



Entry of Innovator and License in Oligopoly

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Received: 6 June 2019 / Revised: 30 January 2020 / Accepted: 13 February 2020 /

Published online: 11 March 2020

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Abstract

When an outside innovating firm has a cost-reducing technology, it can sell licenses of its technology to incumbent firms using a combination of royalty and fixed fee. Alternatively, the innovating firm can enter the market and at the same time sell licenses, or enter the market without license. We examine the credibility of the threat of entry by the innovating firm using a two-step auction under oligopoly with three firms, one outside innovating firm and two incumbent firms. With general demand function, we show that the credibility of the two-step auction depends on the form of the cost function of the new technology, whether it is concave or convex. Also we analyze the optimal strategy for the innovator in a case of linear demand and quadratic cost functions in which the two-step auction is credible.

Keywords License · Entry · Oligopoly · Innovating firm · Two-step auction

JEL Classification D43 · L13

1 Introduction

In Proposition 4 of Kamien and Tauman (1986), it was argued that in an oligopoly when the number of firms is small (or very large), strategy to enter the market and at the same time license the cost-reducing technology to the incumbent firm (license with entry strategy) is more profitable than strategy to license its technology to the incumbent firm without entering the market (license without entry strategy) for the innovating firm. However, their result depends on their definition of license fee. They defined the license fee in the case of licenses without entry by the difference between the profit of an incumbent firm in that case and its profit before it buys a license without entry of the innovating firm. However, it is inappropriate from the game theoretic view point. If an incumbent firm does not buy

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a license, the innovating firm may punish the incumbent firm by entering the market. The innovating firm can use such a threat if and only if it is a credible threat. In a duopoly case with one incumbent firm, when the innovating firm does not enter nor sell a license, its profit is zero; on the other hand, when it enters the market without license, its profit is positive. Therefore, threat of entry without license is credible under duopoly, and then even if the innovating firm does not enter the market, the incumbent firm must pay the difference between its profit when it uses the new technology and its profit when the innovating firm enters without license as a license fee. For example, Hattori and Tanaka (2018) presented analyses of license and entry choice by an innovating firm in a duopoly.

However, in an oligopoly with more than one incumbent firm, the credibility of threat of entry is a more subtle problem. In this paper, we examine definitions of license fees under oligopoly with three firms, one outside innovating firm and two incumbent firms, considering a two-step auction in the case of licenses without entry. Also we suppose that the innovating firm uses a combination of royalty per output and a fixed license fee.

A two-step auction, for example, in the case of a license to one incumbent firm without entry is as follows.

1. The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which is equal to the willingness to pay for the incumbent firms described below, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

2. The second step.

The innovating firm sells a license to one firm at auction *with its entry*.

At the first step of the auction, each incumbent firm has a willingness to pay the following license fee;

the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step, each incumbent firm has an incentive to make a bid when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid. The decision of the innovator not to enter the market in the first step is commitment if the incumbent firms accept the offer.

We need the minimum bidding price because if there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

Threat by such a two-step auction is credible if the total payoff of the innovating firm when it enters the market with a license to one firm is larger than its total payoff when it licenses to one firm without entering the market.

A two-step auction in the case of licenses to two incumbent firms without entry is similar,¹ and at the first step of the auction, the incumbent firm has a willingness to pay the following license fee;

¹Please see Section 6.2.2.

the difference between its profit when both firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

In the first step, each incumbent firm has an incentive to make a bid even if the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

In the next section, we present some literature review. In Section 3, the model of this paper is described. In Section 4, we consider various equilibria of the oligopoly. In Section 5, we present an analysis of a royalty and a fixed license fee under the license with entry strategy by the innovator. In Section 6, we consider a two-step auction and present an analysis of a royalty and a fixed license fee under the license without entry strategy. In Sections 5 and 6, the following results about the optimal royalty rate for the innovator are shown (see Proposition 6.1).²

1. **Entry with license to one firm case**

The optimal royalty rate may be positive or negative.

2. **Entry with licenses to two firms case**

If the outputs of the firms are strategic complements, the optimal royalty rate is positive. If the outputs of the firms are strategic substitutes, it may be positive or negative.

3. **License to one firm without entry case not using two-step auction**

If the outputs of the firms are strategic substitutes, the optimal royalty rate is negative. If the outputs of the firms are strategic complements, it may be positive or negative.

4. **License to one firm without entry case using two-step auction**

If the outputs of the firms are strategic substitutes, the optimal royalty rate is negative. If the outputs of the firms are strategic complements, it is positive.

5. **Licenses to two firms without entry cases using or not using two-step auction**

The optimal royalty rate is positive.

In Section 6, also we examine the credibility of two-step auction and show the following results (see Proposition 6.2).

1. If the cost function of the new technology is linear, the profit of the innovating firm when it enters the market with a license to one firm and its profit when it licenses to one firm without entering the market are equal, that is, entry with license to one firm case and license to one firm without entry case are equivalent.
2. If the cost function of the new technology is strictly convex, two-step auction is credible.
3. If the cost function of the new technology is strictly concave, two-step auction is not credible.

In Section 7, we present an example of linear demand and quadratic cost functions. In this example, two-step auction is credible. We will show that when the cost of the new technology is low, license to two firms without entry strategy is optimal; on the other hand, when it is not low, entry with licenses to two firms strategy is optimal.

2 Literature Review

Various studies focus on technology adoption or R&D investment in duopoly or oligopoly. Most of them analyze the relation between the technology licensor and licensee. The

²About the meanings of strategic substitutability and complementarity, please see Section 4.1

difference of means of contracts, which comprise royalties, upfront fixed fees, combinations of these two, and auctions, is well discussed (Katz and Shapiro 1985). Kamien and Tauman (2002) show that outside innovators prefer auctions, but industry incumbents prefer royalty. This topic is discussed by Kabiraj (2004) under the Stackelberg oligopoly in which the licensor does not have production capacity. Wang and Yang (2004) consider the case when the licensor has production capacity under the Stackelberg duopoly. Sen and Tauman (2007) compared the license system in detail, namely, when the licensor is an outsider and when it is an incumbent firm, using the combination of royalties and fixed fees. However, the existence of production capacity was externally given, and they did not analyze the choice of entry. Therefore, the optimal strategies of outside innovators, who can use the entry as a threat, require more discussion.

Regarding the strategies of new entrants to the market, Duchene et al. (2015) focused on future entrants with old technology and argued that while a low license fee can be used to deter the entry of potential entrants, the firm with new technology is incumbent, and its choice of entry is not analyzed. Also, Chen (2017) analyzed the model of the endogenous market structure determined by the potential entrant with old technology and showed that the licensor uses the fixed fee and zero royalty in both the incumbent and the outside innovator cases, which are exogenously given. Creane et al. (2013) examined a firm that can license its production technology to a rival when firms are heterogeneous in production costs, and showed that a complete technology transfer from one firm to another always increases joint profit under weakly concave demand when at least three firms remain in the industry.

A Cournot oligopoly with fixed fee under cost asymmetry was analyzed by La Manna (1993). He showed that if technologies can be replicated perfectly, a lower cost firm always has an incentive to transfer its technology, and while a Cournot-Nash equilibrium cannot be fully asymmetric, there exists no non-cooperative Nash equilibrium in pure strategies. On the other hand, using cooperative game theory, Watanabe N and Muto (2008) analyzed bargaining between a licensor with no production capacity and oligopolistic firms. Recent research focuses on market structure and technology improvement. Boone (2001) and Matsumura et al. (2013) found a non-monotonic relation between intensity of competition and innovation. Also, Pal (2010) showed that technology adoption may change the market outcome. The social welfare is larger in Bertrand competition than in Cournot competition. However, if we consider technology adoption, Cournot competition may result in higher social welfare than Bertrand competition under a differentiated goods market. Hattori and Tanaka (2015, 2016a) studied the adoption of new technology in Cournot duopoly and Stackelberg duopoly. Rebollo and Sandoń (2012) presented an analysis of the effectiveness of research and development (R&D) subsidies in an oligopolistic model in the cases of international competition and cooperation in R&D. Hattori and Tanaka (2016b) analyzed the problems similar to this paper but consider product innovation, that is, introduction of higher quality good in a duopoly with vertical product differentiation.

3 The Model

There are three firms, Firms A, B, and C. At present, two of them, Firms B and C, produce a homogeneous good. Firm A, which is an outside firm, has a superior cost-reducing technology and can produce the good at lower cost than Firms B and C. We call Firm A the innovating firm, and Firms B and C the incumbent firms. Firm A have the following five options.

1. To enter the market without license to incumbent firms.
2. To enter the market and license its technology to one incumbent firm.
3. To enter the market and license its technology to two incumbent firms.
4. To license its technology to one incumbent firm, but not enter the market.
5. To license its technology to two incumbent firms, but not enter the market.

Let p be the price, x_A , x_B , and x_C be the outputs of Firms A, B, and C. Then, the inverse demand function of the good is written as follows.

$$p = p(x_A + x_B + x_C), \text{ when Firm A enters,}$$

$$p = p(x_B + x_C), \text{ when Firm A does not enter.}$$

It is decreasing ($p'(\cdot) < 0$) and twice continuously differentiable.

The cost functions of Firms A, B, and C before licenses are denoted by $c_A(x_A)$, $c_B(x_B)$, and $c_C(x_C)$. $c_B(\cdot)$ and $c_C(\cdot)$ are the same functions without license. We denote them by $c_B(\cdot)$. If Firm A licenses its technology to two incumbent firms, all cost functions are the same. They are denoted by $c_A(\cdot)$. If Firm A licenses its technology to one incumbent firm, for example Firm C, then the cost functions of Firms A and C are the same. They are denoted by $c_A(\cdot)$. The cost functions are increasing ($c'(\cdot) > 0$) and twice continuously differentiable, and there is no fixed cost; thus, $c_A(0) = c_B(0) = 0$.

We consider the sub-game perfect equilibrium of a game with the following structure.

1. In the first stage, Firm A chooses one of above five options. In the cases of one or two licenses without entry, it determines whether it uses a two-step auction or not.
2. According to a choice in the first stage, Firm A sells no or one or two licenses to Firm B and Firm C through one-step or two-step auctions, and it enters the market or not. Firm A determines the license fees by a combination of royalty and fixed fee.
3. In the third stage, the firms determine their outputs.

4 Ex Post Equilibria of the Oligopoly

We denote the royalty per output and the fixed license fee by r and L . In this section, we consider equilibria of the oligopoly after r and L are determined by the innovator. They are not parts of equilibria, but parameters in this section.

4.1 Entry Without License Case

We suppose that Firm A enters the market without license to incumbent firms. Then, the market becomes a tripoly. The cost function of Firms B and C is $c_B(x_B)$ and $c_B(x_C)$. The profits of Firms A, B, and C are written as

$$\pi_A = p(x_A + x_B + x_C)x_A - c_A(x_A),$$

$$\pi_B = p(x_A + x_B + x_C)x_B - c_B(x_B),$$

$$\pi_C = p(x_A + x_B + x_C)x_C - c_B(x_C).$$

We assume Cournot type behavior of the firms. The first-order conditions for profit maximization are

$$p(x_A + x_B + x_C) + p'(x_A + x_B + x_C)x_A - c'_A(x_A) = 0, \quad (1)$$

$$p(x_A + x_B + x_C) + p'(x_A + x_B + x_C)x_B - c'_B(x_B) = 0, \quad (2)$$

$$p(x_A + x_B + x_C) + p'(x_A + x_B + x_C)x_C - c'_B(x_C) = 0. \tag{3}$$

The second-order conditions are

$$\begin{aligned} 2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_A - c''_A(x_A) &< 0, \\ 2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_B - c''_B(x_B) &< 0, \\ 2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_C - c''_C(x_C) &< 0. \end{aligned}$$

Hereafter, we assume that the second-order conditions in each case are satisfied. Denote the equilibrium profits in this case by π_A^{e0} , π_B^{e0} , and π_C^{e0} . Note that $\pi_B^{e0} = \pi_C^{e0}$.

We define the following notation.

$$\begin{aligned} \theta_A &= 2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_A - c''_A(x_A), \\ \theta_B &= 2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_B - c''_B(x_B), \\ \theta_C &= 2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_C - c''_C(x_C), \\ \sigma_A &= p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_A, \quad \sigma_B = p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_B, \\ \sigma_C &= p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_C. \end{aligned}$$

If Firm A does not enter the market, we have

$$\theta_B = 2p'(x_B + x_C) + p''(x_B + x_C)x_B - c''_B(x_B), \quad \sigma_B = p'(x_B + x_C) + p''(x_B + x_C)x_B,$$

and so on. By the second-order conditions in each case

$$\theta_A < 0, \theta_B < 0, \theta_C < 0.$$

Note that if Firm A licenses its technology to only Firm C, the cost function of Firm C is $c_A(\cdot)$. If Firm A licenses its technology to Firms B and C, their cost functions are $c_A(\cdot)$. Without any license $c_C(\cdot) = c_B(\cdot)$.

If an increase in the output of one firm induces a reduction in the output of a rival firm, the outputs of the firms are strategic substitutes. On the other hand, if an increase in the output of one firm induces an increase in the output of a rival firm, the outputs of the firms are strategic complements.³

From (1), we have

$$\frac{\partial x_A}{\partial x_B} = \frac{\partial x_A}{\partial x_C} = -\frac{p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_A}{2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_A - c''(x_A)} = -\frac{\sigma_A}{\theta_A}.$$

Similarly,

$$\frac{\partial x_B}{\partial x_A} = \frac{\partial x_B}{\partial x_C} = -\frac{p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_B}{2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_B - c''(x_B)} = -\frac{\sigma_B}{\theta_B},$$

and

$$\frac{\partial x_C}{\partial x_A} = \frac{\partial x_C}{\partial x_B} = -\frac{p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_C}{2p'(x_A + x_B + x_C) + p''(x_A + x_B + x_C)x_C - c''(x_C)} = -\frac{\sigma_C}{\theta_C}.$$

If they are negative, the outputs of the firms are strategic substitutes; and if they are positive, the outputs of the firms are strategic complements. By the second-order conditions for the firms $\theta_A < 0$, $\theta_B < 0$, and $\theta_C < 0$. Therefore, if $\sigma_A < 0$, $\sigma_B < 0$, and $\sigma_C < 0$, the outputs of the firms are strategic substitutes; and if $\sigma_A > 0$, $\sigma_B > 0$, and $\sigma_C > 0$, the outputs of the firms are strategic complements.

³The definitions of strategic substitutability and complementarity are according to Bulow et al. (1985).

4.2 License to One Firm Without Entry Case

Suppose that Firm A licenses its technology to one firm, Firm C, but it does not enter the market. Then, the market is a duopoly. The cost function of Firm C is $c_A(x_C)$. The profits of the firms are written as

$$\pi_B = p(x_B + x_C)x_B - c_B(x_B),$$

$$\pi_C = p(x_B + x_C)x_C - c_A(x_C) - rx_C - L.$$

The first-order conditions for profit maximization are

$$p(x_B + x_C) + p'(x_B + x_C)x_B - c'_B(x_B) = 0, \tag{4a}$$

$$p(x_B + x_C) + p'(x_B + x_C)x_C - c'_A(x_C) - r = 0. \tag{4b}$$

Denote the equilibrium profits and the license fee in this case by π_B^{l1} , π_C^{l1} , and L^{l1} . Differentiating the first-order conditions with respect to r yields

$$\theta_B \frac{dx_B}{dr} + \sigma_B \frac{dx_C}{dr} = 0,$$

$$\sigma_C \frac{dx_B}{dr} + \theta_C \frac{dx_C}{dr} = 1.$$

From them, we obtain

$$\frac{dx_B}{dr} = -\frac{\sigma_B}{\Delta}, \quad \frac{dx_C}{dr} = \frac{\theta_B}{\Delta},$$

where

$$\Delta = \theta_B\theta_C - \sigma_B\sigma_C.$$

From the second-order conditions and the stability conditions for oligopoly (see Appendix B), we have $\Delta > 0$ and $\frac{dx_C}{dr} < 0$. Thus, the output of Firm C decreases when the royalty increases. When the outputs of the firms are strategic substitutes, $\sigma_B < 0$, and when the outputs of the firms are strategic complements, $\sigma_B > 0$. We have $\frac{dx_B}{dr} > 0$ in the former case, and $\frac{dx_B}{dr} < 0$ in the latter case. We can consider that an increase in the royalty decreases Firm C's output, and a decrease in Firm C's output increases (or decreases) Firm B's output if the outputs of the firms are strategic substitutes (or complements).

4.3 Licenses to Two Firms Without Entry Case

Suppose that Firm A licenses its technology to two firms, Firms B and C, but it does not enter the market. The cost functions of Firms B and C are $c_A(\cdot)$. In this case, $x_B = x_C$. The profits of the firms and their first-order conditions are similar to them in the previous case with the royalty rate r for Firms B and C.

Denote the equilibrium profits and the license fee in this case by π_B^{l2} , π_C^{l2} , and L^{l2} . We have $x_B = x_C$ and $\pi_B^{l2} = \pi_C^{l2}$. Differentiating the first-order conditions with respect to r yields

$$\theta_B \frac{dx_B}{dr} + \sigma_B \frac{dx_C}{dr} = 1,$$

$$\sigma_C \frac{dx_B}{dr} + \theta_C \frac{dx_C}{dr} = 1.$$

From them and $x_B = x_C$, we obtain

$$\frac{dx_B}{dr} = \frac{dx_C}{dr} = \frac{p'(x_B + x_C) - c''_A(x_B)}{\Delta'}.$$

where

$$\Delta' = \theta_B\theta_C - \sigma_B\sigma_C.$$

From the stability conditions ($\Delta' > 0$),

$$|\theta_B| > |\sigma_B|,$$

$$|\theta_C| > |\sigma_C|.$$

Then,

$$p'(x_B + x_C) - c_A''(x_B) < 0.$$

Thus, we have $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. The outputs of Firms B and C decrease when the royalty increases.

4.4 Entry with a License to One Firm Case

Next suppose that Firm A enters the market and sells a license to one firm, Firm C. The cost function of Firm C is $c_A(x_C)$. The profits of the firms and their first-order conditions are similar to them in the previous cases with the royalty rate r for Firm C.

Denote the equilibrium profits and the license fee in this case by π_A^{e1} , π_B^{e1} , π_C^{e1} , and L^{e1} . Differentiating the first-order conditions with respect to r , we obtain

$$\begin{aligned} \theta_A \frac{dx_A}{dr} + \sigma_A \frac{dx_B}{dr} + \sigma_A \frac{dx_C}{dr} &= 0, \\ \sigma_B \frac{dx_A}{dr} + \theta_B \frac{dx_B}{dr} + \sigma_B \frac{dx_C}{dr} &= 0, \\ \sigma_C \frac{dx_A}{dr} + \sigma_C \frac{dx_B}{dr} + \theta_C \frac{dx_C}{dr} &= 1. \end{aligned}$$

From them

$$\begin{aligned} \frac{dx_A}{dr} &= \frac{\sigma_A(\sigma_B - \theta_B)}{\Gamma}, \\ \frac{dx_B}{dr} &= \frac{\sigma_B(\sigma_A - \theta_A)}{\Gamma}, \\ \frac{dx_C}{dr} &= \frac{\theta_A\theta_B - \sigma_A\sigma_B}{\Gamma}, \end{aligned}$$

where

$$\Gamma = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_C - \theta_B\sigma_A\sigma_C - \theta_C\sigma_A\sigma_B + 2\sigma_A\sigma_B\sigma_C.$$

From the second-order conditions and the stability conditions, θ 's are negative, and $\Gamma < 0$ (see Appendix B). Γ is rewritten as

$$\Gamma = \theta_C(\theta_A\theta_B - \sigma_A\sigma_B) - (\theta_A - \sigma_A)\sigma_B\sigma_C - (\theta_B - \sigma_B)\sigma_A\sigma_C.$$

We assume $\theta_A\theta_B - \sigma_A\sigma_B > 0$,⁴ $|\theta_A| - |\sigma_A| > 0$, $|\theta_B| - |\sigma_B| > 0$, and $|\theta_C| - |\sigma_C| > 0$. We have $\frac{dx_C}{dr} < 0$. When the outputs of the firms are strategic substitutes, $\sigma_A < 0$, $\sigma_B < 0$, $\sigma_C < 0$, and when the outputs of the firms are strategic complements, $\sigma_A > 0$, $\sigma_B > 0$, $\sigma_C > 0$. We have $\frac{dx_A}{dr} > 0$, $\frac{dx_B}{dr} > 0$ in the former case, and $\frac{dx_A}{dr} < 0$, $\frac{dx_B}{dr} < 0$ in the latter case. We can consider that an increase in the royalty decreases Firm C's output, and a decrease in Firm C's output increases (or decreases) the outputs of Firms A and B if the outputs of the firms are strategic substitutes (or complements). Now we assume

$$\left| \frac{dx_C}{dr} \right| > \left| \frac{dx_A}{dr} \right| + \left| \frac{dx_B}{dr} \right|.$$

⁴This is a stability condition in the case of duopoly with Firm A and Firm B.

Since the royalty is imposed to Firm C, it is a plausible assumption. Since $|\theta_C| - |\sigma_C| > 0$, it means the stability condition $\Gamma < 0$.

4.5 Entry with Licenses to Two Firms Case

Next suppose that Firm A enters the market and sells licenses to Firms B and C. The cost functions of Firms B and C are $c_A(\cdot)$. The profits of the firms and their first-order conditions are similar to them in the previous cases with the royalty rate r for Firms B and C.

Denote the equilibrium profits and the license fee by $\pi_A^{e2}, \pi_B^{e2}, \pi_C^{e2}$, and L^{e2} . In this case, $x_B = x_C$ and $\pi_B^{e2} = \pi_C^{e2}$. Differentiating the first-order conditions with respect to r , we obtain

$$\begin{aligned} \theta_A \frac{dx_A}{dr} + \sigma_A \frac{dx_B}{dr} + \sigma_A \frac{dx_C}{dr} &= 0, \\ \sigma_B \frac{dx_A}{dr} + \theta_B \frac{dx_B}{dr} + \sigma_B \frac{dx_C}{dr} &= 1, \\ \sigma_C \frac{dx_A}{dr} + \sigma_C \frac{dx_B}{dr} + \theta_C \frac{dx_C}{dr} &= 1. \end{aligned}$$

From them

$$\begin{aligned} \frac{dx_A}{dr} &= \frac{2\sigma_A(\sigma_B - \theta_B)}{\Gamma'}, \\ \frac{dx_B}{dr} = \frac{dx_C}{dr} &= \frac{\theta_A(\theta_B - \sigma_B)}{\Gamma'}, \end{aligned}$$

where

$$\Gamma' = \theta_A\theta_B\theta_C - \theta_A\sigma_B\sigma_C - \theta_B\sigma_A\sigma_C - \theta_C\sigma_A\sigma_B + 2\sigma_A\sigma_B\sigma_C < 0.$$

By symmetry, we have $\theta_C = \theta_B$ and $\sigma_C = \sigma_B$. Similarly to the previous case, we assume that θ 's are negative and $|\theta_B| - |\sigma_B| > 0$. We get $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. $\frac{dx_A}{dr} > 0$ if $\sigma_A < 0$ which means strategic substitutability, and $\frac{dx_A}{dr} < 0$ if $\sigma_A > 0$ which means strategic complementarity. Using $\theta_C = \theta_B$ and $\sigma_C = \sigma_B$, Γ' is rewritten as

$$\Gamma' = \theta_A\theta_B\theta_B - \theta_A\sigma_B\sigma_B - 2\theta_B\sigma_A\sigma_B + 2\sigma_A\sigma_B\sigma_B = [\theta_A(\theta_B + \sigma_B) - 2\sigma_A\sigma_B](\theta_B - \sigma_B).$$

We assume $\left| \frac{dx_B}{dr} + \frac{dx_C}{dr} \right| > \left| \frac{dx_A}{dr} \right|$. Since the royalty is imposed to Firm B and Firm C, it is a plausible assumption. By $|\theta_B| - |\sigma_B| > 0$, it implies the stability condition $\Gamma' < 0$.

We summarize the results in this section in the following proposition.

Proposition 4.1 (Summary of the results in this section)

1. **License to one firm without entry case**

Firm A licenses its technology to only Firm C, but it does not enter the market. We have $\frac{dx_C}{dr} < 0$. $\frac{dx_B}{dr} > (<)0$ if the outputs of the firms are strategic substitutes (complements).

2. **Licenses to two firms without entry case**

Firm A licenses its technology to Firm B and Firm C, but it does not enter the market. We have $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$.

3. **Entry with a license to one firm case**

Firm A licenses its technology to only Firm C, and it enters the market. We have $\frac{dx_C}{dr} < 0$. $\frac{dx_A}{dr} > (<)0$ and $\frac{dx_B}{dr} > (<)0$ if the outputs of the firms are strategic substitutes (complements).

4. *Entry with licenses to two firms case*

Firm A licenses its technology to Firm B and Firm C, and it enters the market. We have $\frac{dx_B}{dr} < 0$ and $\frac{dx_C}{dr} < 0$. $\frac{dx_A}{dr} > (<)0$ if the outputs of the firms are strategic substitutes (complements).

5 Royalty and License Fees in the Cases of Licenses with Entry

In the case of licenses with entry, the fixed license fee is equal to the usual willingness to pay for the incumbent firms. We follow the arguments by Kamien and Tauman (1986) and Sen and Tauman (2007) about license fee by auction. Hereafter, we abbreviate the argument, $x_B + x_C$ or $x_A + x_B + x_C$, in p , p' and p'' .

5.1 License to One Firm

The willingness to pay for each incumbent firm is equal to

the difference between its profit when only this firm uses the new technology with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because each incumbent firm knows that there will be one licensee regardless of whether or not it buys a license. Then, the fixed license fee is

$$L^{e1} = (\pi_C^{e1} + L^{e1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{e1} = \pi_B^{e1}$. The total payoff of Firm A in this case is written as

$$\varphi^{e1} = \pi_A^{e1} + r x_C + L^{e1} = p x_A - c_A(x_A) + p x_C - c_A(x_C) - (p x_B - c_B(x_B)). \quad (5)$$

Using the first-order conditions for the firms, the condition for maximization of φ with respect to r is written as follows.

$$\frac{d\varphi^{e1}}{dr} = r \frac{dx_C}{dr} + p'(x_C - x_B) \frac{dx_A}{dr} + p'(x_A - x_B) \frac{dx_C}{dr} + p'(x_A + x_C) \frac{dx_B}{dr} = 0.$$

We get the optimal royalty rate for the innovator as follows.

$$\tilde{r}^{e1} = -\frac{p'}{\frac{dx_C}{dr}} \left[(x_C - x_B) \frac{dx_A}{dr} + (x_A - x_B) \frac{dx_C}{dr} + (x_A + x_C) \frac{dx_B}{dr} \right].$$

This may be positive or negative. A negative royalty with a very high fixed fee may be optimal for the licensor. About the analysis of negative royalty, please see Liao and Sen (2005).

5.2 Licenses to Two Firms

The willingness to pay for each incumbent firm in this case is equal to

the difference between its profit when two firms use the new technology with entry of the innovating firm and its profit when only the rival firm buys the license with entry of the innovating firm.

This is because each incumbent firm knows that there will be one licensee when it does not buy a license. In this case, there is a minimum bidding price which is equal to the willingness

to pay for the incumbents because without the minimum bidding price, no firm makes a positive bid. The fixed license fee is

$$L^{e2} = (\pi_C^{e2} + L^{e2}) - \pi_B^{e1}.$$

This means $\pi_C^{e2} = \pi_B^{e1}$. The total payoff of Firm A is written as

$$\begin{aligned} \varphi^{e2} &= \pi_A^{e1} + rx_B + rx_C + 2L^{e2} = px_A - c_A(x_A) + px_B - c_A(x_B) \\ &\quad + px_C - c_A(x_C) - 2\pi_B^{e1}. \end{aligned} \tag{6}$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case because it is determined in the case of entry with a license to one firm. Using the first-order conditions, the condition for maximization of φ with respect to r is written as follows.

$$\frac{d\varphi^{e2}}{dr} = r \left(\frac{dx_B}{dr} + \frac{dx_C}{dr} \right) + p'(x_B + x_C) \frac{dx_A}{dr} + p'(x_A + x_B) \frac{dx_C}{dr} + p'(x_A + x_C) \frac{dx_B}{dr}.$$

The optimal royalty rate is

$$\tilde{r}^{e2} = - \frac{p'}{\frac{dx_B}{dr} + \frac{dx_C}{dr}} \left[(x_B + x_C) \frac{dx_A}{dr} + (x_A + x_B) \frac{dx_C}{dr} + (x_A + x_C) \frac{dx_B}{dr} \right].$$

If the outputs of the firms are strategic complements, $\tilde{r}^{e2} > 0$ because $\frac{dx_A}{dr} < 0$, $\frac{dx_B}{dr} < 0$, and $\frac{dx_C}{dr} < 0$. If the outputs of the firms are strategic substitutes, it may be positive or negative.

6 Royalty and License Fees in the Cases of Licenses Without Entry: Two-Step Auction

6.1 One-Step Auction

If the licenses are auctioned off to the incumbent firms by one-step auction, the fixed license fee is determined by the usual willingness to pay for the incumbent firms described in Kamien and Tauman (1986) and Sen and Tauman (2007).

6.1.1 License to One Firm

The willingness to pay for each incumbent firm is equal to

the difference between its profit when only this firm uses the new technology without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

Then, the fixed license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{l1}.$$

This equation means $\pi_C^{l1} = \pi_B^{l1}$. Denote L in this case by \tilde{L}^{l1} , and denote the total payoff of the innovator by $\tilde{\varphi}^{l1}$ to distinguish it from the total payoff in the two-step auction case, which is denoted by $\hat{\varphi}^{l1}$. $\tilde{\varphi}^{l1}$ is

$$\tilde{\varphi}^{l1} = rx_C + \tilde{L}^{l1} = px_C - c_A(x_C) - (px_B - c_B(x_B)). \tag{7}$$

Using the first-order conditions, the condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is written as

$$\frac{d\tilde{\varphi}^{l1}}{dr} = r \frac{dx_C}{dr} + p'x_C \frac{dx_B}{dr} - p'x_B \frac{dx_C}{dr} = 0.$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$r^{l1} = \frac{p'}{\frac{dx_C}{dr}} \left(x_B \frac{dx_C}{dr} - x_C \frac{dx_B}{dr} \right).$$

Denote it by \tilde{r}^{l1} . If the outputs of the firms are strategic substitutes, $\tilde{r}^{l1} < 0$ because $\frac{dx_B}{dr} > 0$; if the outputs of the firms are strategic complements, it may be positive or negative because $\frac{dx_C}{dr} < 0$ and $\frac{dx_B}{dr} < 0$.

6.1.2 Licenses to Two Firms

The willingness to pay for each incumbent firm in this case is equal to

the difference between its profit when two firms use the new technology without entry of the innovating firm and its profit when only the rival firm buys the license without entry of the innovating firm.

There is a minimum bidding price which is equal to the willingness to pay for the incumbents. The fixed license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{l1}.$$

This means $\pi_C^{l2} = \pi_B^{l1}$. Denote L in this case by \tilde{L}^{l2} , and denote the total payoff of the innovator by $\tilde{\varphi}^{l2}$. It is

$$\tilde{\varphi}^{l2} = r(x_B + x_C) + 2\tilde{L}^{l2} = px_B - c_A(x_B) + px_C - c_A(x_C) - 2\pi_B^{l1}. \tag{8}$$

Note that π_B^{l1} is constant and irrelevant to determination of the royalty rate because it is determined in the case of a license to one firm without entry. The condition for maximization of $\tilde{\varphi}^{l2}$ with respect to r is

$$\frac{d\tilde{\varphi}^{l2}}{dr} = r \left(\frac{dx_B}{dr} + \frac{dx_C}{dr} \right) + p'x_C \frac{dx_B}{dr} + p'x_B \frac{dx_C}{dr} = 0.$$

The optimal royalty rate is

$$\tilde{r}^{l2} = - \frac{p'}{\frac{dx_B}{dr} + \frac{dx_C}{dr}} \left(x_C \frac{dx_B}{dr} + x_B \frac{dx_C}{dr} \right).$$

This is positive.

6.2 Two-Step Auction

We consider a two-step auction for each case.

6.2.1 License to One Firm

In this case, the two-step auction is carried out as follows.

1. The first step.

The innovating firm sells a license to one firm at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price, which

is equal to the willingness to pay for the incumbent firms described below, and the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee. A firm with the maximum bidding price gets a license. If both firms make bids at the same price, one firm is chosen at random. If no firm makes a bid, then the auction proceeds to the next step.

2. The second step.

The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a willingness to pay the following license fee;

the difference between its profit when only this firm uses the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

Then, the fixed license fee is

$$L^{l1} = (\pi_C^{l1} + L^{l1}) - \pi_B^{e1}.$$

This equation means $\pi_C^{l1} = \pi_B^{e1}$. Denote L^{l1} in this case by \hat{L}^{l1} .

In the first step, each incumbent firm has an incentive to make a bid with the license fee \hat{L}^{l1} when the other firm does not make a bid. On the other hand, it does not have an incentive to make a bid when the other firm makes a bid.

We need the minimum bidding price \hat{L}^{l1} because the profit of a non-licensee is π_B^{l1} which is larger than π_B^{e1} . If there is no minimum price, when one of the incumbent firms makes a bid which is slightly but strictly smaller than this price, the other firm does not have an incentive to outperform this bidding.

Denote the total payoff of the innovator in this case by $\hat{\varphi}^{l1}$. Then,

$$\hat{\varphi}^{l1} = r x_C + \hat{L}^{l1} = p x_C - c_A(x_C) - \pi_B^{e1}.$$

Note that π_B^{e1} is a constant number in this case which is determined in the entry with a license to one firm case. The condition for maximization of φ with respect to r is

$$\frac{d\hat{\varphi}^{l1}}{dr} = r \frac{dx_C}{dr} + p' x_C \frac{dx_B}{dr} = 0.$$

Then, we obtain the optimal royalty rate for the innovator as follows.

$$r^{l1} = - \frac{p'}{\frac{dx_C}{dr}} x_C \frac{dx_B}{dr}.$$

Denote it by \hat{r}^{l1} . If the outputs of the firms are strategic substitutes, $\hat{r}^{l1} < 0$ because $\frac{dx_B}{dr} > 0$, and if the outputs of the firms are strategic complements, $\hat{r}^{l1} > 0$ because $\frac{dx_B}{dr} < 0$.

6.2.2 Licenses to Two Firms

We consider the following two-step auction

1. The first step.

The innovating firm sells licenses to two firms at auction *without its entry* conditional on that the bidding price must not be smaller than the minimum bidding price,

which is equal to the willingness to pay for the incumbent firms described below. If both firms make bids, the innovating firm imposes a predetermined (positive or negative) royalty per output on the licensee, and both firms get licenses. If at least one of the firms does not make a bid, then the auction proceeds to the next step.

2. The second step.

The innovating firm sells a license to one firm at auction *with its entry*. Then, the willingness to pay for each incumbent firm in this step is

$$\pi_C^{e1} + L^{e1} - \pi_B^{e1}.$$

At the first step of the auction, each incumbent firm has a willingness to pay the following license fee;

the difference between its profit when two firms use the new technology *without entry* of the innovating firm and its profit when only the rival firm buys the license *with entry* of the innovating firm.

The minimum bidding price should be equal to this willingness to pay. Then, the fixed license fee is

$$L^{l2} = (\pi_C^{l2} + L^{l2}) - \pi_B^{e1}.$$

This means $\pi_C^{l2} = \pi_B^{e1}$. Denote L^{l2} in this case by \hat{L}^{l2} .

In the first step, each incumbent firm has an incentive to make a bid when the other firm makes a bid because if it does not make a bid, the auction proceeds to the next step.

Denote the total payoff of the innovator in this case by $\hat{\varphi}^{l2}$. It is

$$\hat{\varphi}^{l2} = r(x_B + x_C) + 2\hat{L}^{l2} = px_B - c_A(x_B) + px_C - c_A(x_C) - 2\pi_B^{e1}.$$

Note that π_B^{e1} is constant and irrelevant to determination of the royalty rate in this case. The condition for maximization of $\hat{\varphi}^{l2}$ with respect to r is

$$\frac{d\hat{\varphi}^{l2}}{dr} = r \left(\frac{dx_B}{dr} + \frac{dx_C}{dr} \right) + p'x_C \frac{dx_B}{dr} + p'x_B \frac{dx_C}{dr} = 0.$$

The optimal royalty rate is

$$r^{l2} = - \frac{p'}{\frac{dx_B}{dr} + \frac{dx_C}{dr}} \left(x_C \frac{dx_B}{dr} + x_B \frac{dx_C}{dr} \right).$$

Denote it by \hat{r}^{l2} . We see $\hat{r}^{l2} = \tilde{r}^{l2} > 0$, but the total payoff of the innovator with two-step auction and that without two-step auction are different because the fixed license fees in two cases are different.

We summarize the results about the optimal royalty rates for the innovator in the following proposition.

Proposition 6.1 1. Entry with license to one firm case

The optimal royalty rate may be positive or negative.

2. **Entry with licenses to two firms case**

If the outputs of the firms are strategic complements, the optimal royalty rate is positive. If the outputs of the firms are strategic substitutes, it may be positive or negative.

3. **License to one firm without entry case not using two-step auction**

If the outputs of the firms are strategic substitutes, the optimal royalty rate is negative. If the outputs of the firms are strategic complements, it may be positive or negative.

4. **License to one firm without entry case using two-step auction**

If the outputs of the firms are strategic substitutes, the optimal royalty rate is negative. If the outputs of the firms are strategic complements, it is positive.

5. **Licenses to two firms without entry cases using or not using two-step auction**

The optimal royalty rate is positive.

6.3 Credibility of Two-Step Auction

In this subsection, we prove our main results. The innovating firm uses a two-step auction if and only if the threat by the existence of the second step of the auction is credible. The threat is credible if the total payoff of the innovating firm when it enters the market with a license to one firm is larger than its payoff when it does not enter and sells a license to one firm not using a two-step auction. Therefore, from Eqs. 5, 6, 7, and 8 if

$$\varphi^{e1} = \pi_A^{e1} + \tilde{r}^{e1}x_C + L^{e1} \geq \tilde{r}^{l1}x_C + \tilde{L}^{l1} = \tilde{\varphi}^{l1},$$

the two-step auction is credible. On the other hand, if

$$\tilde{r}^{l1}x_C + \tilde{L}^{l1} > \pi_A^{e1} + \tilde{r}^{e1}x_C + L^{e1},$$

the two-step auction is not credible.

We show the following proposition. Note that $c_A(0) = 0$, that is, the fixed cost of the new technology is zero.

Proposition 6.2 1. *If the marginal cost of the new technology is constant, that is, the cost function is linear, entry with a license to one firm case and license to one firm without entry case is equivalent. The marginal cost of the old technology (technology of the non-licensee) needs not be constant.*

2. *If the cost function of the new technology is strictly convex, the two-step auction is credible.*
3. *If the cost function of the new technology is strictly concave, the two-step auction is not credible.*

Proof 1. Note that Firm A can control the output of each firm by the royalty rate. First consider the case of entry with a license to one firm. Let $\bar{x} = x_A + x_C$. Denote the constant marginal cost of the new technology by c . Then, from Eq. 5, the total payoff of the innovator is

$$\varphi^{e1} = p\bar{x} - c\bar{x} - (px_B - c_B(x_B)).$$

If the marginal cost of the new technology is constant, $c'_A = 0$. Thus, $\frac{d\bar{x}}{dr} = \frac{dx_A}{dr} + \frac{dx_C}{dr}$ and $\frac{dx_B}{dr}$ in Section 4.4 are written as

$$\frac{d\bar{x}}{dr} = \frac{p'(2p' + p''x_B - c''_B(x_B))}{\Gamma} = \frac{p'\theta_B}{\Gamma}, \quad \frac{dx_B}{dr} = \frac{-p'(p' + p''x_B)}{\Gamma} = -\frac{p'\sigma_B}{\Gamma}.$$

The condition for maximization of φ^{e1} with respect to r is

$$(p + p'\bar{x} - c - p'x_B)\frac{d\bar{x}}{dr} - (p + p'x_B - c'_B(x_B) - p'\bar{x})\frac{dx_B}{dr} = 0. \tag{9}$$

From the first-order conditions for Firm A and Firm C,

$$p + p'x_A - c = 0, \tag{10}$$

and

$$p + p'x_C - c - r = 0, \tag{11}$$

we have

$$p + p'\bar{x} - c = r - p + c.$$

From this and the first-order condition for Firm B,

$$p + p'x_B - c'_B(x_B) = 0,$$

Equation 9 is rewritten as

$$(r - p + c - p'x_B) \frac{d\bar{x}}{dr} + p'\bar{x} \frac{dx_B}{dr} = 0.$$

Then, the optimal royalty rate is

$$\tilde{r}^{e1} = p - c + p'x_B + p'\bar{x} \frac{\sigma_B}{\theta_B}.$$

The first-order condition for Firm C, Eq. 11, with $r = \tilde{r}^{e1}$ is rewritten as

$$p + p'x_C - c - \left(p - c + p'x_B + p'\bar{x} \frac{\sigma_B}{\theta_B} \right) = p'(x_C - x_B) - p'\bar{x} \frac{\sigma_B}{\theta_B} = 0.$$

With $x_A + x_C = \bar{x}$, this and the first-order condition for Firm A, Eq. 10, imply

$$p + p'\bar{x} - c - p'x_B - p'\bar{x} \frac{\sigma_B}{\theta_B} = 0. \quad (12)$$

Next consider the case of license to one firm without entry not using a two-step auction. Let $\bar{x} = x_C$. Then, from Eq. 7, the total payoff of the innovator in this case is

$$\tilde{\varphi}^{l1} = p\bar{x} - c\bar{x} - (px_B - c_B(x_B)).$$

This is the same expression as φ^{e1} . If $c''_A = 0$, $\frac{d\bar{x}}{dr} = \frac{dx_C}{dr}$, and $\frac{dx_B}{dr}$ in Section 4.2 are written as

$$\frac{d\bar{x}}{dr} = \frac{\theta_B}{\Delta}, \quad \frac{dx_B}{dr} = -\frac{\sigma_B}{\Delta}.$$

The condition for maximization of $\tilde{\varphi}^{l1}$ with respect to r is

$$(p + p'\bar{x} - c - p'x_B) \frac{d\bar{x}}{dr} - (p + p'x_B - c'_B(x_B) - p'\bar{x}) \frac{dx_B}{dr} = 0. \quad (13)$$

From Eqs. 4a and 4b, 13 is rewritten as

$$(r - p'x_B) \frac{d\bar{x}}{dr} + p'\bar{x} \frac{dx_B}{dr} = 0.$$

Then, the optimal royalty rate is

$$\tilde{r}^{l1} = p'x_B + p'\bar{x} \frac{\sigma_B}{\theta_B}.$$

The first-order condition for Firm C, Eq. 4b, with $x_C = \bar{x}$ and $r = \tilde{r}^{l1}$ is rewritten as

$$p + p'\bar{x} - c - p'x_B - p'\bar{x} \frac{\sigma_B}{\theta_B} = 0. \quad (14)$$

Equations 12 and 14 are the same. Therefore, two cases are equivalent.

2. From Eq. 5 φ^{e1} with $\bar{x} = x_A + x_C$ is

$$\varphi^{e1} = p\bar{x} - c_A(x_A) - c_A(x_C) - (px_B - c_B(x_B)).$$

From Eq. 7 $\tilde{\varphi}^{l1}$ with $\bar{x} = x_C$ is written as

$$\tilde{\varphi}^{l1} = p\bar{x} - c_A(\bar{x}) - (px_B - c_B(x_B)).$$

x_C is controllable for the innovator by the rate of royalty.

If the cost function of the new technology, $c_A(\cdot)$, is strictly convex,

$$c_A(x_C) < \frac{x_C}{x_A + x_C} c_A(x_A + x_C) + \left(1 - \frac{x_C}{x_A + x_C}\right) c_A(0) = \frac{x_C}{x_A + x_C} c_A(x_A + x_C),$$

$$c_A(x_A) < \frac{x_A}{x_A + x_C} c_A(x_A + x_C) + \left(1 - \frac{x_A}{x_A + x_C}\right) c_A(0) = \frac{x_A}{x_A + x_C} c_A(x_A + x_C).$$

Then,

$$c_A(x_A) + c_A(x_C) < c_A(x_A + x_C).$$

This means that separation of production between two firms is more efficient than concentration to one firm.⁵ Thus, φ^{e1} is larger than $\tilde{\varphi}^{l1}$ when $x_A + x_C$ in the case of entry with a license and x_C in the case of license without entry are equal, and the maximum value of φ^{e1} is larger than the maximum value of $\tilde{\varphi}^{l1}$. Hence, two-step auction is credible.

3. Similarly to the case of strictly convex cost function, if the cost function of the new technology, $c_A(\cdot)$, is strictly concave, we find

$$c_A(x_A) + c_A(x_C) > c_A(x_A + x_C).$$

This means that concentration of production to one firm is more efficient than separation between two firms.⁶ Thus, $\tilde{\varphi}^{l1}$ is larger than φ^{e1} when $x_A + x_C$ in the case of entry with a license and x_C in the case of license without entry are equal, and the maximum value of $\tilde{\varphi}^{l1}$ is larger than the maximum value of φ^{e1} . Hence, two-step auction is not credible.

□

7 The Optimal Strategies: an Example

First we show

Proposition 7.1 *Entry with licenses to two firms strategy is more preferable than entry without license strategy for Firm A.*

Proof Denote the outputs of Firms A, B, and C and the profit of Firm A in the entry without license case by $x_A^{e0}, x_B^{e0}, x_C^{e0}$, and π_A^{e0} . Note that $x_B^{e0} = x_C^{e0}$. Let r be the royalty per output in the entry with licenses to two firms case. Let us determine the value of r so that $r = c'_B(x_B^{e0}) - c'_A(x_B^{e0}) = c'_B(x_C^{e0}) - c'_A(x_C^{e0})$ hold. Then, we have

$$p + p'x_A - c'_A(x_A) = 0,$$

$$p + p'x_B - c'_A(x_B) - r = p + p'x_B - c'_A(x_B) - c'_B(x_B^{e0}) + c'_A(x_B^{e0}) = 0,$$

$$p + p'x_C - c'_A(x_C) - r = p + p'x_C - c'_A(x_C) - c'_B(x_C^{e0}) + c'_A(x_C^{e0}) = 0.$$

These are equivalent to Eqs. 1, 2, and 3 when $x_A = x_A^{e0}, x_B = x_B^{e0}$, and $x_C = x_C^{e0}$. Therefore, the outputs of all firms in the entry with licenses to two firms case are equal to those in the entry without license case with above royalty. Also the profit of Firm A in the entry with

⁵This property of the cost function is called strict *super-additivity*. Thus, strict convexity of the cost function with zero fixed cost implies strict super-additivity.

⁶This property of the cost function is called strict *sub-additivity*. Thus, strict concavity of the cost function with zero fixed cost implies strict sub-additivity.

licenses to two firms case is equal to that in the entry without license case. The fixed license fee for Firms B and C is

$$L = px_B^{e0} - c_A(x_B^{e0}) - rx_B^{e0} - \pi_B^{e0} = c_B(x_B^{e0}) - c_A(x_B^{e0}) - rx_B^{e0} = c_B(x_C^{e0}) - c_A(x_C^{e0}) - rx_C^{e0}.$$

Then, the total payoff of Firm A in the entry with licenses to two firms case is

$$\begin{aligned} &\pi_A^{e0} + rx_B^{e0} + rx_C^{e0} + c_B(x_B^{e0}) - c_A(x_B^{e0}) + c_B(x_C^{e0}) - c_A(x_C^{e0}) - rx_B^{e0} - rx_C^{e0} \\ &= \pi_A^{e0} + c_B(x_B^{e0}) - c_A(x_B^{e0}) + c_B(x_C^{e0}) - c_A(x_C^{e0}). \end{aligned}$$

Since $c_B(x_B^{e0}) > c_A(x_B^{e0})$ and $c_B(x_C^{e0}) > c_A(x_C^{e0})$, this is larger than π_A^{e0} , and entry with licenses to two firms strategy is more preferable than entry without license strategy for Firm A. □

Analyses of the optimal strategies in general demand and cost functions case, however, seem to be complicated. We will consider an example. We assume that the inverse demand function is

$$p = a - x_A - x_B - x_C,$$

when Firm A enters the market. When it does not enter, $p = a - x_B - x_C$. a is a positive constant. The cost functions of the firms are quadratic. They are $\frac{1}{2}c_Ax_A^2$ for Firm A. For Firm B and Firm C with the old technology, they are $\frac{1}{2}c_Bx_B^2$ and $\frac{1}{2}c_Bx_C^2$. With the new technology, they are $\frac{1}{2}c_Ax_B^2$ and $\frac{1}{2}c_Ax_C^2$. We present summaries of the calculation results.⁷ About details of $\lambda_A, \lambda_B, \lambda_C, \lambda_D,$ and λ_E , please see Appendix A.

License to One Firm Without Entry Case Not Using Two-Step Auction The optimal royalty rate and the total payoff of the innovator are

$$\begin{aligned} \tilde{r}^{l1} &= -\frac{a}{c_B + 2} < 0, \\ \tilde{r}^{l1}x_C + \tilde{L}^{l1} &= \frac{a^2(c_B^2 - c_Ac_B + 2c_B - 2c_A + 1)}{2(c_B + 2)(c_Ac_B + 2c_B + 2c_A + 3)}. \end{aligned}$$

Licenses to Two Firms Without Entry Case Not Using Two-Step Auction The optimal royalty rate and the total payoff of the innovator are

$$\begin{aligned} \tilde{r}^{l2} &= \frac{a}{c_A + 4} > 0, \\ \tilde{r}^{l2}(x_B + x_C) + 2\tilde{L}^{l2} &= \frac{a^2\lambda_A}{(c_A + 4)(c_B + 2)(c_Ac_B + 2c_B + 2c_A + 3)^2}. \end{aligned}$$

Entry Without License Case The profit of the innovator is

$$\pi_A^{e0} = \frac{a^2(c_A + 2)(c_B + 1)^2}{2(c_Ac_B + 2c_B + 3c_A + 4)^2}.$$

⁷More details of calculations such as the equilibrium values of the outputs and profits are available upon request.

Entry with a License to One Firm Case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{e1} = \frac{a(c_A + 1)^2(c_B^2 - c_{ACB} - 2c_A - 2)}{(c_{ACB} + 2c_B + 2c_A + 3)(c_A^2c_B + 4c_{ACB} + c_B + 2c_A^2 + 6c_A + 2)},$$

$$\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1} = \frac{a^2(2c_A^2c_B^2 + 4c_{ACB}c_B^2 + c_B^2 - c_A^3c_B + 2c_A^2c_B + 7c_{ACB} + 2c_B - 2c_A^3 - 2c_A^2 + 2c_A + 1)}{2(c_{ACB} + 2c_B + 2c_A + 3)(c_A^2c_B + 4c_{ACB} + c_B + 2c_A^2 + 6c_A + 2)}.$$

Entry with Licenses to Two Firms Case The optimal royalty rate and the total payoff of the innovator are

$$\tilde{r}^{e2} = \frac{2a(c_A + 1)^2}{(c_A + 2)(c_A^2 + 6c_A + 2)} > 0,$$

$$\pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} = \frac{a^2\lambda_B}{2(c_A + 2)(c_A^2 + 6c_A + 2)(c_{ACB} + 2c_B + 2c_A + 3)^2(c_A^2c_B + 4c_{ACB} + c_B + 2c_A^2 + 6c_A + 2)^2}.$$

License to One Firm Without Entry Case Using Two-Step Auction The optimal royalty rate and the total payoff of the innovator are

$$\hat{r}^{l1} = -\frac{a(c_B + 1)}{(c_B + 2)(c_{ACB} + 2c_B + 2c_A + 2)} < 0,$$

$$\hat{r}^{l1}x_C + \hat{L}^{l1} = \frac{a^2\lambda_C}{\lambda_D}.$$

Licenses to Two Firms Without Entry Case Using Two-Step Auction The optimal royalty rate and the total payoff of the innovator are

$$\hat{r}^{l2} = \frac{a}{c_A + 4} > 0,$$

$$\hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} = \frac{a^2\lambda_E}{(c_A + 4)(c_{ACB} + 2c_B + 2c_A + 3)^2(c_A^2c_B + 4c_{ACB} + c_B + 2c_A^2 + 6c_A + 2)^2}.$$

Comparing $\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}$ and $\tilde{r}^{l1}x_C + \tilde{L}^{l1}$,

$$\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1} - (\tilde{r}^{l1}x_C + \tilde{L}^{l1}) = \frac{a^2c_A(c_B + 1)(c_{ACB}^2 + 5c_{ACB} + 2c_B + 6c_A + 2)}{2(c_B + 2)(c_{ACB} + 2c_B + 2c_A + 3)(c_A^2c_B + 4c_{ACB} + c_B + 2c_A^2 + 6c_A + 2)} > 0.$$

Therefore, two-step auction is credible. About this example, we get the following results.

1. If $0 < c_A < \sqrt{3} - 1$, licenses to two firms without entry strategy are optimal for the innovator. Please see Fig. 1. In this figure

$$\psi_1 = \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - (\tilde{r}^{l1}x_C + \tilde{L}^{l1}),$$

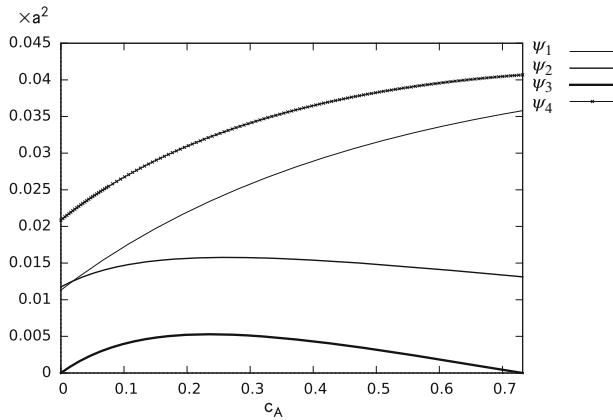


Fig. 1 Optimal strategy for the innovator when $0 < c_A < \sqrt{3} - 1$

$$\begin{aligned} \psi_2 &= \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - (\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}), \\ \psi_3 &= \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - (\pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2}), \\ \psi_4 &= \hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2} - \pi_A^{e0}. \end{aligned}$$

2. If $c_A > \sqrt{3} - 1$, entry with licenses to two firms strategy is optimal for the innovator. Please see Fig. 2. In this figure

$$\begin{aligned} \zeta_1 &= \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - (\tilde{r}^{l1}x_C + \tilde{L}^{l1}), \\ \zeta_2 &= \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - (\pi_A^{e1} + \tilde{r}^{e1}x_C + \tilde{L}^{e1}), \\ \zeta_3 &= \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - (\hat{r}^{l2}(x_B + x_C) + 2\hat{L}^{l2}), \\ \zeta_4 &= \pi_A^{e2} + \tilde{r}^{e2}(x_B + x_C) + 2\tilde{L}^{e2} - \pi_A^{e0}. \end{aligned}$$

In these figures, we assume $c_B = 10$.

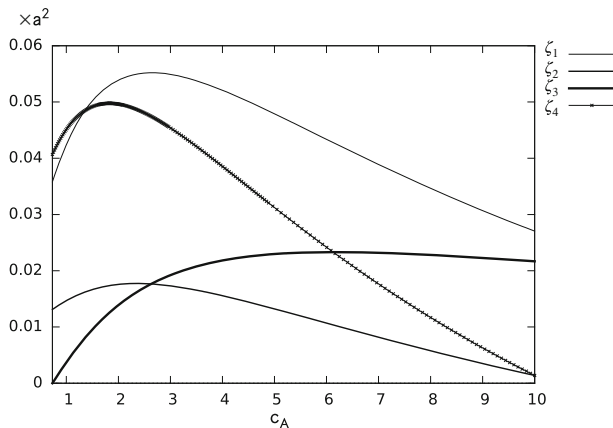


Fig. 2 Optimal strategy for the innovator when $c_A > \sqrt{3} - 1$

8 Concluding Remark

We have analyzed the choice of options for the innovating firm under oligopoly to enter the market with or without licensing its cost-reducing technology to the incumbent firm, or to license its technology without entry, using a combination of a royalty per output and a fixed license fee. We have shown that the results depend on the form of cost functions of the firms. Analyses of optimal strategies in general demand and cost functions case seem to be complicated. It is the theme of the future research. Also, in the future research we want to extend the analysis in this paper to a case of more than three firms. We have a conjecture that if the outputs of the firms are strategic substitutes (or complements), the optimal royalty rate is likely to be negative (or positive).

In this paper, we assume that the goods of the firms are homogeneous. The analysis in a case of differentiated goods is also the theme of a future research.

Funding Information This work was financially supported by Japan Society for the Promotion of Science KAKENHI Grant Number 18K01594 and 18K12780.

Appendix A. Details of Calculations

$$\lambda_A = c_A^2 c_B^3 + 4c_A c_B^3 + 4c_B^3 - c_A^3 c_B^2 + 13c_A c_B^2 + 16c_B^2 - 4c_A^3 c_B - 10c_A^2 c_B + 14c_A c_B + 25c_B - 4c_A^3 - 12c_A^2 + 7c_A + 14,$$

$$\begin{aligned} \lambda_B = & 3c_A^8 c_B^4 + 42c_A^7 c_B^4 + 236c_A^6 c_B^4 + 684c_A^5 c_B^4 + 1095c_A^4 c_B^4 + 962c_A^3 c_B^4 + 438c_A^2 c_B^4 \\ & + 96c_A c_B^4 + 8c_B^4 - 2c_A^9 c_B^3 - 8c_A^8 c_B^3 + 114c_A^7 c_B^3 + 1012c_A^6 c_B^3 + 3364c_A^5 c_B^3 + 5696c_A^4 c_B^3 \\ & + 5160c_A^3 c_B^3 + 2424c_A^2 c_B^3 + 552c_A c_B^3 + 48c_B^3 - 12c_A^9 c_B^2 - 112c_A^8 c_B^2 - 216c_A^7 c_B^2 \\ & + 995c_A^6 c_B^2 + 5454c_A^5 c_B^2 + 10628c_A^4 c_B^2 + 10296c_A^3 c_B^2 + 5103c_A^2 c_B^2 + 1230c_A c_B^2 + 114c_B^2 \\ & - 24c_A^9 c_B - 256c_A^8 c_B - 896c_A^7 c_B - 700c_A^6 c_B + 2970c_A^5 c_B + 8444c_A^4 c_B + 9262c_A^3 c_B \\ & + 4964c_A^2 c_B + 1284c_A c_B + 128c_B - 16c_A^9 - 176c_A^8 - 688c_A^7 - 1060c_A^6 + 92c_A^5 + 2436c_A^4 \\ & + 3252c_A^3 + 1908c_A^2 + 528c_A + 56, \end{aligned}$$

$$\begin{aligned} \lambda_C = & c_A^6 c_B^6 + 12c_A^5 c_B^6 + 54c_A^4 c_B^6 + 112c_A^3 c_B^6 + 105c_A^2 c_B^6 + 36c_A c_B^6 + 4c_B^6 - c_A^7 c_B^5 + 74c_A^5 c_B^5 \\ & + 406c_A^4 c_B^5 + 876c_A^3 c_B^5 + 826c_A^2 c_B^5 + 293c_A c_B^5 + 34c_B^5 - 10c_A^7 c_B^4 - 53c_A^6 c_B^4 + 90c_A^5 c_B^4 \\ & + 1125c_A^4 c_B^4 + 2716c_A^3 c_B^4 + 2641c_A^2 c_B^4 + 976c_A c_B^4 + 119c_B^4 - 40c_A^7 c_B^3 - 264c_A^6 c_B^3 \\ & - 338c_A^5 c_B^3 + 1258c_A^4 c_B^3 + 4179c_A^3 c_B^3 + 4372c_A^2 c_B^3 + 1705c_A c_B^3 + 220c_B^3 - 80c_A^7 c_B^2 \\ & - 552c_A^6 c_B^2 - 1148c_A^5 c_B^2 + 89c_A^4 c_B^2 + 3210c_A^3 c_B^2 + 3928c_A^2 c_B^2 + 1650c_A c_B^2 + 227c_B^2 \\ & - 80c_A^7 c_B - 544c_A^6 c_B - 1272c_A^5 c_B - 852c_A^4 c_B + 1016c_A^3 c_B + 1800c_A^2 c_B + 840c_A c_B \\ & + 124c_B - 32c_A^7 - 208c_A^6 - 496c_A^5 - 460c_A^4 + 32c_A^3 + 324c_A^2 + 176c_A + 28, \end{aligned}$$

$$\lambda_D = 2(c_B + 2)(c_A c_B + 2c_B + 2c_A + 2)(c_A c_B + 2c_B + 2c_A + 3)^2 (c_A^2 c_B + 4c_A c_B + c_B + 2c_A^2 + 6c_A + 2)^2,$$

$$\begin{aligned} \lambda_E = & c_A^6 c_B^4 + 12c_A^5 c_B^4 + 54c_A^4 c_B^4 + 112c_A^3 c_B^4 + 105c_A^2 c_B^4 + 36c_A c_B^4 + 4c_B^4 - c_A^7 c_B^3 - 4c_A^6 c_B^3 \\ & + 34c_A^5 c_B^3 + 250c_A^4 c_B^3 + 584c_A^3 c_B^3 + 568c_A^2 c_B^3 + 205c_A c_B^3 + 24c_B^3 - 6c_A^7 c_B^2 - 44c_A^6 c_B^2 \\ & - 46c_A^5 c_B^2 + 347c_A^4 c_B^2 + 1092c_A^3 c_B^2 + 1152c_A^2 c_B^2 + 448c_A c_B^2 + 57c_B^2 - 12c_A^7 c_B - 96c_A^6 c_B \\ & - 212c_A^5 c_B + 88c_A^4 c_B + 871c_A^3 c_B + 1062c_A^2 c_B + 455c_A c_B + 64c_B - 8c_A^7 - 64c_A^6 \\ & - 160c_A^5 - 76c_A^4 + 254c_A^3 + 384c_A^2 + 182c_A + 28, \end{aligned}$$

Appendix B. Stability Conditions

According to Seade (1980) and Dixit (1986), we consider stability conditions for a duopoly and an oligopoly. Consider the following matrix for a duopoly with Firm B and Firm C.

$$\begin{bmatrix} s_B \theta_B & s_B \sigma_B \\ s_C \sigma_C & s_C \theta_C \end{bmatrix}$$

s_B and s_C are adjustment speeds of the outputs of Firms B and C. For the stability, the trace of this matrix must be negative, and its determinant must be positive. Therefore,

$$s_B \theta_B + s_C \theta_C < 0,$$

and

$$s_B s_C [\theta_B \theta_C - \sigma_B \sigma_C] > 0.$$

Since these are to hold independently of s_B and s_C , we need

$$\theta_B < 0, \theta_C < 0, \theta_B \theta_C - \sigma_B \sigma_C > 0.$$

Consider the following matrix for an oligopoly with Firms A, B, and C.

$$\begin{bmatrix} s_A \theta_A & s_A \sigma_A & s_A \sigma_A \\ s_B \sigma_B & s_B \theta_B & s_B \sigma_B \\ s_C \sigma_C & s_C \sigma_C & s_C \theta_C \end{bmatrix}$$

s_A , s_B , and s_C are adjustment speeds of the outputs of Firms A, B, and C. One necessary condition is that the trace of this matrix is negative. Therefore,

$$s_A \theta_A + s_B \theta_B + s_C \theta_C < 0.$$

Since this is to hold independently of s_A , s_B , and s_C , we need

$$\theta_A < 0, \theta_B < 0, \theta_C < 0.$$

Another necessary condition is that the determinant of this matrix has the sign of $(-1)^3$, that is, it is negative. Thus,

$$\theta_A \theta_B \theta_C - \theta_A \sigma_B \sigma_C - \sigma_A \theta_B \sigma_C - \sigma_A \sigma_B \theta_C + 2\sigma_A \sigma_B \sigma_C < 0.$$

The second-order conditions in each case $\theta_A < 0, \theta_B < 0, \theta_C < 0$ with the stability conditions guarantee the existence of the locally unique stable Nash equilibrium. If the stability conditions are violated, the firms increase (or decrease) their outputs more than a change (increase or decrease) in the rival firm’s output, and then the outputs of the firms diverge from the equilibrium values. For the existence of the globally unique equilibrium, we need that $\theta_A < 0, \theta_B < 0, \theta_C < 0$ globally hold.

References

- Boone J (2001) Intensity of competition and the incentive to innovate. *Int J Ind Organ* 19:705–726
- Bulow J, Geanakoplos J, Klemperer P (1985) Multimarket oligopoly: strategic substitutes and strategic complements. *J Political Econ* 93:488–511
- Chen CS (2017) Endogenous market structure and technology licensing. *Jpn Econ Rev* 68:115–130
- Creane A, Chiu YK, Konishi H (2013) Choosing a licensee from heterogeneous rivals. *GEB* 82:254–268
- Dixit A (1986) Comparative statics for oligopoly. *IER* 27:107–122
- Duchene A, Sen D, Serfes K (2015) Patent licensing and entry deterrence: the role of low royalties. *Economica* 82:1324–1348
- Hattori M, Tanaka Y (2015) Subsidy or tax policy for new technology adoption in duopoly with quadratic and linear cost functions. *Econ Bull* 35:1423–1433
- Hattori M, Tanaka Y (2016a) Subsidizing new technology adoption in a Stackelberg duopoly: cases of substitutes and complements. *Ital Econ J* 2:197–215
- Hattori M, Tanaka Y (2016b) License or entry with vertical differentiation in duopoly. *Econ Bus Lett* 5:17–29
- Hattori M, Tanaka Y (2018) License and entry strategies for an outside innovator under duopoly. *Ital Econ J* 4:135–152
- Kabiraj T (2004) Patent licensing in a leadership structure. *The Manchester School* 72:188–205
- Kamien T, Tauman Y (1986) Fees versus royalties and the private value of a patent. *QJE* 101:471–492
- Kamien T, Tauman Y (2002) Patent licensing: the inside story. *The Manchester School* 70:7–15
- Katz M, Shapiro C (1985) On the licensing of innovations. *Rand J Econ* 16:504–520
- La Manna M (1993) Asymmetric oligopoly and technology transfers. *EJ* 103:436–443
- Liao C, Sen D (2005) Subsidy in licensing: optimality and welfare implications. *The Manchester School* 73:281–299
- Matsumura T, Matsushima N, Cato S (2013) Competitiveness and R&D competition revisited. *Econ Model* 31:541–547
- Pal R (2010) Technology adoption in a differentiated duopoly: Cournot versus Bertrand. *Res Econ* 64:128–136
- Rebolledo M, Sandońs J (2012) The effectiveness of R&D subsidies. *Econ Innovat New Tech* 21:815–825
- Seade JK (1980) The stability of Cournot revisited. *J Econ Theory* 23:15–27
- Sen D, Tauman Y (2007) General licensing schemes for a cost-reducing Innovation. *GEB* 59:163–186
- Wang XH, Yang BZ (2004) On technology licensing in a Stackelberg duopoly. *Aust Econ Pap* 43:448–458
- Watanabe N, Muto S (2008) Stable profit sharing in a patent licensing game: general bargaining outcomes. *Int J Game Theory* 37:505–523

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