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The facility layout problem approached using a fuzzy model and a genetic search

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The problem of facility layout design is discussed, taking into account the uncertainty of production scenarios and the finite production capacity of the departments. The uncertain production demand is modelled by a fuzzy number, and constrained arithmetic operators are used in order to calculate the fuzzy material handling costs. By using a ranking criterion, the layout that represents the minimum fuzzy cost is selected. A flexible bay structure is adopted as a physical model of the system while an effective genetic algorithm is implemented to search for a near optimal solution in a fuzzy contest. Constraints on the aspect ratio of the departments are taken into account using a penalty function introduced into the fitness function of the genetic algorithm. The efficiency of the genetic algorithm proposed is tested in a deterministic context and the possibility of applying the fuzzy approach to a medium-large layout problem is explored.

Keywords: Fuzzy sets, genetic algorithm, layout, optimization, robustness

1. Introduction

The facility layout problem involves the location of all manufacturing resources inside a working area. The ideal approach should be to formulate a close to reality model and to implement an algorithm that permits us to obtain an optimal solution so as to maximize the system's efficiency in the long term (the economic lifetime of the facility layout). Such an approach is impossible because of the complexity of the problem and the extensive literature in this field can be classified according to both the aspects of the problem analysed and the resolution techniques adopted. The most common approach to the problem considers, as a measure of performance, the total cost of the material flow per unit time period and, as a physical model a planar region (workshop) which needs to be subdivided into departments assigned by number and area (block layout). In this context, the design of the material handling system and the placement of the machine tools within the departments are required for a subsequent phase of the design.

The cost of material movement is assumed to be an increasing function of the number and length of product moves. The choice of this cost as the objective function is justified considering its large incidence on the total operative cost. Moreover, a reduction of material movement involves a reduction in space required for aisles, a lower WIP and throughput time, and less congestion of the system.

If we don't explicitly consider the material handling system and the position of work centres in the model, we will end up with a solution which, even if optimal according to the objective function formulated, is not practical. Therefore, it is very important to consider at least some of the

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constraints deriving from the hypothesized material handling system in the block layout model and from the machine tools that must be placed within the departments. Otherwise, the block layout must be extensively modified in the next step of detailed design, nullifying the positive nature of the result obtained. The bay structure adopted in this paper as a physical model for the system, allows us to take the previous aspects into account.

The high computational difficulty related to the layout problem is evident even in the simplest model proposed in literature: departments of equal area and square shape. This model can be formulated as a quadratic assignment problem, but the problem is NP-complete and cannot be solved optimally for more than 15–20 departments. Different heuristic approaches have been proposed in Operation Research literature with the aim of finding good solutions to the problem.

From amongst the models that consider unequal area departments, the first significant formulation was proposed by Armour and Buffa (1963). This approach was implemented into a commercial software (CRAFT), which is still employed. An improvement in respect to CRAFT was the MULTIPLE algorithm (Bozer *et al.*, 1994). Both CRAFT and MULTIPLE are steepest descendent algorithms, and so they converge on the first local minimum encountered in the path. Moreover they are largely affected by the initial solution hypothesized and the shape of the departments of the block layout obtained is often impractical.

An approach based on a mixed-integer programming model was proposed by Montreuil (1990). The author considered unequal area departments of a rectangular shape but the algorithm can solve only problems with six or less departments.

Other approaches, as opposed to the minimization of a flow-distance based cost function, consider the maximization of the adjacencies between departments, taking into account inter-departmental closeness ratings. Among these approaches, one, based on the graph theory, was developed initially by Carrie *et al.* (1978); the cost of material flow is not considered explicitly and the objective is to maximise the adjacencies between those departments which have a material interflow. A review of graph theoretic heuristics is reported in Hassan and Hogg (1987).

In order to overcome one of the problems of the heuristic techniques, i.e. convergence to a first local optimum (the facility layout problem generally exhibits many local minima), approaches based on simulated annealing (Meller and Bozer, 1996) and genetic algorithms have recently been proposed (Suresh *et al.*, 1995; Tate and Smith, 1995; Kochar *et al.*, 1998). In this paper a genetic algorithm, which can be considered a further development of the one proposed by Tate and Smith (1995), is implemented. Its performance and ability to take into account some constraints on the placement and on the shape of the departments will be shown in the next paragraphs.

A review of the different approaches to the layout problem in a deterministic scenario can be found in Meller and Gau (1996).

Another aspect that should be taken into account in layout design is the variability and uncertainty of production scenarios over time. In fact, it is not possible to foresee neither the number and the typology of the parts to be processed exactly, nor, as a consequence, the material flows between departments. As a consequence, layout flexibility must be taken into account, that is to say, the possibility of adapting the system to the changes of scenario by means of layout rearrangement or the intrinsic ability of the system to maintain good performances without any external action. In the literature on the subject these two aspects of flexibility have generally been considered separately. The first formulation of the problem (*dynamic plant layout problem*) considers successive time periods with different scenarios, generally supposed to be certain. The best solution is to minimize the total cost of material flow and of the successive reconfigurations of the layout, in the overall time horizon (Rosenblatt, 1986; Timothy, 1993; Afentakis *et al.*, 1990). The second formulation considers a single time period but the required production is supposed to be uncertain. In this case the objective is called *robust layout*, which is a layout that has a good level of performance for a wide variety of demand scenarios, even though it generally will not be optimal under any specific scenario (Rosenblatt and Lee, 1987; Kouvelis *et al.*, 1992).

Some authors approached the robust layout problem using a stochastic model. Rosenblatt and Lee (1987), for example, consider a scenario where negotiation contracts exist between the producer and the customers, but order levels are not clearly defined. In this situation, material flows between the departments are uncertain, but an analytical approach involving probability distributions of the stochastic variables would be very complex (Shore and Tompkins, 1980). In an interesting, robust approach (Rosenblatt and Lee, 1987), the scenarios corresponding to different levels of market demand are considered: highest (H), most likely (M) and lowest (L). The aforementioned authors measure the robustness of a solution by the number of times that it lies within a pre-specified distance from the optimal solution for each scenario. This approach does not take into account the departments' production capacity limitation. To take into account this aspect of the problem, the combinations of product demands that overcome the production constraints must be considered unfeasible.

A different methodology to approach the uncertainty in the layout problem is the fuzzy theory, which can be formulated in terms of nested bodies of evidence or in terms of fuzzy sets. This approach has been investigated by several researchers. A fuzzy linguistic approach (Evans *et al.*, 1987), based on fuzzy relations, was used in order to specify the location of each department in a manually generated layout and a fuzzy ranking method is proposed to rank the different solutions.

The problem was approached using fuzzy implication and fuzzy consistency (Grobelny, 1987). A grade of satisfaction was defined as an optimality criterion and the aggregation of experts' assessments was studied.

The Analytical Hierarchy Process (AHP) was applied (Dweiri and Meier, 1996) with the aim of assigning a factor of importance and a decision-making fuzzy procedure to generate the activity relation chart. They also modified the commercially available package CORELAP so that it could handle the relation chart and generate the layout.

In the present paper a model that considers fuzzy flows between limited production capacity departments is resolved using a genetic algorithm.

2. The adopted model

The objective function expresses the total material handling cost to be minimised:

$$\text{cost} = \sum_i \sum_j (f_{ij} \cdot c_{ij}) d_{ij} \tag{1}$$

where f_{ij} is the material flow between the departments i and j, c_{ij} is the unit cost (the cost to move one unit load one distance from department i to department j) and d_{ij} is the distance between the centres of departments using a pre-specified metric.

The physical model adopted is called a flexible bay structure (Tate and Smith, 1995). The layout is represented by vertical bays of varying width, each containing one or more rectangular departments. The layout obtained employing this physical scheme is easy to transform into a detailed one. The departments have rectangular form and, as will be shown later, the genetic approach employed allows us to impose constraints on the aspect ratio of the departments in order to simplify the successive placement of the machines in each department. Moreover, the bay structure lets us hypothesize a handling system which operates around a perimeter of the bays where the input/output stations are placed. The problem is to identify the number of bays and the sequence of the departments within each of them, which minimize material movement costs and respect the aspect ratio constraints.

3. Genetic algorithms with deterministic flows

3.1. *Introduction*

Recently genetic algorithms (GA) have been used with success on many NP-hard combinatorial problems. GAs are characterized by a set (*population*) of candidate solutions, a breeding mechanism to create new solutions (*children*) by recombination (*crossover*) of existing solutions (*parents*) and a perturbation method (*mutation*) to avoid a too rapid convergence with a local optimum. Tate and Smith (1995) developed a genetic algorithm with an adaptive penalty function applied to the shape-constrained unequal-area layout problem modelled by the flexible-bay structure.

In this paper a genetic algorithm employing the concepts of evolutionary hybrid algorithms is presented. Evolutionary Hybrid Algorithms (EHA) are based on three general principles:

improvement of the evolutionary process through the dynamic modification of the control parameters, utilization of heuristic operators to improve the characteristics of the population and integration between traditional search operators and standard genetic operators. EHAs modify the control parameters during the evolution process according to specific logical conditions, remove the clones to maintain differentiation of the population and employ search operators. Thus they find local optima which are better than those found by traditional genetic operators.

3.2. *Genetic encoding*

The encoding process utilizes two strings (*chromosomes*) of integers: the first chromosome is a permutation of the departments read from top left to bottom right, and the second chromosome (*breakpoints* identifies the number of the bays and the number of the departments contained, as shown in Fig. 1.

Given the chromosomes, knowing the area A_i of each department and the fixed dimensions H and W of the workshop, it is easy to calculate h_i and w_i, respectively height and width of the department i, and its centre (x_i, y_i).

3.3. *Genetic operators*

In the following the steps of the proposed algorithm are briefly described. A flow-chart of it is shown in Fig. 2.

3.3.1. *Initialisation*

The population consists of 20 elements that, during the initial phase, are generated randomly. The individuals in the population are decoded and evaluated according to some predefined performance criterion (fitness).

3.3.2. *Evaluation*

In order to minimize material movement costs, we must convert the cost value into a fitness value, so that the best individuals have the greatest fitness value. This conversion can be easily obtained by setting the fitness function as equal to the inverse of the material movement cost.

Depending on the location of the bay divisions, the resulting bay structure may create long, narrow departments. To avoid this situation a constraint on the departments' shape must be introduced. Each department j is characterised by its aspect ratio defined as:

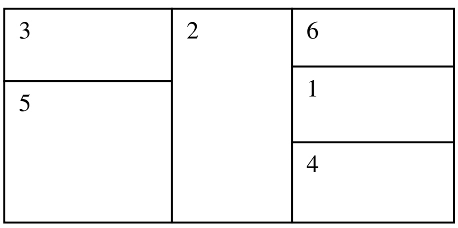
$$\gamma_j = \frac{\max\{h_j, w_j\}}{\min\{h_j, w_j\}}$$

and a parameter γ_{cr} sets the admissible upper boundaries of the aspect ratio. For the generic solution i, let n_i be the number of departments with aspect ratio $\gamma_j > \gamma_{cr}$. By taking this constraint into account in the optimisation process, the adaptive penalty function is adopted (Tate and Smith, 1995):

$$\text{penalty}(i) = n_i^k \cdot (\text{cost}_{\text{feas}} - \text{cost}_{\text{min}}) \tag{2}$$

where

- $\text{cost}_{\text{feas}}$ is the value of the objective function for the best feasible solution ($n_i = 0$) in the current population;
- cost_{min} is the value of the objective function for the best solution in the current population;
- k is a severity parameter.



Permutation: (3, 5, 2, 6, 1, 4)
Breakpoints: (2, 3)

Fig. 1. Flexible-bay structure.

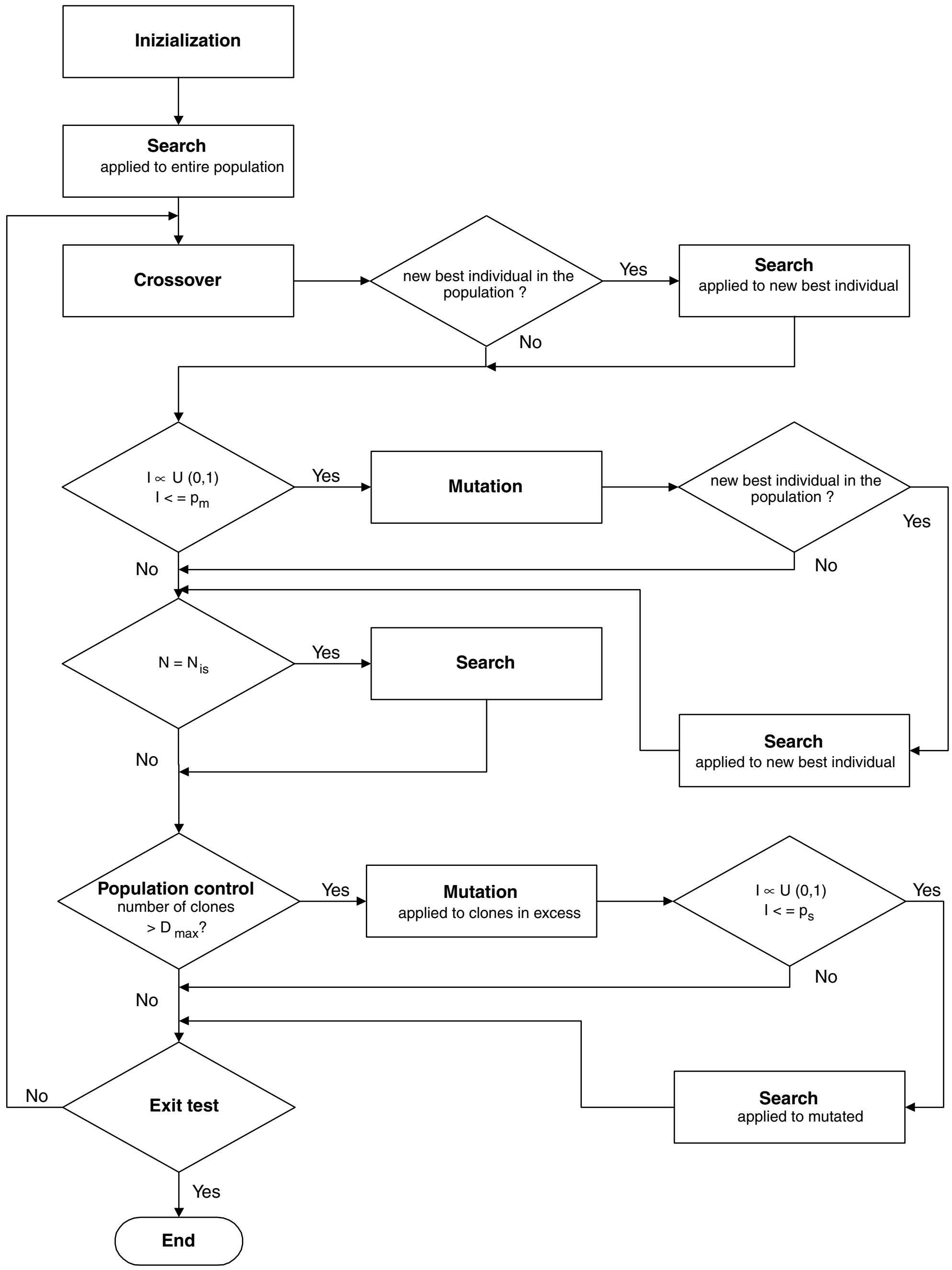


Fig. 2. Flow-chart of the genetic algorithm.

The fitness function will be:

$$F(i) = \frac{1}{[\mathrm{cost}(i) + \mathrm{penalty}(i)]} = \frac{1}{[\mathrm{cost}(i) + n_i^k \cdot (\mathrm{cost}_{\mathrm{feas}} - \mathrm{cost}_{\mathrm{min}})]} \quad (3)$$

This approach allows initially infeasible solutions ($n_i > 0$) but, as the search goes on, penalization will increase. The values of $\mathrm{cost}_{\mathrm{feas}}$ and $\mathrm{cost}_{\mathrm{min}}$ will be modified during the evolutionary process and the penalty function will guide the genetic search to the feasible region.

3.3.3. *Crossover*

The crossover operator is applied to each iteration and consists of the following four steps.

1. Two parents *G1* and *G2* are selected from the population with a probability proportional to their relative fitness.
2. A binary string, with a length equal to the permutation chromosome, is randomly generated.
3. The children *F1* and *F2* are constructed by copying the genes of a parent *G1* in a child *F1*, when the corresponding value of the binary string is equal to 1. To complete *F1*, and avoiding duplication of genes, i.e. the wrong coding, the lacking genes are introduced according to their sequence in *G2*. Analogously for *F2*.
4. The breakpoints chromosome of *F1* is that of *G1* or *G2*, chosen at random. Analogously for the other child *F2*.

Then the fitness of the four elements is evaluated and the best two will enter the population. Figure 3 shows such an operator applied to an example with 12 departments.

3.3.4. *Mutation*

For each generation, the mutation operator is applied, with probability p_{m}, to a randomly selected element of the population. The value of p_{m} increases within an input fixed range (pM_1–pM_2) if there is no improvement in the best solution after M_{it} iterations. The mutation consists of three separate operators, each one randomly activated with a fixed probability. The first operator acts on the permutation chromosome: it swaps couples of randomly selected genes. The number of couples is the input parameter n_{couple}. The two others act on the breakpoints chromosome. The second deletes or inserts a gene (randomly selected) into the breakpoint while the last increases or decreases a unit of a gene (randomly selected).

3.3.5. *Population control*

This operator searches for clones in the population and, if the number of clones exceeds a fixed value Dmax, the mutation operator will be applied to the excess clones.

3.3.6. *Search*

The search operator is a heuristic developed to explore local optimum points. It acts only on the permutation chromosome, swapping a gene with all the others and evaluating the resulting fitness. The best element found will enter the population. The search operator is applied:

– to the entire population after the initialisation phase;
– after the crossover and the mutation, only to the new best element in the population, if generated;
– to the entire population every N_{is} iterations;
– to the mutated clones, if generated, with a probability p_{s}, in the population control phase.

3.3.7. *Updating the fitness*

Each time a new element is introduced in the population, the fitness of all the individuals must be updated.

3.3.8. *Exit test*

The end of the evolution process is determined by a double logic condition: the algorithm ends when the iterations counter is equal to $\mathrm{nit}_{\max}$ or to n_{it}.

G1	5	10	11	2	**4**	**1**	9	**8**	**3**	7	**12**	6	2	6	8	
G2	9	5	10	1	6	7	4	12	11	8	3	2	1	2	5	9
Binary string	1	1	1	0	1	1	0	0	1	0	0	1				
F1	5	10	11	9	4	1	7	12	3	8	2	6	1	2	5	9
F2	9	5	10	**4**	6	7	**1**	**8**	11	**3**	**12**	2	1	2	5	9

Fig. 3. Crossover order based operator; generation of children *F1* and *F2* from parents *G1* and *G2*.

The value of $\mathrm{nit}_{\max}$ is constant while the initial value of n_{it} is lower than $\mathrm{nit}_{\max}$ and is modified during the process to avoid early convergence. When the algorithm finds a new better solution, n_{it} is increased to $n_{\mathrm{it}} + \Delta\mathrm{it}$.

3.4. *Experimental tests*

The algorithm was tested against the classical problem of Armour and Buffa (20 departments), which is the most commonly benchmarking problem used in literature.

The values of the control parameters previously described are reported in table 1. They are the same employed by Tate and Smith (1995) for the common parameters. The best results of 10 runs for each value of γ_{cr} (as in Tate and Smith) are shown in Table 2.

The comparison between our results and those obtained by Tate and Smith, shows the effectiveness of the algorithm proposed.

4. A fuzzy approach to the layout problem.

In this paper the uncertainty associated with each product demand is represented by fuzzy numbers. Consequently the flow between the departments F_{ij} is a fuzzy number that could be derived by summing all the fuzzy requests (expressed in unit loads) of the products that are transferred from department *i* to department *j*. However, a limited production capacity of the departments restricts the possible scenarios only to the ones that respect such constraints. This aspect of the problem in a fuzzy layout model can be approached by "fuzzy arithmetic with requisite constraints".

Table 1. Control parameters of the genetic algorithm

Control Parameter	*Value*
n_{it}	500000
$\mathrm{nit}_{\max}$	700000
Δ_{it}	50000
k	3
N_{is}	5000
p_{s}	0.3
$D_{\max}$	2
n_{couple}	3
pM_1	0.2
pM_2	0.4
M_{it}	25000
p_{m1}	0.2
p_{m2}	0.25
p_{m3}	0.25

Table 2. Results for Armour and Buffa test problem

γ_{cr}	*Best result GA Tate and Smith*	*Best result EHA*	*Improvement (%)*
1000	1638.5	1638.5	0.00
50	3009.5	3009.5	0.00
25	3635.9	3535.1	2.77
15	4296.1	4140.5	3.62
10	4633.3	4440.7	4.16
7	5255	4793.5	8.78
5	5524.7	5397.6	4.11
4	5743.1	5370.9	6.48
3	5832.6	5594.3	4.09
2	6171.1	6023.2	2.40
1.75	7205.4	6453.1	10.44
1.70667	6662.9	6029.3	4.58

In particular, triangular fuzzy numbers are considered. The membership function of a triangular fuzzy number A is $A(x)$:$\mathcal{R} \rightarrow$ (0,1]:

$$A(x) = \begin{cases} (x-l)/(m-l) & \text{when } x \in [l,m] \\ (u-x)/(u-m) & \text{when } x \in [m,u] \\ 0 & \text{otherwise} \end{cases}$$

Let $A = (l,m,u)$ be a shorthand symbol representing this special form.

4.1. *Fuzzy arithmetic with requisite constraints*

Fuzzy arithmetic is based either on the α-cut representation, in terms of arithmetic operations on closed intervals of real numbers, or on the extension principle (Klir, 1995). The results obtained by the mere application of these principles are sometimes meaningless. Traditional mathematic operators applied to fuzzy numbers, in fact, do not take into account the meaning of the operands and the context the operators are referred to. To overcome this inconsistency, a "fuzzy arithmetic with requisite constraints theory" was formulated (Klir, 1997).

Employing the α-cut representation, the four basic arithmetic operators for fuzzy numbers are defined for all α ∈ (0,1] by:

$$^{\alpha}(A \otimes B) = \{a \otimes b | \langle a, b \rangle \in (^{\alpha}A \times ^{\alpha} B)\} \quad (4)$$

where $\otimes$ denotes any of the basic arithmetic operators and $\times$ the Cartesian product.

On the contrary, employing the extension principle, by:

$$(A \otimes B)(c) = \sup_{\forall a,b|c=a\otimes b} \min\{A(a), B(b)\} \quad (5)$$

for all $c \in \mathcal{R}$,

The information known about the results which must be returned by the arithmetic operators, can be expressed by a relation between the operands. This relation can be either fuzzy or crisp. By calling such a relation R, a constrained operator can be defined. This operator will be named *constrained arithmetic operator* and expressed by:

$$^{\alpha}(A \otimes B)_R = \left\{a \otimes b | \langle a,b\rangle \in (^{\alpha}A \times {}^{\alpha}B) \cap {}^{(}a,b)\right\}\} \quad (6)$$

which is the generalization of (4), or by:

$$(A \otimes B)_R(c) = \sup_{\forall a,b|c=a\otimes b} \min\{A(a), B(b), R(a,b)\} \quad (7)$$

which is the generalization of (5).

An in-depth explanation of equations (6) and (7) can be found in Klir (1997).

In order to explain the meaning of such an operator, let us consider two triangular fuzzy numbers $A=[l_A,m_A,u_A]$ and $B=[l_B,m_B,u_B]$ and the fuzzy arithmetic expression $A/(A+B)$. In this operation we have an application of the fuzzy arithmetic which is affected by an equality constraint. This constrained operation on the two fuzzy numbers A and B may conveniently be expressed as follow (E denotes the relation representing the equality constraint):

$$^{\alpha}A/(A+B)]_E = \{a/(a+b)|\langle a,b\rangle \in^{\alpha} A \times^{\alpha} B\}$$

For example, let us consider the two triangular fuzzy numbers $A=[1,2,3]$ and $B=[2,3,4]$. If we do not consider the equality constraint, the α-cut, with $\alpha=0$, of the operation $A/(A+B)$ is the interval [1/(3+4), 3/(1+2)] = [1/7, 1]. On the contrary, if we take the equality constraint into account, the result is the interval [1/(1+4), 3/(3+2)] = [1/5, 3/5].

When the constraint is considered, therefore, a smaller fuzzy interval is obtained which does not contain meaningless values and, as a consequence, expresses less vagueness because we are using a larger amount of information.

4.2. *The fuzzy constrained layout model*

Let F_{ij} be a fuzzy number that represents the flow between departments i and j. Knowing the fuzzy demand Q_k with a membership function of $Q_k(q_k)$ (where q_k is the demand value of product k in terms of unit loads), we can easily evaluate F_{ij} using equations (4) or (5), summing the fuzzy demands for each product that is transferred from department i to j in the manufacturing process. Nevertheless, in this way, the resulting fuzzy number F_{ij} could have a non zero membership value in correspondence with flow values that exceed the production capacity of the departments. Therefore the constrained arithmetic operator needs to be employed.

Let s_{kj} be the maximum capacity of department j to process product k, and so the q_k/s_{kj} ratio is the percentage of production capacity of department j taken up to process the product k. Let Y_j^k be a boolean variable equal to 1, if product k is manufactured in department j, or otherwise equal to 0. The production capacity constraint can be expressed by the following relation:

$$\sum_{k=1}^{N} \frac{q_k \cdot Y_j^k}{s_{kj}} \leq 1 \ \forall j \quad (8)$$

Hence, from all possible q_k, the only admissible ones are the $q_k \in \Re^+$ that satisfy the equation (8).

This constraint can be expressed by means of the crisp relation:

$$\begin{aligned} R(q_1, q_2, q_3, \ldots, q_N) &= 1 \quad \text{if} \sum_{k=1}^{N} \frac{q_k \cdot Y_j^k}{S_{kj}} \leq 1 \ \forall j \\ R(q_1, q_2, q_3, \ldots, q_N) &= 0 \quad \text{otherwise} \end{aligned} \quad (9)$$

Equations (6) and (7) respectively become:

$$\begin{aligned} {}^{\alpha}F_{ij}(q) = \Bigg\{ \sum_{k=1}^{N} q_k X_{ij}^k | \langle q_1, q_2, q_3, \ldots, q_N\rangle \\ \in ({}^{\alpha}Q_1 \times {}^{\alpha}Q_2 \times {}^{\alpha}Q_3 \times \ldots {}^{\alpha}Q_N) \cap \\ {}^{\alpha}R(q_1, q_2, q_3, \ldots, q_N) \Bigg\} \end{aligned} \quad (10)$$

$$F_{ij}(q) = \sup_{\forall q_k | q = \sum_{k=1}^{N} q_k \cdot X_{ij}^k} \min[Q_1(q_1), Q_2(q_2), \ldots Q_N(q_N), R(q_1, q_2, q_3, \ldots q_N)] \quad (11)$$

where X_{ij}^k is an element of a matrix $\mathbf{X}^k$ that represents the times in which product k is transferred from department i to department j during its production cycle.

Using these equations, the fuzzy flows between the departments are compatible with the capacity of the facilities. These flows cannot, however, be used in the optimisation model because we cannot be sure that for each ${}^{\alpha}F_{ij}(q)$ the upper bounds set are compatible with the production capacity constraints. For this reason the constraints expressed by relation R must be considered in the fuzzy function of the cost.

As a consequence the total cost of material movement is expressed, in fuzzy form, by (10), as:

$$
{}^{\alpha}C(c) = \left\{ \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\sum_{k=1}^{N} q_k X_{ij}^k\right) c_{ij}d_{ij} | \langle q_1, q_2, q_3, \ldots, q_N \rangle \in ({}^{\alpha}Q_1 \times {}^{\alpha}Q_2 \times {}^{\alpha}Q_3 \times \cdots {}^{\alpha}Q_N) \cap {}^{\alpha} R(q_1, q_2, q_3, \ldots, q_N) \right\} \tag{12}
$$

or, by means of (11), as:

$$
C(c) = \sup_{\forall q_k | c = \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\sum_{k=1}^{N} q_k \cdot X_{ij}^k\right) c_{ij}d_{ij}} \min[Q_1(q_1), Q_2(q_2), \ldots Q_N(q_N), R(q_1, q_2, q_3, \ldots q_N)] \tag{13}
$$

Since the cost related to the generic layout is a fuzzy number, it is necessary to define a ranking criterion in order to select the best solution.

4.3. *Ranking fuzzy numbers*

Although several researchers have approached the ranking problem, there is no fully reliable procedure that can be applied to all the possible cases.

If A and B are two fuzzy numbers defined for $x \in \Re$.

Let ${}^{\alpha}\underline{a} = inf_{x\in\Re}\{x|A(x) \geq \alpha\}$ and ${}^{\alpha-}a = sup_{x\in\Re}\{x|A(x) \geq \alpha\}$, we will define $A \geq B$ ($B \geq A$) (strong relation) when, for each α-cut, the following two relations are both satisfied

$$
{}^{\alpha}\underline{a} \geq^{\alpha} \underline{b} \text{ and } {}^{\alpha}\overline{a} \geq {}^{\alpha}\overline{b} \qquad ({}^{\alpha}\underline{b} \geq {}^{\alpha}\underline{a} \text{ and } {}^{\alpha}\overline{b} \geq {}^{\alpha}\overline{a}) \tag{14}
$$

When these relation are not satisfied, i.e. MAX (A,B) ≠ A (MAX(A,B) ≠ B), a weak ranking relation must be established to determine whether A is weakly bigger or equal to B or vice versa. A ranking method based on the integral value was suggested by Tian-Shy and Mao-Jiun (1992). This method is related by Fortemps and Roubens (1996) to the ranking method based on area compensation (Roubens, 1990).

The genetic approach employed in this paper, requires not only population ranking, but also the assignment of a fitness value to each element of the population. Since this value is related to the material movement cost, it is necessary to express this fuzzy cost by a crisp number. For this purpose, a method based on the integral value is adopted to express the defuzzification function:

$$
F(A) = \frac{1}{2}\int_0^1 [{}^{\alpha}\underline{a} +^{\alpha} \overline{a}]d\alpha \tag{15}
$$

Moreover, by means of this formulation and by employing a coefficient $\beta\in[0,1]$, we can consider the grade of pessimism/optimism of the decision maker as being:

$$
F(A) = \int_0^1 [\beta \cdot {}^{\alpha}\underline{a} + (1-\beta) \cdot^{\alpha} \overline{a}]d\alpha \tag{16}
$$

The value $\beta = 0.5$ obviously identifies the neutral decision maker.

The value of the defuzzification function $F(A)$ is the centre of the mean value of the fuzzy number A, in terms of the Dempster–Shaffer theory (Fortemps and Roubens, 1996). Moreover function (16) has a robustness property, according to:

$$
\{\forall A, A' \in \Re | d(A, A') < \varepsilon\} \Rightarrow \{|F(A) - F(A')| < \varepsilon\} \tag{17}
$$

where:

$$
d(A, A') = \max_{\alpha\in(0,1]} \max \left\{ |{}^{\alpha}\underline{a} -^{\alpha} \underline{a}'|, |{}^{\alpha}\overline{a} - {}^{\alpha}\overline{a}')| \right\} \tag{18}
$$

This robustness property ensures that if A and A' are close in terms of (18), the values of the defuzzification function are also close and therefore the use of this function in the calculation of the fitness is justified.

Another aspect developed in this paper is the consideration of an acceptable risk level in the layout design. This can be obtained by integrating equation (16) between a chosen value h and 1. In fact, in reference to a fuzzy-set interpretation of the possibility theory (Zadeh, 1978), reconsidered by Klir and Wierman (1999), if $d\alpha$

is the basic probability of nesting set ${}^{\alpha}A$, then $h = \int_0^h d\alpha$ is an upper bound of the probability of set: $S_A = \{x|0 < A(x) \leq h\}$. Therefore h can be seen as an accepted grade of risk. So equation (16) can be written:

$$F(A) = \frac{1}{h}\int_h^1 [\beta \cdot {}^{\alpha}\underline{a} + (1-\beta) \cdot {}^{\alpha}\overline{a}]d\alpha \tag{19}$$

Equation (19) can be used to compare the fuzzy cost of the layout solutions, taking into account the grade of pessimism/optimism for the decision maker and the accepted risk.

5. Genetic algorithms with fuzzy flows

For each individual generated by the genetic algorithm (Fig. 2), the fuzzy cost and the related fitness value are determined by employing the procedure shown in Fig. 4.

As in the deterministic case, the fitness function can be expressed by:

$$F(i) = \frac{1}{[\text{cost}(i) + \text{penalty}(i)]} = \frac{1}{[\text{cost}(i) + n_i^k \cdot (\text{cost}_{\text{feas}} - \text{cost}_{\min})]} \tag{20}$$

where cost(i) is the cost of material flow which, in the fuzzy contest, is obtained by applying defuzzification function (19) to fuzzy cost expression (12).

In a practical way, in order to obtain fuzzy cost C, it is necessary to determine the lower and upper bounds for each α-cut. It has been assumed that the production capacity is not saturated in correspondence to the lower values of the request for each α-cut.

Therefore the calculation of the lower bound is trivial, because relation (9) is always equal to one for all combinations of ${}^{\alpha}\underline{q}_k \forall k$ and so:

$$ {}^{\alpha}\underline{c} = \left\{ \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\sum_{k=1}^{N} {}^{\alpha}\underline{q}_k X_{ij}^k\right) c_{ij} d_{ij} \right\} \tag{21}$$

The calculation of the upper bound is carried out using the following model of Linear Programming:

$$ {}^{\alpha}\overline{c} = \max \sum_{i=1}^{M}\sum_{j=1}^{M}\left(\sum_{k=1}^{N} q_k \cdot X_{ij}^k\right) \cdot c_{ij} \cdot d_{ij} \tag{22}$$

subject to:

$$\sum_{k=1}^{N} \frac{q_k \cdot Y_j^k}{s_{kj}} \leq 1 \quad j = 1, \ldots, M \tag{23}$$

$$ {}^{\alpha}\underline{q}_k \leq q_k \leq {}^{\alpha}\overline{q}_k \quad k = 1, \ldots, N \tag{24}$$

In this way, for each α-cut, it is possible to obtain the maximum cost relating to the combination of the material flows which respect the capacity constrains.

Once the ${}^{\alpha}\underline{c}$ and ${}^{\alpha}\overline{c}$ values for each α-cut have been calculated, the fuzzy cost C obtained is defuzzificated by numerical integration of equation (19) and this constitutes the value of cost(i) in the fitness function (20).

6. Test problems

A first test, without considering the constraints on the departments' production capacity, was carried out simply to verify the correctness of the genetic algorithm in the fuzzy context. The test was derived from the original crisp problem (par. 3.4), representing the flows using a triangular membership function with the centre value equal to the crisp value, and spreads of ±20%. The departments' aspect ratio, γ_{cr}, was set equal to 3 and no constraints on the departments' capacity were imposed. The layout configuration obtained, as expected, is the same as the crisp problem: Permutation (11,16,15,13,17,14,10,9,12,3,19,5,1,2,4,7,8,6,18,20), Breakpoint (2,5,9,12,18).

The fuzzy cost of the solution is shown in Fig. 5. The cost (5594.30) corresponding to value one of the membership function is equal to the cost obtained for the original (crisp) problem: thus the correctness of the software is confirmed.

In order to apply the previously described fuzzy constrained layout model, a new problem with requisite constraints was formulated. The number of departments (20) and their areas are set equal to the ones of the crisp problem (par. 3.4) and the aspect ratio is imposed equal to 3. The production cycles of nine typologies of products are shown in Table 3. The fuzzy demand of the generic product k is expressed by a trapezoidal fuzzy number. The related membership function is defined by the four values reported in Table 4, corresponding to Q_k

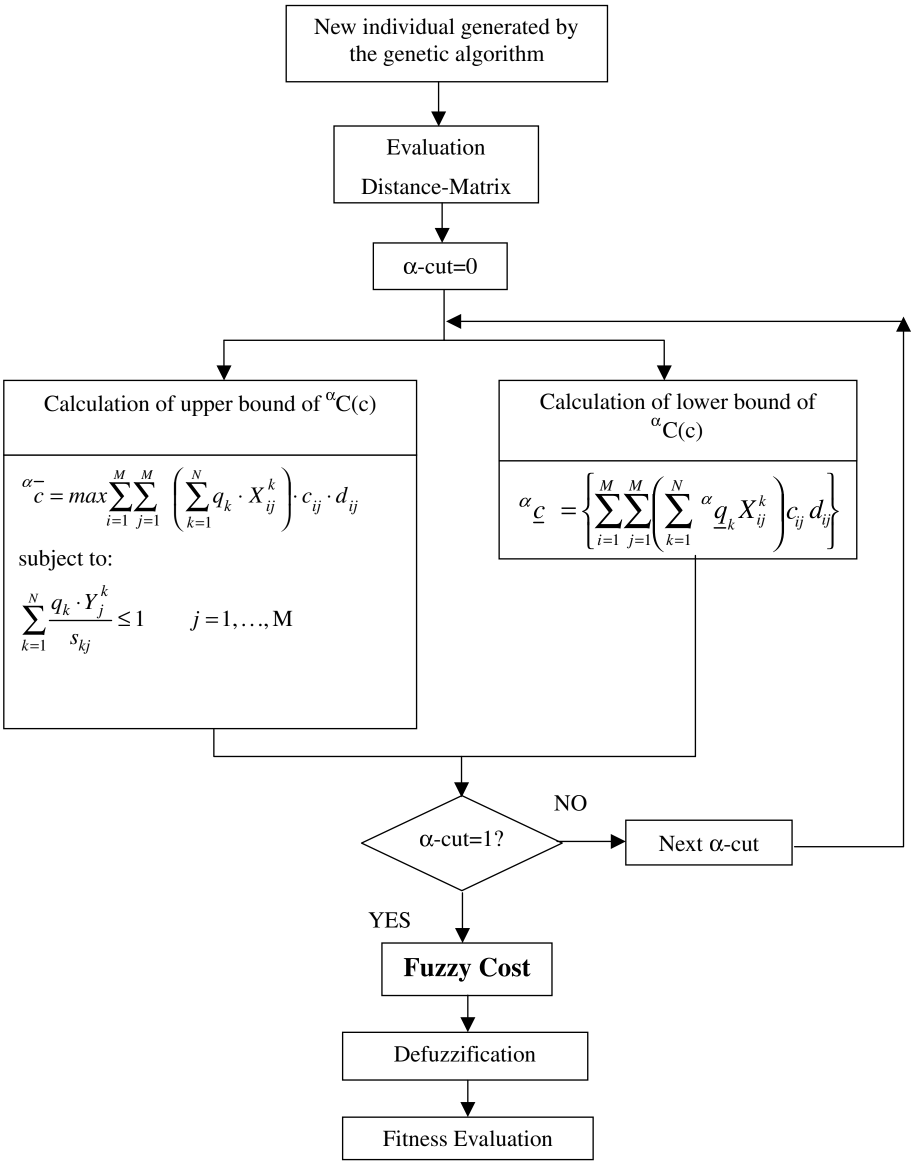


Fig. 4. Procedure for fuzzy cost evaluation.

$(a)=0$, $Q_k(b)=1$, $Q_k(c)=1$, $Q_k(d)=0$. All cost coefficients c_{ij} are fixed equal to 1. The departments' production capacities (s_{kj}) are fixed equal to 800. This value has been chosen in order not to saturate the departments' production capacity in correspondence with the lower request values; this situation is realistic, as already noted in paragraph 5.

By employing the procedure described in the previous paragraphs, the solution obtained was: Permutation (11,15,14,10,1,20,2,4,5,3,18,9,7,6,13, 19,8,12,16,17), Breakpoint (5,10, 14, 18). The related fuzzy cost is shown in Fig. 6.

The effect of the capacity production constraints on the right spread of the fuzzy cost (corresponding to the greatest flows) is evident. In fact, as Table 4 shows, the trapezoidal fuzzy numbers representing the product demand, have a right spread greater than the left spread. On the contrary, when the constraints are applied, the fuzzy cost has a lower right spread.

Fig. 5. Fuzzy cost for the problem with fuzzy flows represented by triangular membership function.

Table 3. Working sequences

Product	*Sequence*
A	1-2-3-6-4-7-8-13-14-11-15-17-19-18-5-20
B	1-3-9-2-7-10-12-15-14-18-6-5-4-20—16-13
C	1-10-11-12-4-18-7-5-2-19-8-6-9-15-20
D	1-11-16-17-12-6-7-14-9-4-19-3-10-15-20-18-13
E	1-14-3-10-2-4-5-8-20-7-15-11-16-6-12-17
F	15-5-10-14-2-20-19-16-8-12-3-17-6-11-18-9
G	8-19-12-3-6-9-15-18-20-11-13-16-10-14-17-7-5
H	9-17-18-12-10-15-19-20-2-4-5-3-11-8-7-6-1
I	19-10-15-13-16-20-11-2-3-5-12-6-7-9-4-14

Table 4. Fuzzy requests of nine products expressed by trapezoidal membership function

	A	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*
a	20	40	30	30	20	40	30	30	30
b	90	80	80	80	60	80	60	60	70
c	110	120	120	120	140	120	140	140	130
d	170	180	230	200	180	230	180	300	290

Conclusions

A valid approach to the layout problem should be to formulate a model which is capable of maximizing the systems efficiency over the long term (the economic lifetime of the facility layout), taking the uncertainty of input data and, in particular, market demand into account. Moreover, the block layout solutions must be easily convertible to a detailed layout. The aim of this work has been to obtain a robust layout, which is to say a layout that performs well in a wide variety of demand scenarios. The element of uncertainty associated with each product demand is modelled as a fuzzy number, leading to a fuzzy material handling cost. The facility layout problem is a NP-hard problem, and it is further complicated by the introduction of uncertainty which adds new dimensions to the search space. In order to solve such an enhanced model with near real instances and in reasonable time, the choice of a powerful

Fig. 6. Fuzzy cost for the constrained test problem.

meta heuristic is required. Genetic algorithms are employed nowadays in many fields, thanks to their ability to perform parallel search in complex spaces avoiding premature convergence to local optima. A key aspect in genetic algorithms is related to the data structures employed. In the context of facility layout, the flexible bay structure (Tate and Smith, 1995) is particularly suited to a chromosome representation and allows the decision maker to control the department shape, using an adaptable penalty function.

To evaluate the fitness function of solutions generated by the proposed algorithm, the fuzzy material handling cost has to be defuzzified, thus allowing the implementation of a ranking criterion. The defuzzification function employed allows us to take the decision maker's degree of pessimism/optimism and his accepted degree of risk into account.

In addition to this, each department has a limited capacity, so that particular combinations of product demands may exceed capacity and lead to infeasible scenarios. This aspect is dealt with using a fuzzy constrained arithmetic operator, which evaluates the fuzzy material handling costs considering only the possible flows between the departments which are compatible with the capacity constraints.

The facility layout model proposed has allowed to structure an effective genetic algorithm for the optimization of medium-large layout problems, taking into account the element of uncertainty related to product demands, the finite production capacity of the departments and some constraints on their shape and location, leading to a realistic and robust layout.

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