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Strategic Compatibility Choice, Network Alliance, and Welfare

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Abstract Based on a simple model of compatibility choice under differentiated Cournot duopoly with network externalities, we consider how the levels of a network externality and product substitutability affect the choice of compatibility. In particular, if the level of network externality is larger than that of product substitutability, there are multiple equilibria involving imperfect and perfect compatibility. Furthermore, we demonstrate the conditions for constructing such a network alliance so that firms provide perfectly compatible products. The network alliance is stable and socially optimal.

Keywords Compatibility · Network externality · Fulfilled expectation · Cournot duopoly · Horizontally differentiated product · Network alliance

JEL Classifications D21 · D43 · D62 · L15

1 Introduction

In network industries, e.g., airlines, railways, telecommunications, application software, operating systems, and Internet services, we can observe not only network externalities but also compatibility and standardization of products and services. In particular, compatibility is a characteristic of products and services that interact with other products and services to enhance performance for users. Thus, the problems of compatibility and standardization are important for firms (providers), consumers (users), and the policy maker. However, the choice of compatibility between products and services tends to be made by providers rather than by users, unless an intervention by the policy maker occurs.

Since the publication of the seminal paper by Katz and Shapiro (1985), many researchers have examined social and private incentives to achieve product compatibility, which refers to

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the trade-off between compatibility and standardization (or between incompatibility and perfect compatibility) in the presence of a network externality.[1]

We observe that there are several levels (i.e., imperfect, partial, and perfect) of compatibility between products and services in network industries. For example, estimating network effects and compatibility in the Polish mobile market, Grajek (2010) finds that there are strong network effects and that the estimated level of compatibility is very low despite full interconnection of the mobile telephone network. Furthermore, Gandal (1995) empirically analyzes complementary network externalities in personal computer software markets because users need to exchange data files between spreadsheets and database management systems. As an example, he shows a statistical analysis package such as TSP, which is fully compatible with Lotus files, but not with Dbase files.

Taking the framework of Economides (1996), we develop a simple model of one-way compatibility choice under differentiated Cournot duopoly with network externalities. Using the model, we consider how the level of a network externality and product substitutability affect the compatibility choice.

Chen and Chen (2011), whose work is closely related to ours, analyze compatibility choice under a horizontally differentiated duopoly with network externalities. However, based on their specific assumption, they consider only the case of a strategic substitute relationship, and they conclude that a firm's optimal choice is an incompatible product in the market, whereas socially optimal compatibility involves establishing the perfectly compatible standard. In such a case, a social dilemma arises.[2] In the current paper, we relax their assumption and demonstrate that if the level of a network externality is larger than that of product substitutability, then there are multiple equilibria involving imperfect and perfect compatibility. Thus, a social dilemma need not always arise. Furthermore, we consider a network alliance between firms as a collusive behavior. We demonstrate that the stable network alliance is socially optimal where firms provide perfectly compatible products.

2 The Model

2.1 Setup

We develop a Cournot duopoly model in a network industry, where a firm provides a horizontally differentiated product with a network externality. Based on the framework of Economides (1996) and Häckner (2000), we assume that a linear inverse demand function of product i is given by:

$$p_i = A - q_i - \gamma q_j + N\left(S_i^e\right), i, j = 1, 2, i \neq j, \tag{1}$$

where A is the intrinsic market size, q_i (q_j) is the output level of firm i (j), and $\gamma \in (0, 1)$ is the level of product substitutability. In other words, it represents the level of product differentiation, i.e., if $\gamma \Rightarrow 0(1)$, the products are more horizontally differentiated (homogenous). $N\left(S_i^e\right)$ is a network externality function, where S_i^e represents the expected network size for product i. To

[1] See also Katz and Shapiro (1994), Matutes and Regibeau (1996), Shapiro and Varian (1999), Gandal (2002), Farrell and Klemperer (2007), Shy (2011), Farrell and Simcoe (2012).

[2] See also Toshimitsu (2014).

simplify the analysis, we assume that $N\left(S_i^e\right) = nS_i^e$, where $n \in [0, 1)$ represents the level of a network externality.

As mentioned above, the level of compatibility is endogenously determined by firms (providers). Compatibility is an important factor affecting an expected network size. Furthermore, we assume one-way compatibility in this model. Using Chen and Chen (2011), we assume the following formulation of an expected network size with compatibilities.

$$S_i^e = q_i^e + \phi_j q_j^e, i, j = 1, 2, i \neq j, \tag{2}$$

where $\phi_j \in [0, 1]$ denotes the level of compatibility, which is the strategic variable of firm j. If $\phi_j = 1(0)$, then product j operates (does not operate) with product i perfectly, i.e., perfect compatible (incompatible) product. q_i^e in eq. (2) represents the *own effect* on the expected network size. $\phi_j q_j^e$ implies that the *demand spillover effect* on the expected network size of product i depends on the level of compatibility of firm j. In other words, firm j makes the use of its product (e.g., content or system) for the rival firm's consumers (users) possible. Thus, we may appreciate that $\phi_j = 1(0)$ implies an Open (a Closed) strategy of firm j.

Considering the concept of a fulfilled expectation presented by Katz and Shapiro (1985), we assume that consumers develop expectations for network sizes after the firms make their output decisions.[3] This implies that the firms can commit to their output level, so that consumers believe the output levels and then form expectations regarding the network size, i.e., $q_i^e = q_i$, $i = 1, 2$. Thus, eq. (2) can be rewritten as follows:

$$S_i^e = S_i = q_i + \phi_j q_j, \quad i, j = 1, 2, \quad i \neq j, \tag{3}$$

where S_i is the actual network size of firm i's product.

Hereafter, we consider a two-stage game: in the first stage, firms choose the level of compatibility; in the second stage, firms determine the level of output (i.e., Cournot competition). We derive a subgame perfect Nash equilibrium (SPNE) by backward induction. Furthermore, apart from some references to firm j, we mainly consider firm i.

Based on eqs. (1) and (3), we derive the following inverse demand function of firm i's product[4]:

$$p_i = A - (1-n)q_i - \left(\gamma - n\phi_j\right)q_j, \quad i, j = 1, 2, \quad i \neq j. \tag{4}$$

Regarding eq. (4), we assume that $1 - n > |\gamma - n\phi_j|$, such that the own-price effect exceeds the cross-price effect; i.e., $\left|\frac{\partial p_i}{\partial q_i}\right| > \left|\frac{\partial p_i}{\partial q_i}\right|$. This condition implies that $1 > n + \gamma$. Furthermore, we assume that production costs are zero, because we readily observe low and even negligible marginal running costs in Internet businesses. Thus, the profit function is $\pi_i = p_i q_i$..

Hereafter, we consider a two-stage game: in the first stage, the firms choose the degree of product compatibility; in the second stage, the firms determine the output (i.e., Cournot

[3] See Katz and Shapiro (1985) and Economides (1996). Strictly speaking, we consider subgame perfect Nash equilibria in which consumers observe output levels (capacities) before making actual consumption decisions. Because consumers have to make their choice given the choices of all other consumers in the Nash equilibrium, each consumer's beliefs about the behavior of the other consumers are confirmed. In the appendix, we examine the case where consumers form expectations regarding network size before the firms' output decisions.

[4] This formulation is similar to that of Crémer et al. (2000) and Ji and Daitoh (2008). These researchers assume a homogenous product market.

duopolistic competition). We derive an SPNE by backward induction. Furthermore, unless we specifically refer to firm j, we address firm i.

2.2 The Cournot–Nash Equilibrium

Based on eq. (4), the first-order condition of profit maximization is given by:

$$\frac{\partial \pi_i}{\partial q_i} = p_i - (1-n)q_i = A - 2(1-n)q_i - (\gamma - n\phi_j)q_j = 0.$$

Thus, we derive the following reaction function for firm i:

$$q_i = \frac{A}{2(1-n)} - \frac{\gamma - n\phi_j}{2(1-n)} q_j, \quad i, j = 1, 2, \quad i \neq j. \tag{5}$$

Given eq. (5), the strategic relationship between the firms depends on the levels of product substitutability and compatibility with network externalities, i.e., $\frac{\partial q_i}{\partial q_j} < (>) 0 \Leftrightarrow \gamma > (<) n\phi_j$, i, $j = 1, 2$, $i \neq j$. This implies that a strategic substitute (complement) relationship emerges if the level of product substitutability is larger (smaller) than that of compatibility with a network externality of the rival firm. In particular, although the two products are substitutable, a relationship of strategic complementarity is sustained under Cournot competition if the level of compatibility with a network externality is sufficiently large, i.e., if $n\phi_j > \gamma$. Hereafter, we refer to compatibility with a network externality, i.e., $n\phi_j$, as network compatibility.

Taking eq. (5), we derive the following Cournot–Nash equilibrium:

$$q_i = \frac{A\{2(1-n) - (\gamma - n\phi_j)\}}{D}, \quad i, j = 1, 2, \quad i \neq j, \tag{6}$$

where $D \equiv 4(1-n)^2 - (\gamma - n\phi_1)(\gamma - n\phi_2) > 0$.

In view of eq. (6), the effect of an increase in the level of compatibility of firms i and j on the output of firm i are given by:

$$\frac{\partial q_i}{\partial \phi_i} = -\frac{n(\gamma - n\phi_j)}{D} q_i > (<) 0 \Leftrightarrow \gamma < (>) n\phi_j, \tag{7}$$

$$\frac{\partial q_i}{\partial \phi_j} = \frac{2n(1-n)}{D} q_j > 0. \quad i, j = 1, 2, \quad i \neq j. \tag{8}$$

Equation (7) shows that the effect of an increase in the level of compatibility of firm i depends on the levels of product substitutability and network compatibility of firm j. If the level of network compatibility of firm j is larger (smaller) than that of product substitutability, the output of firm i increases (decreases). Furthermore, eq. (8) implies that an increase in the level of compatibility of firm j increases the output of firm i, because an increase in the level of compatibility of firm j enhances the network size of firm i and also increases consumers' willingness to pay for product i.

2.3 SPNE in the Strategic Compatibility Choice

The profit function of firm i is expressed as: $\pi_i = (1-n)(q_i)^2$, $i = 1, 2$. Taking eqs. (7) and (8), we obtain the following:

$$\frac{\partial\pi_i}{\partial\phi_i} = 2(1-n)(q_i)^2 \frac{n\left(n\phi_j - \gamma\right)}{D} > (\leq)0 \Leftrightarrow \gamma < (\geq) n\phi_j, \tag{9}$$

$$\frac{\partial\pi_i}{\partial\phi_j} = 4(1-n)q_i q_j \frac{n(1-n)}{D} > 0. \tag{10}$$

where ϕ_i, $\phi_j \in [0, 1]$, $i, j = 1, 2$, $i \neq j$.

From eq. (9), if the level of network compatibility for the rival firm is larger (smaller) than that of product substitutability, then the level of compatibility increases (decreases) firm i's profit by the complement (substitute) effect caused by firm j. In this case, firm i chooses a perfect compatibility (incompatibility) i.e., $\phi_i = 1(0)$, $i = 1, 2$.However, if the level of network compatibility equals that of product substitutability, i.e., firm i is indifferent to choosing any level of compatibility from incompatibility to perfect compatibility, i.e., $\phi_i \in [0, 1]$, $i = 1, 2$..

Because we obtain the same results with respect to firm j, we derive the SPNE in the noncooperative compatibility choice as follows.

Proposition 1 *The following SPNE exists.*

(i) *If* $\gamma > n$, *then* $\phi_i^* = 0$, $i = 1, 2$.
(ii) *If* $\gamma \leq n$, *then* $\phi_i^* = 0$ *and* $\phi_i^* = 1$, $i = 1, 2$.

With respect to *(i)*, if the level of a network externality is smaller than that of product substitutability, then both firms provide an incompatible product. With respect to *(ii)*, if the level of a network externality is equal to or larger than that of product substitutability, then two equilibria exist. For this case, we should note the following. If $\gamma < n$, then there are three SPNE; $\phi_i^* = 0$, $\phi_i^* = \frac{\gamma}{n}$, and $\phi_i^* = 1$, $i = 1, 2$. However, at the equilibrium, $\phi_i^* = \frac{\gamma}{n}$, it holds that $\pi_i^*\left(\frac{\gamma}{n}, \frac{\gamma}{n}\right) = \pi_i\left(\phi_l, \frac{\gamma}{n}\right)$, for any $\phi_i \in [0, 1]$, $i = 1, 2$. Thus, this equilibrium is unstable. Hence, both firms provide either an incompatible product or a perfectly compatible product.

Furthermore, regarding the parameters, n representing the strength of a network externality is a complementary effect, while γ representing the level of product substitutability is a substitutionary effect. In other words, the former implies the degree of a firm's cooperative behavior, while the latter that of a firm's competitive one. Thus, a strong network externality means that firms are likely to determine the level of compatibility cooperatively.

Now, we should consider the effect of the outcomes of the SPNE in Proposition 1 on social welfare. Chen and Chen (2011, Proposition 5) have already demonstrated that a perfect compatibility is socially optimal (*S*), i.e., $\phi_i^S = 1$, $i = 1, 2$. From the perspective of social optimality, because there are multiple equilibria in the case of a strong network externality, i.e., $n \geq \gamma$, it is not necessary for both firms to choose perfect compatibility. Furthermore, socially optimal compatibilities do not arise in the case of a weak network externality.

2.4 Network Alliance: Socially Preferable Collusion

We consider the possibility of a network alliance among firms. This implies that both firms cooperatively determine the level of compatibility to maximize the aggregate profits, i.e., $\Pi(\phi_1, \phi_2) \equiv \pi_1 + \pi_2$. That is, the FOC to maximize the aggregate profits is given by:

$$\frac{\partial \Pi}{\partial \phi_i} = 2(1-n)\left\{ q_i \frac{\partial q_i}{\partial \phi_i} + q_j \frac{\partial q_j}{\phi \phi_i} \right\} = \frac{2n(1-n)A}{D^2} q_i H_j > 0. \tag{11}$$

where $H_j \equiv 2(1-n)\{h_1 + h_2\} + (\gamma - n\phi_j)^2 > 0$, $h_i \equiv 1 - n - (\gamma - n\phi_i) > 0$, $i, j = 1, 2$, $i \neq j$. Because we obtain the same results with respect to firm j, we derive the SPNE under cooperative (collusive) compatibility (C) as follows.

Proposition 2 *Cooperative (collusive) compatibility is* $\phi_i^C = 1$, $i = 1, 2$. *That is, because both firms provide perfectly compatible products, a social dilemma dose not arise.*

Equation (11) demonstrates that the firms are likely to construct a collusive network alliance with respect to the products (e.g., Shapiro and Varian 1999, Ch. 8). In this case, we have the following question: Can the network alliance be sustainable (stable)? Without loss of generality, let us assume that firm i chooses incompatibility, i.e., $\phi_i = 0$, when firm j chooses perfect compatibility, i.e., $\phi_j = 1$, under the network alliance. Taking eq. (6), we derive the following relationship:

$q_i(0, 1) > (<) q_i(1, 1) \Leftrightarrow \pi_i(0, 1) > (<) \pi_i(1, 1) \Leftrightarrow \gamma > (<) n$, $i = 1, 2$.

If the level of a network externality is smaller (larger) than that of product substitutability, that is, with a weak (strong) network externality, firm i has (do not have) an incentive to deviate from the alliance. Let us summarize the result as follows.

Proposition 3 *With a strong network externality,* i.e.,$n > \gamma$, *the network alliance is sustainable and socially preferable.*

Proposition 3 implies that given a strong network externality, collusive behavior regarding the decision of compatibility may not be ani-competitive but can improve consumer surplus and thus social welfare. In other words, product standardization among firms raises the prices, whereas it increases consumer surplus by the complementarity effect through a strong network externality. As an example, we can observe various alliances (linkups) of airlines, railways and telecommunications companies. Therefore, if our results hold, such collusive network alliances do not necessarily reduce consumer surplus. In other words, a network alliance can improve social welfare.

3 Conclusion

Intuitively, a perfectly compatible product standard is socially optimal or, at least, is preferable for consumers. However, competing firms in network product markets may choose an incompatible product (e.g., *i*-OS vs. Android).

In this paper, we have considered how the presence of a network externality affects the one-way compatibility decision of competing firms and have demonstrated that there are multiple equilibria, i.e., incompatible products and perfectly compatible products. In particular, perfectly compatible decisions are socially preferable. However, it is uncertain whether the socially preferable equilibrium arises. In other words, a social dilemma case may arise where each firm's profit increases but consumer surplus decreases. In this case, we have considered a network alliance and demonstrated the conditions for constructing a stable and socially preferable network alliance.

Various limitations arise in relation to our model, as well as the specificity of the demand and network functions. For example, although we have considered the case of Cournot duopoly, we should analyze the case of Bertrand competition and extend it to the case of oligopoly. Furthermore, we have addressed the case of one-way and horizontal product compatibility. In the future, we will extend our model to the cases of two-way compatibility and vertical product compatibility.

Appendix: The case of consumers *ex ante* expectations

Taking eqs. (1) and (2), the profit function of firm i is given by:

$$\pi_i = \left\{A - q_i - \gamma q_j + N\left(S_i^e\right)\right\} q_i, \quad i, j = 1, 2, \quad i \neq j. \tag{12}$$

The FOC of profit-maximization of firm i is:

$$\frac{\partial \pi_i}{\partial q_i} = p_i - q_i = A - 2q_i - \gamma q_j + N\left(S_i^e\right) = 0, \quad i, j = 1, 2, \quad i \neq j. \tag{13}$$

At the point of a fulfilled expectation, i.e., when $q_i^e = q_i$ and $q_j^e = q_j$, in view of eqs. (2) and (13), we obtain the following:

$$A - (2 - n) q_i - \left(\gamma - n\phi_j\right) q_j = 0, \quad i, j = 1, 2, \quad i \neq j, \tag{14}$$

Thus, we derive the following fulfilled expectation Cournot equilibrium:

$$q_i^{**} = \frac{\left\{2 - n - \left(\gamma - n\phi_j\right)\right\} A}{\Delta}, \quad i, j = 1, 2, \quad i \neq j, \tag{15}$$

where $\Delta \equiv (2-n)^2 - (\gamma - n\phi_1)(\gamma - n\phi_2) > 0$. Using eq. (13), the profit of firm i can be expressed as: $\pi_i^{**} = \left(q_i^{**}\right)^2$, $i = 1, 2$..

Given eq. (15), the effects of an increase in the level of compatibility of the firms on the equilibrium output are given by:

$$\frac{\partial q_i^{**}}{\partial \phi_i} = \frac{n\left(n\phi_j - \gamma\right)}{\Delta} q_i^{**} > (<) 0 \Leftrightarrow n\phi_j > (<) \gamma, \tag{16}$$

$$\frac{\partial q_i^{**}}{\partial \phi_j} = \frac{n(2 - n)}{\Delta} q_j^{**} > 0, \quad i, j = 1, 2, \quad i \neq j. \tag{17}$$

We proceed to the game of compatibility choice. Based on eqs. (16) and (17), we derive the following:

$$\frac{\partial \pi_i^{**}}{\phi \phi_i} = \frac{2n\left(n\phi_j - \gamma\right)}{\Delta}\left(q_i^{**}\right)^2 > (<)0 \Leftrightarrow n\phi_j > (<)\gamma, \tag{18}$$

$$\frac{\partial \pi_i^{**}}{\partial \phi_j} = \frac{2n(2-n)}{\Delta} q_i^{**} q_j^{**} > 0, \tag{19}$$

where $\phi_j, \phi_j \in [0, 1]$, $i, j = 1, 2$, $i \neq j$.

Thus, as shown in the text, we derive the same result as in Proposition 1 in the text. Therefore, by the same procedure as in the text, we can derive the same results as in Propositions 2 and 3.

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