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ARTICLE

Methods of Representation as Inferential Devices

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Abstract
In this article I am going to reconstruct Stephen Toulmin's procedural theory of concepts and explanations in order to develop two overlooked ideas from his philosophy of science: *methods of representations* and *inferential techniques.* I argue that these notions, when properly articulated, could be useful for shedding some light on how scientific reasoning is related to representational structures, concepts, and explanation within scientific practices. I will explore and illustrate these ideas by studying the development of the notion of instantaneous speed during the passage from Galileo's geometrical physics to analytical mechanics. At the end, I will argue that methods of representations could be considered as constitutive of scientific inference; and I will show how these notions could connect with other similar ideas from contemporary philosophy of science like those of models and model-based reasoning.

Keywords Representation · Scientific inference and reasoning · Concepts

1 Introduction: The Classical View of Reasoning

The classical view of reasoning, mainly developed within philosophy and with great influence in early cognitive psychology, puts logic and probability as the central engines of human reasoning, and deductive inference as its paradigmatic case (see Henle 1962). Within this tradition, reasoning is pictured as an individual ability, depending exclusively on some 'hardwired' cognitive mechanisms, and characterizable by some set of domain-general rules operating over a sentence-like representational system. During the recent decades, however, this view has been progressively abandoned by many cognitive scientists (cf. Mercier and Sperber 2017) and, to a minor extent, by philosophers (e.g., Harman 1986; Wartofsky 1987; Clark 2006). The main reason for that is that this highly idealized view

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of thinking neglects some clearly relevant factors involved in reasoning, notably, its socio-cultural dimensions and its dependence of external tools.

A good theory of reasoning should not only explain the internal processes taking place in the mind/brain of cognitive agents, but also the way in which these processes are influenced by the socio-cultural context in which the agents are embedded and the role of tools and external devices that the agents use while performing different (high-level) cognitive tasks. Following this direction, cognitive scientists started to propose alternative theories trying to account for this 'situated' character of reasoning. To name some of them: Mercier and Sperber's "argumentative theory" (Mercier and Sperber 2017) which proposes that the key element for understanding the origins and function of reasoning is that of social interaction; Hutchins's "distributed approach", which sees cognitions as a process distributed across people and artifacts, and dependent both in internal and external representations (Hutchins 2010); or "the extended mind thesis" which proposes that high-level cognitive processes are driven, to a great extent, by elements that are external to the cognitive agent herself (Clark 2006; Clark and Chalmers 1998).[1]

This tendency was echoed in philosophy of science (where the classical view was deeply rooted in) and has encouraged some philosophers to propose more realistic accounts of scientific thinking. Some notable examples are Hacking's "styles of reasoning" (Hacking 1994), which describe scientific reasoning as a multi-dimensional notion depending on domain-specific forms of reasoning that are socially and culturally developed; Nersessian and Magnani's work on model-based reasoning, which shows how scientific models are a central part of scientific reasoning abilities and practices (e.g., Magnani 2004; Nersessian 2010); or Wartofsky's "constructive mentalism", which claims that understanding scientific cognition implies inquiring into the historical processes that give rise to the specific forms of culturally-situated cognitive practices (Wartofsky 1987).

Even before all these proposals, and within the framework of a systematic criticism of classical logic as an adequate theory of reasoning and argumentation, Stephen Toulmin developed an "ecological" view of scientific theorizing which understands scientific thinking throughout the use of two main notions: "method of representation" (MR) and "inferential technique" (IT). For Toulmin, scientific reasoning cannot be fully explained by reducing it to a set of fundamental cognitive capacities working with an amodal type of input and independently of any socio-cultural context. On the contrary, it can only be understood as a socially-embedded cognitive activity, that depends on the mastering of different methods for representing information that make possible specific forms of inferences.

Toulmin uses these two notions in his two most important works on philosophy of science (Toulmin 1953, 1972a), and they play a central role in his explanatory-based approach to science. However, he does not provide analytical definitions for them, neither he uses them in a systematic manner throughout his works. Something that can overshadow the potential of these notions as proper categories of analysis for the philosophy of science.

In what follows, I intend to explore the notions of MR and IT and articulate them with Toulmin's procedural view of concepts and explanation. I will try to show that they can constitute interesting analytical categories for studying scientific practice, and in particular, that they can shed some light on the complex relation between scientific reasoning, models and conceptual change.

[1] From a more fundamental perspective, all these theories are a reaction to the formalist view of high-level cognition promoted by the computationalist approach of Fodor and Pylyshyn. See, for example, Oaksford and Chater (1991) for an explanation of the limitation of the aforementioned approach.

The article is structured in the following way. In Sect. 2 I characterize MRs and ITs, and I explain their relations with other central notions in Toulmin's philosophy of science like explanation, conceptual use and conceptual change. In Sect. 3 I illustrate the ideas developed with a case study from the history of mathematical physics. More specifically, I will show how the development of an explicit notion if instantaneous speed in physics was possible thanks to the introduction of a new MR that allowed for a new way of reasoning about motion. Section 4 finally shows how Toulmin's ideas relate (and anticipated in some cases) some important notions from contemporary philosophy of science like "models" or "model-based reasoning".

2 Representational Methods and Inferential Techniques

Firstly, it is important to keep in mind that Toulmin's philosophical project was, to a big extent, a reaction to some foundationalist views in epistemology—notably Frege's program (Toulmin 1972a, 52–55)—that had a deep impact in philosophy of science at the beginning of the twentieth century. These views—that Toulmin called "absolutists"—assumed that the job of philosophy was to look for the ahistorical and immutable principles of rationality that would underly scientific knowledge. In particular, Toulmin was a fierce opponent of logical positivists. According to him, they made the mistake of identifying scientific rationality with "logicality", that is, to suppose that "the *rationality* of a science could be explained in terms of the *logical* attributes of the propositional systems intended to express its intellectual content at one time or another" (Toulmin 1974, 404). Toulmin thought, on the contrary, that scientific rationality was historically and culturally situated, and as such, dynamic in nature. In this sense, he proposed that elucidating the principles underlying a rational enterprise like science, required to dig into the complexities of the "intellectual ecology" that characterize it in different periods of times (Toulmin 1972a, Ch. 4.3).

Secondly, Toulmin defended an explanatory-centered view of science in which its main role was to provide explanations of phenomena as a way for obtaining understanding—in contrast to the traditional way of thinking of science as allowing for prediction. In order to understand what scientists consider rational in some specific period of time, philosophers have to analyze the "explanatory practices" used in scientific disciplines in that period. Toulmin thought that these practices were articulated around *ideals of natural order* (or *explanatory ideals*), that is, background explanatory structures that determine some "natural way" in which some class of phenomena is supposed to behave (Toulmin 1961, 79). They establish what is the "normal"—and so the "expected"—behavior of the phenomena studied. When some phenomenon deviates from the ideal, it needs to be explained, and for doing that, scientists propose laws that encode specific explanatory procedures to account for them. For instance, one of the explanatory ideals which articulate classic geometrical optics is the principle of rectilinear propagation of light. Refraction is a phenomenon that deviates from the ideal, and so, begs for an explanation. In this sense, Snell's law of refraction is an ad-hoc law which accommodates the deviant phenomenon to the background ideal of natural order.

Toulmin used to call his approach to explanation "procedural" because the focus was not in abstract patterns of arguments (like in the deductive-nomological model or in Kitcher's "patterns of reasoning" view) but on concepts-in-use. And he conceived conceptual use in science as depending on the mastering of standardized procedures for representing and modeling natural phenomena. For instance, mastering the concept of *refraction* in geometrical optics does not only involve knowing its abstract definition, but it requires to master the different symbolic techniques (geometrical and algebraic in this case) that are used for representing and reasoning about cases of refraction. Most scientific concepts require, in order for someone to master them, knowing some symbolic techniques for representing information (Toulmin 1972b, 161). In this sense, and because Toulmin thinks that concepts and conceptual systems are the fundamental units of science, understanding scientific practices (notably reasoning and explanation) require to understand the MRs used and developed by scientific communities.

But, what exactly are MRs? While Toulmin used this notion extensively in his two most ambitious works on philosophy of science (Toulmin 1953, 1972a), he did not provide an analytical definition of it. Roughly put, MRs are "intellectual techniques" that allow scientists to construct and use models of phenomena. In general, any standardized symbolic system that scientists use for representing phenomena—diagrams, pictures, mathematical formulae or even computer programs—counts as a MR. Toulmin suggests that the crucial role that MRs play in scientific theorizing is related to how they make up for the representational limitations of natural language. He writes:

> "representation techniques" include all those varied procedures by which scientists demonstrate—i.e. exhibit, rather than prove deductively—the general relations discoverable among natural objects, events and phenomena: so, comprising not only the use of mathematical formalisms, but also the drawing of graphs and diagrams, the establishment of taxonomic "trees" and classifications, the devising of computer programmes, etc. (Toulmin 1972a, 162–163).

Some of the most important features of MRs are: (1) they are associated with the explanatory ideals of scientific disciplines; (2) they are generative, since they establish the rules and the symbolic resources that will constitute particular models that scientists will use to represent, understand and reason about phenomena; (3) they are part of the "collective methods of thought" (Toulmin 1972a, viii), and as such they are "communal" in nature—their use and "validity" depends on the agreement of the scientific community; (4) they play a central role in discovery; and (5) they are essential to scientific reasoning because they bring with them new ITs. In the remainder of the article, I will focus on (4) and (5) and their mutual relation.

Regarding the notion of "inferential technique", it can be roughly defined as the set of procedures that allow scientists to draw model-based inferences within the context of a particular MR. More specifically, when scientists try to explain some phenomenon, they represent it by building a model using some specific symbolic resources. Many of the inferences that scientists are going to make using this model, depend on rule-based procedures for manipulating these symbolic structures. As we will see, ITs are important because they challenge the classical view of reasoning that assume that all (rational) inferential mechanisms are to be found in some abstract system of formal rules like classical logic (see Toulmin 1972a, 487–488, or 1953, 25).

Regarding the relation between (4) and (5), Toulmin thinks that MRs play a central role in discovery because they allow for the introduction of new ways of thinking about phenomena:

> The heart of all major discoveries in the physical sciences is the discovery of novel methods of representation, and so fresh techniques by which inferences can be drawn (Toulmin 1953, 34).

h = height of the object which casts a shadow

α = altitude of the light source above

l = length of the shadow = $h^*\sin(90°- \alpha)/\sin(\alpha)$

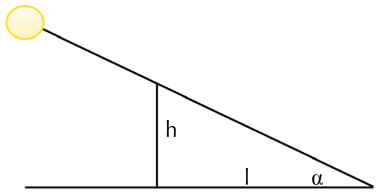

Fig. 1 Geometrical diagram used for calculating the length of the shadow in basic geometrical optics

The above quotation only makes sense when we consider Toulmin's ideas regarding the "ecological" relation between conceptual use, representation and inference. As it was suggested before, Toulmin's view of concept possession is also procedural. Scientific concepts are neither linguistic entities nor abstract ideas, instead, they obtain their content by being part of "communal" practices that involve the mastering of representational techniques for explanatory purposes. This is, concepts are not constructed in the head of users thanks to some kind of intellectual "grasping" of abstract ideas (as it would be from a Fregean perspective), but they are acquired when the user learns how to produce explanations by following the symbolic—and inferential—procedures established by the MR of the scientific discipline in which the concept takes place. Furthermore, Toulmin believes that this procedural character of scientific concepts is not lost when scientist's do their thinking "in their heads". But internalized thinking that tokens some scientific concept reflect the external symbolic procedures that characterize the MR (Toulmin 1972b, 163).

Toulmin illustrates these ideas by analyzing the concept *light* in geometric optics. As with many other concepts, *light* has a scientific version and an everyday version of it. Propositions that include the concept *light* in the everyday sense, are not necessarily supported by any MR, and so its inferential role[2] is different from its scientific version. In its everyday use, *light*'s inferential role is associated with concepts like *vision*, *shadow*, *darkness*, *color*, etc. For instance, it can take part of inferences like "If there is not light in the room, I will not be able to see". But within the context of geometrical optics *light* shows a different inferential role due to its association with a MR. In particular, the introduction of the principle of rectilinear propagation allows to model optical phenomena with geometrical methods and brings a "fresh way of drawing inferences" (Toulmin 1953, 25) based on the reading/manipulation of geometrical diagrams and other symbolic procedures. For instance, our reasoning about the length of a shadow cast by a wall depends on the construction of geometrical diagrams and the application of arithmetic and trigonometrical techniques to it. In other words, this kind of reasoning build on a set of model-based inferences that exploit formal properties of the model in order to draw some conclusion about the target phenomenon (Fig. 1).

These symbolic procedures are not heuristic tools that facilitate reasoning. They constitute the inferential technique of geometrical optics and, according to Toulmin, they cannot be fully translated to any language-based logical scheme:

[2] The *inferential role* of a concept is determined by the set of inferences that the concept may allow or participate in within a certain inferential practice.

> [I]f the novel techniques of inference-drawing here used have not been recognized by logicians for what they are, that is probably because in geometrical optics one learns to draw inferences, not in verbal terms, but by drawing lines (Toulmin 1953, 26).

In this sense, the content of the scientific concept *light* becomes associated with the kind of procedures aforementioned:

> The view of optical phenomena as consequences of something travelling and the diagram-drawing techniques of geometrical optics are introduced hand-in-hand: to say that we must regard light as travelling is to say that only if we do so can we use these techniques to account for the phenomena being as they are (Toulmin 1953, 26).

Now, these ideas bring Toulmin closer to an inferentialist view of (scientific) meaning. But with the peculiarity that for him, the kind of inferences that will contribute to the meaning of scientific concepts are not merely propositional, but model-based. Furthermore, Toulmin's ideas regarding inference and MRs have as corollary that there is not a unique (cross-disciplinar) form of scientific inference, but different ITs are constituted by situated practices involving different systems of representation and procedures of explanation. The central epistemic value for these practices is not only to explain the data but to understand phenomena in new and productive ways. Since different disciplines use different representational methods and explanatory ideals, ITs are diverse.

Now, coming back to (4), Toulmin seems to think about scientific discovery mainly as conceptual innovation—at the level of individual concepts or at the level of "conceptual populations" (Toulmin 1970). In this sense, and because of the previously explained relation between concepts, inference and representation, conceptual change could involve changes at the level of the MR and its associated IT used within a scientific discipline. Toulmin endorse this idea in different ways throughout various of his works. However, he doesn't give any detailed example of it in the history of science. In what follows, I will analyze the case of development of the concept of instantaneous speed during the passage from geometrical physics to analytical mechanics. With this, I will try to illustrate how representational techniques are involved in conceptual development in science and how MRs, concepts, and IT are interweaved.

3 From Geometrical Physics to Mathematical Physics

Conceptual change has classically been understood as a problem circumscribed to the linguistic dimension of science. Philosophers of science have tended to focus on language-related issues, like referential stability or the possibility of translations between successive theories, disregarding the relations between scientific language and other elements of scientific theorizing like models and reasoning.[3] Toulmin's approach to conceptual change is centered in the latter relation. For him, the "symbolic aspect" of scientific practice (the one related to concepts) comprises both the language and the MRs (Toulmin 1972a, 163). In this sense, conceptual changes should be studied by analyzing the dynamic relation between MRs, concepts and procedures of explanation. However, even if he discusses the issue, Toulmin does not analyze any detailed example of this phenomenon, so I will apply his ideas to the study of the development of the notion of instantaneous speed during

[3] Except for the cases of Nersessian (1999, 2010) and Thagard (1992).

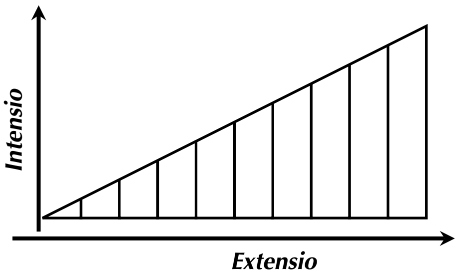

Fig. 2 Representation of an uniformly difform quality, like, for example, uniformly accelerated motion

the passage from Galilean geometrical physics to analytical mechanics. For doing that, I will follow two similar analyses of this episode proposed by Blay (1992, 1999) and Panza (2002). This case study will allow me to illustrate two main points: the first one concerns the strong dependency relationship between the MR and reasoning, and the second one concerns the role of the MR in the development of concepts.

The MR that characterized Galileo's physics was based on two main mathematical tools: the "method of the configuration of qualities" (an application of two-dimensional geometry to kinematics) and the eudoxian theory of proportions. The first one was developed in the Middle-Ages following the work of Nicolas Oresme. Oresme's intention was to represent the variations of *qualities* (phenomena enduring in time like velocity, temperature or luminosity). According to this method, qualities are measured through its *intensio* and *extensio*. The *intensio* (rate of change of the quality) is measured in *degrees* represented as straight lines associated with different points in a horizontal line representing the *extensio* of the quality.

For example, when we consider motion, *speed* is seen as an intensive quantity in relation to the extensive quantity *time.* At different points at the extensio line, speed has a correspondent degree, which can be represented by a perpendicular line. Oresme saw that the figures formed by delimiting the lines of extensio and intensio could represent different types of motion, and what is more, that the area of that figures was equivalent to the total space traversed by the body in motion (Clagett and Oresme 1968, 15). As Schemmel explains (2008, 65), this equivalence is due to the medieval idea that speed is space traversed in a specific period of time. In this sense, a uniform quantity may be represented by a rectangle, a uniformly difform quality by a triangle or a trapezium (see Fig. 2) and a difformly difform quality by various kinds of irregular figures.

Galieo's reasoning about motion was based on this very MR,[4] that implied a great deal of diagrammatic manipulation and visual thinking. Along with it, eudoxian theory of proportions was used as the tool for studying the mutual dependencies of magnitudes by comparing ratios. This last method was the precursor of the functional analysis of motion, which defines velocity as $v = \frac{\Delta s}{\Delta t}$. But Galileo could not arrive to this last definition, because the theory of proportions included an "homogeneity constraint" establishing that only magnitudes of the "same kind" could be compared in a ratio (see Def. 3 and 4, Book V, *The Elements*). Since time and speed are represented by lines, and space by areas, it was impossible to form ratios between them. Hence Galileo could not define speed explicitly,

[4] There are, however, some differences concerning the interpretation of the extension of the quality in Oresmes and Galileo (see Palmerino 2010 or Schemmel 2008 for a detailed explanation).

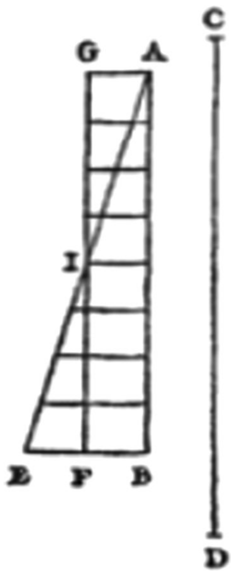

Fig. 3 Diagrammatic representation used for reasoning about the mean speed theorem (Galilei 1954, 173)

but he had to use a more complex formula for expressing proportional relations between different quantities[5]: $\frac{v_1}{v_2} = \frac{s_1}{s_2}.\frac{t_1}{t_2}$ (using contemporary algebraic notation).

This initial constraint for finding a simple definition to the notion of velocity clearly illustrates what Toulmin claims when he affirms that the explicit definition of some concepts depends on the availability of a MR that supports their inferential role. In general, as various scholars have observed (cf. Giusti 1994; Palmieri 2003; Palmerino 2010; Sellés 2006), the MR that Galileo used allowed for the development of some concepts but also imposed serious constraints to the development of others:

> [W]hen a mathematical theory is chosen to describe the phenomena (but very often there is no freedom in this choice, and Galileo had only the theory of proportions at his disposal), the mathematical language will condition not only the manner of exposing, but also sometimes the way of conceiving the very nature of things, to the point that it is not always easy to separate what belongs to the author's thought from what is instead determined by the underlying mathematical theory, which organizes the phenomena of nature according to its own structures (Giusti 1994, 493; my translation).

Giusti's observation is perfectly aligned with Toulmin's ideas about how MRs set the limits of conceptual development in scientific practices.[6] As we just saw, Galileo's analysis of motion is seriously limited by the mathematical (representational) structure that guides his reasoning. In particular, because it prevents Galileo from reasoning with the notion of continuity and with a proper notion of *instantaneous speed*. As Blay explains (1992, 133–151), within the framework of geometrical physics (the tradition of Galileo,

[5] Guicciardini (2013).

[6] Following Roux (2010, 3), we can say that Galileo's case shows how mathematical language is not "conceptually neutral".

Descartes and Newton) there was an operational, yet non-explicit, notion of instantaneous speed. And it was necessary to wait until the development of analytical mechanics to have the representational tools that will make possible one.

Galileo used the notion of *degree of speed* as an informal tool for capturing the general idea of instantaneous speed. We can see this notion operating in Galileo's proof of the Mean Speed Theorem, which establishes a relation between uniform motion and uniformly accelerated motion. The Theorem I, Proposition I of the *Dialogue* expresses the following:

> [T]he time in which any space is traversed by a body starting from rest and uniformly accelerated is equal to the time in which that same space would be traversed by the same body moving at a uniform speed whose value is the mean of the highest speed and the speed just before acceleration began (Galileo 1954, 173).

Galileo's reasoning in the proof is built around the diagrammatic representation of a particular case (Fig. 3).

The line AB represents the time in which a body travers the space CD falling from rest from point C. All the lines parallel to EB represent the *degrees of speed* at different instants starting form A, while EB itself represents the highest value of speed gained during AB. F bisects EB and FG is drawn parallel to AB until reaching GA, which is parallel to FB. The formed figures AGFB and AEB represent the two different forms of motions mentioned in the above quotation. And their relation—which is the central point of the theorem—is determined by studying the geometrical properties of the figures that represent them. In particular, the areas of each figure is assumed to be equal to the *infinite* "aggregate" (*totidem velocitatis momentum*) of degrees of speed. Since both areas are equal, then the equality of both overall speeds is inferred. And as a consequence, Galileo concludes that the space traversed by each motion is also the same.

This proof clearly illustrates Toulmin's ideas regarding how scientific reasoning depends upon representational techniques. Galileo's reasoning is clearly model-based, in the sense that it exploits formal properties of the model—according to the rules established in the MR—in order to make inferences about the phenomenon represented. One of the central points of Galileo's reasoning is—as it was just said—the idea that the area of a geometrical figure is made up of an aggregate of an infinity of lines (see Clavelin 1968, 316), and in that way the geometrical model represents the increase of momentum in time. This is so due to the representational properties of the model, since geometrical entities are considered *continua* like momentum and time (for a detailed explanation see Ducheyne 2008). A similar thing happens with the notion of degree of speed, that tries to capture the idea of instantaneous speed. Galileo didn't have an explicit definition of instantaneous speed, because within the framework of the MR he used for analyze motion, speed was considered as an intensive magnitude increasing by "successive additions of degrees" (Blay 1999, 72), so his reasoning does not make use of an explicit definition of instantaneous speed but it depends entirely on the formal properties of the MR. This was also the case in Oresme's proof of the same theorem. As Panza writes:

> Though it explicitly deals with speed as an instantaneous (or punctual) quality, the proof of this theorem is not founded on any explicit definition, either of speed in general, or of instantaneous (or punctual) speed. It simply works because of a diagrammatic formalism associated to the metaphysical idea of speed (or quality, in general) (Panza 2002, 260).

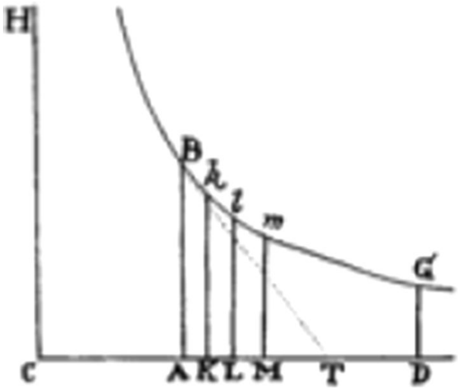

Fig. 4 Proposition V, Theorem III, *Principia I* (Newton 1999, 642)

In the *Principia*, Newton takes a big step towards the development of the formal notion of instantaneous speed and with that to the mathematizing of motion,[7] which implied the development of a new MR that will play a crucial role in the conceptual development of analytical mechanics. *Grosso modo*, Newton developed the concept of motion within the framework of dynamics, this is, explaining motion in relation to the notion of forces as a cause. According to Newton, motion is a relation between two magnitudes represented by a curve, which can be described within the framework of Cartesian geometry. The use of analytical methods in geometry for analyzing motion implied a substantial advance in relation to the Galilean MR, but still, Newton was tied to geometrical ideas when reasoning about motion. As Panza emphasizes:

> As long as motion is, for Newton, a mathematical object, it is essentially a geometrical one. […] One of the aims of analytical mechanics in 18th century is that of transforming the Newtonian science of motion in an analytical science, i.e. to pass from a geometrisation of the science of motion to a new theory of motion where the latter is just an analytical object (Panza 2002, 263).

Regarding the notion of instantaneous speed, as Blay showed, Newton worked with an *operant* notion of it, but again, without explicit definition. For example, in the proof of the Proposition V, Theorem III, in the *Principia I*, he uses the idea of instantaneous speed with a variable segment which represents the speed in an instant as the areas of the figures under the motion curve (see Fig. 4). The reasoning is still geometrical, but it's also based on an infinitesimal idea that he cannot really represent formally: supposing that the area can be divided into *innumerable equals intervals* (see Blay 1992, 135–137).

However, since an explicit definition of the notion of instantaneous speed is not at disposal, it is impossible to explicitly reason with that concept. And this could be easily seen as a conceptual limitation due to the MR in use. And as Panza said (2002, 246), this limitation is not an *expository choice*, but a consequence of the very method that Newton was using to reason about motion.

[7] It is common in the literature to talk about mathematization in Galileo, but I believe, following Blay and Panza, that there is an important difference between geometrization of motion in the tradition of Galileo–Descartes–Newton and the kind of mathematization in the development of analytical mechanics (see Panza 2002; Blay 1992).

Another central change that Newton made was to eliminate the metaphysical interpretation of motion as a quality (ibid., 263). That implied a big advance in the explanatory structure behind the Galilean interpretation of motion, and it will have a deep impact on the MR, notably because the new (analytical) notion of motion will not be affected by the homogeneity constraint of the theory of proportions.

The crucial step for leaving behind the geometric MR for understanding motion and all the conceptual limitations that came with it was taken by Varignon's (1654–1722). His interpretation of the concept of speed at each instant which is valid for both rectilinear and curvilinear motion was a crutial step towards the consolidation of a new MR in physics: analytical mechanics. Varignon built on Leibniz's *differential*—using the method developed by l'Hôpital in *Analyse des infiniment petits* (1969)—in order to define the velocity at instant t as valid for every infinitesimal interval of time dt. His argument was straightforward: since $t+dt \sim t$, speed doesn't vary and so $v(t) \sim v(t+dt)$. This allowed Varignon to give an explicit (algebraic) definition of instantaneous speed as: $v = \frac{ds}{dt}$, as well as of other crucial functional relations like $dv = \frac{d^2s}{dt}$ and $y = \frac{dv}{dt}$ ("force of instantaneous acceleration").

Using again Toulmin's analytical tools, we can see in this example how the explicit definition of the notion of instantaneous speed depended on the MR in use; clearly illustrating Toulmin's idea regarding the role of MRs in conceptual development. The mathematical language used in physical theories (and more generally, the symbolic structure used in a MR) is not *conceptually neutral* (Roux 2010), but quite the opposite. In fact, it is not a "language" in which we can express any content, or any 'abstract idea' but, in many cases, it is the condition of possibility of the very emergence of the content of these ideas and of their proper systematic use within a particular IT. Furthermore, with the new MR of analytical mechanics comes a whole new IT for reasoning about motion: algebraic-based reasoning instead of geometrical (case-based) reasoning. As Blay explains:

> [T]he figure, essential to the development and the organization of the geometric-infinitesimal thinking, gradually becomes simple diagram with Varignon. This is to say, that the figure loses its traditional value of intellection in order to take a secondary value of mere illustration (Blay 1992, 161; my translation).

Furthermore, one of the main advantages of the new algebraic-based IT is that it allows reasoning with generality, and is not tied to any particular figure like in the Galilean-style of physics. As Panza explains:

> This representation enables Varignon to eliminate any constraint of homogeneity, since it allows him to compare spaces, times, speeds and accelerative forces by means of a comparison of segments […]. These identities make the solution of a number of cinematic and dynamic problems independent of a geometric analysis of the data are expressed by means of suitable equations, the solutions can also be expressed by other equations derived by using the algorithm of the calculus (Panza 2002, 265).

In summary, this brief case study illustrates the two main features of MRs. Firstly, they are involved in conceptual change and conceptual use. Galileo's way of understanding motion imposed serious limits to the development of an explicit notion of instantaneous speed. However, he managed to reason with an operational version of this concept by exploiting the geometrical properties of his MR. Furthermore, as it was explained above, proposing an explicit notion of instantaneous speed required for the development of a new MR for thinking about motion. Secondly, this case study shows how scientific reasoning can rely heavily on representational techniques. As it was explained before, the two MRs studied—the geometrical and the analytical one—provide their own symbolic resources to build models for analyzing motion. These models work as *inferential devices* that make possible different patterns of inferences, and in this sense, different "styles of reasoning" about motion in physics.

4 Models, Model-based Reasoning, and Inferential Techniques

As I said before, Toulmin's notion of IT was a pioneering attempt to propose a more realistic alternative to the traditional view of scientific reasoning as an amodal and domain-general process based on logic and probability. However, looking beyond its historical merit, one could argue that Toulmin's view is not useful today because it is redundant with the new "environmental" perspectives that proliferate in contemporary philosophy of science. Particularly with the notion of *model-based reasoning* developed mainly by Nersessian (1999, 2010), Giere (1999, 2004, 2006) and Magnani (2002, 2004).[8] I will argue that, on the contrary, Toulmin's approach is not redundant but complementary to the model-based reasoning approach (MBRA). For that, I will briefly compare these two views in order to show their strong coincidences and their relevant differences.

The first common point concerns the role of the notion of "model" in these two views. Both Toulmin and the MBRA consider models as the key element for understanding scientific practice, emphasizing their role in scientific reasoning, concept formation, and discovery in general (see Nersessian 1999, 2010; Giere 2006; Morrison and Morgan 1999).

One important account of models within the MBRA was developed by Giere (1999). He defends a "representational" view of models[9] which sees them as the structures which enable us to 'access' phenomena, playing a central role in scientific theorizing. They are not just tools for interpreting how mathematical formulae and theoretical principles connect with actual phenomena, but they are the very target of these principles and formulae. Scientists can speak of formulae as describing phenomena thanks to the supposed representational properties of models.

In Giere's view, models are constructed according to theoretical principles (like Newton's laws, Darwin's principles, etc.), which at the same time gain meaning thanks to models, since we cannot understand them literally as laws of nature. These principles work as *general templates* (Giere 2004, 745) for the construction of models:

> [S]cientists generate models using principles and specific conditions. The attempt to apply models to the world generates hypotheses about the fit of specific models to particular things in the world. Judgments of fit are mediated by models of data generated by applying techniques of data analysis to actual observations. Specific hypotheses may then be generalized across previously designated classes of objects (Giere 2006, 60–61).

Toulmin also understands models as representational. However, his central unit of analysis of science is not strictly models but MRs, this is (again) the generative techniques

[8] Toulmin's ideas on representation, specially those developed in his 1972 book, are also a clear antecedent of Suarez's inferential view of scientific representation (Suárez 2004). However, discussing this particular relation exceeds the scope of this article.

[9] Against what he calls an "instantial view", which understands models as instantiations of the axioms and mathematical structures of theories and focuses on the problem of truth and reference in the relationship between models and target phenomena.

and procedures that underlies the construction of particular models. In Toulmin's view, the epistemological dimension is center stage. Explanation, conceptual use and reasoning are all interrelated aspects of a model-based practice whose central goal is to provide understanding. MRs, hand in hand with ITs, constitute new and productive ways of thinking about already known phenomena, and in that way, they provide scientific understanding:

> By making the journeys (inferences) so licensed, the physicist finds his way around phenomena: by thinking of the systems he studies in terms of appropriate models, he sees his way around them and comes to understand them (Toulmin 1953, 104).

Toulmin, as Giere, does not see theoretical principles as *laws of nature* that could be literally interpreted as talking directly about phenomena. They gain their meaning only in association with MRs and explanatory procedures that influence the construction of models that allow us to reason (indirectly) about phenomena (Toulmin 1953, 26–27):

> If the layman is told only that matter consists of discrete particles, or that heat is a form of motion, or that the Universe is expanding, he is told nothing or rather, less than nothing. If he were given a clear idea of the sorts of inferring techniques the atomic model of matter, or the kinetic model for thermal phenomena, or the spherical model of the Universe is used to interpret, he might be on the road to understanding; but without this he is inevitably led into a cul-de-sac (Toulmin 1953, 39).

It seems fair to see Toulmin as one of the pioneers of the representational view of models, and in particular, his approach is very close to Giere's approach, but, I think, with some additional virtues. Notably, it does not understand models as abstract and independent objects but as part of a situated, cognitive practice connecting understanding, inference, and explanation. In Giere's view, these elements are (supposedly) connected, but he does not specify how, while Toulmin's procedural view of concepts and explanation does.

Coming back to reasoning, within the MBRA, models also play a central role in it, and it's at this point where the deep similarities between the MBRA and Toulmin's approach become clear. Nersessian (1999) defines "model-based reasoning" as an inferential process that involves the construction and manipulation of various kinds of representations, not mainly sentential and/or formal, but associated with multiple formats of representation of information. As she explains:

> In model-based reasoning, inferences are made by means of creating models and manipulating, adapting, and evaluating them. A model, for my present purposes, can be characterized loosely as a representation of a system with interactive parts and with representations of those interactions. Model-based reasoning can be performed through the use of conceptual, physical, mathematical, and computational models, or combinations of these (Nersessian 2010, 12).

ITs are the model-specific patterns of reasoning that characterize the cognitive practice of scientists working within a specific discipline which use a specific MR. These patterns are rule-based procedures that make use of multi-modal elements of the model and follow the explanatory schemes involved in the MR. It is easy to see that ITs and model-based reasoning are deeply similar notions, but still, they are not exactly the same.

The main difference between these two notions is that the study of model-based reasoning is generally oriented to the specification of some domain-general cognitive mechanisms that are involved in the cognitive manipulation of models (for example: analogical reasoning, mental modeling, manipulative abduction[10], or visual reasoning). While ITs, on the other hand, are concerned with how the different model-specific symbolic systems, schemes of explanations, and concepts interact in order to conform to a particular historically and socially-situated 'procedure of reasoning'.[11] In this sense, Toulmin's analytical tools could be useful for studying the procedures of reasoning that are not the direct product of our "hardwired" cognitive capacities, but those which depends upon the collective use of normative symbolic systems, like scientific language and scientific systems of representation.

5 Conclusion

In this article, I revisited two overlooked notions of Toulmin's philosophy of science that are a central part of his procedural theory of concepts and explanation. I showed that Toulmin's ideas parallel some important new currents in philosophy of science regarding models, conceptual change, and symbolic reasoning. And that they point in the same direction as some new trends in cognitive science that understand reasoning as socially and culturally situated. I explained in what specific sense MRs could be considered as constitutive of scientific inference and how these elements are of central importance to conceptual development in science. I further showed how MRs (and models) in science play more than a mere *representational* role, since they are central to inference. It is in this sense that I speak of MRs as *inferential devices*, this is: symbolic systems that, when correctly manipulated, allow users to arrive to conclusions that could not be inferred otherwise. Furthermore, since inferential practices in science are specific of MRs, the diversity of MRs across scientific disciplines implies a diversity of inferential practices. This point could be developed as an argument in favor of some forms of scientific pluralism, especially those forms associated with the notion of *style of reasoning* (see Ruphy 2011).

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[10] See Magnani (2001, Ch. 3).
[11] In this sense, inferential techniques are close to Hacking's notion of "style of reasoning" (see Hacking 1994).

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