Journal of Intelligent Manufacturing, 16, 103–114, 2005
© 2005 Springer Science+Business Media, Inc. Manufactured in The Netherlands.

A parametric model for cell formation and exceptional elements' problems with fuzzy parameters★

FEYZAN ARIKAN* and ZÜLAL GÜNGÖR

Industrial Engineering Department, Gazi University, 06570Maltepe, Ankara, Turkey
E-mail: farikan@gazi.edu.tr

Received September 2003 and Accepted April 2004

This paper introduces a new fuzzy mathematical model based on the fuzzy parametric programming (FPP) approach for the cellular manufacturing system (CMS) design. The aim of the proposed model is to handle two important problems of CMS design called cell formation (CF) and exceptional elements (EE) simultaneously in fuzzy environment. The model is capable to express vagueness of all the system parameters and gives the decision-maker (DM) alternative decision plans for different grades of precision. So, it is expected to provide a more realistic CMS design for real life problems. To illustrate the model proposed here, an example with fuzzy extension in data set is adopted from literature and computational results are presented.

Keywords: Cellular manufacturing, cell formation, exceptional elements, fuzzy mathematical programming, fuzzy numbers

1. Introduction

Cellular manufacturing (CM), is a philosophy and innovation to improve manufacturing productivity and flexibility since it allows small batch-type production to gain economic advantages to those of mass production and still retain the flexibility of job-shop production. One of the first and most important steps in designing CM Systems, called cell formation (CF), is to create completely independent machine cells and identify part families and allocate part families to machine cells. CF solutions often contain exceptional elements (EE). The EE can be defined as machines required by two or more part families or conversely as parts that require processing on machines in two or more cells. The EE create interactions between two manufacturing cells. The interaction between cells which is an important obstacle to reach the benefits of CMS such as reduction in set-up time, work-in-process inventory, material handling cost, improvement in material flow, space utilisation etc. ruin the objective of creating independent cells. So CF and EE problems both have crucial importance in the design of CMS.

In the existing literature, exceptional element-problem is dealt with after cell formation and these two problems are generally handled in crisp environment. The former approach results with the complete dependence of the solution effectiveness on the technique used in the cell formation stage. The latter is contradictory to the fuzzy nature of the problem.

*Author for correspondence.

★This paper was presented in the 2nd Group Technology/Cellular Manufacturing-World Symposium, Ohio University, Ohio, USA, July 28–30.

In literature, there are a lot of studies and approaches, which can be found in the survey papers (Moiser and Taube, 1985; Wemmerlöv and Hyer, 1986; Singh, 1993; Heragu, 1994; Offodile *et al.,* 1994; Cheng *et al*., 1995; Selim *et al*., 1998) dealt with CF. In the first studies related with EE, since researchers concentrated especially on the development of CF procedures, treatment of EE was a secondary objective. Although the other researchers such as Seifoddini (1989), Kern and Wei (1991) and Shafer *et al.* (1992) have addressed elimination of EE as their primary focus, the efficiency of their solutions depended on the efficiency of the CF technique used at the beginning.

CMS design problem, as a real life problem, needs to be investigated in fuzzy environment due to the fuzzy design parameters. The early studies of fuzzy CMS are Chu and Hayya's (1991), Zang and Wang's (1992) and Arieh and Triantaphyllou's (1992) studies some examples of the late studies belong to Masnata and Settineri (1997), Gill and Bector (1997), Güngör and Arıkan (2000). These are generally focused on the fuzziness stemmed from the part features and they used the traditional solution methodologies developed for the CM. In this study it is concentrated on the fuzzy mathematical programming. While the CMS design literature is investigated in the context of fuzzy mathematical programming, it is found two studies interested CF and/or EE problem. First study (Shanker and Vrat, 1999) presented two fuzzy linear programming models for "post clustering" stage and considered the fuzziness of part demand, budgetary limit on purchase new machines and the aspiration level of the objective function. The other study (Tsai *et al*., 1997) proposed a mathematical model which handle EE automatically during CF. But in the model, only the aspiration level of the objective function and the number of machine types allowed in each cell are considered as fuzzy. The fuzziness of some parameters such as demand, capacity and financial data that have very important role to attain the implementable solution as well as optimal are not considered.

In this study, the proposed model handles the EE problem during the clustering stage and considers the fuzziness of part demand, machine capacity and the fuzziness of EE's elimination costs which are coefficients of objective function instead of the aspiration level attained to the objective function. Since these fuzzy parameters constitute both objective function and constraint coefficients, the well-known fuzzy mathematical programming (FMP) approaches such as fuzzy linear programming (FLP) which only consider the fuzziness of right-hand-side coefficients can not be used as the way of solution for the proposed fuzzy model. According to the classification of single objective FMP approaches (Lai and Hwang, 1992) in the literature, the fuzzy parametric programming (FPP) (Carlsson and Korhonen, 1986) comes in handy since it takes into consideration the fuzziness of whole parameters in a single objective mathematical model. So the fuzzy CMS problem interested in this study is modelled by the FPP.

By using the FPP approach, the study proposes a FPP model that provides both implementable and optimal solutions for different grades of precision of fuzzy parameters, which is demonstrated with a numerical example.

In the remainder of the paper; first, the FPP approach is reviewed with a brief theoretical background. Then, under the heading of "the proposed model", the model construction steps are investigated. In Section 4, how the model works is demonstrated with a numerical illustration. Finally, computational results, conclusions and future directions are presented.

2. Fuzzy parametric programming

FPP, one of FMP approaches, proposed by Carlsson and Korhonen (1986). FPP, separated from other FMP approaches by being able to consider all coefficients in a mathematical model as fuzzy as being in real life, is reviewed briefly in this section. Prior to the review, some basic definitions in FST (Zadeh, 1965) are introduced.

Definition:

A fuzzy subset $\tilde{A}$ of a universe of discourse X is defined by a membership function $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$ which associates with each element x of X a number $\mu_{\tilde{A}}(x)$ in the interval [0,1], where $\mu_{\tilde{A}}(x)$ represents the grade of membership of x in $\tilde{A}$. Formally $\tilde{A}$ can be written as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}, \quad x \in X \tag{1}$$

Definition:

Intersection of two fuzzy subsets denoted $\tilde{A} \cap \tilde{B}$:

$$\tilde{A} \cap \tilde{B} = \{(x, \mu_{\tilde{A}\cap\tilde{B}}(x) | x \in X\},$$
$$\text{where } \mu_{\tilde{A}\cap\tilde{B}}(x) = (\mu_{\tilde{A}}(x) \wedge \mu_{\tilde{B}}(x)) \quad (2)$$

Let consider the following mathematical model:

$$\begin{aligned} &\max \; z = \mathbf{cx} \\ &\text{s.t.} \quad x \in X = \{x | -\mathbf{Ax} \leq \mathbf{b}, \mathbf{x} \geq 0\} \end{aligned} \quad (3)$$

where **c** is an n-vector, **A** is an (mxn)-matrix and **b** is an m-vector. When a model of this type is applied to a practical problem, the parameters cannot be known exactly and they are better or worse estimate from existing data or subjective knowledge. On the other hand, the solution of the model is the optimal for the related parameter set which however they are determined. But in real life, it is important for a solution being optimal and "implementable" at the same time. The implementability grades of a solution are given by membership functions, which, in turn, can be derived from the grades of imprecision in the parameters (Carlsson and Korhonen, 1986).

It is assumed that the intervals for possible values of fuzzy parameters are specified by the user as $[c^0, c^1)$, $[A^0, A^1)$, $[b^0, b^1)$. The lower bounds represent "risk-free" values in the sense that a solution most certainly should be implementable. The upper bounds, on the other hand, represent parameter values which are most certainly unrealistic, "impossible" and the solution obtained by using these values is not implementable. While moving from "risk-free" toward "impossible" parameter values, it is moved from solutions with a high grade to solutions with a low grade on implementing. The purpose of FPP is to find and optimal compromise "in-between" as a function of grades of imprecision in parameters (Carlsson and Korhonen, 1986).

The precision of an optimal solution, μ_S, is defined by the intersection of membership functions belong to imprecise parameters denoted by μ_C, μ_A, μ_b

$$\mu_S = \left(\mu_{c_j} \wedge \mu_{A_{ij}} \wedge \mu_{b_i}\right), \quad i = 1, \ldots, m, \; j = 1, \ldots, n \quad (4)$$

The expression means that the inherited precision in the optimal solution equals the precision of the most "risky" of the parameters. In model (3) the best value for the objective function at a fixed level of μ_S is reached when

$$\mu_S = \left(\mu_{c_j} = \mu_{A_{ij}} = \mu_{b_i}\right), \quad i = 1, \ldots, m, \; j = 1, \ldots, n \quad (5)$$

Since model tends to use the "risky" values of the parameters. The expression just above means that the best value of the objective function, at a fixed level of precision, can be found by using parameter values of the same level of precision (Carlsson and Korhonen, 1986).

2.1. *FPP model formulation*

For model (4), fuzzy parameters denoted by $p \in [p^0, p^1)$ and it is assumed that they have linear monotonically decreasing membership functions:

$$\mu_p = [(p - p^0)/(p^0 - p^1)],$$
$$\text{where } p \in [p^0, p^1) \text{and } \mu_p = 1 \text{ if } p < p^0, \; \mu_p = 0 \text{ if } p > p^1 \quad (6)$$

p parameter value is evaluated from the expression above as $p = p^1 + \mu_p(p^0 - p^1)$. Then by using this structure for model (3), the FPP model is as follows:

$$\begin{aligned} &\max \quad z = [c^1 + \mu(c^0 - c^1)]x \\ &\text{st.} \quad -[A^1 + \mu(A^0 - A^1)]x \leq b^1 + \mu(b^0 - b^1) \quad x \geq 0 \\ &\text{where } \mu_c = \mu_A = \mu_b = \mu \end{aligned} \quad (7)$$

In the model (7), it is assumed they are identical for each parameter and linear in form. Of course there can be many possible forms for a membership functions such as piece-wise linear (triangular, trapezoidal), exponential, hyperbolic for real life situations.

3. The proposed model

The model construction steps based on FPP approach are summarised in Fig. 1 as a flow chart. The steps are explained in detail below.

Step 1. Problem Definition: Herein the cellular manufacturing design (CMS) problem is considered which is defined as the creation of manufacturing cells and dealing with EE simultaneously. To be

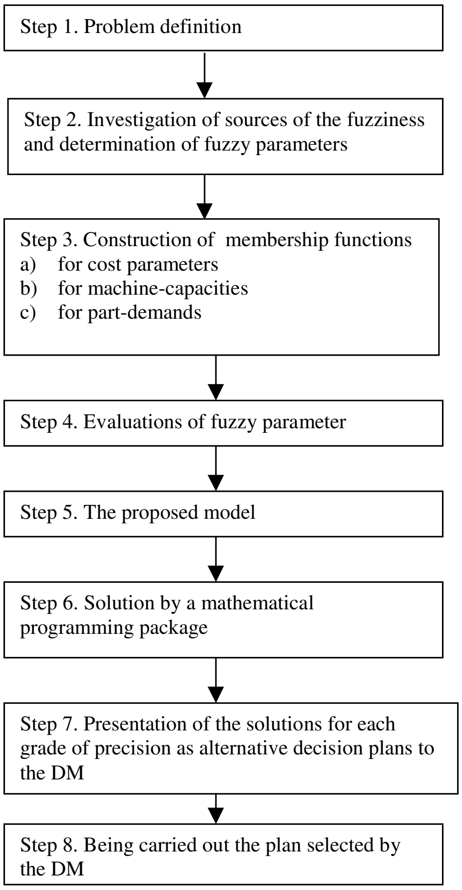

Fig. 1. The steps of the proposed model construction.

dealt with the EE during CF, the combination of three alternative elimination strategies are considered, which are duplication of bottleneck machines, intercellular transfer and/or subcontracting of exceptional parts.

The special feature of the problem is the necessity of being investigated in a fuzzy environment because of fuzzy system parameters. So, sources of the fuzziness are investigated as a second step of the construction of the proposed model.

Step 2. Investigation of sources of the fuzziness: In a mathematical model developed for designing CMS, model parameters such as "satisfactory profit", "satisfactory demand", "insufficient machine capacity", "approximate cost of subcontracting an exceptional part", "reasonable budget for machine duplication", are difficult to capture by determinism. This is mainly because of three reasons: (a) substantial time gap between design and implementation, (b) high cost in acquiring system parameters with precision, (c) lack of statistical observation at the design stage.

The imprecise parameters and the reasons of impreciseness for the problem are given in detail below (Table 1) (Shanker and Vrat, 1999). In contrast to the reviewed literature the proposed model considers the fuzziness of all parameters mentioned in the table.

Step 3. Construction of membership functions: To express the fuzziness mathematically, firstly the membership functions for each fuzzy parameter defined in step 2 should be constructed. While choosing the types of membership functions, possible behaviours and trends of parameters in real life and their risk trade-off in possible lower and upper limits are considered.

In the following, for fuzzy parameters as costs of EE elimination (A_i, I_j, S_j), machine capacities, C_i, and part demands, D_j, fully trade-off membership functions are constructed. In this study they are assumed linear monotonically increasing, convex exponential and piece-wise linear, respectively. One of the aims of this assumption is to present the modelling and solution ways when the membership functions are different in form and have non-linear structures as being in real life.

(a) *Membership functions for EE elimination costs*

The linear membership functions for fuzzy parameters of machine duplication cost (A_i), intercellular transfer cost (I_j) and subcontracting cost (S_j) are denoted by μ_{A_i}, μ_{I_j}, μ_{S_j} respectively. The upper bounds in the intervals $[A_i^0, A_i^1)$, $[I_j^0, I_j^1)$, $[S_j^0, S_j^1)$ represent "risk-free" values. The membership functions defined as follows for$\forall i$ and $\forall j$:

$$\mu_{A_i} = (A_i^0 - A_i)/(A_i^0 - A_i^1), \quad \text{where} \quad A_i \in [A_i^0, A_i^1) \quad \text{and} \quad \mu_{A_i} = 0 \ \text{if} \ A_i < A_i^0, \quad \mu_{A_i} = 1 \ \text{if} \ A_i > A_i^1 \tag{8}$$

$$\mu_{I_j} = (I_j^0 - I_j)/(I_j^0 - I_j^1), \quad \text{where} \ I_j \in [I_j^0, I_j^1) \quad \text{and} \quad \mu_{I_j} = 0 \ \text{if} \ I_j < I_j^0, \ \mu_{I_j} = 1 \ \text{if} \ I_j > I_j^1 \tag{9}$$

$$\mu_{S_j} = (S_j^0 - S_j)/(S_j^0 - S_j^1), \quad \text{where} \ S_j[S_j^0, S_j^1) \quad \text{and} \quad \mu_{S_j} = 0 \ \text{if} \ S_j < S_j^0, \ \mu_{S_j} = 1 \ \text{if} \ S_j > S_j^1 \tag{10}$$

Table 1. Reasons for considering uncertainty in CMS design parameters (Shanker and Vrat, 1999, p. 2549)

CMS design parameter	*Reasons for considering uncertainty in parameter estimates*
(1) Part demand	(a) Time gap between design and implementation (b) High cost in acquiring system parameters with precision (c) Insufficient market survey at design stage (d) Product specifications not yet finalised (e) Unknown product mix (f) Competitors' competence and preparedness
(2) Subcontracting cost of EE	(a) Time gap between design and implementation (b) Market uncertainty due to competition (c) Unwillingness of subcontractors to quote prices without order near future (d) Inflation
(3) Purchase price of bottleneck machines	(a) Time gap between design and implementation (b) Changes in government policy (c) Changes in import restrictions (d) Unwillingness of suppliers to quote prices without order near future (e) Inflation
(4) Intercellular transfer cost	(a) Material handling equipment, yet undecided (b) Likely changes in plant layout (c) Inflation (d) Location and size of stores (e) Undecided process plans and sequence of operations
(5) Available capacity of machine	(a) Undecided machine type (b) Failure, location of faults and maintenance (c) Duplication possibilities of machines

Since the lower bounds represent minimum cost values, which cannot be implementable, of which the grade of membership is zero (Fig. 2).

(b) *Membership functions for machine capacity*

The convex exponential membership function for fuzzy parameters of available capacity for each machine, C_i, is denoted by μ_{C_i}. The lower bound in the interval $[C_i^0,\ C_i^1)$ represents the "risk-free" value for each i. The membership function defined as follows for $\forall i$:

$$\mu_{C_i} = \{1 - \exp[0.8(C_i - C_i^1)/(C_i^0 - C_i^1)]\}/[1 - \exp(0.8)], \quad \text{where } C_i \in [C_i^0, C_i^1) \quad \text{and}$$
$$\mu_{C_i} = 1 \ \text{if } C_i < C_i^0,$$
$$\mu_{C_i} = 0 \ \text{if } C_i > C_i^1 \qquad (11)$$

The upper bound of the function represents the full capacity of machine type i without any failure which cannot be implementable, of which the grade of membership is zero. (Fig. 3) Convex exponential structure provides rapid changes in the membership values while the fuzzy capacity of the system close to the unimplementable and also implementable capacity values according to the interval considered as being in real life.

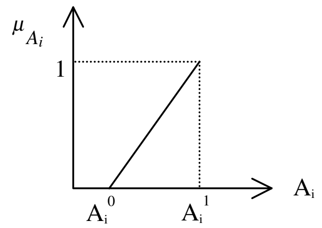

Fig. 2. Membership function for machine duplication cost. A_i^1, "risk-free" value; A_i^0, "unimplementable" value.

(c) *Membership functions for part demand*

The piece-wise linear membership function for fuzzy parameters of part demand, D_j, is denoted

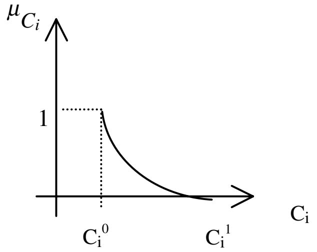

Fig. 3. Membership function for machine capacity. C_i^1, "unimplementable" value; C_i^0, "risk-free" value.

by μ_{D_j}. The function is defined in the interval of $[D_j^0, D_j^1, D_j^2)$ and $[D_j^0, D_j^1]$ represents the "risk-free" value-interval for each j. The membership function defined as follows for $\forall\, j$:

$$\mu_{D_j} = \left\{ \begin{array}{cc} 1 & D_j^0 \le \tilde{D}_j < D_j^1 \\ (D_j^2 - \tilde{D}_j)/(D_j^2 - D_j^1) & D_j^1 \le \tilde{D}_j < D_j^2 \end{array} \right\} \quad (12)$$

D_j^2 values for each j represent the very high demands that cannot implementable. (Fig. 4)

Step 4. Evaluations of fuzzy parameters: Fuzzy parameter values are derived from the mathematical expressions (8)–(12) of membership functions defined just above. Each parameter value can be calculated by using the following formulations:

$$A_i = A_i^0 - \mu_{A_i}(A_i^0 - A_i^1), \quad \text{where } A_i \in [A_i^0, A_i^1) \text{ for } \forall i \quad (13)$$

$$S_j = S_j^0 - \mu_{S_j}(S_j^0 - S_j^1), \quad \text{where } S_j \in [S_j^0, S_j^1) \text{ for } \forall j \quad (14)$$

$$I_j = I_j^0 - \mu_{I_j}(I_j^0 - I_j^1), \quad \text{where } I_j \in [I_j^0, I_j^1) \text{ for } \forall j \quad (15)$$

$$C_i = C_i^1 + (1/0.8)\ln\{1 - \mu_{C_i}[1 - \exp(0.8)]\}(C_i^0 - C_i^1), \quad \text{where } C_i \in [C_i^0, C_i^1) \text{ for } \forall i \quad (16)$$

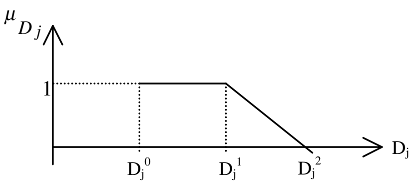

Fig. 4. Membership function for part demand. D_j^2, "unimplementable" value; $[D_j^0, D_j^1]$, "risk-free" value-interval.

$$D_j = D_j^2 - \mu_{D_j}(D_j^2 - D_j^1), \quad \text{where } D_j \in [D_j^1, D_j^2) \text{ for } \forall j$$

$$D_j = D_j \quad \text{where } D_j \in [D_j^0, D_j^1) \text{ for } \forall j \quad (18)$$

Since the levels of precision are fully trade-off ($\mu_{A_i} = \mu_{S_j} = \mu_{I_j} = \mu_{C_i} = \mu_{D_j} = \mu$), the expressions (13)–(18) can be written as a function of μ and then each of them is used to built up the proposed FPP model.

Step 5. The proposed model: After the definition of the design problem and its features, the parametric model of the cell formation and exceptional elements problem with fuzzy parameters is constructed as follows. Fuzzy parameters denoted by using ~ in the model.

Index set: i: Machine index, j: Part index, k: Cell index

Decision Variables

- X_{ik} — 1 if machine i assigned to cell k; 0 otherwise,
- Y_{jk} — 1 if part j assigned to cell k; 0 otherwise,
- U_{ijk} — 1 if $X_{ik} = 1$ ve $Y_{jk} = 0$, 0 otherwise,
- V_{ijk} — 1 if $Y_{jk} = 1$ ve $X_{ik} = 0$, 0 otherwise,
- Z_{ijk} — number of intercellular transfers required by part j as a result of machine type i not being available within cell k,
- O_{ijk} — units of part j to be subcontracting as a result of machine type i not being available within cell k,
- R_{ik} — number of machine type i to be purchased for cell k (integer),
- Q_i — number of machine type i needed to process corresponding parts in a machine cell (integer),
- M_{ijk} — number of machine type i dedicated to cell k for producing exceptional part j.

Parameters

- a_{ij} — 1 if part j needs to be processed by machine i; 0 otherwise,
- $\tilde{A}_i$ — purchase price of machine type i,
- $\tilde{C}_i$ — periodic capacity of machine type i,
- $\tilde{D}_j$ — periodic demand for part j,
- $\tilde{I}_j$ — incremental cost for moving a unit of part j within two cells,

NM minimum number of machine types allowed in each cell,
MM maximum number of machine types allowed in each cell,
P_{ij} processing time of machine type i needed to produce part j,
$\tilde{S}_j$ incremental cost of subcontracting a unit of part j for an operation,
SP set of pairs (i,j) such that $a_{ij}=1$,
$U\tilde{C}_{ij}$ utilisation capacity of machine type I for parts j $(P_{ij}\tilde{D}_j/\tilde{C}_i)$.

The Model
Objective function

$$\min \sum_k \sum_i (A_i^0 - \mu(A_i^0 - A_i^1))R_{ik} + \sum_k \sum_{(i,j)\in SP} (I_j^0 - \mu(I_j^0 - I_j^1))Z_{ijk} + \sum_k \sum_{(i,j)\in SP} (S_j^0 - \mu(S_j^0 - S_j^1))O_{ijk} \quad (19)$$

st.

$$\sum_{k=1}^{c} X_{ik} = 1 \quad \forall i \quad (20)$$

$$\sum_{k=1}^{c} Y_{jk} = 1 \quad \forall j \quad (21)$$

$$NM \leq \sum_{k=1}^{m} X_{ik} \leq MM \quad \forall k \quad (22)$$

$$X_{ik} - Y_{jk} + U_{ijk} - V_{ijk} = 0 \quad \forall (i,j) \in SP \quad \forall k \quad (23)$$

$$Z_{ijk} + O_{ijk} + ((C_i^1 + (1/0.8)\ln(1 - \mu(1 - \exp(0.8))) {}^*(C_i^0 - C_i^1)M_{ijk}/P_{ij}) = \tilde{D}_j U_{ijk} \forall (i,j) \in SP \ \forall k \quad (24)$$

$$\sum_{(i,j)\in SP} M_{ijk} \leq R_{ik} \quad \forall i \ \forall k \quad (25)$$

$$Q_i \leq \sum_{(i,j)\in SP} U\tilde{C}_{ij}\left(1 - \sum_k V_{ijk}\right) + 1 \quad \forall i \quad (26)$$

$$\sum_k \sum_{(i,j)\in SP} P_{ij}Z_{ijk}/((C_i^1 - (1/0.8)\ln(1-\mu(1-\exp(-0.8))) * (C_i^0 - C_i^1) \leq Q_i - \sum_{(i,j)\in SP} U\tilde{C}_{ij}\left(1 - \sum_k V_{ijk}\right) \forall j \quad (27)$$

$$X_{ik}, Y_{jk}, \mathrm{U}_{ijk}, V_{ijk} = 0 \quad \text{or} \quad 1 \quad (28)$$

$$R_{ik}, \quad Q_i \qquad \text{integer} \quad (29)$$

Objective function of the model (19), minimises three types of fuzzy cost associated with EE. They are bottleneck duplication cost, intercellular transfer cost for EE and subcontracting cost, respectively. Constraints (20) and (21) ensure that each machine and part is assigned to only one cell. Constraint (22) prevents the assignment of less than NM and more than MM machines to each cell. Constraint (23) ensures that an *EE* either is a bottleneck machine or an exceptional part. Constraint (24) guarantees that the fuzzy demand of exceptional part j can be shared by the duplicated machine i, transfer within cells, or subcontracting. Constraint (25) determines the number of machine type i to be purchased for cell k as integer. Constraint (26) determines the number of machine type i needed in each cell. Constraint (27) ensures that the number of intercellular transfers between machines of type i do not exceed the available fuzzy machine capacity. Equations (28) and (29) stand for 0,1 integer and integer constraints, respectively.

Step 6. Model Solving: The proposed FPP model is solved by a mathematical programming package. While we solve the model, membership functions' structures determine our solution strategy. If each fuzzy parameter has identical membership functions just like mentioned in model (7) then the FPP model can be easily solved by using the traditional way even if the membership functions are non-linear. But in the proposed model, membership functions for each fuzzy parameter are different in form. So, the model is solved for predetermined different values of the membership function and the set of solutions represent alternative decision plans for varying grade of precision. They prepare reasonable and realistic foundation that enables DM to arrive at appropriate conclusions.

Herein, another important point during model solving is that for $\mu = 1$, the membership function takes continuous values in the interval $[D_j^0, D_j^1)$ for each j due to the structure of the function of fuzzy part demand. In such a situation, while the other parameter's "risk-free" values remain same, the $[D_j^0, D_j^1)$ region should be investigated of which detailed explanation is illustrated in the next Section 4.

Steps 7 and 8 are carried out just as mentioned in the flow chart.

Table 2. The data set for fuzzy cell formation problem (M : Machine, P : Part)

M/P^*	1	2	3	4	5	6	7	8	9	10	A_i^0	A_i^1	C_i^0	C_i^1
1	2.95	0	2.2	0	0	0	0	0	0	4.61	45,000	50,784	140,000	160,000
2	2.76	5.18	1.89	3.89	0	5.14	0	0	0	0	60,000	67,053	140,000	160,000
3	5.54	4.29	0	0	0	0	0	0	0	0	37,000	43,944	140,000	160,000
4	2.91	0	0	1.97	2.59	4.01	0	2.7	0	0	61,000	67,345	140,000	160,000
5	0	0	0	4.28	0	4.51	0	0	0	0	35,000	42,414	140,000	160,000
6	1.92	0	0	0	0	0	2.23	0	5.52	0	70,000	75,225	140,000	160,000
7	0	0	0	0	3.4	0	1.16	4.72	0	2.49	50,000	52,741	140,000	160,000
8	0	5.32	0	0	0	0	0	3.75	3.85	0	59,000	63,523	140,000	160,000
9	0	0	0	0	0	0	4.04	0	0	1.83	49,000	50,632	140,000	160,000
S_j^0	3.8	4	3	4	4.5	3	4.2	4.3	4.6	4				
S_j^1	4.2	4.3	3.5	4.4	5	3.9	4.4	4.6	5	5				
I_j^0	3.60	2.70	2.75	3.20	2.80	3.40	2.70	2.50	3.30	3.20				
I_j^1	3.85	2.95	2.88	3.45	2.80	3.65	2.95	2.75	3.55	3.20				
D_j^0	33,000	30,000	20,000	11,000	18,000	17,000	46,000	46,000	16,000	23,000				
D_j^1	35,000	32,000	22,000	12,500	19,000	18,500	47,000	47,000	17,000	25,000				
D_j^2	40,000	36,000	25,000	14,000	20,000	19,000	48,500	49,000	17,500	26,000				

4. Data set for numerical computations and parameterised of fuzzy coefficients

To illustrate the proposed FPP model, the data set adapted from the Shafer *et al.*'s (1992) study. Table 2 lists the processing times of each part and the part-machine matrix structure. Other related interval data for the fuzzy parameters as the costs involved, the part demand and the machine capacity in the table were generated randomly.

Fuzzy parameter values for $\forall\ i$ and $\forall\ j$ ($i=1, 2, \ldots, 9$ and $j=1, 2, \ldots, 10$), were calculated for each μ level (0, 0.1, 0.2, … , 1) and by using the parameter sets gathered from the calculations, 11 different problems are constructed. For the interval $(D_j^0, D_j^1]$ of which the level of membership is constant and 1, it is considered by 1000 units increments in part demand, so there are 2,2,1,1,1,1,1,1,1,2 different part demand levels for each j, respectively. The permutation of these levels results the eight different parameter sets in the interval $(D_j^0, D_j^1]$ while the other parameter values are constant for $\mu = 1$ (Table 3).

The calculation of each parameter for $\mu = 0.1$ is illustrated below:

$$A_1 = 45,000 - 0.1(45,000 - 50,784) = 45,578.4\$, \quad \text{where } A_1 \in [45,000,\ 50,784) \quad \text{for } i = 1$$

$$S_1 = 3.8 - 0.1(3.8 - 4.2) = 3.84\$, \quad \text{where } S_1 \in [3.8,\ 4.2) \quad \text{for } j = 1$$

$$I_1 = 3.6 - 0.1(3.6 - 3.85) = 3.63\$, \quad \text{where } I_1 \in [3.6,\ 3.85) \quad \text{for } j = 1$$

Table 3. $\mu=1$, $D_{jx}\ 10^3 \in (D_j^0, D_j^1]$, alternative demand combinations for each part

j	*D1*	*D2*	*D3*	*D4*	*D5*	*D6*	*D7*	*D8*
1	33	33	33	33	34	34	34	34
2	30	31	30	31	30	31	30	31
3	21	21	21	21	21	21	21	21
4	12	12	12	12	12	12	12	12
5	18	18	18	18	18	18	18	18
6	18	18	18	18	18	18	18	18
7	46	46	46	46	46	46	46	46
8	46	46	46	46	46	46	46	46
9	16	16	16	16	16	16	16	16
10	23	23	24	24	23	23	24	24

Table 4. $D_j \in (D_j^1, D_j^2]$ for $\forall\ j$ the FPP model solutions, $O_{ijk} = 0$

μ	*min Z* ($)	Z_{ijk}	R_{ik}	*EE*	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9
0	368 844.20	$Z_{691}=2427$ $Z_{883}=6333$	$R_{13}=R_{22}=R_{41}=R_{61}=R_{73}=R_{82}=1$	9	2	3	3	2	1	1	2	2	2
0.1	374 715.91	$Z_{692}=2727$ $Z_{883}=6904$	$R_{11}=R_{23}=R_{42}=R_{62}=R_{71}=R_{83}=1$	9	2	3	3	2	1	1	2	2	2
0.2	380 149.07	$Z_{691}=2972$ $Z_{882}=7394$	$R_{13}=R_{22}=R_{41}=R_{61}=R_{73}=R_{82}=1$	9	2	3	3	2	1	1	2	2	2
0.3	385 406.23	$Z_{692}=3173$ $Z_{881}=7820$	$R_{13}=R_{21}=R_{42}=R_{62}=R_{73}=R_{81}=1$	9	2	3	3	2	1	1	2	2	2
0.4	389 849.68	$Z_{113}=4892$ $Z_{693}=3338$	$R_{13}=R_{21}=R_{63}=R_{72}=1$ $R_{43}=2$	11	1	3	3	1	1	1	2	3	2
0.5	393 705.51	$Z_{111}=4838$ $Z_{693}=3472$	$R_{11}=R_{23}=R_{61}=R_{72}=1$ $R_{41}=2$	11	1	3	3	1	1	1	2	3	2
0.6	397 796.64	$Z_{112}=4735$ $Z_{261}=166$ $Z_{692}=3580$	$R_{12}=R_{21}=R_{62}=R_{73}=1$ $R_{42}=2$	11	1	3	3	1	1	1	2	3	2
0.7	401 751.56	$Z_{112}=4590$ $Z_{263}=335$ $Z_{692}=3665$	$R_{12}=R_{23}=R_{62}=R_{71}=1$ $R_{42}=2$	11	1	3	3	1	1	1	3	3	2
0.8	405 338.40	$Z_{111}=4407$ $Z_{263}=482$ $Z_{692}=3730$	$R_{11}=R_{23}=R_{61}=R_{72}=1$ $R_{41}=2$	11	1	3	3	1	1	1	3	3	2
0.9	408 759.37	$Z_{113}=4192$ $Z_{261}=610$ $Z_{693}=3778$	$R_{13}=R_{21}=R_{63}=R_{72}=1$ $R_{43}=2$	11	1	3	3	1	1	1	3	3	2
1.0	411 866.48	$Z_{111}=3949$ $Z_{263}=722$ $Z_{691}=3811$	$R_{11}=R_{23}=R_{61}=R_{72}=1$ $R_{41}=2$	11	1	3	3	1	1	1	3	3	2

$$C_i = 160,000 + (1/0.8)\ln[1 - 0.1(1 - \exp(0.8))] \times (140,000 - 160,000)]$$
$$= 157,109.8\,\text{min.,} \quad \text{where}$$
$$C_i \in [140,000,\ 160,000) \quad \text{for } \forall i = 1,2,\ldots,9$$

$$D_1 = 40,000 - 0.1(40,000 - 35,000)$$
$$= 39,500\text{unit} \quad \text{where}$$
$$D_1 \in [35,000,\ 40,000) \quad \text{for } j = 1$$

$$D_1 = D_j \quad \text{where } D_1 \in [32,000,\ 35,000) \quad \text{for } j = 1$$

5. Computational results

Eleven problems for different levels of μ and eight problems for $\mu = 1$ are modelled and solved by GAMS—The General Algebraic Modelling System (Brooke *et al.*, 1988).

Table 4 summarises computational results for different levels of μ. According to the table, the relationship between the total cost of EE elimination and μ is inversely proportional. It shows the necessity to avoid the unrealistic solutions while μ is increasing. For the first four levels, related block-diagonal arrangements of solutions stay same. If the arrangement for $\mu=0.3$ in Table 5 is investigated; nine EE are eliminated by duplication of machine type 1, 2, 4, 6, 7 and 8. Also, part type 9 and 8 are transferred to related cells since the machine type 6 and 8 are not being available within cell 2 and cell 1, respectively.

The block diagonal arrangements of parts and machines for the next six levels of μ are same (Table 6). The solutions of each precision level show that even if the number of EE is increased from 9 to 11, it is worth noting that the total cost is minimised for the related cell configuration. Machine duplication and intercellular transfer of exceptional parts are the elimination alternatives that are chosen. Bottleneck machines are 1, 2, 4, 6 and 7, and exceptional parts for $\mu=0.4$ and 0.5 are 1 and 9, and for $\mu=0.6$... 1 part type 6 is added to these two.

Table 5. $D_j \in (D_j^1, D_j^2]$ for $\forall\ j$ and the cell formation for $\mu=0$, 0.1,0.2, 0.3, (EE 9, Voids 12)

M/P	*4*	*5*	*6*	*8*	*1*	*2*	*3*	*9*	*7*	*10*
4	1	1	1	1	1					
5	1		1							
7		1		1					1	1
1					1		1			
2	1		1		1	1	1			
3					1	1				
8				1		1		1		
6					1			1	1	
9									1	1

Table 6. $D_j \in (D_j^1, D_j^2]$ for $\forall\ j$ and the cell formation for $\mu=0.4, \ldots, 1$ (EE 11, Voids 15)

M/P	*7*	*10*	*1*	*2*	*3*	*5*	*8*	*9*	*4*	*6*
1		1	1		1					
6	1		1					1		
9	1	1								
2			1	1	1				1	1
3			1	1						
7	1	1				1	1			
8				1			1	1		
4			1			1	1		1	1
5									1	1

The GAMS solutions and the cell configurations for eight problems for $\mu = 1$ in the interval of $(D_j^0, D_j^1]$ are given in Tables 7 and 8, respectively. Eleven EE are eliminated by duplication of machine type 1, 4, 6, 7 and 8. Also part type 1 and 9 are transferred to related cells since the machine type 1, 2, 4 and 6 are not being available within related cells.

6. Conclusion

This study proposes an efficient FPP model for the realistic CMS design problem, which handle the cell formation and EE simultaneously. The proposed model considers the fuzziness in the EE elimination costs, part demands and the machine capacities, which makes the study different from the others in literature. Model constructed by using the fully trade-off membership functions of fuzzy parameters. Theoretical aspects of FPP provide the opportunity of being modelled the problem as flexible as being in real life

The solutions of FPP for each level of μ, gives the alternative decision plans for the fuzzy parameters in different risk levels. To avoid the unrealistic and risk-free plans, the solution of FPP model for $\mu=0.5$ is taken as a reference decision plan. The other solutions are very important for the DM to enable him to arrive at

Table 7. $\mu = 1$, $D_j \in (D_j^0, D_j^1]$ for $\forall\, j$ the FPP model solutions, $O_{ijk} = 0$ (EE 10, Voids 14)

Prob. #	*min Z* ($)	Z_{ijk}	R_{ik}	*EE*	Q_1	Q_2	Q_3	Q_4	Q_5	Q_6	Q_7	Q_8	Q_9
1,2,3,4	392 637.65	$Z_{113} = 1203$ $Z_{693} = 2115$	$R_{13} = R_{22} = R_{63} = R_{71} = 1$ $R_{43} = 2$	11	1	3	3	1	1	1	2	3	2
5,6,7,8	397 722.43	$Z_{113} = 2203$ $Z_{693} = 2463$	$R_{13} = R_{22} = R_{63} = R_{71} = 1$ $R_{43} = 2$	11	1	3	3	1	1	1	2	3	2

Table 8. $\mu = 1$, $D_j \in (D_j^0, D_j^1]$ for $\forall\, j$ the cell formation

M/P	*7*	*10*	*4*	*6*	*1*	*2*	*3*	*5*	*8*	*9*
1		1			1		1			
6	1				1					1
9	1	1								
4			1	1	1			1	1	
5			1	1						
2			1	1	1	1	1			
3					1	1				
7	1	1						1	1	
8						1			1	1

appropriate conclusions in the real life's imprecise environment.

7. Future directions

Since the mathematical model considers the only objective function as the minimisation of the EE elimination costs, some different cell configurations were found by FPP solutions. Even if the increase in EE concludes the minimum cost, it is obvious that the worse cell configuration makes the scheduling more complicated. So it is worth considering the maximum cell utilisation as another objective function for the model.

The other directions could be the addition of the fuzzy budget constraint to support the duplication of bottleneck machine alternative, and also the considerations of the fuzziness in part processing times for an application study.

References

Arieh, B.D., and Triantaphyllou. (1992) Quantifying data for group technology with weighted fuzzy features. *International Journal of Production Research*, **30**(6), 1285–1299.

Brooke, A., Kendrick, D. and Meeraus, A. (1988) *GAMS-The General Algebraic Modelling System, A User's Guide*, The Scientific Press, USA.

Carlsson, C. and Korhonen, P. (1986) A parametric approach to fuzzy linear programming. *Fuzzy Sets and Systems*, **20**, 17–30.

Cheng, C. H., Kumar, A. and Motwani, J. A. (1995) Comparative examination of selected cellular manufacturing clustering algorithms. *International Journal of Operations and Production Management*, **15**, 86–97.

Chu, C. H. and Hayya, J. C. (1991) A fuzzy clustering approach to manufacturing cell formation. *International Journal of Production Research*, **29**(7), 1475–1487.

Gill, A. and Bector, C. R. (1997) A fuzzy linguistic approach to data quantification and construction of distance measures for part family formation problem. *International Journal of Production Research*, **35**(9), 2565–2578.

Güngör, Z. and Arıkan, F. (2000) Application of fuzyy decision making in part machine grouping. *International Journal of Production Economics*, **63**, 181–193.

Heragu, S. S. (1994) Group technology and cellular manufacturing. *IEEE Transactions on System, Man and Cybernetics*, **24**(2), 203–214.

Kern, G. M. and Wei, J. C. (1991) The cost of eliminating exceptional elements in group technology cell formation. *International Journal of Production Research*, **29**(8), 1535–1547.

Lai, Y. J. and Hwang, C. L. (1992) *Fuzzy Mathematical Programming-methods and applications*, Springer-Verlag, Berlin, Germany.

Masnata, A. and Settineri, I. (1997) An application of fuzzy clustering to cellular manufacturing. *International Journal of Production Research*, **35**(4), 1077–1094.

Mosier, C. and Taube, L. (1985) The facets of group technology and their impacts on implementation-A state of the art survey. *Omega*, **13**(5), 381–391.

Offodile, O. F., Mehrez, A. and Grznar, J. (1994) Cellular manufacturing: A taxonomic review framework. *Journal of Manufacturing Systems*, **13**(3), 196–220.

Seifoddini, H. (1989) Duplication process in machine cell formation in group technology. *IIE Transactions*, **21**, 382–388.

Selim, H. M., Askin, R. G. and Vakharia, A. J. (1998) Cell formation in group technology: Review, evaluation and directions for future research. *Computers Industrial Engineering*, **34**(1), 3–20.

Shafer, S. M., Kern, G. M. and Wei, J. C. (1992) A mathematical programming approach for dealing with exceptional elements in cellular manufacturing. *International Journal of Production Research*, **30**(5), 1029–1036.

Shanker, R. and Vrat, P. (1999) Some design issues in cellular manufacturing using the fuzzy programming approach. *International Journal of Production Research*, **37**(11), 2545–2563.

Singh, N. (1993) Design of cellular manufacturing systems: An invited review. *European Journal of Operational Research*, **69**, 284–291.

Tsai, C. C., Chu, C. H. and Barta, T. A. (1997) Modelling and analysis of a manufacturing cell formation problem with fuzzy mixed integer programming. *IIE Transactions*, **29**, 533–547.

Wemmerlöv, U. and Hyer, N. L. (1986) Procedures for the part family/machine group identification problem in cellular manufacturing. *Journal of Operations Management*, **6**(2), 125–147.

Zadeh, L. A. (1965) Fuzzy sets. *Information and Control*, **8**, 338–353.

Zang, C. and Wang, H. P. (1992) Concurrent formation of part family and machine cells based on fuzzy set theory. *Journal of Manufacturing Systems*, **11**, 61–67.