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Mathematics, PISA, and culture: An unpredictable relationship

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Abstract Recent studies have indicated, particularly in the European context, that students' mathematical successes on international tests of student achievement may not be attributable to the quality of classroom instruction, although, as is shown, this is unlikely to be the case in Flanders, the autonomous Dutch-speaking region of Belgium. Flemish students' mathematics performance on such tests have placed them at the head of the European rankings, warranting Flanders as a site of research interest that has been largely ignored by the international community. In this paper, drawing on analyses of four sequences of five lessons, taught by teachers construed locally as competent, I explore the nature of Flemish mathematics teaching. Framed by anecdotal reports that it reflects the structuralism of the now largely abandoned Bourbakian new mathematics movement humanised by the Dutch tradition of realistic mathematics education, the analyses focus on examining not only the extent to which these traditions are manifested in Flemish classrooms but the ways in which they interact. The dominant tradition seems to be that of mathematical structuralism mediated by teachers' use of realistic problems; a tradition not unlikely to underpin Flemish students' repeated successes. The results are discussed in relation to research highlighting the significance on students' achievement of the broader cultural milieu in which they and their teachers operate.

Keywords Bourbaki · Finland · Flanders · Mathematics instruction · PISA · Realistic mathematics education · TIMSS

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Abbreviations

IEA	International Association for the Evaluation of Educational Achievement
OECD	Organisation for Economic Cooperation and Development
PIRLS	Progress in International Reading Literacy Study
PISA	Programme of International Student Assessment
RME	Realistic mathematics education
TIMSS	Trends in International Mathematics and Science Study

Introduction

For more than a century, those responsible for a nation's educational system have monitored those of their economic or military competitors (Waldow et al. 2014). This process has not only been accelerated by the results of large scale international tests of achievement but created "a growing number of rich nations" becoming "obsessed with what other rich nations do, extrapolating from their competitors' success in pursuit of the accolade of 'world class' (Alexander 2010, p. 801). While such obsessions may be naïve, much can be learnt from systematic investigations of the education-related practices of countries other than one's own. In this respect, typically due to perceptions of curricular similarity, much research has focused on understanding how a difficult subject like mathematics can be made more accessible to more learners. In this paper, drawing on a case study examination of mathematics teaching in Flemish Belgium, I argue that those minded to engage in subject-related policy borrowing would find their interests better served by examining systems that are not only successful on more than one form of international test but also culturally similar. But first, I summarize why cultural dissonance makes comparison difficult and why success on one form of international test may be an insufficient indicator of teaching quality. In so doing, I draw on two conceptually different tests of mathematics achievement; PISA, the Programme of International Student Assessment, and TIMSS, the Trends in International Mathematics and Science Study.

The mathematics component of the various TIMSS assessments, premised on an internationally agreed but essentially hypothetical mathematics curriculum, has focused on the mathematical behaviors expected of students in relation to that subject matter (Mullis et al. 2009). That is, TIMSS, sponsored by the International Association for the Evaluation of Educational Achievement (IEA), typically assesses students' technical competence at grades 4 and 8. The mathematics component of PISA, undertaken at age 15, aims to look beyond classroom mathematics towards those situations people face when going about their daily lives and which necessitate the application of mathematical skills in less structured contexts (OECD 2003). In short, PISA addresses "the capacity of students to put mathematical knowledge into functional use in a multitude of different situations in varied, reflective and insight based ways" (Schleicher 2007, p. 351). This paper, due to their occurring at similar times in the student experience, is framed against student achievement on PISA and TIMSS grade 8. Finally, and this is an issue to which I return later, one factor significant in understanding students' success on tests like PISA and TIMSS is the extent to which mathematics test items require reading competence. For example, TIMSS items are typically short and deploy familiar curriculum-related vocabulary, whereas PISA test items are located in extensive text requiring interpretation (Rindermann 2007). Indeed, while the OECD may argue that PISA assesses mathematical applicability, answering its questions correctly "may be more a test of 'common sense'…than the results of mathematical schooling" (Prais 2003, p. 143).

For more than three decades considerable American research investment has focused on comparing US classrooms with those in different Pacific Rim countries. Much of the early work was undertaken by Stigler and his colleagues (Stigler et al. 1982; Stigler and Perry 1988) leading to the first TIMSS video study in 1995 (Stigler et al. 1999). Today, this tradition continues through the work of, for example, Cai and his colleagues (Cai and Wang 2010; Cai et al. 2014). However, despite being of interest to mathematics education researchers, the policy- and practice-related impact of such research is limited, due to the not insignificant difficulty of disentangling the consequences of didactics from, say, the uncertain but pervasive influences of cram schools (Bray 2010) or cultural norms deriving from the Confucian culture (Tweed and Lehman 2002) thought to distinguish such communities from those of mainstream USA. Furthermore, exacerbating the problem, collective descriptions such as the Confucian example mask not only the fact that even within China can be found 56 ethnic groups, many of which draw on guiding philosophies different from Confucianism (Wong 2004), but also considerable differences with respect to how mathematics teaching is construed (Clarke 2006). Even within Western Europe can be found similar interpretive difficulties. For example, due to its repeated PISA successes, many systems have looked to Finland for policy-related insights (Laukkanen 2008), an interest encouraged by the OECD's (2010, p. 6) bizarre assertion that "bringing all countries up to the average performance of Finland, OECD's best performing education system in PISA, would result in gains in the order of USD 260 trillion" over the "lifetime of the generation born in 2010". However, highlighting the cultural distance between Finland and other Western European states, the Finns have described themselves as having an "authoritarian, obedient and collectivist mentality" closer to those of Japan or Korea than their Nordic neighbours (Simola 2005, p. 457).

Moreover, Finland's PISA successes mask not only a general mathematical underachievement but also the largely hidden role of Finnish culture in the construction of Finnish students' PISA-related achievement. For example, Finland has participated in two TIMSS, in 1999 and 2011, and on both occasions its grade 8 students' algebraic and geometric competence barely reached the international mean (Mullis et al. 2000, 2012). Also, a longitudinal study of 2400 Finnish engineering undergraduates found only 30 % able to subtract one simple fraction from another and divide the answer by an integer (Tarvainen and Kivelä 2006). Finally, recent analyses of Finnish mathematics teaching, including one exploiting PISA's own assessment framework to investigate how teachers facilitate students' acquisition of PISA-related competence, found no evidence of students being encouraged to develop higher order mathematical skills (Andrews 2013; Andrews et al. 2014), reflecting earlier and long-standing concerns about students' acquisition of mathematical (to be distinguished from mathematical literacy) problem solving competence (Martio 2009). In sum, Finnish students' PISA achievements seem unrelated to teaching quality and mask significant failures with respect to those mathematical competences, typically assessed by TIMSS, that are necessary for the economically essential university study of mathematics-related disciplines. In other words, while Finland's PISA successes may be enviable, its relative TIMSS failures make it a site of limited interest to those interested in improving the quality of mathematics teaching and learning elsewhere.

With respect to the impact of culture on Finnish students' PISA successes it is not insignificant that Finland has a largely homogeneous population (Itkonen and Jahnukainen 2007) and a genuinely comprehensive school system based on deep-seated egalitarian principles (Andrews 2014), factors contributory to educational achievement (Välijärvi et al. 2002). Moreover, Finnish PISA successes, unlikely without high levels of student reading competence, may be better understood in relation to the 400 year Lutheran tradition that Finns may only partake of Christian sacraments, and therefore marry, if they are demonstrably literate (Linnakylä 2002), an expectation reflected in modern Finland having the densest library network in the world (Sahlberg 2007). In other words, as Pacific Rim nations' educational traditions may be difficult for Western eyes to penetrate, Finland is similarly problematic. It is culturally unlike other European nations, its collective mathematics achievement lies beneath a threshold that would make it of interest to outsiders and the mathematics-related successes it has enjoyed seem to be consequences of irreplicable cultural factors linked to what it is to be Finnish.[1]

So, what education systems have achieved both PISA and TIMSS successes and, importantly, are sufficiently culturally similar to those Western, typically multicultural, nations concerned with improving their students' mathematical competence as to make classroom analyses worthwhile? Flanders, the autonomous Dutch-speaking region of Belgium, may be one. But first, some background. Belgium, which came into being in the early 1830s, forms a physical barrier between France to the south and Netherlands to the north. Its origins lie in many decades of religious turmoil leading to the separation of a large Catholic community from Protestant Netherlands, a process confirmed by the treaty of London in 1839. Initially conceived as a French-speaking nation, Belgium comprised a high proportion of Dutch-speakers, whose continual lobbying for linguistic and cultural autonomy resulted in Belgium becoming fully federated in 1993 (De Rynck 2005). Today, Flanders has full responsibility for all educational matters. With respect to its geopolitical importance, the headquarters of both NATO and the European Union are found in Brussels, which, although administratively independent, lies within Flanders. The largest Flemish city, Antwerp, has the second largest freight port in Europe and is the centre of the international diamond trade. With respect to population, area and per capita GDP, Flanders is not dissimilar to the US state of Maryland.

[1] In fact, it is more a consequence of being a Finnish-speaking Finn than just being Finnish. See Andrews (2014) for a detailed summary of this phenomenon.

Flanders as a potential research site

Flanders has participated in three of the five reported TIMSSs and all reported PISAs. Its mathematical achievements, not always well-known because PISA typically reports Belgium as a whole,[2] have been unparalleled in Europe. Its students' performance on both forms of test are shown in Table 1. It can be seen that of the six possible comparisons between Flanders and Finland, Flanders surpassed Finland on four occasions and that these margins were statistically significant, which was not the case when Finland's PISA scores surpassed Flanders'. On two of the three occasions when Flanders did not achieve the highest European PISA ranking, it was second to Finland in 2006 and 2009 and Liechtenstein[3] in 2012. The only other systems scoring more highly were drawn from Hong Kong, Japan, Korea, Macau, Shanghai, Singapore, and Taiwan. Thus, acknowledging the arguments above concerning the policy-related research limitations of Pacific Rim nations, homogeneous populations or city states (Meyer 2013), Flanders has emerged as a European system that has been mathematically successful on both PISA and TIMSS. Also, as is discussed below, multicultural Flanders has a number of demographic and educational characteristics that make it a site of potential, and as yet largely untapped, research interest.

The above prompt some important questions; are there characteristic patterns of Flemish mathematics teaching and, if so, do they offer insights into how mathematics may be taught effectively in, as is shown below, multicultural communities operating a multiplicity of school types and curricular tracks? Such questions, while important, rest on at least three assumptions. Firstly, as was discussed above, they assume that success on international tests of achievement is due to a resonance between teaching and test objectives, which evidence from Finland shows may not be the case. Secondly, they assume a degree of predictability in a country's didactical traditions. That is, they assume the existence of a *mathematics teaching script*, or "the culturally determined patterns of belief and behaviour, frequently beneath articulation, that distinguish one set of teachers from their culturally different colleagues" (Andrews and Sayers 2013, p. 133). Thirdly, and returning to the issues discussed above, they assume that an analysis of mathematics teaching in one cultural context would be relevant to those working in another. In this respect, Flanders is of particular importance because it reflects, in at least three ways, educational and social demographics commonplace in the West.

Firstly, Flanders has experienced socioeconomic and ethnic segregation comparable to many Western states, due to post war immigration from Southern Europe, Turkey and North Africa (Agirdag et al. 2012a). Many of these communities have congregated in the working class districts of larger cities (Dronkers and van der

[2] Prior to PISA 2012 the Flemish authorities had published their own PISA summaries, produced at the University of Gent by Inge de Meyer and her colleagues (De Meyer 2008; De Meyer and Warlop 2010; De Meyer et al. 2002, 2005).

[3] Liechtenstein, due to its favourable tax regimes attracting many international companies to base their headquarters there, has the highest per capita GDP in the world. With a population of 35,000 it is also the world's fourth smallest state, making it a site of limited relevance with respect to the comparative education debate.

Table 1 Flemish TIMSS and PISA scores plus European and world rankings

	TIMSS grade 8				PISA				
	1995	1999	2003	2011	2000	2003	2006	2009	2012
Flanders	565	558	537	–	543	553	543	537	531
Finland	–	520	–	514	536	544	548	541	519
Highest	643	604	605	613	557	553	549	600	613
International mean	513	487	467	500	500	500	500	500	500
Flanders European rank	1	1	1	–	1	1	2	2	2
Flanders international rank	5	6	6	–	3	1	5	7	9

Velden 2013, leading not only to higher proportions of immigrant children in particular schools but also a migration of those white Flemings with sufficient economical capital to take advantage of the right to choose the schools their children attend (Van Houtte and Stevens 2009). As is often the case elsewhere, the educational achievement of these minority immigrant communities is lower than that of native European students (Dronkers and van der Velden 2013) leading to increased feelings of isolation, helplessness (Agirdag et al. 2012b), inter-peer aggression and classroom delinquency (Leflot et al. 2013).

Secondly, Flanders operates a differentiated school system in which schools, defined by what is known as their initiating bodies, are fully funded by the government (Cherchye et al. 2010) and expected to provide education free to the child and conform to the curricular regulations specified by the Flemish government. In broad terms, there are public schools initiated by either the Flemish government or the municipalities and private schools initiated by private bodies (Van Heule 2000). In practice, 65 % of all Flemish primary (Ballet and Kelchtermans 2008) and 75 % of all Flemish secondary schools (Agirdag et al. 2012a) are private schools initiated by the Catholic church.

Thirdly, with respect to secondary education, the core curriculum, including mathematics, is the same for everyone. However, under teachers' and parents' guidance, students elect to follow vocationally oriented, humanities oriented or classically oriented tracks, and there is a general understanding that these form an academic hierarchy (Op't Eynde et al. 2006). Significantly, due to their de-facto right to choose, a higher proportion of students begin secondary school in the higher tracks than would be the case in selective systems with the consequence that expectations, and consequently achievement, of Flemish students is typically higher than in other stratified European systems (Prokic-Breuer and Dronkers 2012). That being said, school type and track influence significantly the mathematical learning of Flemish students (Cherchye et al. 2010; Opdenakker et al. 2002; Pugh and Telhaj 2007; Pustjens et al. 2007).

However, the status of Flemish teachers is high; they enjoy a positive image among their compatriots, who understand that teaching is a challenging and demanding occupation (Verhoeven et al. 2006). Teachers are not only motivated by intrinsic, altruistic and interpersonal features of their role (De Cooman et al. 2007) but are flexible and amenable to innovation (Ballet and Kelchtermans 2008). On a related theme, the Flemish authorities regard the well-being of all participants as being a high priority. Thus, various studies have been commissioned, to "evaluate whether schools take seriously their responsibility concerning the well-being of their personnel" (Aelterman et al. 2007, p. 286). Such commissions have focused on headteachers (Devos et al. 2007), pupils (Engels et al. 2004) and teachers (Aelterman et al. 2007). In other words, Flemish teachers are valued by all strata of Flemish society, with evidence highlighting their strong motivation, flexibility and desire to do their best for their students.

In sum, and acknowledging that no two cultures are so perfectly matched as to remove all doubts about the viability of policy-borrowing-related comparison, Flanders seems to be a site of significant research interest. Its educational achievements, at least as far as the tests on which it has participated, are high. The education system is highly stratified, which distinguishes it from, say, Finland. It is a genuinely multicultural community, which makes it a site of particular research interest for those concerned with improving educational performance in other multicultural communities, and it is economically comparable to many of those nations most likely to want to learn from its policies and practices. However, like Finland, its teachers are valued in both word and deed.

Interestingly, despite its international successes, little is known about Flemish mathematics teaching. A few studies undertaken shortly after federation,[4] found a tradition privileging declarative knowledge and lower-order procedural skills (Janssen et al. 2002) and, despite strong curricular expectations, few opportunities for students to solve problems or discuss different solutions to the same problem (Verschaffel et al. 1994; Verschaffel and De Corte 1997). Lessons were dominated by the teacher orchestrating whole class activities (Verschaffel et al. 1999). That being said, conversations with Flemish colleagues and other anecdotal evidence (Dykstra 2006) indicate that Flemish mathematics teaching is a pedagogical marriage of two differing mathematics education traditions. The first, introduced into Belgian[5] curricula during the era of the new mathematics of the 1960s and, in accordance with international trends abandoned less than 20 years later (Dykstra 2006), privileged the structural rigour of Bourbakian mathematics. The second, which has become the underpinning principle of both teacher education and school textbooks, draws on the Dutch tradition of realistic mathematics education (RME) (Dykstra 2006). Indeed, while current curricular expectations are broadly "in line with the developments on the international scene" they do not go as far as the Dutch in "emphasizing the constructive and realistic view on mathematics education and in de-emphasizing the place of some mechanistic and structuralistic elements of mathematics" (Verschaffel et al. 2005, p. 53). In other words, the Flemish mathematics curriculum, while acknowledging the solution of real-world problems, appears to have retained some elements of Bourbakian structuralism. In the following, to frame later analyses and discussion, literature on Bourbaki and RME respectively is summarized.

[4] As indicated earlier, Flanders was not granted sovereignty over educational matters until Belgium became fully federated in 1993.

[5] The historical summary of Bourbaki relates to Belgium before federation.

Bourbaki and his influence on school mathematics

Since its disappearance from current curricular discourse, relatively little has been written on Bourbaki and his relationship to school mathematics, particularly from the perspective of didactics (Kilpatrick 2012). In the following, I describe the Bourbakian influence on school mathematics and the didactical traditions associated with it. In so doing, I acknowledge that this can be little more than a crude characterization.

Nicolas Bourbaki was the name under which a circle of French mathematicians, particularly active during the second quarter of the twentieth century, published material intended to address a perceived lack of structural integrity and intellectual rigour in university mathematics (Munson 2010; Weintraub and Mirowski 1994). The collective authorship was adopted to protect the reputations of a relatively young membership against establishment reactions (Clark 2005), although with time the group acquired an authority by dint of its increasingly acknowledged, but continually anonymous, "'polycephalic' nature" (Aubin 1997, p. 304).

The group's aim was the promotion of an axiomatic mathematics, based on set theory, from which all topics would be derived (Guedj 1985; Landry 2007). Through this work Bourbaki expressed his desire "to purify mathematics of any reliance on the external world" (Aubin 1997, p. 298) and "uphold the primacy of the pure over the applied, the rigorous over the intuitive" (Weintraub and Mirowski 1994, p. 247/8). As Cartan (1980), one of the original group members, wrote of the axiomatic method;

> instead of declaring which objects are to be investigated, one has only to list those properties of the objects to be used in the investigation. These properties are then brought to the fore expressed by axioms; whereupon it ceases to be important to explain what the objects *are,* that are to be studied. Instead, the proof can be constructed in such a way as to hold true for every object that satisfies the axioms. It is quite remarkable how the systematic application of such a simple idea has shaken mathematics so completely (Cartan 1980, p. 176/7).

Such perspectives, in which "natural numbers were a structured set of elements that had lost all relation to referents" (Aubin 1997, p. 306), influenced greatly how Piaget came to view mathematics and mathematical learning (Munson 2010). Indeed, Piaget and the Bourbakists were attracted to each other by the commonality of structure; mental structures on the one hand and mathematical structures on the other (Gispert and Schubring 2011). This structural convergence provided the intellectual underpinning of the New Math movement, which, in an attempt to bridge the gap between school and university mathematics, privileged principles over procedures, highlighted structures, sets and patterns, and emphasized experiential over rote learning (Klein 2003; Vanpaemel et al. 2012). Thus, mirroring the practices of university mathematics, the new mathematics focused on "the careful construction of mathematical concepts, beginning with sets and logic" (Maasz and Schlöglmann 2006, p. 1). In so doing, little attention was paid to either basic skills or the applications of mathematics (Klein 2003; Maasz and Schlöglmann 2006).

Bourbaki's influence was particularly strong in Belgium and France (Furinghetti et al. 2013), where, acknowledging the primary role of structure, active approaches to teaching were adopted (Gispert and Schubring 2011). In Belgium in the 1950s teachers were encouraged to exploit various concrete materials to underpin students' understanding of the structures under scrutiny and "to help children…make the transition from intuitive experiences towards abstract understanding" (Vanpaemel et al. 2012, p. 425). Such activities underpinned Papy's mandated curriculum, the main features of which were "the unity of mathematics, the progressive introduction of fundamental mathematical structures, the use of (pseudo) concrete situations, the student's ability to construct mathematical demonstrations, and a basic familiarity with logical concepts" (Vanpaemel et al. 2012). In this respect, while Bourbaki generally eschewed emotions with respect to mathematics, it was acknowledged that intuition, based on the deep and immediate knowledge of the logic of the structures familiar to mathematicians, had a role to play (Aubin 1997).

Finally, in this section, many of the authors above criticized the Bourbakian school mathematics narrative, for, inter alia, confusing and alienating parents (Kilpatrick 2012; Klein 2003). Also, the Bourbakian denouncement of Euclidean geometry prompted the introduction of transformation geometry, which turned out to be "a rather empty set" (De Lange 2002, p. 147), while those who advocated Bourbakian school mathematics were naive to believe not only that "rigorous proof was reflective of mathematical practice" but also that "formal derivations are, in themselves, an aid to understanding and thus constitute a useful didactic device" (Hanna 1990, p. 7). In sum, many saw the new mathematics as a failure, being "unteachable, unlearnable, unmathematical" (Freudenthal 1979, p. 321).

Realistic mathematics education

In many ways, the realistic mathematics education (RME) movement was a reaction to the Bourbakian mathematics experienced by Dutch students, whereby learning, driven by a well-defined deductive curriculum, entailed the acquisition of isolated and decontextualized knowledge and skills (Elbers and De Haan 2005; Wubbels et al. 1997). Freudenthal argued that "what humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics" (Freudenthal 1968, p. 7). Implicit in the above are several key elements. Firstly, learning mathematics necessitates doing mathematics; it is not passive but "something that must be done" (De Lange 2002, p. 145). Secondly, through the processes of mathematizing, students come to reinvent mathematics (Gravemeijer and Doorman 1999; Gravemeijer 2004; Rasmussen and King 2000). Thirdly, the subject matter of mathematics should draw on solutions to problems derived from reality (Gravemeijer 1994; Van den Heuvel-Panhuizen 2005).

Based on the use of problems located in some sense of reality, and I return to this issue below, RME distinguishes between horizontal and vertical mathematizing (Van den Heuvel-Panhuizen 2003). Horizontal mathematizing refers to learners' use of informal strategies and other mathematical tools in their solution of problems (Barnes 2005; Van den Heuvel-Panhuizen 2003). It represents a journey from the world of life to the world of symbols (Freudenthal 1991). Vertical mathematizing reflects the sorts of activities students undertake when working within the mathematical system itself (Barnes 2005; Van den Heuvel-Panhuizen 2003). It represents a journey a learner may take within the mathematical world of symbols (Freudenthal 1991) and plays a major role in abstraction, as mathematical entities are structured, organized or developed into new entities, typically more abstract than the original (Hershkowitz et al. 2001). Through this process of progressive mathematization, whereby students pass from informal solutions, through some level of schematization, to an awareness of the general principles (Van den Heuvel-Panhuizen 2003), students reinvent mathematics (Gravemeijer 1998, 2004). Importantly, instruction does not start from the results of others' mathematical activity (Gravemeijer 2004; Van Dijk et al. 2003) but allows students to construct mathematical concepts through opportunities to develop and reflect on their own solution strategies (De Lange 2002; Elbers and De Haan 2005).

In its original conception, real-world problems served as both the basis for and the application of subject knowledge (Elbers and De Haan 2005; Gravemeijer 2004; Streefland 1993). However, this contextualization of mathematics created definitional inconsistency. For example, an activity focused on the intersections of the medians of a triangle would not typically be considered realistic, even though some children find such tasks personally interesting (Wubbels et al. 1997). Consequently, realistic has now come to be construed as pertaining to that which is "experientially real to the student" (Gravemeijer and Doorman 1999, p. 111), allowing for a problem's context to be the real world, the "fantasy world of fairy tales and even the formal world of mathematics" (Van den Heuvel-Panhuizen 2005, p. 2). Problems in RME act as "anchoring points" (Gravemeijer and Doorman 1999, p. 111) or "stepping stones" (Gravemeijer 2004, p. 107) in students' reinvention of mathematics. Carefully sequenced, these problems not only account for "students' current mathematical understanding" but can be justified "in terms of the potential mathematical end points of the learning sequence" (Dolk et al. 2002, pp. 164–165). However, RME orients teachers away from tasks that concretize a given concept towards "applied problems that can be suitable as points of impact for a process of progressive mathematization" (Gravemeijer 2004, p. 116). In other words, RME seems to eschew manipulatives as a means of reifying concepts.

When working on problems students are expected not only to produce their own solutions but to share those solutions with others (Keijzer and Terwel 2003; Kroesbergen and Van Luit 2002; Wubbels et al. 1997). This public sharing, focused on differences in the ways students obtain their solutions, highlights a planned lack of stress on standard algorithms (Van Putten et al. 2005). Throughout, the role of the teacher is to facilitate children's reinvention of mathematics through their management of both private and public discourse (Wubbels et al. 1997). That is, guided reinvention offers a way out of the generally perceived dilemma of how to bridge the gap between informal knowledge and formal mathematics (Gravemeijer and Doorman 1999, p. 111).

Finally, working within an RME framework does not come naturally to teachers and may create professional uncertainty in those who are “not supposed to lack confidence” (De Lange 2002, p. 150). More generally, the approaches espoused by the advocates of RME have prompted questions about the teaching of routine skills and, particularly, the role of functions and algebraic manipulation in the preparation of students for further study of mathematics. The removal of such material may have demystified mathematics but seems to have made the transition from school to university mathematics problematic (De Lange 2002, p. 149). Indeed, mathematical structures, which Streefland (1993) argues have been brought to the surface by RME, are now “hidden under much text and context or is formed by a story that is not mathematical” (De Lange 2002, p. 149).

The present study

Bringing all the above together, this paper, through analyses of four sequences of videotaped lessons taught by teachers construed locally as effective, Flemish mathematics teaching is opened up to scrutiny. In so doing three broad aims are addressed. These are

- to determine whether Flemish mathematics teaching is, as the Flemings have informally suggested it is, a juxtaposition of RME and Bourbaki;
- to consider whether Flemish students’ PISA and TIMSS achievements could be attributable to such teaching;
- to ascertain whether Flanders, as a system that has proved successful on two distinct forms of international test of mathematical achievement, has the potential to be a site of research interest for those concerned with exploring how one system’s practices may appropriately inform another’s.

Methods

The data analyzed here derive from a European Union-funded project focused on how teachers conceptualize and present mathematics to students in the age range 10–14 in England, Finland, Flanders, Hungary and Spain. In each country, data were video recordings of five successive lessons on each of four topics acknowledged as representative of all participating countries’ curricula. Four teachers from each country were involved. Each was selected against local criteria of effectiveness. These decisions, concerning lesson sequences and teacher effectiveness, were intended to minimize the likelihood of teachers presenting show-piece lessons, enable us to discern how teachers manage students’ learning from one lesson to the next and, importantly, glean insights into how a particular system construes teacher competence. The decision not to impose a common set of teacher effectiveness criteria emerged during the first project meeting when it transpired that vocabulary and concepts assumed to be internationally understood had widely varying and locally contextualized meanings (Andrews 2007). In other words, we began to realize that if mathematics teaching is a cultural construction then teacher competence would be similarly constructed. As it turned out, a criterion found to be common across participating countries concerned teachers' being involved, through project universities, in the professional development of both in-service and pre-service teachers.

With respect to the four teachers discussed here, one, Emke,[6] had not only been involved in teacher education activities at the project university—as were her project colleagues—but had also been filmed as part of a video-based teacher development project. Three of the teachers, Emke, Marc and Pauline, were in their early thirties and had experienced RME-focused teacher education. The fourth, Karin, was in her mid-fifties and had started her career during the Bourbakian era. However, she was not unaware of RME-related developments; she had attended in-service courses, was aware of changes to textbooks and worked as a school-based teacher educator.

In all cases, guided by the objective of capturing all utterances made by the teacher and as much board-work as possible, a tripod-mounted camera was placed near the rear of the room. Teachers wore radio microphones, while strategically placed telescopic microphones captured as much student-talk as possible. After filming, tapes were digitally compressed for coding, copying and distribution. The first two lessons of each sequence were transcribed and translated by English-speaking colleagues to create subtitled recordings. This allowed colleagues from any project country to analyze a substantial proportion of another country's output. With respect to this paper, three phases of analysis were undertaken. Firstly, each subtitled video was viewed several times to get a feel for how the episodes of the lesson played out; secondly, each subtitled video was scrutinized for practices commensurate with either Bourbaki or RME; thirdly, non-subtitled videos were scrutinized for evidence supportive of the inference gleaned earlier. However, in so doing it is acknowledged that some matters remain imponderable. For example, while the role of the teacher is explicit in RME, particularly from the perspective of managing public discourse, no such issues were identified in the Bourbaki-related literature. However, it would be foolish to assume that whole class activities focused on the sharing of student ideas would be unique to RME, particularly in the light of a Bourbakian focus on the use of manipulatives as a way of reifying concepts. Therefore, in the analysis presented below, only those incidents that could unequivocally be construed as either RME- or Bourbaki-focused were included. The following draws on episodes from both titled and not-titled videos.

Results

The results are presented in four sections. The first offers a simple frequency analysis of the episodes of teachers' lessons. This is followed by qualitative analyses of episodes emphasizing RME, Bourbaki or both respectively. Each of the latter three sections is similarly structured; examples drawn from each teacher's episodes

[6] The names of all teachers in this paper are pseudonyms.

are presented and discussed against the relevant literature. Therefore, Karin, due to her episodes comprising no RME-related emphases, appears only in the section pertaining to Bourbaki.

A quantitative analysis of lesson episodes

One organizing feature of the larger study's quantitative analyses was the episode as the unit of analysis, which was here defined as that part of a lesson during which a teacher's observable didactical intentions remained constant (Andrews 2007). For the purposes of this paper each episode, previously identified by Flemish project colleagues, was examined for discernible RME or Bourbakian emphases. This process highlighted four important points. The first was that teachers privileged the two sets of mathematical principles in very different ways and proportions. The second was that overall twice as many episodes comprised a unique focus on Bourbaki as on RME. The third was that episodes could accommodate both traditions. The fourth was that those episodes in which neither Bourbaki nor RME were observed typically involved seatwork in which neither was evident. For example, an exercise asking students to solve the equation $3x - 2 = x + 4$ is, of itself, reflective of neither tradition. It is just an equation; any relation to either RME or Bourbaki would have emerged only in the ways teachers chose to exploit it, typically as part of a whole class period of working.

RME

Emke's teaching of percentages

This first episode derives from Emke's first lesson on percentages. Students had been asked to bring a percentages-related artefact from home. The lesson began as follows:

Emke	Who can give me an example? A large one, so that everybody is able to see it. Yes, Katia?
Katia	A yoghurt pot
Emke	A yoghurt pot, yes. Please show it…. And?
Katia	With 9 % fruit
Emke	9 % fruit. What would that mean? Katia has a yoghurt pot containing 9 % fruit. Sofie?
Sofie	There are 9 % of 100 fruit in it
Emke	9 % of 100 fruit in it. I do not understand this very well. Who can explain it? 9 % fruit are in it. Does that mean there are nine pieces of fruit in it?
Class	No
Emke	Maybe 9 g?
Class	No
Emke	Would it make a difference if it is a large or a small pot? Or would it remain 9 %?
Joost	Yes, I think it would stay the same

Emke That those 9 % will always remain, yes. The amount will change. Afterwards, we will learn what 9 % actually means

Commentary: Percentages are mathematical tools for managing aspects of the real world. Moreover, as a curriculum topic it is arbitrary in that *per centum* (the original Latin expression) could just as easily have been based on tenths and known as *per decem*. Percentages, therefore, only have meaning when presented in relation to some aspect of the real world from which they are drawn, a function acknowledged in Emke's introduction that highlights it as *realistic*. Moreover, her use of artefacts drawn from home linked the mathematics that was to follow with the experiences of students' everyday lives (Streefland 1993; Gravemeijer 1994). Importantly, artefacts used in this way do not offer conceptual reifications but highlight their real world connection, automatically excluding them from a Bourbakian interpretation.

Throughout the episode, Emke's questions—What would that mean? Who can explain it? Would it make a difference if it is a large or a small pot?—facilitated not only her exploration of her students' current understanding of the topic but also their engagement with its conceptual underpinning. Such behaviors were resonant with RME expectations (Dolk et al. 2002), as were her extended public discussions (Keijzer and Terwel 2003; Wubbels et al. 1997). However, as indicated above, the latter activity is not, of itself, excluded from a Bourbakian teaching repertoire.

Marc's teaching of polygons

At the start of his third lesson on polygons, Marc wheeled his bicycle to the front of the room and asked his students how they could determine how far it would travel if the wheel turned one full circle? This yielded various suggestions including running a long piece of string along the circumference of the tire before measuring it against a tape and marking the tire before doing a complete turn and measuring how far he had gone, also with a long measuring tape. On completion of this task Marc invited his class to undertake a short investigation into the relationship between the circumference and diameter, which was managed by means of children using both rule and string to measuring the diameter and circumference of different circles they had been asked to prepare at home and bring to the lesson. On completion of this task, which involved only three circles, results were fed collectively into a table on the board and a discussion ensued over the results and what inferences could be drawn from them.

Over the coming lessons, bicycles and their wheels made several appearances. For example, in his final lesson, he asked students to form four groups before posing the following problem, which was presented to each group on a piece of paper.

> About 125 years ago, bicycles had a small back wheel with a diameter of 50 cm and a large front wheel with a diameter of 150 cm. How many metres does back wheel cover with each rotation of the front wheel?

After 5 min Marc invited the class to reconvene and the solution was discussed publicly with considerable student input. Throughout these tasks, Marc invited students to work in pairs or small groups and then share publicly the outcomes of their work.

Commentary: Marc's use of the bicycle not only motivated the initial problem but structured the activities that followed it. In so doing, he clearly located his work within the RME tradition (Kroesbergen and Van Luit 2002; Streefland 1993). The nature of the problems posed, whether the initial bicycle task or the subsequent, appeared to attend to students' current mathematical understanding (Dolk et al. 2002) as part of a programme designed to facilitate student' reinvention of mathematics (Gravemeijer 1998). Indeed, both the investigation and the final problem allude not only to vertical mathematization but not insignificant results concerning the relationship between a circle's diameter and circumference and the effect of a scalar multiplication of one unit linked linearly to another. Further, his asking his class to work in groups and then share the results of their labors resonated closely with RME expectations (Keijzer and Terwel 2003; Kroesbergen and Van Luit 2002; Wubbels et al. 1997), although his focus was on unique and well-defined objectives and not any lack of stress on standard algorithms (Van Putten et al. 2005).

Pauline's teaching of equations

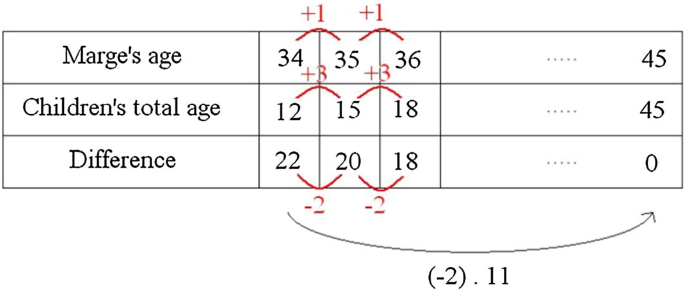
The final example in this section derives from Pauline's introduction to her sequence of lessons on linear equations. As they arrived for the lesson, she greeted her students with a smile and informed them that they were shortly to work on a problem concerning characters from *The Simpsons*, an act which brought smiles to the faces of many. She informed the class that the children's mother, Marge, was aged 34, while the children, Bart, Lisa and Maggie, were 7, 5 and 0[7] years respectively. She wrote these details in a table she had prepared as her students had entered the room and announced:

> Bart is 7 years old and Lisa is 5 years old when their little sister Maggie is born. Their mother, 34 years old, wonders if there will be ever a year in which she will have exactly the same age as her three children together.

Students were then invited to complete the table in their exercise books and, after several minutes, a public discussion revealed an answer of 11 years. During this discussion Pauline focused attention on the ways in which the children's combined ages increased by three annually while their mother's increased only by one. Finally, she asked what figures would be in the final column and, having received answers of 45, 45 and 0, asked why. A volunteer told her that the zero was a consequence of subtracting 2 eleven times from 22. By now Pauline's annotated table looked as shown in Fig. 1.

Finally, Pauline discussed the use of an unknown to represent the number of years that would pass before the two sums would be equal. This extensive bout of questioning led to her writing that if x represented the number of years necessary for the two sums to be equal then Marge would reach $34 + x$ years, while the children would reach $7 + x$, $5 + x$ and $0 + x$ respectively. Finally, $7 + x + 5 + x + x = 34 + x$ was written on

[7] In fact, the children's ages are said to be 10, 8 and 1, but these do not allow for an integer solution.



Marge's age	34	35	36	 45
Children's total age	12	15	18	 45
Difference	22	20	18	 0

Fig. 1 Pauline's Simpsons problem solution table

the board, at which point Pauline announced that the topic under scrutiny was equations, and, with input from students, simplified the equation to $12 + 3x = 34 + x$.

Commentary: This was a realistic problem in that all participants knew it was not about the real world but imaginably real and, judging from their initial response, motivational (Van den Heuvel-Panhuizen 2005). The initial task, to complete the table and determine the number of passing years, was done individually and collectively, in accordance with RME traditions (Keijzer and Terwel 2003; Kroesbergen and Van Luit 2002; Wubbels et al. 1997), as was her introduction of the unknown and extended discussion leading to the creation of the equation. In both cases, as with Marc's episode, her objectives appeared unrelated to stress on standard algorithms (Van Putten et al. 2005). Also, as with both Marc and Emke, Pauline's tasks seemed not only to account for students' current mathematical understanding but could be warranted "in terms of the potential mathematical end points of the learning sequence" (Dolk et al. 2002, pp. 164–165), as part of a systematic reinvention of mathematics (Gravemeijer 1998). However, her demonstration of the intersecting straight line graphs seemed not to fit with the typical expectations of RME, not least because her students were passive throughout (Freudenthal 1968). Moreover, having derived the equation at the end of the episode, it seemed odd that she elected not to ask students to try to solve it and then discuss, in a typical RME manner.

Bourbaki

Karin's teaching of polygons

While Karin's episodes showed no RME-related evidence, they frequently evidenced a Bourbakian emphasis. Early in her first lesson four students were invited to the board to draw accurately one of a right-angled trapezium, a rectangle, a parallelogram or a rhombus. Each student used a set square to help them, reflecting an emphasis on precision that permeated all the board-work observed in Karin's lessons. In particular, Adam, the student assigned to draw the rhombus, chose to do so by first constructing diagonals. Once the drawings had been completed, Karin engaged in a lengthy exposition focused, initially, on the quadrilaterals' common properties before showing how each derived from the general in increasingly particular ways. In so doing, she highlighted the fact that a focus only on either angles or edges would not necessarily define either their particular characteristics or the hierarchy within whey they are positioned. Following this, apparently drawing on Adam's approach to the rhombus, Karin drew several pairs of diagonals before showing not only how different configurations yielded in unique ways some of the quadrilaterals just discussed but also the insufficiency of diagonals for defining all quadrilaterals. Throughout her work, Karin asked few questions but exploited extensively both the formal and informal vocabulary of geometry, as in, for example, her synonymous use of both parallel and evenwijdig. In addition, her students meticulously copied everything that was on the board, whether bidden or not.

Commentary: During this episode, as she did throughout her lesson sequence, Karin's teaching was located within the mathematical world of the Bourbakists (Aubin 1997; Weintraub and Mirowski 1994), with no concessions to the application of what she taught (Klein 2003; Maasz and Schlöglmann 2006). It is known that the hierarchical nature of quadrilaterals is problematic for learners (Fujita and Jones 2007) and throughout this episode Karin focused attention not only on the conceptual basis of this hierarchy but also the logic underpinning it. In so doing she emphasised the important understanding that no one approach to defining quadrilaterals, whether by angle, diagonal or edge, is sufficient. This explicit emphasis on the structural and argumentative relationships resonates with a Bourbakian view on mathematical learning (Klein 2003; Vanpaemel et al. 2012). Moreover, Karin's use of informal vocabulary not only facilitated her students' transition from intuitive to abstract understanding of mathematical formalisms (Vanpaemel et al. 2012) but went considerably beyond the naming of quadrilaterals often found in English-speaking countries (Haggarty and Pepin 2002; Vincent and Stacey 2008).

Emke's teaching of percentages

During the closing minutes of her second percentages lesson, during which students had worked through many tasks focused on establishing the conceptual basis for percentages as multiplicative structures, Emke sketched the picture shown in Fig. 2, where, for the benefit of the reader, a square refers to one hundred.

The conversation went as follows:

Emke What exercise have I written on the board? Elise?
Elise Two for every hundred
Emke Of?
Elise Four hundred (Emke writes the formulation (2 for every 100) of 400, on the board)

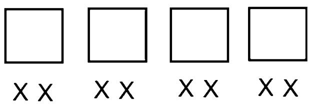


Fig. 2 Emke's sketched percentage-related problem

Emke	Do you know the solution, Jan?
Jan	Eight
Emke	Eight, that's right (She adds the solution to the formulation). Is there still another way to write it?
Anna	Two at every hundred (She writes the formulation on the board)
Emke	Two at every hundred of four hundred is
Anna	Eight (Emke adds the solution to the formulation)
Emke	Is there still another way, Tanja?
Tanja	2 % of four hundred
Emke	2 % of four hundred is… (She writes, *2 % of four hundred is*, on the board)?
Tanja	Eight (Emke completes the written formulation)
Emke	Is there a shorter way to write it? Is there anyone who can write it in a shorter form? Daniel, can you…?
Daniel	(Daniel comes to the board and writes) 2 % van 400 = 8
Emke	First we replaced *for every hundred* with the word percent and now we have replaced that by the percent sign. So 2 % of four hundred is eight

Commentary: In this final episode of her second lesson Emke formalized the proportional reasoning embedded in her earlier percentage tasks and introduced a formal vocabulary and symbolism, actions commensurate with Bourbakian structural ambitions (Maasz and Schlöglmann 2006; Klein 2003). In so doing, while acknowledging her having exploited a series of structured tasks to reach this point, an ambition that accords with RME principles (Dolk et al. 2002), her attention seemed focused on students' achievement of a single, well-defined, goal concerning their understanding of percentages as a multiplicative structure, a goal resonant with a Bourbakian view on mathematical learning (Klein 2003; Vanpaemel et al. 2012). Also, the sequence of activities she undertook, all of which exploited number base materials, seemed designed to "help children…make the transition from intuitive experiences towards abstract understanding" (Vanpaemel et al. 2012, p. 425). Significantly, despite the explicit real-worldness of the topic, the task, and those that preceded it, was located in a mathematical world of multiplicative structures, very much in keeping with the Bourbakian belief about the primacy of the pure over the applied (Aubin 1997; Weintraub and Mirowski 1994), and the subordination of the applications of mathematics (Klein 2003; Maasz and Schlöglmann 2006).

Marc's teaching of polygons

Shortly after the start of his second lesson on polygons, Marc distributed a worksheet on which were several definitions intended to help students to classify triangles, quadrilaterals and higher order polygons. He invited students to select individually a quadrangular object from a list that the class had collectively constructed—he highlighted the example of a desk—and applied his scheme to their choice. The conversation went as follows:

Marc (Pointing to the list) Choose one of those and think about it. For example, think of a desk. Think of a desk. I look to see if my desk has parallel sides. Is it a quadrilateral with at least one pair of parallel sides? If it has at least one pair, then it is a trapezium. Does a desk have one pair of parallel sides, Lieve?

Lieve No, it has two pairs of parallel sides

Marc Yes (He demonstrates), two this way and two that way, I have a… Fien?

Fien Parallelogram

Marc Parallelogram. Yes? Now, (referring to the scheme on the paper) I look at the… Ines?

Ines Sides

Marc Sides. Very good. Now we proceed to the next step. And now, we split up into two figures, which makes it more difficult. I have a parallelogram; on the one hand I can measure the sides; on the other hand I can look at the angles. So, I look at the sides. Please follow the scheme. Is it a parallelogram from which the four sides, we just measured, are equal? 'Yes'? Then, I have a rhombus… After I measured the sides, I measure the angles of that rhombus. And so I have to ask: belonging to these four sides, which are equal, are there four right angles? If this is the case, I have more than a rhombus, I have a square. Right? So, we start with a parallelogram, I measure the sides and after that I measure the angles… Now, we can have a look at the other side. I'll do just the inverse. I start with the angles. Please, have a look. Parallelogram, I measure the angles (Marc waits)

Marc Does the parallelogram have four right angles? How many degrees does a right angle have? How many degrees? Jonas?

Jonas 90 degrees

Marc 90 degrees. Yes, so I have a…?

Isaac Rectangle

Marc Rectangle. It is all in the name. A figure with right angles. But it could also be a

Dirk Square

Marc Yes. Is it a square? Look at the next step. If I measure the sides; all four angles are right and if all sides are equal too, we have a…?

Many Square

Marc Square. Yes? So, I hope the scheme can help you to overcome problems. One pair of parallel sides, trapezium; two pairs of parallel sides, parallelogram and after that I split up. I either look at the sides for a rhombus and then look at the angles, for a square. Or, I look first at the angles, for a rectangle, and then that at the sides, for a square

Commentary: In this episode, as with an earlier one focused on the classification of triangles, Marc presented criteria for the classification of quadrilaterals. Using a desk as an example, he demonstrated how the scheme, which began with the broad criterion of a single pair of parallel sides, became more narrowly focused and ended with the most tightly defined quadrilateral, the square. In doing so, he undertook, within a discursive milieu, a formal and structurally integrated presentation of mathematical terms and theory in accordance with Bourbakian traditions (Klein 2003). In particular, his use of the desk, which drew on an earlier discussion of quadrangular shaped objects, resonated with the Bourbakian use of concrete materials to make the transition from intuitive to abstract understandings (Vanpaemel et al. 2012, p. 425), although it could also be argued that it reflected an RME perspective in its being *experientially real* (Gravemeijer and Doorman 1999). Interestingly, although this was not the explicit intention of the lesson, his work was located in a Bourbaki-like set theory, with each level of quadrilateral clearly located in the one above (Munson 2010). Finally, although Euclidean geometry had been eliminated from school mathematics, Marc's classification of quadrilaterals, which had a Euclidean underpinning, was presented in a way that emphasized the Bourbakian expectation of a "careful construction of mathematical concepts" (Maasz and Schlöglmann 2006, p. 1).

Pauline's teaching of equations

Midway through her first lesson on equations, Pauline wrote $a = b$ on the board and initiated a discussion, linked to the notion of a balance scale, concerning what operations could be applied to the equation while maintaining equality. Students volunteered the four rules of arithmetic, a summary of which can be seen in Table 2.

The following lesson, during her formal exposition on the solution of $6(x - 5) - 8 = x - 3$, students were asked to justify the actions they proposed. In so doing, students volunteered terms like associativity, commutativity and distributivity as their means of warranting their proposed actions.

Commentary: In these two short cameos can be seen evidence of an expectation that students are not only exposed to the structures, notation and vocabulary of formal mathematics but that they use them as part of their everyday classroom discourse, a process very close to the original Bourbakian principles (Guedj 1985; Landry 2007). During the former, Pauline formalized some fundamental arithmetical principles that students had previously derived intuitively, in accordance with earlier Belgian perspectives on Bourbakian mathematics (Vanpaemel et al. 2012). During the latter, previously learnt vocabulary, pertaining to the structural properties of arithmetical operations, was invoked to support students in their articulation of warrants for the operations exploited in solving a non-trivial equation, highlighting not only mathematics as self-contained but pure from any applications (Guedj 1985; Weintraub and Mirowski 1994).

The juxtaposition of Bourbakian and RME

In the following, a single episode from one of Emke's lessons is presented to illustrate the ways in which the two traditions appeared to interact.

Emke's teaching of percentages

Following her introductory activity with the percentage-related artefacts, Emke invited students to use base 10 number blocks to model the conceptual underpinning of percentages. The first task (of several over two lessons) went as follows:

Emke Everybody should place four hundred squares in front of him. Not stacked, but next to one another. There should be some distance between them, so that you can see very clearly that you have four times one hundred, right?… Now put five in front of each hundred (she waits while students follow her instructions). So, what did you do? What have you done? Yes, Elke?

Elke I've put five unit cubes in front of each hundred square

Emke And which value has a hundred cube?

Elke One hundred

Emke One hundred. So what have you done?

Joost I've put five unit cubes in front of each hundred

Emke Yes. You can also say, I've put five per 100, or five at 100. Or you can also say, I've put five on 100. These are all different ways of saying the same thing. How should we write it down? You have put five units for every hundred, of how many? How many do I have in total?

Adam Four hundred (Emke writes, including the brackets (five for every hundred) of four hundred

Commentary: This was the first of several episodes during which Emke used the apparatus to highlight percentages as multiplicative rather than additive. During this first example her objective seemed to be to ensure that students were able to follow the procedure correctly and begin the process of recording what she later came to call a formulation. Indeed, much of the remainder of the first lesson focused almost solely on the formulation, which was always presented, for example, as (five for every hundred) of four hundred. In so doing she exploited continuously number base blocks "to help children…make the transition from intuitive experiences towards abstract understanding" (Vanpaemel et al. 2012, p. 425), a key element of school Bourbakian mathematics, which would not have found resonance with RME expectations (Gravemeijer 2004). However, the logical sequencing of Emke's activities, leading to the episode described above in relation to Bourbaki, seemed very much in accord with the RME tradition of choosing problems "to support a logical sequence of cognitive development" (Gravemeijer 2004: 107), which in this case were focused explicitly on Bourbakian multiplicative structures and proportional reasoning (Guedj 1985; Landry 2007). However, her frequent public discussion did not, in this or any other case, privilege, or even encourage, students' non-standard solution processes, as would be expected in an RME-focused classroom (Van Putten et al. 2005). Her aim, throughout this and other episodes, was the creation of a collective understanding of the same Bourbakian structural relationship (Aubin 1997), which was unrelated to any obvious application of mathematics (Klein 2003; Maasz and Schlöglmann 2006). In sum, this particular episode demonstrated various characteristics, some of which were clearly RME and some of which were clearly Bourbakian.

Discussion

Before discussing the findings in relation to the aims presented above, the reader is reminded that all four teachers were construed locally as effective and may not be typical. In accordance with earlier studies of Flemish mathematics teaching, all maintained tight control over their lessons through long periods of whole class orchestration (Dykstra 2006) and presented rare opportunities for independent problem solving (Verschaffel and De Corte 1997). However, their efforts, in differing ways, seemed focused on a deep conceptual understanding that took them beyond mere declarative knowledge and lower-order procedural skill (Janssen et al. 2002).

In relation to the first project aim, the extent to which Flemish mathematics teaching is a juxtaposition of RME and Bourbakian structuralism, the evidence indicates an interesting interaction. For example, Emke's introductory activity was realistic not only in her use of artefacts to motivate both students and mathematics but also in her invocation of a public discussion during which many perspectives and ideas were shared. Such behavior reflected a key element of the realistic tradition (Keijzer and Terwel 2003; Wubbels et al. 1997). However, her remaining tasks, with their single and essentially non-negotiable goals and exploitation of number base blocks, were focused explicitly on an understanding of percentage as independent of any real world and were unequivocally Bourbakian (Guedj 1985; Landry 2007). Moreover, while her sequence of tasks could be construed as resonating with the RME tradition of choosing problems "to support a logical sequence of cognitive development" (Gravemeijer 2004, p. 107), her aim was the creation of a collective understanding of the same Bourbakian structural relationship (Aubin 1997; Klein 2003), unrelated to any obvious RME privileging of mathematical application (Elbers and De Haan 2005; Gravemeijer 2004; Streefland 1993). In short, although Emke appeared to work towards the RME goal of mathematical reinvention and provided the stepping stones so to do (Gravemeijer 2004), her broad aims seemed very much more aligned with Bourbakian perspectives on the nature or mathematics than with those of RME. That is, Emke's practice, at least as far as her teaching of percentages was concerned, reflected Bourbakian learning outcomes mediated by RME didactics.

Marc's practice was similarly complex. His bicycle-based problems were clearly "experientially real to the student" (Gravemeijer and Doorman 1999, p. 111). His quadrilateral classification tasks were clearly Bourbakian in their focus on "students' understanding of the structures under scrutiny" and reflected his desire to help them "make the transition from intuitive experiences towards abstract understanding" (Vanpaemel et al. 2012, p. 425). However, they were made real by means of his use of concrete materials like the desk in ways that did not concretize a given concept (Gravemeijer 2004). In other words, many of Marc's tasks were realistic, a conclusion supported by his constant use of individual- paired- and group-work; after which extensive public discussions supported his structured reinvention of mathematics. However, throughout his lessons his focus was on facilitating students' awareness of the structural and argumentative relationships of an axiomatic geometry, ambitions resonant with Bourbakian mathematics. Thus, Marc's practice, at least as far as his teaching of polygons was concerned, also reflected Bourbakian learning outcomes mediated by RME didactics.

Pauline's practice was different from that of both Emke and Marc. The realistic problem with which she introduced the topic was probably closer in spirit to the original Dutch conception of RME than all the tasks posed by either Emke or Marc. Moreover, the manner in which she initiated its solution, by inviting students to complete the Simpsons' age-sum table before discussing the opportunities for mathematical reasoning that emerged from it, was a genuine attempt to probe her students' understanding and share perspectives (Keijzer and Terwel 2003; Wubbels et al. 1997). However, on the conclusion of this initial episode, her teaching fell into a routine, focused solely on problems set in a world of mathematics. That being said, the problems she posed were cognitively challenging. For example, she based her formal exposition on equation solving around $6(x - 5) - 8 = x - 3$. Moreover, such problems were always collectively solved by means of many questions focused on ensuring her students had a deep conceptual understanding of the mathematics they studied. She always appeared to work from first principles, facilitating students' exploitation of what they had learnt previously. In sum, Pauline's practice, at least as far as her teaching of equations was concerned, seemed deep-set in a Bourbakian conception of mathematics, with only a rare concession to RME.

Karin's teaching seems the simplest to summarize. All her lessons were located in a world of mathematics and focused on elements of Bourbakian structuralism (Aubin 1997; Weintraub and Mirowski 1994). Her aim, it seemed, was to secure students' structural knowledge through precise exposition and a strong mathematical logic (Vanpaemel et al. 2012). In this respect, her use of informal language to support students' learning of these mathematical structures permeated all her lessons and could be seen as her supporting students' in their shift from intuitions to warranted mathematical abstractions (Vanpaemel et al. 2012). In sum, Karin offered no evidence of any RME-related emphasis, acting entirely in accordance with Bourbakian objectives.

So, is there sufficient evidence to support claims concerning a pedagogical marriage between Bourbaki and RME? The answer is a tentative yes. The figures of Table 3 show, in absolute terms, that of those episodes yielding identifiable perspectives, Bourbaki clearly dominated. That is, emphases on mathematical

Table 2 Pauline's summary of equations-related laws of arithmetic

$$a = b \quad \Rightarrow \quad a + c = b + c$$

$$a - c = b - c$$

$$a : c = b : c$$

$$a \,.\, c = b \,.\, c$$

structures unrelated to the real-world were more visible than realistic problems; all four teachers appeared to privilege a mathematical world focused on a deep conceptual understanding of mathematical structures and argumentation. However, such figures typically mask important underlying differences. For example, Karin's episodes yielded no evidence of any RME influence, whereas the three younger teachers appeared to exploit realistic tasks as the means of introducing students to a form of mathematical knowledge commensurate with the structural ambitions of Bourbaki, something unseen in Karin's lessons. That is, for Emke, Marc and Pauline, teachers who would have learnt school mathematics within a Bourbakian tradition but were professionally educated within an expectation of RME, had, in individual ways, developed a composite, a marriage, of the two traditions. As for Karin, it is possible that her childhood, teacher education and early professional experiences of mathematics were framed against a particular set of Bourbakian principles that continue to guide her professional decision making. In other words, it is not inconceivable that Karin, despite her involvement in school-based teacher education and in-service experiences focused on RME, remains unconvinced of the validity of RME approaches to a subject she regards as essentially Bourbakian. It is equally possible that she is one of the many teachers who lack confidence at a time when they are "not supposed to lack confidence" (De Lange 2002, p. 150). Thus, the evidence above indicates that among the younger generation of Flemish teachers may be discerned a model of teaching, or even a cultural script, whereby Bourbakian goals are mediated by an RME-driven reinvention of mathematics. However, a small number of case study teachers, each of whom may have been atypically competent, can offer no more than an indication of what may be a more general phenomenon (Table 2).

So, what of the second aim? Can the practices and curricular emphases described above explain Flemish success on international tests? Any answer is, in essence, speculative but it is not unreasonable to assume that Flemish students' TIMSS successes, successes achieved on a conventional assessment of curriculum mathematics, would be a consequence of teaching focused on a deep understanding of the mathematical structures that underpin the procedures they learn. In this respect, and despite any variation in the ways in they were seen to encourage this understanding, all four teachers focused extensively on "the relationships and interconnections of ideas that explain and give meaning to mathematical procedures" (Eisenhart et al. 1993, p. 9). In other words, while it may not be sufficient for high levels of mathematical achievement, project teachers' emphases would seem to be necessary.

Table 3 Episodic frequencies of RME- and Bourbaki-related tasks

	RME	Bourbaki	Both	Neither	Total
Pauline	3	12	0	12	27
Emke	13	7	5	12	37
Marc	7	8	3	8	26
Karin	0	18	0	7	25
Total	23	45	8	39	

As for PISA, the data above may be less persuasive. That Flemish students understand and can deploy the mathematics necessary for solving PISA-related problems is probably not in doubt. But what evidence is there that they have the reading competence necessary for reading and interpreting the text-heavy problems typically associated with PISA? In this respect, the figures of Table 4 show that Flemish students' reading competence is among the highest in Europe, typically behind only Finland. This competence is further evidenced by the results of the single PIRLS (2006) on which Flanders participated showing Flemish grade 4 students not only achieving one of the highest absolute scores (547 on a mean of 500) but doing so with one of the smallest distributions of the 40 participating nations. That is, PIRLS points towards a community that not only reads well, but a community in which all read well. Thus, evidence from different sources suggests that the combination of reading competence and a deep-seated conceptual understanding of mathematics is likely to explain Flemish students' PISA-related mathematics performance.

And what of the third aim, is Flanders warranted as a site of future research interest? Much of the evidence yielded by the four teachers presented here, all of whom were construed locally as effective, alludes to a mathematics teaching tradition worthy of further investigation. Admittedly, Karin did not show any evidence of an RME influence, but her three younger colleagues, while sharing Karin's Bourbakian goals, exploited realistic tasks as the means of introducing new material. This juxtaposition appears to confirm anecdotal claims that Flemish mathematics comprises Bourbakian goals humanized by RME and, as such, is clearly worthy of further investigation.

However, the discussion above in relation to reading indicates that a system's successes may not be entirely attributable to classroom practice. Indeed, evidence from Finland shows that repeated PISA successes may be due less to the quality of classroom interactions than deep-seated cultural expectations of personal literacy (Andrews 2013; Andrews et al. 2014). In other words, and drawing on the Finnish example, there may be explanations beyond the didactical for Flemish success, highlighting further the need to understand the role of culture in the construction of a system's educational policies, practices and performance. This need for further culture-focused research is brought into focus by particular characteristics of Flemish education policy that research shows typically limits educational achievement. For example the more a system stratifies or segregates its students, the greater the achievement segregation (Gorard and Smith 2004; Shapira 2012). Admittedly, Flemish stratification may be moderated by students being permitted to choose, initially at least, their secondary schools (Op't Eynde et al. 2006; Prokic-Breuer and Dronkers 2012), but, in relation to other highly stratified systems, the extent of

Table 4 Flemish students' PISA reading performance

	PISA 2000	PISA 2003	PISA 2006	PISA 2009	PISA 2012
Flemish reading	532	530	522	519	518
European rank	2	2	2	2	3

Flemish success would appear to be a rarity (Buchmann and Park 2009). The totality of these, and other, matters lead to a conclusion that without further research it would be difficult to understand the construction of Flemish educational achievement. That being said, a mathematics education tradition of Bourbaki moderated by RME poses some interesting research and policy challenges for educational analysts.

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