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Equivalent-circuit model for electrostatic micro-torsion mirror

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Abstract Equivalent circuit model for micro-torsion mirror using open-source circuit simulator is reported, which gives a convenient technique for the design of circuits with electrostatic actuators. In the simulation, linear equations for electrical system, mechanical system, and transducer between both systems are considered. Mathematical operations of these equations are interrupted as appropriate circuits, so that a seamless multi-physics mixed simulation becomes possible.

Keywords Equivalent circuit · Micro mirror · Actuator · MEMS · Transducer · Linear response

1 Introduction

There has been an increase in R&D activity on the integration of MEMS (micro-electro-mechanical systems) and electric circuits and on the integration through combinations of different types of MEMS. As more and more successful products emerge through these works, the MEMS equivalent circuit model will play an important role as a new design tool capable of compensating for the shortcomings in the conventional design methods that focus primarily on MEMS mechanical components.

So it will be recognized that creating a better environment for MEMS research through these works on the capability of replacing MEMS with equivalent electric circuits will be essential to the growth of fine MEMS.

Researches on MEMS simulations have been remarkably progressed since Senturia et al. proposed [1, 2]. While we may easily integrate sensors in electrical circuits because they can be regarded as electronic elements [3, 4]. There are problems on the integration of actuators and circuits. Then, seamless interface between micromechanics and electronics is needed to perform multi-physics simulation in a straightforward manner on a single simulation platform [5, 6].

The objectives of the research on the MEMS equivalent circuit are follows. Firstly, it will be useful when integrating MEMS with one another, and will enhance device model so that we can construct a fine MEMS equivalent circuit model that can connect MEMS devices to one another and support high integration. Then, it will give seamless interface between micromechanics and electronics which is needed to perform multi-physics simulation in a straightforward manner on a single simulation platform. Finally, it will develop mutual transformation of equivalent circuit models and three-dimensional CAD models to enable mutual transformation of the shape data and material property values of equivalent circuit models and three-dimensional CAD models.

In the present paper, the equivalent circuit model for micro-torsion mirror using open-source circuit simulator is reported, which demonstrates mixed simulation between micro actuators and electrical circuit [7]. In the simulation, electrostatic force, equation of motion and Kirchhoff's law are considered. Linear response in the equation of motion is interrupted as an electrical circuit which consists of capacitors, resistors, and coils, so that a seamless multi-physics mixed simulation becomes possible.

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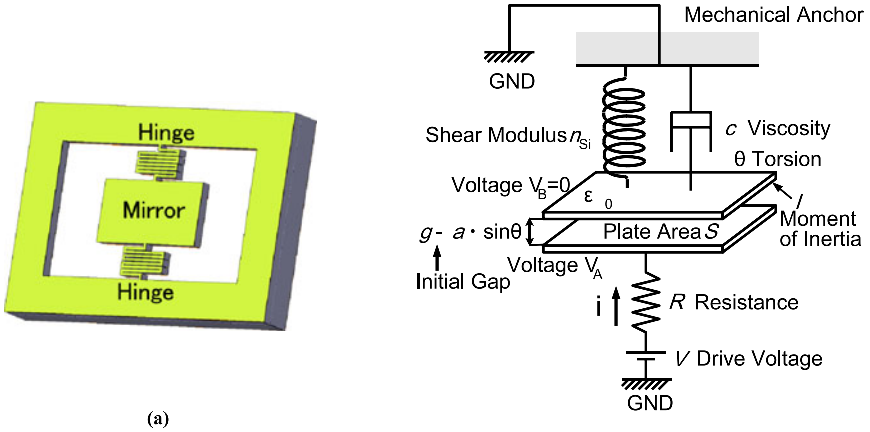


(a) **(b)**

Fig. 1 Torsion-mirror electrostatic micro actuator: (**a**) typical structure, (**b**) analytical model

2 Equivalent circuit model

2.1 Mechanics and electronics systems

It is well known that there is the analogy between mechanical and electric circuit systems. They are expressed as the linear response system, which become following differential equations of second order. Generally, the equation of mechanical system is following, that is, the inertia force, viscosity force and restoring force are balanced with the external force;

$$m\frac{d^2x}{dt^2}+c\frac{dx}{dt}+kx=F \tag{1}$$

As for the electrical system, the potentials of the inductance, the resistance, and the capacitance are balanced with the power supply voltage;

$$L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{1}{C}q=V \tag{2}$$

The linear response theory gives us the response of the system, so that we can analyze its frequency response.

We can basically make the equivalent circuit models of actuators by using not only the linear response theory but also the energy conversion theory which gives energy transducers between two systems. By making this procedure, the mechanical system can be replaced by an electrical system in order to integrate actuators in the electric circuits. Furthermore, it is possible to analyze non-liner transient and electrostatic pull-in effect by introducing feedback loop and conditional branch, respectively.

2.2 Analytical model of micro-torsion mirror

The micro-torsion mirror device is widely used in projectors and optical communication devices and so on. Typical structure and the analytical model for a electrostatic micro-torsion mirror are illustrated as Fig. 1(a) and (b), respectively. In the analytical model, a torsion-plate (mirror) is suspended with a spring (shear modulus n_s). The plate is electrically biased through an electrical resistance in order to tilt and cause change of the reflection angle of light. The plate moves with a viscosity c.

In general, the equations for the electro mechanical coupled system can be deduced from the analytical mechanics theory, that is, the Lagrange's equation by the mechanics and the electrodynamics including the dissipation. As a result, three types of equations are obtained as follows [7],

1. Electrostatics equations;

$$\begin{cases} N=\frac{1}{2}\varepsilon_0 bV_A^2(\frac{\cos\theta}{\sin^2\theta})\{\frac{r\sin\theta}{1-r\sin\theta}+\ln(1-r\sin\theta)\} \\ C=-\frac{\varepsilon_0 b}{\sin\theta}\cdot\ln(1-r\sin\theta) \end{cases} \tag{3}$$

where $r=a/g$, N is the moment by electrostatic force, θ is the torsion of the plate, a and b are the sides of the square plate, C is the capacitance, V is the drive voltage of the plate, and ε_0 is the dielectric constant of the gap.

2. Equation of motion;

$$I\cdot\ddot{\theta}+2\cdot c\cdot I_p\cdot\dot{\theta}+n_{\mathrm{Si}}\cdot\theta=N \tag{4}$$

where, I is the moment of inertia of plate, c is the viscosity of air, and n_{Si} is the shear module of silicon. I_p is the polar moment of inertia of the spring defined as;

$$I_p=\frac{w\cdot h^3\cdot n_{\mathrm{Si}}}{3\cdot l} \tag{5}$$

3. Kirchhoff's laws;

$$\begin{cases} V=R\cdot i+V_A \\ q=\frac{d}{dt}\{C\cdot(V_A-V_B)\} \end{cases} \tag{6}$$

where, i is the current of the circuit, q is the charge of the capacitor. The typical parameters are listed in Table 1.

Table 1 List of parameters

Parameter	Symbol	Value used in Simulation
Dielectric Constant	ε_0	8.85×10^{-12} F/m
Shear Modulus (Si)	n_{Si}	73 GPa
Density (Si)	ρ	2.33×10^3 kg/m^3
Plate Torsion (Angle)	θ	
Electrostatic Force (Moment)	N	See (1)
Induced Electrical Charge	q	See (1)
Plate Area	$S = 2a \cdot b$	$(100\ \mu\mathrm{m})^2$ $a = 50\ \mu\mathrm{m}$, $b = 100\ \mu\mathrm{m}$
Suspension Width	w	10 μm
Suspension/Plate Thickness	h	2 μm
Moment of Rotation	$I = M(1/3a^2 + d^2)$	
Polar Moment of Inertia	$I_p = wh^3 n_{\mathrm{Si}}/3\,1$	
Plate Mass	$M = \rho S h$	
Viscosity	c	1×10^{-7} Ns/m
Electrostatic Initial Gap	g	2 μm
Stopper Angle	$\theta_{\mathrm{LIM}} = \sin^{-1}(\mathrm{g/a})$	
Capacitance	C	See (5)
Drive Voltage to Fixed Plate	V_A	MAX 20 V
Drive Voltage to Movable Plate	V_B	GND 0 V
Input Resistance	R	50 Ω or 2 M Ω

In the above equations, the equation of motion [(4) and (5)] describe the mechanical system, and the Kirchhoff's laws [(6)] describes the electrical system. While the electrostatic equation [(3)] connects both systems.

2.3 Linear response of the system

According to the linear response theory, the torsion plate expressed by (4), has an amplitude (A) and phase (α) of the small vibration, a specific frequency (ω_0) and Q factor (Q) of the resonance mode. They are expressed as,

$$\begin{cases} A = \frac{N_0}{I} \frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}} \\ \omega_0 = \sqrt{n_{\mathrm{Si}}/I} \\ Q = \frac{I}{4cI_p} \frac{\omega^2 + \omega_0^2}{\omega} \\ \tan\alpha = \frac{2\gamma\omega}{\omega_0^2 - \omega^2} \end{cases} \tag{7}$$

where, $\gamma = c \cdot I_p/I$. The amplitude and phase shifts of the vibration of the torsion-mirror around resonance frequency are shown in Figs. 2 and 3, respectively [8].

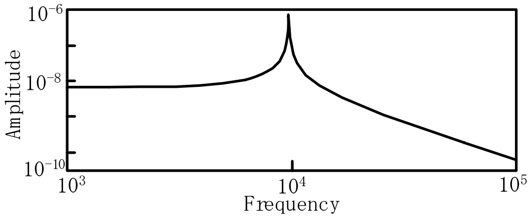


Fig. 2 Amplitude of vibration of the torsion-mirror around he resonance frequency

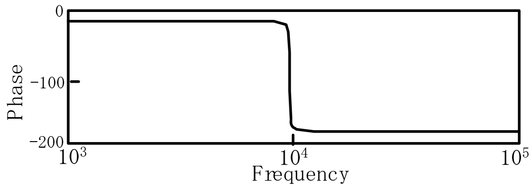


Fig. 3 Phase of vibration around the resonance frequency

2.4 Implementation by circuit simulator

Qucs is a free software licensed under the General Public License (GPL). Using Qucs as a platform has an advantage in developing an equivalent circuit model in a circuit. It can be downloaded from Internet and comes with the complete source code [9]. Qucs consists of several standalone programs interacting with each other through the GUI. The conversion tool is used by the GUI to import and export datasets, net lists and schematics from and to other CAD/EDA software.

As shown in Fig. 4, we can use a capacitor as a mathematic integrator by regarding the input electrical current as an angle acceleration $\ddot{\theta}$, and the output voltage as an angle velocity $\dot{\theta}$.

Built-in current-source of Qcus called EDD (equation defined device), which has been originally prepared to emulate non-linear I–V curve, could make the equivalent circuit model possible. The output current of an EDD is described by an equation using the input voltages as variables. The procedures to build the equivalent circuit by Qcus are follows.

As an example using EDD is shown in Fig. 5, which is an equivalent circuit model for a viscoelastic suspension. As shown in Fig. 5(a), we can draw the circuit symbol by Qcus and implement it in a electrical circuit instead of the EDD subroutine shown in Fig. 5(b). Figure 6 is another example which shows a equivalent circuit model for parallel-plate electrostatic micro-torsion mirror by use of the EDD.

2.5 Equivalent cirquit model

Figure 7 shows an equivalent circuit model for the overall electrostatic micro-torsion mirror including the analytical model described above. Three EDD blocks are used to implement the subroutine of electrostatic force (EDD$_1$), the subroutine of the equation of motion (EDD$_2$), and the pull-in contact subroutine (EDD$_3$). In Fig. 7, CCVS is the current controlled Voltage Source, VCCS is the voltage controlled current source, and VCVS is the voltage controlled voltage source.

2.6 Nonlinear effect due to pull-in contact

The pull-in contact subroutine (EDD$_3$) in Fig. 8 uses an auxiliary path which emulates that the torsion plate spontaneously pull into the counter electrode when electrostatic attractive force exceeds the mechanical restoring force, where mechanical stopper is actually inserted to avoid electrical short circuit. The condition of pull-in contact can be implemented by using an if—clause of the program as follows,

$$i_d = \begin{cases} V_{\mathrm{A}}/R_d & \text{if } (\theta > \theta_{\mathrm{LIM}}\ \&\,\&\ N_E > N_M) \\ & \text{or } (\theta < -\theta_{\mathrm{LIM}}\ \&\,\&\ N_E < N_M) \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

where θ_{LIM} is the stopper angle (i.e. $a \cdot \sin\theta_{\mathrm{LIM}} = g$). If the torsion θ is greater than the stopper angle θ_{LIM} and the moment by electrostatic force N_E is greater than the moment by the mechanical restoring force N_M $(= n_{\mathrm{Si}} \cdot \theta)$, or if the torsion θ is less than the stopper angle $-\theta_{\mathrm{LIM}}$ and the moment by the mechanical restoring force N_M is greater than the moment by electrostatic force N_E, then the capacitor is discharged by the drain current of i_d. Otherwise, the charge

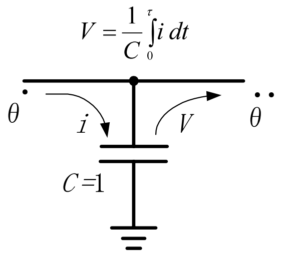
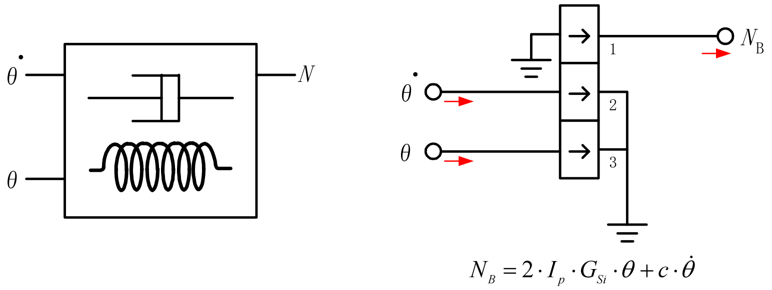
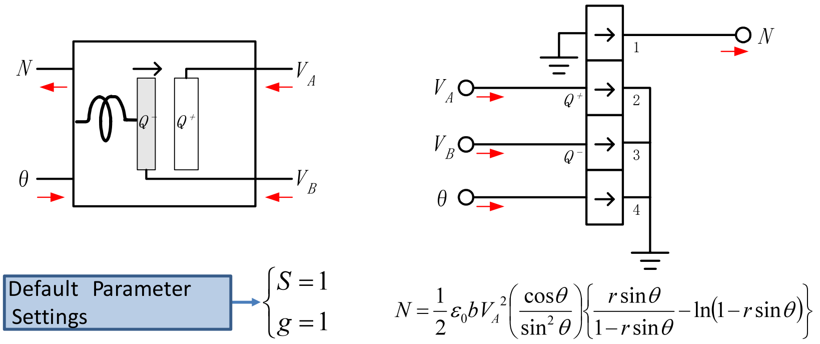


Fig. 4 Electrical circuit implementation of mathematic integral using an ideal electrical capacitor



(a) Circuit Symbol (b) EDD Program

Fig. 5 Equivalent circuit model for viscoelastic suspension



(a) Circuit Symbol (b) EDD Program

Fig. 6 Equivalent circuit model for electrostatic micro-torsion mirror

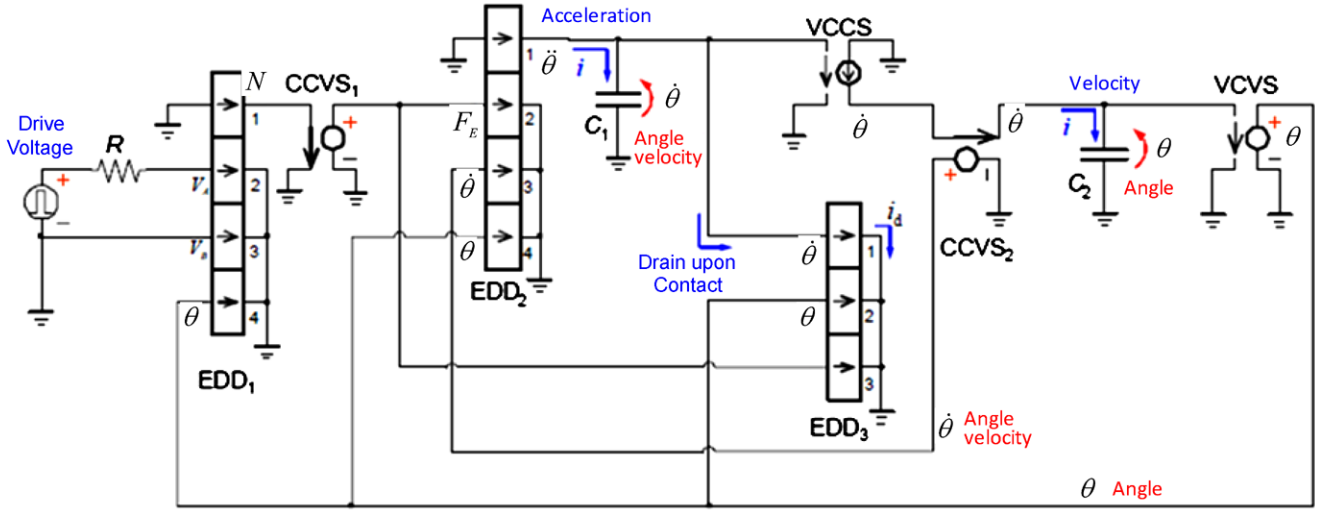


Fig. 7 Equivalent circuit model for the over-all electrostatic micro-torsion mirror

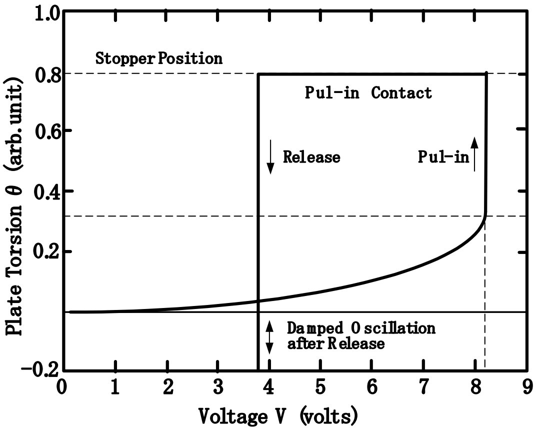


Fig. 8 Non-liner transient analysis by including electrostatic pull-in effect

is maintained by virtually throttling the drain current to be zero. Here, R_d is the drain resistance.

As a result, the EDD$_3$ subroutine provides a hysteresis in the torsion-voltage characteristics as shown in Fig. 8, which could realize the pull-in effect of the torsion-mirror.

3 Conclusions

Overall model for the electrostatic micro torsion mirror could be made. Equivalent sub-circuit models for equation of motion, electrostatic force, electrical circuit, and dissipation energy of a micro torsion mirror have been developed by using the EDD subroutine of open-source circuit simulator Qucs, which implemented the linear response characteristics of the system. The nonlinear transient effects such as electrical pull-in contact using the bypassing subroutine. These models can be applicable to seamless simulations between electrostatic actuators and electrical circuit when we build blocks for the multi-physics analysis of micro torsion mirrors and electric circuits.

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