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# Don't Blame the Idealizations

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Published online: 5 April 2013 - Springer Science+Business Media Dordrecht 2013

Abstract Idealizing conditions are scapegoats for scientific hypotheses, too often blamed for falsehood better attributed to less obvious sources. But while the tendency to blame idealizations is common among both philosophers of science and scientists themselves, the blame is misplaced. Attention to the nature of idealizing conditions, the content of idealized hypotheses, and scientists' attitudes toward those hypotheses shows that idealizing conditions are blameless when hypotheses misrepresent. These conditions help to determine the content of idealized hypotheses, and they do so in a way that prevents those hypotheses from being false by virtue of their constituent idealizations.

Keywords Ceteris paribus · Idealization · Scientific representation · Scapegoat · Scope restrictor - Validity limit

## 1 Introductory Remarks

The ideal gas law and the Lotka-Volterra equation are paradigm examples of what I shall call *idealized hypotheses*. The ideal gas law is exactly true of gases composed of noninteracting point-masses in random motion that undergo only perfectly elastic collisions, and the Lotka-Volterra equation is exactly true of predator–prey populations in which, among other conditions, prey reproduce at a constant rate while predators consume prey and die at constant rates. I call the conditions 'gas particles are noninteracting point-masses' and 'predator mortality rates are constant' idealizing conditions or constituent idealizations (for their respective hypotheses), because they distort and simplify.

Real-world phenomena often contravene idealized hypotheses. These hypotheses seem to misrepresent phenomena, and the source of falsity in each case seems to be, at least in part, the constituent idealizations for each hypothesis. For example, gas behaviors at high pressures violate the ideal gas law, and the ideal gas law apparently misrepresents high

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pressure gas behavior, at least in part, by virtue of idealizing the volume and attractive forces of gas particles.

Some take this appearance for reality. Laymon, for example, claims that ''idealizations (because they are false) introduce bias or distortion into our computations'' (Laymon [1985](#page-15-0), 148 and [1984,](#page-15-0) 116; Elgin [2007](#page-14-0), 38; Slater [2008](#page-15-0), 534). This interpretation takes appearances at face value and involves two theses: first, that idealized hypotheses misrepresent any phenomenon that contravenes them; second, that at least some idealized hypotheses misrepresent phenomena that contravene them by virtue of the hypotheses' constituent idealizations.

There is, however, a competing inclination to interpret idealizing conditions as restricting the domains of application for idealized hypotheses rather than injecting falsity into them (Wimsatt [1987,](#page-15-0) 28–29). There is disagreement about how such restriction happens. Rohrlich and Hardin [\(1983](#page-15-0)) maintain that hypotheses' constituent idealizations impose validity limits and that these limits restrict the domain of phenomena for which hypotheses are valid. Advocates of the *ceteris-paribus* approach, in contrast, take idealized hypotheses to have (often implicit) *ceteris-paribus* clauses, based upon their constituent idealizations, which exempt them from representing phenomena that involve interfering factors (see McMullin [1985,](#page-15-0) 268–270; Lange [2002](#page-15-0)). There are other similar approaches (Cartwright [1989](#page-14-0); Giere [2006](#page-14-0); Suppe [1989\)](#page-15-0).

While all of these charitable interpretations deny that idealized hypotheses misrepresent whatever phenomena contravene them, the interpretations are less clear about whether, when idealized hypotheses do misrepresent, they sometimes do so by virtue of their constituent idealizations. The second thesis of the face-value approach to interpreting idealized hypotheses is false only if some method for restricting the scope of idealized hypotheses is guaranteed to exempt those hypotheses from misrepresenting any phenomena for which their constituent idealizations would be to blame. But, so far as I know, no advocate for any charitable interpretation has provided an argument establishing this. Furthermore, there are worries that methods of charitably interpreting idealized hypotheses are too liberal, exempting such hypotheses from misrepresenting at all (see Smith [2002](#page-15-0)). If these worries are well-founded, charitable methods exempt idealizing conditions from blame for misrepresentation only at the cost of rendering idealized hypotheses trivially true; and if the worries are misplaced, the extant literature shows at most that some idealizing conditions, for some idealized hypotheses, are not to blame when those hypotheses misrepresent. This is a far cry from refuting the second thesis of the face-value approach and, moreover, depends entirely upon whether the particular method of charitable interpretation under consideration is correct.

The plethora of competing methods for charitably interpreting idealized hypotheses makes doubtful the success of any argument, based upon only one such method, against the second thesis of the face-value approach. Nor will abstract appeals to principles of charity help, since even if those principles favor interpreting idealized hypotheses as not always misrepresenting whatever phenomena contravene them, they do not help to determine sources of blame for misrepresentations. There is, however, reason to believe that the second thesis of the facevalue approach is mistaken regardless of which interpretive method is correct. Attention to the ways in which practicing scientists tend to restrict domains of application for idealized hypotheses, along with some reasonable auxiliary assumptions, shows that no idealized hypothesis misrepresents because of its constituent idealizations. Those idealizations help to determine the hypothesis' content, and they do so in a way that prevents hypotheses from being false by virtue of their constituent idealizations.

The argument I shall give in defense of this thesis does not rule out the possibility that idealized hypotheses sometimes misrepresent. Nor does it appeal to any particular method of charitable interpretation. Instead, it extracts a distinction implicit in all (extant) charitable interpretations, between a hypothesis' apparent content and its actual content, in order to argue that if scientists are principled when restricting the scope of idealized hypotheses, idealizing conditions are never to blame when idealized hypotheses misrepresent. The

main virtue of this approach is that it avoids the need to defend as correct any particular charitable interpretation. The argument, if sound, thereby provides a principled constraint on extant methods of charitable interpretation, so that any method ascribing blame to idealizing conditions should be rejected. If unsound, the argument focuses inquiry on what must be true of scientific practice when those conditions are to blame. However, this approach provides no guidance for how to determine the restricted scopes of idealized hypotheses. Accordingly, while what follows constrains the details for such guidance, providing the details themselves remains a separate project. (But see Jones [2009](#page-15-0) for an effort to provide details.)

I begin my argument with some terminological preliminaries. I then motivate, in turn, an adequacy condition for blaming idealizing conditions and an adequacy condition for interpreting idealized hypotheses. These conditions entail, contrary to the face-value approach, that idealizing conditions are never to blame when idealized hypotheses misrepresent.

#### 2 Vehicles of Representation and Sources of Blame

I take scientific hypotheses to be statements or their mathematical equivalents. So, for example, in this sense neither an ideal gas nor a collection of gas models are hypotheses, but  $PV = nRT$  is. While this departs from the current fashion of understanding scientific hypotheses as nonlinguistic, it is not necessarily incompatible with that approach (Worrall [1984](#page-15-0)). Since statements are truth-apt, this approach allows me to treat the content of a hypothesis as, roughly, what the hypothesis says about the world or how it represents the world as being. To a first approximation, the content of a hypothesis is its truth conditions.

Scientific hypotheses are vehicles of representation. A hypothesis represents when it characterizes something as being a certain way; it is true of that thing when this representation is successful and false of it when a failure. Moreover, a hypothesis is a *non*fictional (vehicle of) representation when it "point[s] to a certain part of the world and say[s] 'if you want to know *about that part of the world I am pointing to*, for a certain sort of purpose, I promise to help you in that respect and not let you down''' (Winsberg [2009](#page-15-0), 90). Hypotheses that are *fictional* representations make no such promise. I shall argue that when idealized hypotheses do make this promise, their constituent idealizations are never to blame when they break that promise. Whether idealizing conditions are to blame when fictional representations break whatever promises they make is a topic I leave for another occasion. This does not, I trust, render my thesis entirely irrelevant. Several of our best scientific hypotheses, such as Coulomb's law and the Standard Model of particle physics, at least seem to be both idealized and non-fictional; and it remains unclear whether scientific models are properly understood as fictional representations at all (Giere [2009](#page-14-0)). (For further discussion of fictional representation, see Frigg and Hunter [2010;](#page-14-0) Suárez [2010](#page-15-0). I hereafter omit the qualifier 'non-fictional.')

I have nothing more to say about the nature of (non-fictional) representation beyond insisting that statements can represent, that ''represent'' is not a success term, and that a hypothesis misrepresents something only if the hypothesis represents it. For example, since the hypothesis ''Barack Obama is an American citizen'' does not represent the hotness of the sun (or the fact that the sun is hot), it should follow that it does not misrepresent the sun's temperature. These are fairly standard presumptions in the literature on scientific representation. (See Suárez [2010](#page-15-0) for an overview.)

The targets of representation for scientific hypotheses are *phenomena*. Following Frigg and Hartmann, I use ''phenomenon'' as ''an umbrella term covering all relatively stable and general features of the world that are interesting from a scientific point of view'' [\(2009](#page-14-0), Sect. 1.1). Nothing here depends upon whether phenomena are facts, things, properties, events, or capacities; whether they are experimental or model organisms; and so on. The nature of phenomena is irrelevant to the focus in what follows, namely, the vehicles of representation and, in particular, what can make them misrepresent.

There is a sense in which phenomena are what make hypotheses misrepresent. This is the sense relevant to truth-maker theory, concerning, typically, a relation between a linguistic entity, such as a hypothesis, and something nonlinguistic that makes the negation of the linguistic entity true or the entity itself false (For examples, see Armstrong [2004;](#page-14-0) Tennant [2010\)](#page-15-0). I shall focus, however, on a second sense in which something can make a hypothesis misrepresent. The discovery of Neptune supplies a paradigmatic illustration. Prior to Neptune's discovery, Newtonian gravitational theory predicted a particular orbit for Uranus. This prediction misrepresented Uranus' exact orbit, and there is a sense in which the wobbles in Uranus' orbit contravened the Newtonian prediction and thereby made that prediction misrepresent. Yet there is also a sense in which what made the prediction misrepresent was the auxiliary assumption that there are no planets beyond Uranus. This is a blame-oriented, rather than a truth-maker-oriented, sense in which something can make a hypothesis misrepresent.

This blame-oriented sense concerns a relation between two linguistic entities. For a Quine-Duhem problem, the relation involves a prediction and some premise, such as a lawstatement or an auxiliary assumption, in a good argument for that prediction; and a solution to the problem requires identifying the premise(s) to blame for the prediction's falsity. When one statement makes some other statement misrepresent a phenomenon in the blame-oriented sense, say (by stipulation) that the former statement is a *source of falsity* for the latter, that the latter is *false by virtue of* the former, or that the latter *misrepresents* because of the former. For example, although wobbles in Uranus' orbit were falsity-makers for the original Newtonian prediction about the planet's exact orbit, the source of falsity for that prediction was the auxiliary assumption that there are no planets beyond Uranus; the prediction was false by virtue of this auxiliary assumption, misrepresenting the exact orbit because of that assumption.

I have no precise account for what it is for a statement to be a source of falsity for a hypothesis. Something being a source of falsity differs from something being a falsitymaker for a hypothesis (in the truth-maker-oriented sense) in the same way that psilocybin's being a reason for a person's hallucinating differs from a bad trip's being a reason for the hallucination. My thesis in this paper is that, while idealizing conditions might be appropriately cited when explaining how an interpretation of an idealized hypothesis' actual content has gone awry, they are never among the elements to be cited in explaining how theory-making has misfired—idealizing conditions are never sources of falsity for idealized hypotheses; they never, in the blame-oriented sense, make idealized hypotheses false (I admit that intuition and example do not lift the explicatory burden of giving this thesis a more precise meaning; I hope to satisfy this burden in future work, trusting here that example and intuition suffice to allay substantive confusions).

Whether a hypothesis misrepresents a phenomenon (or represents it at all) depends upon what the content of that hypothesis happens to be. But a hypothesis' actual content need not be what it seems to be. Appearances can be deceiving, especially concerning language. The waitress who quips that a ham sandwich left the restaurant without paying, for example, is not saying that a sandwich literally exited the restaurant and did not pay its bill. There are pragmatic reasons for this kind of discrepancy between appearance and actuality. For instance, a briefer expression is often more efficient to communicate than one that makes explicit the various qualifications and limitations that constitute the claim's actual content, at least when there is a background presumption that whoever needs to act upon the claim knows how to decipher the content it conveys.

Scientific language is no different than ordinary language in this regard. An examination of standard science textbooks and journal articles reveals that statements of scientific hypotheses tend to be unencumbered by qualifications or restrictions. Consider, for example, the ideal gas law. Halliday et al.'s *Fundamentals of Physics* identifies the equation  $PV = nRT$  as the ideal gas law and notes that the law holds for any single gas or any mixture of different gases for which the gas density is low ([2001,](#page-14-0) 456); but this proviso is presented as restricting only the range of gas phenomena the law represents correctly. Bettelheim et al.'s Introduction to General, Organic and Biochemistry explicitly states that the ideal gas law is valid for (and thereby presumably represents) ''not only any pressure, volume, and temperature, but also … any quantity of gas'' ([2010,](#page-14-0) 148). More advanced texts, however, offer provisos that seem to restrict the range of gas phenomena the law represents (whether correctly or incorrectly). For instance, Kotz et al.'s Chemistry and Chemical Reactivity notes that the ideal gas law ''describes the behavior of a so-called ideal gas'' as well as ''real gases at pressures around one atmosphere or less and tem-peratures around room temperature" [\(2009](#page-15-0), 524). The tenor of the subsequent discussion suggests that the law *only* describes this restricted range of gas behavior. If the authors are not using ''describes'' as a success term–the context does not make it clear–it follows that the ideal gas law does not represent gas behavior at high pressures or low temperatures.

The philosophical literature makes this more confusing. There, scientific hypotheses are variously taken to be material conditionals, counterfactuals, *ceteris-paribus* claims, and so on. Derden distinguishes between *equations*, such as  $PV = nRT$ , and laws, such as the ideal gas law, which he takes to be a material conditional with the ideal gas *equation* as its consequent and a series of idealizing conditions as its antecedent (Derden [2003](#page-14-0), 243–244). Lange specifies the law of definite proportions as being a *ceteris-paribus* claim according to which any chemical compound consists of elements in proportions unvarying by mass unless the compound is like ruby or like polyoxyethylene or something like that; but standard presentations of the law do not include these exceptions (Lange [2002](#page-15-0), 408). Rohrlich takes hypotheses from physics to have tacit validity limits specifying their domain of validity, because ''an equation in physics is strictly meaningless unless its validity domain is known'' (Rohrlich [2002,](#page-15-0) 320).

There is good reason for this disagreement about what the actual content of idealized hypotheses happens to be. Teaching a scientific hypothesis in abbreviated form seems to be pedagogically more effective than teaching its qualifications and limitations at the same time as its actual content. When we learn, generalizations come first; only later do we grasp the exceptions and nuances (see Bransford et al. [1999;](#page-14-0) Donovan and Bransford [2005\)](#page-14-0). It is useful, accordingly, to distinguish between a hypothesis' apparent content and its actual content.

The *apparent content* of a hypothesis is, roughly, the unrestricted, unqualified, literal content of the hypothesis. The *actual content* of the hypothesis, in contrast, is the content the hypothesis would have were all of its implicit qualifications and exceptions made explicit. While some charitable approaches to interpreting the actual content of idealized hypotheses might seem to guarantee that a hypothesis' implicit qualifications and exceptions include all constituent idealizations for the hypothesis, the notion of actual content alone does not. For example, if idealized hypotheses should be interpreted at face value, they lack implicit qualifications and exceptions and thereby are often false by virtue of their actual content misrepresenting realworld phenomena. Charitable interpretations of idealized hypotheses tend to presume a distinction between apparent and actual content. For example, Derden expresses the distinction as one between equations and laws; interpretations that appeal to *ceteris-paribus* clauses or validity limits express it as one between hypotheses stated without these provisos and hypotheses stated with the provisos made explicit. So the distinction itself can be understood as generalizing a distinction wrought in different ways by different methods of charitable interpretation.

Certain widespread attitudes of practicing scientists toward the confirmation status of idealized hypotheses suggest that this distinction is significant. Consider, for example, Coulomb's law. This is widely regarded as extremely well-confirmed over a wide range of charge separation distances, ranging from  $10^{-11}$  meters to several kilometers (Gadre and Shirsat [2001,](#page-14-0) 3). Halliday et al.'s *Fundamentals of Physics* conveys the general attitude of the scientific community:

Coulomb's law has survived every experimental test; no exceptions to it have ever been found. It holds even within the atom, correctly describing the force between the positively charged nucleus and each of the negatively charged electrons …. This simple law also correctly accounts for the forces that bind atoms together to form molecules, and for the forces that bind atoms and molecules together to form solids and liquids [\(2001](#page-14-0), 509; emphasis added; see also Tu and Luo [2004](#page-15-0), S136).

However, some phenomena contravene the law. For instance, two electrostatically interacting and similarly charged conducting balls switch from repelling each other to attracting each other at short separation distances (Saranin [1999\)](#page-15-0); and electrostatic interactions between atoms in proteins do not become arbitrarily large as the atomic separation distance becomes small, but instead become dominated by strong repulsive forces (Jackman et al. [1994,](#page-15-0) 197). If the law's actual content does not differ from its apparent content, the law misrepresents these phenomena and thereby seems to be falsified by them. Hence, if interpretations of the content of idealized hypotheses should accommodate attitudes of practicing scientists toward the confirmation status of those hypotheses, properly interpreting Coulomb's law requires distinguishing the law's actual content from its apparent content (see also Jones [2009](#page-15-0), 125–126). This is so regardless of the proper way for excavating the law's actual content.

Considerations like this not only motivate a distinction between apparent and actual content for hypotheses but also suggest that idealized hypotheses should not always be interpreted at face value. It does not show that these hypotheses should never be interpreted in this way, much less that their constituent idealizations should never be blamed for their falsity. Nonetheless, these claims follow from three others.

- 1. A hypothesis' actual content misrepresents a phenomenon only if it represents that phenomenon.
- 2. If a hypothesis' actual content misrepresents a phenomenon, it does so by virtue of the hypothesis' constituent idealizations only if both (a) the hypothesis' apparent content

misrepresents that phenomenon and (b) the hypothesis' apparent content is false by virtue of the hypothesis' constituent idealizations.

3. A hypothesis' actual content represents a phenomenon only if it is not the case that both (a) the hypothesis' apparent content misrepresents that phenomenon and (b) the hypothesis' apparent content is false by virtue of the hypothesis' constituent idealizations.

For suppose that the actual content of some hypothesis, act(H), misrepresents some phenomenon P. And suppose, for *reductio*, that act(H) does so by virtue of its constituent idealizations  $ci(H)$ . Then, via  $(2)$ , the hypothesis' apparent content, app $(H)$ , misrepresents P and app $(H)$  does so by virtue of ci $(H)$ . Since, via  $(1)$ , act $(H)$  represents P, it follows from (3) that either app(H) does not misrepresent P or that app(H) is not false by virtue of ci(H), which contradicts the entailment from (2). Hence, whenever the actual content of a hypothesis misrepresents a phenomenon, this misrepresentation is not by virtue of the hypothesis' constituent idealizations.

(1) is a stipulation. (2) follows from some plausible relations between a hypothesis' apparent and actual content; (3), from some observations about scientific practice. I briefly defend (2) in the next section and offer a lengthier defense of (3) in the section following. After that, I consider the argument's merits and conclude with some remarks about what can make idealized hypotheses false if not their constituent idealizations.

#### 4 An Adequacy Condition for Blaming Idealizations

While there are a plethora of approaches for how to interpret a hypothesis' actual content, none is widely accepted. If interpreted at face value, the actual content of a scientific hypothesis is exactly what appears in typical textbook or journal presentations of that hypothesis. This approach endorses a kind of literalism, taking scientists to be more like Alice (from Wonderland) than the March Hare, meaning by hypotheses exactly what they say about them. (I am not aware of anyone who *explicitly* endorses this approach; but it is a natural default position.) Charitable interpretations for hypotheses' actual content, such as those appealing to validity limits or *ceteris-paribus* clauses, reject this literalism. They agree with face-value interpretations, however, that the only way for a hypothesis' actual content to misrepresent a phenomenon is for its apparent content to do so.

It is not feasible to demonstrate this claim for every extant approach; there are too many. But consider three cases. If hypotheses are interpreted at face value, their actual and apparent content are identical, and there is obviously no way for the former to misrepresent something the latter does not. If hypotheses contain validity limits, their actual content is just their apparent content prefixed with a condition that certain validity limits are satisfied. So a hypothesis' actual content misrepresents only when those limits are satisfied and its apparent content misrepresents. Similarly, if hypotheses are interpreted as ceteris-paribus claims, their actual content is just their apparent content prefixed with a ceteris-paribus clause. So a hypothesis' actual content misrepresents only when its *ceteris-paribus* clause is satisfied and its apparent content misrepresents.

Suppose, then, that a hypothesis' actual content misrepresents some phenomenon by virtue of the hypothesis' constituent idealizations. It follows, by an appeal to consensus among face-value and charitable interpretions, that the hypothesis' apparent content misrepresents the phenomenon too. Furthermore, the source of this falsity must be the hypothesis' constituent idealizations. For if the hypothesis' apparent content misrepresents the phenomenon for some reason unrelated to the hypothesis' constituent idealizations, this can only be in virtue of some mysterious pre-established harmony between those idealizing conditions making the actual content false and other reasons making the apparent content false. This is implausible. For example, it would be odd for the ideal gas law's actual content to misrepresent ammonium gas at low temperatures and medium pressures by virtue of idealizing intermolecular forces, and yet for the law's apparent content to misrepresent that gas by virtue of, say, incorrectly representing the relationship between pressure, volume, and temperature. These considerations entail a constraint for legitimately blaming idealizing conditions for the falsity of an idealized hypothesis' actual content, namely, that if a hypothesis' actual content misrepresents a phenomenon, it does so by virtue of the hypothesis' constituent idealizations only if the hypothesis' apparent content misrepresents that phenomenon and the hypothesis' apparent content is false by virtue of the hypothesis' constituent idealizations.

### 5 An Adequacy Condition for Interpretation

Although there is no consensus about the details of how properly to interpret idealized hypotheses, there is a constraint that any adequate interpretation must accommodate. There are certain ways in which practicing scientists tend to restrict the ranges of phenomena for which idealized hypotheses are representations. If these restrictions are not *ad hoc*, if scientists restrict hypotheses' scopes in a principled manner, then every reason for a hypothesis' apparent content being false by virtue of its constituent idealizations is also a reason for restricting the representational scope of the hypothesis' actual content. I shall argue that this entails, as an adequacy condition for interpreting scientific hypotheses, that a hypothesis' actual content represents a phenomenon only if either the hypothesis' apparent content does not misrepresent the phenomenon or the hypothesis' apparent content is not false by virtue of the hypothesis' constituent idealizations.

Recall the discussion of Coulomb's law. The law's actual content must differ from its apparent content. But the actual content cannot be more expansive, with a wider domain of application; that kind of interpretation does not accommodate the widespread attitude of practicing scientists toward the law's confirmation status. So the law's actual content must have a narrower, restricted domain of application. Only this kind of interpretation accommodates scientists' attitudes. Interpreting the law's content at face value thereby mistakes appearance for reality and raises the temptation to blame the law's constituent idealizations. This temptation should be resisted, because even if these idealizations appear blameworthy, even if the apparent content of Coulomb's law misrepresents electrostatic interactions between atoms in proteins by virtue of some idealizing conditions, the law's actual content does not misrepresent those interactions at all.

This line of reasoning can be made more precise and generalized with the help of two technical notions. Let the *scope restrictors* be the class of all conditions such that, when a hypothesis' apparent content misrepresents some phenomenon by virtue of one of these conditions, its actual content does not represent that phenomenon. Competing interpretations of idealized hypotheses' content make different proposals about the nature of scope restrictors. They are *ceteris-paribus* conditions according to the *ceteris-paribus* interpretation and validity limits according to the validity limit interpretation, while the class is empty according to the face-value interpretation. When a hypothesis' actual content is narrower than its apparent content, the scope restrictors determine how to restrict the hypothesis' apparent content in order to obtain its actual content.

Let the *scapegoat idealizations* be the class of all conditions such that a hypothesis' apparent content misrepresents by virtue of one of these conditions and this condition is a constituent idealization for the hypothesis. Like all idealizing conditions, scapegoat idealizations distort and simplify (see Jones [2005\)](#page-15-0). They represent phenomena of interest as having features they do not have by representing some quantity as effectively absent or some proportion as effectively negligible. For example, the condition  $\frac{dg}{dh} = 0$  simplifies: when combined with an equation that represents some phenomenon as depending upon variations of gravity with height, the condition allows one to derive an equation that does not take into account gravitational variation; and when the former equation is unknown, the condition allows one to register the fact that one is not taking into account some feature of the phenomenon. Conditions that make the magnitude of some quantity approach an extreme limiting value, such as *limit* ( $R \to \infty$ ) in which the Earth's radius is made to be infinitely large, and conditions, such as *limit*  $((v/c) \rightarrow 0)$  from relativity theory, in which some proportion between two quantities approaches or attains some extreme limiting value, perform similar functions. The conditions in these examples, moreover, are all scapegoat idealizations, idealizing conditions that are sources of falsity for the apparent content of idealized hypotheses.

If scientists are principled when restricting the scope of scientific hypotheses, every scapegoat idealization is a scope restrictor. This follows from the ways in which a condition that distorts and simplifies can be a source of falsity for an idealized hypothesis' apparent content. There are two standard ways. First, the condition might represent as absent some factor that is present. Such conditions might idealize friction or other forces as effectively absent, idealize systems as effectively closed to gamma rays or other outside influences, or idealize objects as effectively absent. When a factor makes a difference to a particular phenomenon, a hypothesis about the phenomenon based upon a condition idealizing that factor often will appear to be false. For example, the apparent content of Coulomb's law entails that electrostatic interactions between atoms in proteins become arbitrarily large as the atomic separation distance becomes small. The law thereby idealizes as effectively absent any factor that might interfere with those interactions. But strong repulsive forces become dominant at small separation distances, so that the electrostatic interactions do not become arbitrarily large: the law's apparent content misrepresents these interactions.

A second way in which an idealizing condition can be a source of falsity for an idealized hypothesis' apparent content is by representing as negligible some (non-negligible) proportionality relation between physical quantities. For instance, one idealizing condition for non-relativistic Newtonian mechanics is that an object's velocity is negligibly small compared to the speed of light in a vacuum:  $(v/c) \ll 1$ . The apparent content of nonrelativistic Newtonian mechanics misrepresents phenomena for which the ratio  $v/c$  is not negligible. When a proportionality relation is not negligible, hypotheses that idealize that relation often appear to be false.

There is perhaps one further way in which an idealizing condition can be a source of falsity for an idealized hypothesis' apparent content. Hypotheses often have false apparent content when based upon conditions that idealize a board as perfectly flat, a collision as perfectly elastic, a ball as perfectly spherical, consumers as perfectly knowledgeable about prices, and so on. Each of these conditions distorts, in a simplifying way, the magnitude of some physical quantity. It is not clear whether this is a special case of falsity due to idealized interfering factors or proportionality relations or, instead, a *sui generis* source of falsity. For example, perhaps idealizing consumers as omniscient about prices amounts to assuming that their ignorance of prices is absent; but perhaps it does not. Regardless of how these conditions should be classified, there seem to be no other kinds of scapegoat idealization.

Attention to scientific practice shows that each kind of scapegoat idealization has at least one paradigm instance that is also a scope restrictor. Coulomb's law is a convenient source of examples, because the scientific community widely regards that law as not falsified by the several phenomena that contravene it. In particular, the scientific community regards Coulomb's law as not falsified by electrostatic interactions between atoms in proteins at small separation distances or the attractive behavior of two electrostatically interacting, similarly-charged conducting balls at short separation distances. The actual content of Coulomb's law must not represent these phenomena, lest the law be falsified by them by virtue of misrepresenting them.

The condition 'no strong repulsive force is present' is a constituent idealization of Coulomb's law, and in particular it is a scapegoat idealization of the first kind (see Jackman et al. [1994\)](#page-15-0). The first phenomenon fails to be part of the law's representational scope only if this condition is also a scope restrictor for the law. The condition 'the ratio of the characteristic size of the interacting bodies to their separation distance is negligibly small' is also a constituent idealization for the law, and in particular it is a scapegoat idealization of the second kind (see Smythe [1968](#page-15-0), 2). The second phenomenon fails to be part of the law's representational scope only if this condition is also a scope restrictor for Coulomb's law.

If scapegoat idealizations of the third kind form a distinctive class, it is not clear that there are any such idealizing conditions for Coulomb's law. Fortunately, there is another appropriate source of examples for scapegoat idealizations of this kind. The scientific community widely regards the Standard Model of particle physics as not falsified by any available evidence (Gaillard et al. [1999](#page-14-0)). Nonetheless, the Standard Model seems to have consequences that are inconsistent with evidence about phenomena for which gravity matters. For instance, observations of gravitational lenses and the deflection of starlight during solar eclipses provide evidence that spacetime is curved near massive objects. But the Standard Model is a quantum field theory and, as such, contains the flatness of spacetime as an essential component (Hartmann [1998\)](#page-15-0). The condition 'spacetime is flat' is a constituent idealization for the Standard Model and a scapegoat idealization of the third kind. Phenomena for which gravity matters do not fall within the Standard Model's representational scope only if this condition is also a scope restrictor for the Standard Model (see Jones [2009](#page-15-0), 127–128).

If the scientific community restricts the representational scope of idealized hypotheses in a principled way, a scapegoat idealization of a particular kind should qualify as a scope restrictor in a paradigm case only if any scapegoat idealization of the same kind qualifies as a scope restrictor. Being principled requires treating like cases alike. For example, since the condition 'the ratio of the characteristic size of the interacting bodies to their separation distance is negligibly small' is a scapegoat idealization that treats a particular proportionality relation as negligibly small, and since that condition is a scope restrictor for Coulomb's law with respect to the attractive behavior of two electrostatically interacting, similarly-charged conducting balls at short separation distances, every scapegoat idealization that treats a particular proportionality relation as negligibly small should be a scope restrictor with respect to any phenomenon. Since, for each kind of scapegoat idealization, there is a paradigm case in which that condition is also a scope restrictor, it follows that every scapegoat idealization is a scope restrictor.

Glashow confirms this generalization, claiming that ''Newtonian mechanics is abso-lutely true—with a well-defined envelope defined by c and h" (Glashow [1999,](#page-14-0) 77). The constant c, for example, figures in one of the validity limits for Newtonian mechanics, according to which the ratio between a body's speed and the speed of light in a vacuum is

negligibly small. This is a scapegoat idealization: the apparent content of Newtonian mechanics misrepresents phenomena for which this ratio is not negligibly small. Assuming that Glashow's attitude is correct, the validity limit is also a scope restrictor for Newtonian mechanics. (This does not mean that *every* condition for a hypothesis is a scope restrictor, because not all conditions for a hypothesis are *idealizing* conditions and hypotheses can be false by virtue of these non-idealizing conditions. I substantiate this claim in the concluding section.)

This examination of scientific practice yields the result that, for each kind of scapegoat idealization, there is a paradigm instance of that idealization that is also a scope restrictor. Presuming that scientists restrict the representational scope of idealized hypotheses in a principled way, it follows that every scapegoat idealization is a scope restrictor. The definitions of ''scope restrictor'' and ''scapegoat idealization'' thereby entail the third supporting premise for the thesis that no idealized hypothesis misrepresents because of its constituent idealizations.

For suppose some hypothesis' apparent content misrepresents some phenomenon P and that it does so by virtue of the hypothesis' constituent idealizations. Then, by the definition of ''scapegoat idealization,'' the hypothesis' apparent content misrepresents P by virtue of some scapegoat idealization. Since every scapegoat idealization is a scope restrictor, it follows that the hypothesis' apparent content misrepresents P by virtue of some scope restrictor. Hence, by the definition of "scope restrictor," the hypothesis' actual content does not represent P. It follows, by conditional proof and contraposition, that a hypothesis' actual content represents a phenomenon only if it is not the case that both the hypothesis' apparent content misrepresents that phenomenon and the hypothesis' apparent content is false by virtue of the hypothesis' constituent idealizations.

### 6 Content in Scientific Practice Revisited

The thesis that idealizing conditions are never to blame when idealized hypotheses misrepresent provides a principled justification for not interpreting idealized hypotheses at face value. Its supporting argument relies upon several contentions about scientific practice, and it is worth making explicit some costs of rejecting them. The support for the first premise is linguistic; those who reject it risk changing the topic rather than engaging with the argument. The substantial premises, the ones that merit strict scrutiny, are the second and third.

The second premise is, I suspect, the least worrisome. My argument for it relies upon two contentions:

- A. If a hypothesis' actual content misrepresents a phenomenon, its apparent content does too.
- B. If a hypothesis' actual content is false of some phenomenon by virtue of the hypothesis' constituent idealizations, its apparent content is too.

I lack conclusive arguments for either contention. They are obviously true if there is no difference between a hypothesis' actual and apparent content; so advocates of face-value interpretations should find neither problematic. Moreover, attention to scientific practice shows that there is a difference, and marking it conservatively favors both (A) and (B). Finally, the cost of denying either is high. Rejecting (A) requires not only rejecting some of the most widely endorsed approaches for interpreting the actual content of scientific hypotheses but also attributing a kind of esotericism to practicing scientists. For if (A) is false, Coulomb's law (say) might misrepresent some electric phenomenon even though a person aware of all such phenomena, but exposed only to common statements of the law, would be unable to discover the law's falsity. This is quite implausible, given the likely pedagogical rationale for the distinction between actual and apparent content: making explicit a hypothesis' qualifications and limitations should not reveal new reasons for which the hypothesis can be false. Similarly, rejecting (B) requires supposing that even when a hypothesis' actual content misrepresents a phenomenon because of its constituent idealizations, the hypothesis' apparent content might give no indication that these idealizations are to blame.

The argument for the third premise invokes the notions of a scope restrictor and a scapegoat idealization as well as a substantial contention:

C. Every scapegoat idealization is a scope restrictor.

I suspect that (C) is of most concern to those who interpret idealized hypotheses at face value. I offer, in its defense, an analogical argument:

- C1 For each kind of scapegoat idealization, there is a paradigm instance of that kind that is also a scope restrictor
- C2 For each kind of scapegoat idealization, if there is a paradigm instance of that kind that is also a scope restrictor, then every instance of that kind is a scope restrictor

Examples support  $(C1)$ , and  $(C2)$  follows from the principle that similar cases should be treated similarly.

Rejecting (C1) requires either rejecting my analysis of the supporting examples or identifying idealizing conditions that are not scope restrictors and yet make hypotheses false without representing as absent some factor that is present, representing as negligible some non-negligible proportionality relation between physical quantities, or misrepresenting the magnitude of some physical quantity in a simplifying way. This first option, however, fails to accommodate the general attitude of practicing scientists toward Coulomb's law and the Standard Model. Regarding the second option, even if there are such conditions, my examples establish a version of  $(C1)$  that quantifies over some prominent kinds of scapegoat idealization. The restricted contention supports the result that none of the prominent kinds of idealizing conditions are to blame when an idealized hypothesis' actual content misrepresents. This also provides a reason not to interpret idealized hypotheses at face value, albeit in less dramatic fashion.

Rejecting (C2) requires either denying that scientists restrict hypotheses' scopes in a principled way or identifying relevant dissimilarities between some scapegoat idealizations and others such that the dissimilarities provide good reason for maintaining that not all scapegoat idealizations are scope restrictors. The first option, however, sacrifices charitable interpretation for an ad hoc preservation of appearances. Regarding the second, consider an objection that might seem promising but, on closer inspection, loses much of its luster. Suppose there is a principled distinction between fundamental hypotheses and phenomenological ones. The examples I give of scapegoat idealizations that are also scope restrictors involve Coulomb's law and the Standard Model, and there is probably a good case to be made that these are fundamental. Proponents of interpreting idealized hypotheses at face value, in contrast, give examples like the ideal gas law and the Lotka-Volterra equation, and there is probably a good case to be made that these hypotheses are phenomenological. So one might argue that (C2) is false because, even if no fundamental hypothesis misrepresents because of its constituent idealizations, some phenomenological hypotheses do: some scapegoat idealizations for phenomenological hypotheses are scope restrictors.

There are several problems with the particulars of this objection, even granting the presumed distinction between fundamental hypotheses and phenomenological ones. First, it is not clear that the distinction can bear the weight the objection requires. Cartwright ([1983\)](#page-14-0), for example, argues that the fundamental hypotheses, rather than the phenomenological ones, typically misrepresent. Regardless of whether that argument succeeds, the objection relies upon the intuition that some phenomenological hypotheses misrepresent because of their constituent idealizations. Although many philosophers seem to have this intuition, it is not clear that it accurately captures the attitudes of practicing scientists, if only because the working scientists with whom I have spoken report that they consider the ideal gas law to have a limited range of application and deny that the law is falsified by available evidence (see Jones [2009,](#page-15-0) 131).

Furthermore, it is not clear that some scapegoat idealizations for phenomenological laws are not scope restrictors. For example, a scapegoat idealization for both Coulomb's law and the ideal gas law is the condition 'no interfering factors are present'. If this is a scope restrictor for one hypothesis, it seems that it should be a scope restrictor for the other too. More generally, the constituent idealizations for fundamental hypotheses are often idealizing conditions for phenomenological ones, and there is not anything obviously special about scapegoat idealizations for phenomenological hypotheses that might prevent them from being constituent idealizations of fundamental ones. Maintaining that there is a privileged class of scapegoat idealizations for phenomenological hypotheses that are not scope restrictors looks suspiciously ad hoc, especially when scientists themselves do not clearly follow suit.

I suspect that there will be similar problems for other attempts to show that not all scapegoat idealizations are scope restrictors. Scientific practice itself does not suggest any obvious distinctions of the relevant kind, and the attitudes of practicing scientists do not clearly support any intuitions that would put such a distinction to work in the right way. Moreover, the scapegoat idealizations that are scope restrictors for some hypotheses are prevalent, appearing as constituent idealizations for many scientific hypotheses. While the examples I use to support (C) conveniently involve hypotheses widely regarded as exceptionless, they do not seem to be especially distinctive in any other way. Still, one might worry that, despite these dim prospects for discovering a relevant difference between kinds of scapegoat idealization, (C) must be false, lest it follow that idealized hypotheses never misrepresent at all. I address this worry in the final section.

#### 7 Concluding Remarks

Just as whether water makes a wheel rust depends upon the material content of the wheel, whether idealizing conditions make a hypothesis false depends upon the actual content of the hypothesis. If the preceding considerations are correct, then when the actual content of an idealized hypothesis misrepresents some phenomenon, its constituent idealizations are never a source of that falsity.

This result is trivial, of course, if it implies that the actual content of an idealized hypothesis cannot misrepresent any phenomenon. Fortunately, it is possible to show that the result does not have this implication. For it is possible to construct a scenario and an idealized hypothesis about the facts of that scenario, such that none of the hypothesis'

constituent idealizations are sources of falsity and yet the hypothesis' actual content misrepresents a certain phenomenon.

Consider a fictional, nonrelativistic Newtonian world containing a simple pendulum subject only to forces due to gravity G and damping D from the surrounding environment. Suppose that, among other conditions, the pendulum's mass M is constant and its rod is rigid by virtue of the distance R between its pivot and center of mass being constant. Since the world is nonrelativistic, the ratio of the pendulum's velocity  $V$  to the speed of light C is negligibly small, so that its momentum  $P = MV$ . Hence, since the world is also Newtonian, the pendulum's net force  $\mathbf{F} = dP/dt = M(dV/dt)$ . The net torque equals the sum of the torques due to each of the forces acting on the pendulum. Supposing, contrary to the facts of this fictional world, that there is no damping on the pendulum, it follows that the pendulum's net torque  $\tau = RMG(\sin \theta)$ , where  $\theta$  is the angle of the pendulum's displacement from its equilibrium position. These conditions yield an equation of motion for the pendulum:

E1. 
$$
dV/dt = -(G/R)(\sin \theta)
$$

Ex hypothesi, equation  $(E1)$  is an idealized hypothesis: its constituent idealization is that there is no damping force on the pendulum. By construction of the example, this idealizing condition is the entire reason for why the pendulum's exact behavior contravenes this equation.

Let a literal interpretation of equation (E1) be the hypothesis' apparent content. This content misrepresents the pendulum's exact behavior. The no-damping idealization, however, sets a validity limit for the equation, given by the pendulum's quality factor Q, a dimensionless parameter representing the pendulum period's resistance to disturbance. Pendulums subject to small amounts of damping are higher quality, because their oscillations die more slowly; while pendulums subject to high amounts of damping (such as pendulums immersed in oil) are lower quality (see King [2009:](#page-15-0) 43–45). The validity limit for equation (E1) is  $(I/Q) \ll 1$ .  $(Q^{-1} \rightarrow 0$  as the damping  $\gamma \rightarrow 0$ .) Since the apparent content of (E1) is false of the pendulum's exact behavior, the condition '1/Q is negligibly small' is a scapegoat idealization for that behavior. The idealization is thereby also a scope restrictor for that behavior, guaranteeing that the actual content of equation (E1) does not represent the pendulum's exact behavior. Accordingly, the no-damping condition restricts, rather than distorts, the equation's actual content, preventing that content from misrepresenting the pendulum's exact behavior. This seems to confirm the worry that idealized hypotheses cannot misrepresent if, among other things, every scapegoat idealization is a scope restrictor.

Consider, however, an equation of motion for the above pendulum that is idealized and yet does not have an apparent content that is false entirely by virtue of the equation's constituent idealizations. Suppose that, in addition to (falsely) assuming that there is no damping force on the pendulum, scientists in the fictional world hypothesize that an object's momentum is a product of its mass M, velocity  $V$ , and temperature T, so that an object's amount of motion depends not only upon how much stuff is in the object and how fast that stuff is moving but also upon how hot that stuff is. These scientists, accordingly, deduce that the pendulum's momentum  $P = cMVT$ , where c is a proportionality constant to ensure consistency of dimensions and is set equal to 1 by an appropriate choice of dimensional units. Supposing also that scientists correctly take the pendulum's temperature to be constant, their faulty hypothesis about momentum-temperature dependence entails that the pendulum's net force  $\mathbf{F} = dP/dt = MT(dV/dt)$ . This yields the following equation of motion for the pendulum:

<span id="page-14-0"></span>Ex hypothesi, equation (E2) is an idealized hypothesis: its constituent idealization is, as with  $(E1)$ , that there is no damping force on the pendulum. Like equation  $(E1)$ , equation (E2)'s apparent content misrepresents the pendulum's exact behavior and the equation's constituent idealization is a source of this falsity. Since the apparent content of (E2) is false of the pendulum's exact behavior, the condition '1/Q is negligibly small' is a scapegoat idealization for that behavior. The idealization is thereby also a scope restrictor for that behavior, guaranteeing that the actual content of equation  $(E2)$  does not represent the pendulum's exact behavior. Nonetheless, the equation's actual content misrepresents something about the pendulum.

The apparent content of equation (E2) entails that the pendulum's period depends, in part, upon its temperature: heating or cooling the pendulum should alter its period if the equation is correct. Since this is false, the apparent content of equation (E2) misrepresents the factors that affect the pendulum's period. But the (lone) constituent idealization for the equation is not a source of this falsity. Temperature is not an interfering factor for this dependence phenomenon, because the assumption that pendulum motion depends upon temperature is not a simplifying assumption. Nor is there a scapegoat idealization for this dependence that might prevent the actual content of (E2) from misrepresenting that dependence. Accordingly, the actual content of equation (E2) misrepresents the factors that affect the pendulum's period. Even though the equation is idealized, its actual content is false by virtue of something other than the equation's constituent idealizations.

## References

Armstrong, D. M. (2004). Truth and truthmakers. New York: Cambridge University Press.

- Bettleheim, F. A., Brown, W. H., Campbell, M. K., & Farrell, S. O. (2010). Introduction to general, organic and biochemistry (9th ed.). Belmont, CA: Brooks/Cole.
- Bransford, J. D., Brown, A. L., & Cooking, R. R. (1999). How experts differ from novices. In M. S. Donovan, J. D. Bransford, & J. W. Pellegrino (Eds.), How people learn: Brain, mind, experience, and school (pp. 19–38). Washington, D.C.: National Academies Press.
- Cartwright, N. (1983). How the laws of physics lie. New York: Oxford University Press.
- Cartwright, N. (1989). Nature's capacities and their measurement. New York: Oxford University Press.
- Derden, J. (2003). A different conception of scientific realism: The case for the missing explananda. The Journal of Philosophy, 100, 243–267.
- Donovan, M. S., & Bransford, J. D. (2005). Pulling threads. In M. S. Donovan & J. D. Bransford (Eds.), How students learn: Science in the classroom (pp. 569–590). Washington, D.C.: National Academies Press.
- Elgin, C. (2007). Understanding and the facts. Philosophical Studies, 132, 33–42.
- Frigg, R., & Hartmann, S. (2009). Models in science. In E.N. Zalta (Ed.), The Stanford encyclopedia of philosophy (Summer 2009 Edition). [plato.stanford.edu/archives/sum2009/entries/models-science/.](http://plato.stanford.edu/archives/sum2009/entries/models-science/)
- Frigg, R., & Hunter, M. (Eds.). (2010). Beyond mimesis and convention: Representation in art and science. New York: Springer.
- Gadre, S. R., & Shirsat, R. N. (2001). *Electrostatics of atoms and molecules*. India: Orient Blackswan.
- Gaillard, M. K., Grannis, P. D., & Sciulli, F. J. (1999). The standard model of particle physics. Reviews of Modern Physics, 71, S96–S111.
- Giere, R. N. (2006). Scientific perspectivism. Chicago: The University of Chicago Press.
- Giere, R. (2009). Models and fictions. In M. Suárez (Ed.), Fictions in science: Philosophical essays on modeling and idealization (pp. 248–258). New York: Routledge.
- Glashow, S. L. (1999). Does quantum field theory need a foundation? In T. Y. Cao (Ed.), Conceptual foundations of quantum field theory (pp. 74–88). Cambridge: Cambridge University Press.
- Halliday, D., Resnick, R., & Walker, J. (2001). Fundamentals of physics, extended sixth edition. New York: Wiley.
- <span id="page-15-0"></span>Hartmann, S. (1998). Idealization in quantum field theory. In N. Shanks (Ed.), Poznan studies in the philosophy of science and the humanities: Idealization in contemporary physics (Vol. 64, pp. 99–122). Amsterdam: Rodopi.
- Jackman, R. L., Cottrell, T. J., & Harris, L. J. (1994). Protein engineering. In Y. H. Hui & G. G. Khachatourains (Eds.), Food biotechnology: Microorganisms (pp. 181-236). New York: Wiley-Interscience.
- Jones, M. R. (2005). Idealization and abstraction: A framework. In M. R. Jones & N. Cartwright (Eds.), Idealization XII: Correcting the model—idealization and abstraction in the sciences (pp. 173–218). Amsterdam: Rodopi.
- Jones, N. (2009). General relativity and the standard model: Why evidence for one does not disconfirm the other. Studies in History and Philosophy of Modern Physics, 40, 124–132.
- King, G. C. (2009). Vibrations and waves. New York: Wiley.
- Kotz, J. C., Treichel, P. M., & Townsend, J. (2009). Chemistry and chemical reactivity (7th ed.). Belmont, CA: Brooks/Cole.
- Lange, M. (2002). Who's afraid of ceteris-paribus laws? Or: How I learned to stop worrying and love them. Erkenntnis, 57, 281–301.
- Laymon, R. (1984). The path from data to theory. In J. Leplin (Ed.), Scientific realism (pp. 108–123). Berkeley: University of California Press.
- Laymon, R. (1985). Idealizations and the testing of theories by experimentation. In P. Achinstein & O. Hannaway (Eds.), Observation, experiment, and hypothesis in modern physical science (pp. 147–173). Cambridge, MA: Cambridge University Press.
- McMullin, E. (1985). Galilean idealization. Studies in History and Philosophy of Science, 16, 247–273.
- Rohrlich, F. (2002). The validity limits of physical theories. Physics Letters A, 295, 320–322.
- Rohrlich, F., & Hardin, L. (1983). Established theories. Philosophy of Science, 50, 603–617.
- Saranin, V. A. (1999). On the interaction of two electrically charged conducting balls. Soviet Physics Uspekhi, 42, 385–390.
- Slater, M. (2008). How to justify teaching false science. Science Education, 92, 526–542.
- Smith, S. (2002). Violated laws, ceteris paribus clauses, and capacities. Synthese, 130, 235–264.
- Smythe, W. R. (1968). Static and dynamic electricity (3rd ed.). NY: McGraw-Hill.
- Suárez, M. (2010). Scientific representation. *Philosophy Compass*, 5, 91–101.
- Suppe, F. (1989). The Semantic conception of theories and scientific realism. Chicago: University of Illinois Press.
- Tennant, N. (2010). Inferential semantics for first-order logic: Motivating rules of inference from rules of evaluation. In J. Lear & A. Oliver (Eds.), The force of argument: Essays in honor of Timothy Smiley (pp. 223–257). New York: Routledge.
- Tu, L.-C., & Luo, J. (2004). Experimental tests of Coulomb's law and the photon rest mass. *Metrologia*, 41, S136–S146.
- Wimsatt, W. C. (1987). False models as a means to truer theories. In M. Nitecki & A. Hoffman (Eds.), Neutral models in biology (pp. 23–55). New York: Oxford University Press.
- Winsberg, E. (2009). Science in the age of computer simulation. Chicago: University of Chicago Press.
- Worrall, J. (1984). An unreal image. The British Journal for the Philosophy of Science, 35, 65–80.