)
5	$\tilde{5}$	Strongly more important/preferred	(4,5,6)
7	$\tilde{7}$	Very strongly more important/preferred	(6,7,8)
9	$\tilde{9}$	Extremely more important/preferred	(8,9,10)

[a] Fundamental scale used in pair wise comparison (Saaty 1989)

Zimmermann 1996). A fuzzy set is defined by a membership function (all the information about a fuzzy set is described by its membership function). The membership function maps elements (crisp inputs) in the universe of discourse (interval that contains all the possible input values) to elements (degrees of membership) within a certain interval, which is usually [0, 1]. Then, the degree of membership specifies the extent to which a given element belongs to a set or is related to a concept. The most commonly used range for expressing degree of membership is the unit interval [0, 1]. If the value assigned is 0, the element does not belong to the set (it has no membership). If the value assigned is 1, the element belongs completely to the set (it has total membership). Finally, if the value lies within the interval [0, 1], the element has a certain degree of membership (it belongs partially to the fuzzy set). A fuzzy set, then, contains elements that have different degrees of membership in it.

In this study, triangular fuzzy numbers (TFNs), $\tilde{1}$ to $\tilde{9}$, are used to represent subjective pair wise comparisons of selection process (equal to extremely preferred) in order to capture the vagueness (Table 2). A fuzzy number is a special fuzzy set $F = \{(x, \mu_F(x)), x \in R\}$, where x takes it values on the real line, $R : -\infty < x < +\infty$ and $\mu_F(x)$ is a continuous mapping from R to the closed interval [0, 1]. A TFN denoted as $\tilde{M} = (l, m, u)$, where $l \leq m \leq u$, has the following triangular type membership function;

$$\mu_F(x) \begin{cases} 0 & x < l \\ \frac{x-l}{m-l} & l \leq x \leq m \\ \frac{u-x}{u-m} & m \leq x \leq u \\ 0 & x > u \end{cases}$$

Alternatively, by defining the interval of confidence level α, the TFN can be characterized as:

$$\forall\alpha \in [0,1] \quad \tilde{M}_\alpha = \left[l^\alpha, u^\alpha\right] = [(m-l)\,\alpha + l, \\ -(u-m)\,\alpha + u]$$

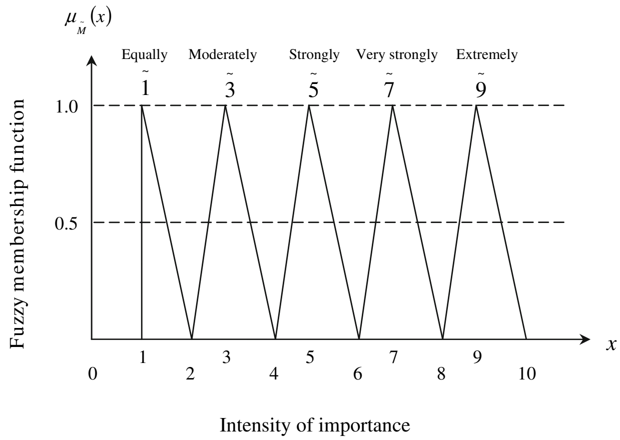

Fig. 2 Fuzzy membership function for linguistic values for attributes or alternatives

Some main operations for positive fuzzy numbers are described by the interval of confidence, by Kaufmann and Gupta (1988) as given below:

$$\forall m_L, m_R, n_L, n_R \in R^+, \tilde M_\alpha = \left[m_L^\alpha, m_R^\alpha\right],$$
$$\tilde N_\alpha = \left[n_L^\alpha, n_R^\alpha\right], \alpha \in [0, 1]$$
$$\tilde M \oplus \tilde N = \left[m_L^\alpha + n_L^\alpha, m_R^\alpha + n_R^\alpha\right]$$
$$\tilde M \ominus \tilde N = \left[m_L^\alpha - n_L^\alpha, m_R^\alpha - n_R^\alpha\right]$$
$$\tilde M \otimes \tilde N = \left[m_L^\alpha n_L^\alpha, m_R^\alpha n_R^\alpha\right] \quad \tilde M / \tilde N = \left[m_L^\alpha / n_L^\alpha, m_R^\alpha / n_R^\alpha\right]$$

The TFNs, $\tilde 1$ to $\tilde 9$, are utilized to improve the conventional nine-point scaling scheme. In order to take the imprecision of human qualitative assessments into consideration, the five TFNs $(\tilde 1,\tilde 3,\tilde 5,\tilde 7,\tilde 9)$ are defined with the corresponding membership function. All attributes and alternatives are linguistically depicted by Fig. 2. The shape and positions of linguistically terms are chosen in order to illustrate the fuzzy extension of the method.

Steps of fuzzy ANP approach

Below, the fuzzy ANP-based methodology is presented step-by-step.

Step I. Model construction and problem structuring

The top most elements in the hierarchy of determinants are decomposed into dimensions and attribute-enablers. The decision model development requires identification of dimensions and attribute-enablers at each level and the definition of their interrelationships. The ultimate objective of hierarchy is to find out the weights of alternatives. In this study, we determined two evaluation determinants (*ICS-Improved Customer Satisfaction and IPF-Increased Profitability*) that are aggregated in *Machine Tool Weighted Index (MTWI)* selection step. To define this hierarchy, we utilized the Saaty's suggestions of using a network for categories of benefits, costs, risks and opportunities (Saaty 1996). Instead of Saaty's categories, we used evaluation determinants. Both determinants are very important in MTS. In order to analyze the combined influence of both determinants on MTS, a *MTWI* is calculated to find out the weights of machine tool alternatives. This index also takes the influences of dimensions and its attribute-enablers into consideration (see Fig. 1).

Step II. Pair wise comparison matrices between component/ attributes levels

By using TFNs $(\tilde 1,\tilde 3,\tilde 5,\tilde 7,\tilde 9)$, the decision-maker is asked to respond to a series of pair wise comparisons with respect to an upper level "control" criterion. These are conducted with respect to their relevance importance towards the control criterion. In the case of interdependencies, components in the same level are viewed as controlling components for each other. Levels may also be interdependent. Through pair wise comparisons by using TFNs $(\tilde 1,\tilde 3,\tilde 5,\tilde 7,\tilde 9)$, the fuzzy judgment matrix $\tilde A \quad (a_{ij})$ is constructed as given below;

$$\tilde A = \begin{bmatrix} 1 & \tilde a_{12} & \ldots\ldots & \tilde a_{1n} \\ \tilde a_{21} & 1 & \ldots\ldots & \tilde a_{2n} \\ \ldots & \ldots & \ldots\ldots\ldots & \\ \ldots & \ldots & \ldots\ldots\ldots & \\ \tilde a_{n1} & \tilde a_{n2} & \ldots\ldots & 1 \end{bmatrix} \tag{1}$$

where, $\tilde a_{ij}^\alpha = 1$, if $i = j$ and $\tilde a_{ij}^\alpha = \tilde 1, \tilde 3, \tilde 5, \tilde 7, \tilde 9$ or $\tilde 1^{-1}$, $\tilde 3^{-1}, \tilde 5^{-1}, \tilde 7^{-1}, \tilde 9^{-1}$, if $i \neq j$.

For solving fuzzy eigenvalue: A fuzzy eigenvalue, $\tilde\lambda$ is a fuzzy number solution to $\tilde A \tilde x = \tilde\lambda \tilde x$ (Eq. 1), where $\tilde\lambda_{\max}$ is the largest eigenvalue of A. Saaty (1981) provides several algorithms for approximating $\tilde x$.

Where is *nxn* fuzzy matrix containing fuzzy numbers $\tilde a_{ij}$ and $\tilde x$ is a non-zero *nx1,* fuzzy vector containing fuzzy number $\tilde x_i$.To perform fuzzy multiplications and additions by using the interval arithmetic and $\alpha - cut$, the equation $\tilde A \tilde x = \tilde\lambda \tilde x$ is equivalent to

$$\left[a_{i1l}^\alpha x_{1l}^\alpha, a_{i1u}^\alpha x_{1u}^\alpha\right] \oplus \cdots \oplus \left[a_{inl}^\alpha x_{nl}^\alpha, a_{inu}^\alpha x_{nu}^\alpha\right] = \left[\lambda x_{il}^\alpha, \lambda x_{iu}^\alpha\right]$$

where,

$$\tilde A = \left[\tilde a_{ij}\right], \tilde x^t = \left(\tilde x_1, \ldots, \tilde x_n\right),$$

$$\tilde a_{ij}^\alpha = \left[a_{ijl}^\alpha, a_{iju}^\alpha\right], \tilde x_{ij}^\alpha = \left[x_{il}^\alpha, x_{iu}^\alpha\right], \tilde\lambda^\alpha = \left[\lambda_l^\alpha, \lambda_u^\alpha\right] \tag{2}$$

for $0 < \alpha \leq 1$ and all i, j, where $i = 1, 2 \ldots n, j = 1, 2 \ldots n$. $\alpha - cut$ is known to incorporate the experts or decision maker(s) confidence over his/her preference or the judgments. Degree of satisfaction for the judgment matrix $\tilde{A}$ is estimated by the index of optimism μ. The larger value of indexμindicates the higher degree of optimism. The index of optimism is a linear convex combination (Lee 1999) defined as:

$$\tilde{a}_{ij}^{\alpha} = \mu a_{iju}^{\alpha} + (1 - \mu)\, a_{ijl}^{\alpha}, \quad \forall \mu \in [0, 1] \tag{3}$$

While α is fixed, the following matrix can be obtained after setting the index of optimism, μ, in order to estimate the degree of satisfaction.

$$\tilde{A} = \begin{bmatrix} 1 & \tilde{a}_{12} & \ldots & \tilde{a}_{1n}^{\alpha} \\ \tilde{a}_{21}^{\alpha} & 1 & \ldots & \tilde{a}_{2n}^{\alpha} \\ .. & .. & \ldots & .. \\ .. & .. & \ldots & .. \\ \tilde{a}_{n1}^{\alpha} & \tilde{a}_{n2}^{\alpha} & \ldots & 1 \end{bmatrix}$$

The eigenvector is calculated by fixing the μvalue and identifying the maximal eigenvalue.

After defuzzification of each pair wise matrix, *the consistency ratio* (CR) for each matrix is calculated. The deviations from consistency are expressed by the following equation consistency index, and the measure of inconsistency is called *the consistency index* (CI);

$$CI = \frac{\lambda_{\max} - n}{n - 1} \tag{4}$$

The consistency ratio (CR) is used to estimate directly the consistency of pair wise comparisons. The *CR* is computed by dividing the *CI* by a value obtained from a table of *Random Consistency Index* (RI);

$$CR = \frac{CI}{RI} \tag{5}$$

If the *CR* less than 0.10, the comparisons are acceptable, otherwise not. *RI* is the average index for randomly generated weights (Saaty 1981).

Step III. Pair wise comparison matrices of inter-dependencies

In order to reflect the interdependencies in network, pair wise comparisons among all the attribute-enablers are calculated.

Step IV. Super-matrix formation and analysis

The super-matrix formation allows a resolution of the effects of interdependence that exists between the elements of the system. The super-matrix is a partitioned matrix, where each sub-matrix is composed of a set of relationships between two levels in the graphical model. Raising the super-matrix to the power $2k + 1$, where k is an arbitrary large number, allows convergence of the interdependent relationships between the two levels being compared. The super-matrix is converged for getting a long-term stable set of weights.

Table 3 Notations used to calculate desirability index

Notations	Definition
P_{ja}	the relative importance weight of dimension j on determinant a
A_{kja}^{D}	the relative importance weight for attribute-enabler k of dimension j. and determinant a for the dependency (D) relationships between attribute-enabler's component levels
A_{kja}^{I}	the stabilized relative importance weight for attribute-enabler k of dimension j, and determinant a for the independency (I) relationships within attribute-enabler's component level
S_{ikja}	is the relative impact of machine tool alternative i on attribute-enabler k of dimension j of MTS network
K_{ja}	the index set of attribute-enablers for dimension j of determinant a
J	the index set for attribute j

Step V. The finding out the weights of alternatives

The equation of *desirability index,*D_{ia} for alternative i and determinant a is defined as;

$$D_{ia} = \sum_{j=1}^{J} \sum_{k=1}^{K_{ja}} P_{ja} A_{kja}^{D} A_{kja}^{I} S_{ikja} \tag{6}$$

Table 3 shows the notations used to calculate desirability index.

Step VI. Calculation of Machine Tool Weighted Index (MTWI)

To finalize the analysis of MTS. MTWI values are calculated for the alternatives, and then they are normalized to rank the alternatives (MTWI_i for an alternative i is the product of the desirability indices,D_{ia}).

Numerical example

Above, a fuzzy ANP-based approach has been presented to evaluate a set of machine tool alternatives (CNC vertical machining centers) to find out the ultimate one. In this section, a numerical example is presented to prove this approach's applicability. In determining the alternatives, first,

Table 4 Fuzzy comparison matrix for the determinants

Determinant	ICS	IPF
ICS	1	$\tilde{5}$
IPF	$\tilde{5}^{-1}$	1

Table 5 $\alpha - cuts$ fuzzy comparison matrix for the determinants ($\alpha = 0.5$)

Determinant	ICS	IPF
ICS	1	[4, 6]
IPF	[1/6, 1/4]	1

we made a list of a possible set of the alternatives (12) in the market and then narrowed down it to the reasonable number (3) by using a pre-selection method (sequential elimination method: alternative vs. alternative) for the ANP study. The pre-selection method is used to eliminate the non-dominant alternatives among the others so that the ANP cannot be a time-consuming and complicated process to reach the ultimate solution. No reference machine used. We utilized the features of the most commonly used CNC machine tools in metal-working industry, and developed a general model so that it can be effectively used in practice by a decision-maker (s) in any field of manufacturing.

Second, we carried out the fuzzy ANP study using TFNs, $\tilde{1} - \tilde{9}$ to express the preference in the pair wise comparisons. Then, we obtained the fuzzy comparison matrix for the relative importance of the determinants (*ICS* and *IPF*) shown in Table 4.

Then, the lower limit and upper limit of the fuzzy numbers with respect to the α were defined as follows by applying Eq. 2;

$$\tilde{1}_\alpha = [1, 3 - 2\alpha],$$
$$\tilde{3}_\alpha = [1 + 2\alpha, 5 - 2\alpha] .\ \tilde{3}_\alpha^{-1} = \left[\frac{1}{5 - 2\alpha}, \frac{1}{1 + 2\alpha}\right],$$
$$\tilde{5}_\alpha = [3 + 2\alpha, 7 - 2\alpha] .\ \tilde{5}_\alpha^{-1} = \left[\frac{1}{7 - 2\alpha}, \frac{1}{3 + 2\alpha}\right],$$
$$\tilde{7}_\alpha = [5 + 2\alpha, 9 - 2\alpha] .\ \tilde{7}_\alpha^{-1} = \left[\frac{1}{9 - 2\alpha}, \frac{1}{5 + 2\alpha}\right],$$
$$\tilde{9}_\alpha = [7 + 2\alpha, 11 - 2\alpha] .\ \tilde{9}_\alpha^{-1} = \left[\frac{1}{11 - 2\alpha}, \frac{1}{7 + 2\alpha}\right]$$

Later, we substituted the values, $\alpha = 0.5$ and $\mu = 0.5$ above expression into fuzzy comparison matrix, and obtained the entire $\alpha - cuts$ fuzzy comparison matrix shown in Table 5 (Eq. 3 was used to calculate eigenvector for pair wise comparison matrix). Because the dimension of the matrix, n is 2, we did not need to calculate the *CI* and the*CR*. A total of 1 matrix was built(Table 6).

We also followed the same way to build pair wise comparison matrix for the dimensions under each determinant and made all fuzzy calculations. Then, for the determinant *Improved Customer Satisfaction* (*ICS*) first we calculated eigenvalue of the matrix A by solving the characteristic equation of A, det $(A - \lambda I) = 0$ and found out all λ values for $A(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7, \lambda_8)$. The largest eigenvalue of

Table 6 Pair wise comparison matrix for the relative importance of the determinants

Determinants	ICS	IPF	e-Vector
ICS	1.000	5.000	0.831
IPF	0.208	1.000	0.169

pair wise matrix, $\lambda_{\max}$ was calculated to be 8.784. The dimension of the matrix, n is 8 and the random index,RI (n) is 1.41 (RI - *function of the number of attributes*, Saaty 1981). Finally, we calculated the *CI* and the *CR* of the matrix as follows and a total of 2 matrices were built (Tables 7, 8, 9);

$$CI = \frac{\lambda_{\max} - n}{n - 1} = \frac{8.784 - 8}{7} = 0.112,$$
$$CR = \frac{CI}{RI} = \frac{0.112}{1.41} = 0.079 < 0.100$$

Fuzzy comparison matrices of attribute-enablers under *ICS* were also calculated using the same way and only shown under *ICS* and *IPR* in Tables 10, 11 and 12. A total of 16 matrices were built.

Then, to reflect the interdependencies in network, we built pair wise comparison matrices for each attribute-enabler and made all fuzzy calculations. A total of 40 matrices were built. Only here, pair wise comparison matrix for sub-attributes under *ICS, IPR* and *SPS* using TFNs as given in Tables 13, 14 and 15.

Similarly, fuzzy comparison matrices for the alternatives (MT1, MT2 and MT3) for each attribute-enabler were constructed and only pair wise comparison for SPS under *ICS* and *IPR,* shown in Tables 16, 17 and 18. A total of 40 matrices were built.

Then, we created the super-matrix, M, detailing results of the relative importance measures for each of the attribute-enablers for *ICS* determinant of MTS clusters. Since there are 20 pair wise comparison matrices, one for each of the interdependent attribute-enablers in the *ICS* hierarchy, there will be 20 non-zero columns in this super-matrix. Each of non-zero values in the column in super-matrix, M, is the relative importance weight associated with the interdependently pair wise comparison matrices. In this model, there are 2 super-matrices, one for each of the determinants (*ICS*

Table 7 Fuzzy comparison matrix for the dimensions for the determinant *ICS*

ICS								
Dimensions	IPR	HFL	EUS	BAD	BPA	IRL	MSE	SMS
IPR	1	$\tilde{1}$	$\tilde{3}$	$\tilde{7}$	$\tilde{9}$	$\tilde{7}$	$\tilde{9}$	$\tilde{9}$
HFL	$\tilde{1}^{-1}$	1	$\tilde{1}$	$\tilde{5}$	$\tilde{9}$	$\tilde{9}$	$\tilde{9}$	$\tilde{7}$
EUS	$\tilde{3}^{-1}$	$\tilde{1}^{-1}$	1	$\tilde{3}$	$\tilde{1}$	$\tilde{3}$	$\tilde{3}$	$\tilde{9}$
BAD	$\tilde{7}^{-1}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	1	$\tilde{1}$	$\tilde{3}$	$\tilde{5}$	$\tilde{7}$
BPA	$\tilde{9}^{-1}$	$\tilde{9}^{-1}$	$\tilde{1}^{-1}$	$\tilde{1}^{-1}$	1	$\tilde{1}$	$\tilde{3}$	$\tilde{7}$
IRL	$\tilde{7}^{-1}$	$\tilde{9}^{-1}$	$\tilde{3}^{-1}$	$\tilde{3}^{-1}$	$\tilde{1}^{-1}$	1	$\tilde{1}$	$\tilde{1}$
MSE	$\tilde{9}^{-1}$	$\tilde{9}^{-1}$	$\tilde{3}^{-1}$	$\tilde{5}^{-1}$	$\tilde{3}^{-1}$	$\tilde{1}^{-1}$	1	$\tilde{1}$
SMS	$\tilde{9}^{-1}$	$\tilde{7}^{-1}$	$\tilde{9}^{-1}$	$\tilde{7}^{-1}$	$\tilde{7}^{-1}$	$\tilde{1}^{-1}$	$\tilde{1}^{-1}$	1

Table 8 $\alpha - cuts$ fuzzy comparison matrix for the determinant *ICS* ($\alpha = 0.5$)

ICS								
Dimensions	IPR	HFL	EUS	BAD	BPA	IRL	MSE	SMS
IPR	1	[1, 2]	[2, 4]	[6, 8]	[8, 10]	[6, 8]	[8, 10]	[8, 10]
HFL	[1/2, 1]	1	[1, 2]	[4, 6]	[8, 10]	[8, 10]	[8, 10]	[6, 8]
EUS	[1/4, 1/2]	[1/2, 1]	1	[2, 4]	[1, 2]	[2, 4]	[2, 4]	[8, 10]
BAD	[1/8, 1/6]	[1/6, 1/4]	[1/4, 1/2]	1	[1, 2]	[2, 4]	[4, 6]	[6, 8]
BPA	[1/10, 1/8]	[1/10, 1/8]	[1/2, 1]	[1/2, 1]	1	[1, 2]	[2, 4]	[6, 8]
IRL	[1/8, 1/6]	[1/10, 1/8]	[1/4, 1/2]	[1/4, 1/2]	[1/2, 1]	1	[1, 2]	[1, 2]
MSE	[1/10, 1/8]	[1/10, 1/8]	[1/4, 1/2]	[1/6, 1/4]	[1/4, 1/2]	[1/2, 1]	1	[1, 2]
SMS	[1/10, 1/8]	[1/8, 1/6]	[1/10, 1/8]	[1/8, 1/6]	[1/8, 1/6]	[1/2, 1]	[1/2, 1]	1

Table 9 Pair wise comparison matrix for the relative importance of the dimensions for the determinant $ICS(CR = 0.079)$

ICS									
Dimensions	IPR	HFL	EUS	BAD	BPA	IRL	MSE	SMS	e-Vector
IPR	1.000	1.500	3.000	7.000	9.000	7.000	9.000	9.000	0.336
HFL	0.750	1.000	1.500	5.000	9.000	9.000	9.000	7.000	0.273
EUS	0.375	0.750	1.000	3.000	1.500	3.000	3.000	9.000	0.139
BAD	0.146	0.208	0.375	1.000	1.500	3.000	5.000	7.000	0.089
BPA	0.113	0.113	0.750	0.750	1.000	1.500	3.000	7.000	0.071
IRL	0.146	0.113	0.375	0.375	0.750	1.000	1.500	1.500	0.038
MSE	0.113	0.113	0.375	0.208	0.375	0.750	1.000	1.500	0.030
SMS	0.113	0.146	0.113	0.146	0.146	0.750	0.750	1.000	0.023
								λ_{max}	8.784
								CI	0.112
								RI	1.41
								CR	$0.079 < 0.100\ ok.$

Table 10 Fuzzy comparison matrix of attribute-enablers under *ICS* and *IPR*

ICS				
IPR	SPS	MPW	CFD	TSP
SPS	1	$\tilde{1}$	$\tilde{3}$	$\tilde{7}$
MPW	$\tilde{1}^{-1}$	1	$\tilde{1}$	$\tilde{3}$
CFD	$\tilde{3}^{-1}$	$\tilde{1}^{-1}$	1	$\tilde{1}$
TSP	$\tilde{7}^{-1}$	$\tilde{3}^{-1}$	$\tilde{1}^{-1}$	1

Table 11 $\alpha - cuts$ fuzzy comparison matrix of attribute-enablers under *ICS* and *IPR* ($\alpha = 0.5$)

ICS				
IPR	SPS	MPW	CFD	TSP
SPS	1	[1, 2]	[2, 4]	[6, 8]
MPW	[1/2, 1]	1	[1, 2]	[2, 4]
CFD	[1/4, 1/2]	[1/2, 1]	1	[1, 2]
TSP	[1/8, 1/6]	[1/4, 1/2]	[1/2, 1]	1

Table 12 Pair wise comparison matrix for the relative importance of the attribute-enablers of the dimension, *IPR* for the determinant *ICS (CR = 0.079)*

ICS					e-Vector
IPR	SPS	MPW	CFD	TSP	
SPS	1.000	1.500	3.000	7.000	0.474
MPW	0.750	1.000	1.500	3.000	0.272
CFD	0.375	0.750	1.000	1.500	0.163
TSP	0.146	0.375	0.750	1.000	0.092
				λ_{max}	4.213
				CI	0.071
				RI	0.90
				CR	$0.079 < 0.100$ *ok*.

Table 13 Fuzzy comparison matrix for attribute-enablers for *SPS* under *ICS* and *IPR*

SPS	MPW	CFD	TSP
MPW	1	$\tilde{3}$	$\tilde{5}$
CFD	$\tilde{3}^{-1}$	1	$\tilde{1}$
TSP	$\tilde{1}^{-1}$	$\tilde{5}^{-1}$	1

Table 14 $\alpha - cuts$ fuzzy comparison matrix for attribute-enablers for *SPS* under *ICS* and *IPR* ($\alpha = 0.5$)

SPS	MPW	CFD	TSP
MPW	1	[2, 4]	[4, 6]
CFD	[1/4, 1/2]	1	[1, 2]
TSP	[1/6, 1/4]	[1/2, 1]	1

Table 15 Pair wise comparison matrix for the relative importance of the attribute-enablers for *SPS* under *ICS* and *IPR (CR = 0.085)*

SPS	MPW	CFD	TSP	e-Vector
MPW	1.000	3.000	5.000	0.643
CFD	0.375	1.000	1.500	0.216
TSP	0.208	0.750	1.000	0.141
			λ_{max}	3.099
			CI	0.050
			RI	0.58
			CR	$0.085 < 0.100$ *ok*.

Table 16 Fuzzy comparison matrix for the alternatives under *ICS*, *IPR* and *SPS*

ICS			
SPS	MT1	MT2	MT3
MT1	1	$\tilde{5}$	$\tilde{9}$
MT2	$\tilde{5}^{-1}$	1	$\tilde{3}$
MT3	$\tilde{9}^{-1}$	$\tilde{3}^{-1}$	1

Table 17 $\alpha - cuts$ fuzzy comparison matrix for criteria ($\alpha = 0.5$) for alternatives under *ICS*, *IPR* and *SPS*

ICS			
SPS	MT1	MT2	MT3
MT1	1	[4, 6]	[8, 10]
MT2	[1/6, 1/4]	1	[2, 4]
MT3	[1/10, 1/8]	[1/4, 1/2]	1

Table 18 Pair wise comparison matrix for the relative importance of MTS alternatives under *ICS*, *IPR* and *SPS (CR= 0.071)*

ICS				e-Vector
SPS	MT1	MT2	MT3	
MT1	1.000	5.000	9.000	0.745
MT2	0.208	1.000	3.000	0.182
MT3	0.113	0.375	1.000	0.074
			λ_{max}	3.082
			CI	0.041
			RI	0.58
			CR	$0.071 < 0.100$ *ok*.

and *IPF*) of the best MTS hierarchy network, which need to be evaluated. Then, super-matrix, M, is converged for getting a long-term stable set of weights. For this power of super-matrix is raised to an arbitrarily large number. In our case study, convergence is reached at 37th power. Table 19 shows the values after convergence.

To weight the alternative, we used Eq. 6, and made all calculations. Table 20 shows the calculations for the desirability indices for machine tool alternatives that are based on the *ICS* control hierarchy by using the weights obtained from the pair wise comparisons of machine tool alternatives, dimensions and attribute-enablers from the converged super-matrix. The weights are used to calculate a score for the determinant of MTS desirability for each alternative being considered. For example, the desirability index of machine tools; MT1, MT2 and MT3 under the first determinant *ICS*, where index $a = 1$, is calculated respectively by using Eq. 4 as illustrated in Table 20.

To find out the weights of machine tool alternatives, machine tool weighted index (MTWI) is calculated for each alternative (MT1, MT2 and MT3) as given in Table 21.

The final results are given in Table 22. The table indicates that the weights of alternatives together with PR ratio analysis realized for them. As seen in Table 22, the bold value indicates the largest PR value, and the corresponding alternative is the ultimate selected machine tool, *MT1*.

Conclusion

The objective of the research was, to use an integrated approach to MTS problem through fuzzy ANP. This approach aims to evaluating various kinds of conventional and CNC machine tools, especially utilized for general use in manufacturing systems. In order to reach to final solution, a PR ratio analysis is realized using the results of the fuzzy ANP, and investment costs for the alternatives. The investment cost of each alternative used in PR analysis is evaluated separate criterion rather than the elements in the ANP hierarchy, to compare the benefits and costs of alternatives in a good manner.

Using of fuzzy ANP approach to evaluating machine tool alternatives results in the following two major advantages; (1) Fuzzy numbers are preferable to extend the range of a crisp comparison matrix of the conventional ANP method, as human judgment in the comparisons of selection criteria and machine tool alternatives is really fuzzy in nature, (2) Adoption of fuzzy numbers can allow decision-maker to have freedom of estimation regarding the MTS.

This approach used here arrives at a synthetic score, which may be quite useful for decision-maker. The ANP methodology powered by fuzzy logic is a robust multiple criteria method for synthesizing the determinants, dimensions and attribute-enablers governing the finding out weights of the alternatives. It integrates various determinants, dimensions and attribute-enablers in a decision model in order to capture their relationships and interdependencies across and along the hierarchies. It is also effective as both quantitative and qualitative characteristics can be considered simultaneously without sacrificing their relationships.

As compared to the AHP, the analysis using the ANP is relatively cumbersome, because a great deal of pair wise comparison matrices is constructed. In our study, 101 matrices were built by assuming that fuzzy, $\alpha - cuts$ fuzzy and the relative importance matrices are counted as 1. Acquiring the relationships among determinant dimensions and attribute-enablers required very long and exhaustive effort. On the other hand, advantage of the ANP method is to capture interdependencies across and along the decision hierarchies. It means that the ANP provides more reliable solution than the AHP. Although the AHP is easier to apply than the ANP due to the fact that its holistic view and interdependencies accounted in the ANP, we preferred using it, because MTS problem has been critically important for most companies for a long time. Making wrong decision in selecting the proper machine tool might put a company into risk in terms of losing market share, cost and time.

This study is to aim to evaluate a set of alternatives in terms of evaluation criteria, and also uses fuzzy logic in order to model the vagueness and uncertainty of the DM. Use of fuzzy logic provides to get more reliably judgments of the DM than the crisp-based methods. The strengths of fuzzy models are their ability to approximate very complex, multi-dimensional processes and their insensitivity to noisy data. Their identification is computationally intensive but, once established, they provide quick responses. That is why we used the fuzzy ANP to solve the MTS problem. Unfortunately, fuzzy logic calculations require a considerably time to construct and process the pairwise comparisons, if especially the number of alternatives and criteria are more. If so, software like SuperDecisions should be used. In addition, prescreening process could be good way of narrowing down the size of the problem. However, the approach proposed here does not consider all the possible factors and criteria associated with MTS problem. The attribute-enablers, criteria and interactions between the attribute-enablers presented in the framework are specific to a typical manufacturing organization. The proposed methodology can easily be adapted to different situations by adjusting the different levels of the hierarchy and their related attributes.

In future research, a knowledge-based system (KBS) or expert system (ES) can be adapted to this approach to interpret the outputs automatically via a user interface. A KBS or ES creates a rule-based database to interpret the analysis results, and makes its comments using an inference engine, and presents them to the user whenever needed.

Table 19 Super-matrix for *improved customer satisfaction* (*ICS*) after convergence (M^{37})

ICS	SPS	MPW	CFD	TSP	TCT	CRT	AST	MDM	ARA	DDL	DNC	CNC	UPG	RPT	TDF	CPI	BFR	RDS	RCC	OTS	PRC	SAC	STR	ORS	RMN
SPS	0.413	0.413	0.413	0.413																					
MPW	0.337	0.337	0.337	0.337																					
CFD	0.148	0.148	0.148	0.148																					
TSP	0.102	0.102	0.102	0.102																					
TCT					0.387	0.387	0.387																		
CRT					0.323	0.323	0.323																		
AST					0.290	0.290	0.290																		
MDM								0.209	0.209	0.209															
ARA								0.411	0.411	0.411															
DDL								0.380	0.380	0.380															
DNC											0.323	0.323	0.323												
CNC											0.387	0.387	0.387												
UPG											0.290	0.290	0.290												
RPT														0.259	0.259	0.259									
TDF														0.360	0.360	0.360									
CPI														0.382	0.382	0.382									
BFR																	0.394	0.394	0.394						
RDS																	0.441	0.441	0.441						
RCC																	0.165	0.165	0.165						
OTS																				0.441	0.441	0.441			
PRC																				0.394	0.394	0.394			
SAC																				0.165	0.165	0.165			
STR																							0.375	0.375	0.375
ORS																							0.369	0.369	0.369
RMN																							0.257	0.257	0.257

Table 20 MTS desirability indexes for *improved customer satisfaction* (*ICS*) ($a = 1$)

Dimension	Attribute enabler	P_{j1}	A^D_{kj1}	A^I_{kj1}	S_{1kj1}	S_{2kj1}	S_{3kj1}	Machine tool alternative		
								MT1	MT2	MT3
1	1	0.336	0.474	0.413	0.745	0.182	0.074	0.0490	0.0120	0.0049
	2	0.336	0.272	0.337	0.683	0.237	0.080	0.0210	0.0073	0.0025
	3	0.336	0.163	0.148	0.660	0.249	0.091	0.0053	0.0020	0.0007
	4	0.336	0.092	0.102	0.739	0.153	0.108	0.0023	0.0005	0.0003
2	5	0.273	0.237	0.387	0.683	0.237	0.080	0.0171	0.0059	0.0020
	6	0.273	0.080	0.323	0.080	0.237	0.683	0.0006	0.0017	0.0048
	7	0.273	0.683	0.290	0.745	0.182	0.074	0.0403	0.0098	0.0040
3	8	0.139	0.529	0.209	0.120	0.087	0.792	0.0018	0.0013	0.0122
	9	0.139	0.355	0.411	0.274	0.064	0.662	0.0056	0.0013	0.0134
	10	0.139	0.116	0.380	0.739	0.153	0.108	0.0045	0.0009	0.0007
4	11	0.089	0.506	0.323	0.128	0.101	0.770	0.0019	0.0015	0.0112
	12	0.089	0.402	0.387	0.249	0.091	0.660	0.0034	0.0013	0.0091
	13	0.089	0.091	0.290	0.660	0.249	0.091	0.0016	0.0006	0.0002
5	14	0.071	0.662	0.259	0.080	0.237	0.683	0.0010	0.0029	0.0083
	15	0.071	0.274	0.360	0.739	0.153	0.108	0.0052	0.0011	0.0008
	16	0.071	0.064	0.382	0.128	0.101	0.770	0.0002	0.0002	0.0013
6	17	0.038	0.760	0.394	0.274	0.064	0.662	0.0031	0.0007	0.0075
	18	0.038	0.145	0.441	0.745	0.182	0.074	0.0018	0.0004	0.0002
	19	0.038	0.095	0.165	0.249	0.091	0.660	0.0001	0.0001	0.0004
7	20	0.030	0.643	0.441	0.660	0.249	0.091	0.0056	0.0021	0.0008
	21	0.030	0.216	0.394	0.249	0.091	0.660	0.0490	0.0120	0.0049
	22	0.030	0.141	0.165	0.128	0.101	0.770	0.0210	0.0073	0.0025
8	23	0.023	0.760	0.375	0.683	0.237	0.080	0.0053	0.0020	0.0007
	24	0.023	0.145	0.369	0.739	0.153	0.108	0.0023	0.0005	0.0003
	25	0.023	0.095	0.257	0.080	0.237	0.683	0.0171	0.0059	0.0020
Total desirability indices (D_{i1}) of *ICS* for machine tool alternatives								0.172	0.054	0.085

Table 21 Machine tool weighted index (MTWI) for MTS alternatives

Alternatives	Determinants		Calculated weights for alternatives	
	Improved Customer Satisfaction (ICS) 0.831	Increased Profitability (IPF) 0.169	MTWI	Normalization
MT1	0.172	0.198	0.176	0.557
MT2	0.054	0.085	0.059	0.187
MT3	0.085	0.063	0.081	0.256
Total				1.000

Table 22 PR analysis to find out the ultimate machine tool alternative

Alternatives	Investment Cost (IC) (x1000 $)	Fuzzy ANP score (%) (FAS)	PR=FAS/IC
MT1*	175	55.7	**0.318**
MT2	125	18.7	0.150
MT3	200	25.6	0.128

* Selected machine tool alternative
Bold value indicates the largest PR value

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