Journal of Applied Mechanics and Technical Physics, Vol. 48, No. 6, pp. 897–904, 2007

PREDICTING THE NONLINEAR HEREDITARY VISCOELASTICITY OF POLYMERS

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UDC 539.434:677.494

A mathematical model for the nonlinear hereditary viscoelasticity of polymer materials is proposed to predict deformation processes of various complexity — from simple relaxation and simple creep to complex deformation–relaxation and reverse relaxation processes with alternative loading and unloading.

Key words: *polymers, nonlinear hereditary viscoelasticity, stress–strain state.*

Introduction. An important direction of materials science is the study of the deformation properties of polymer materials under nondestructive loads close to operating loads by means of mathematical modeling of deformation based on experimental data processing.

At present, numerous studies of the viscoelastic properties of polymers have been performed. Nevertheless, the great variety of materials has led to the need to develop new methods for studying their deformation properties. Complication of the structure of polymer materials has a significant effect on their viscoelastic properties, which has motivated the development of new mathematical models for the indicated properties and the use of computational methods of experimental data processing. The development of new methods for analyzing the mechanical properties of polymers with their complex structure taken into account allows more reliable predictions of deformation processes.

Recent efforts in materials science have been directed toward promoting the automation of means of quality assurance and product tests. The solution of problems of controlling technological processes is indissolubly connected with improvements in methods and devices for laboratory modeling of material behavior during processing and operation and mathematical modeling methods of experimental data processing and analysis.

Analysis of Deformation Properties. Existing approaches to the analysis of the deformation properties of polymer materials are based on descriptions of generalized experimental curves of relaxation and creep by using normalized relaxation functions and retardation functions, which are usually taken to be integral normal distribution curves on a logarithmic reduced-time scale. These methods of strain analysis and prediction allow one to carefully study polymer materials of simple macrostructure, such as synthetic fibers. The mechanical properties of polymers of more complex structure and articles made of them are difficult to study because their spectrum of relaxation and retardation times is complicated by the superposition of the elementary spectra corresponding to the elements constituting the material. This has stimulated the development of mathematical models for deformation properties based on new, simpler (if possible) relaxation functions and retardation functions corresponding to the complicated spectra. Theory for experimental data analysis and processing has been constructed taking into account both the requirement of the minimum number of mathematical model parameters and their physical validity.

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The variety of modifications of viscoelastic equations is due to the choice of integral kernels. The most promising are the Abel', Rabotnov, Rzhanitsyn, Vul'fson–Koltunov, and Havriliak–Negami kernels [1]. The indicated types of kernels possess the linearity property, i.e., they ignore the effect of strain and stress on the acceleration or deceleration of viscoelastic processes.

The development of the nonlinear theory of viscoelasticity has been based on the Aleksandrov–Gurevich assumption of a decrease in the activation energy under the action of external loading, which corresponds to the acceleration of relaxation and creep processes. It has been proposed to use a logarithmic reduced-time scale. The complication of the concept of time has made it possible to simplify the relaxation kernels. Nonlinear integral kernels were studied by Persoz, Moskvitin, and Bugakov [1]. These integral kernels contained nonlinearity in the form of a strain or stress function, which significantly complicated the problem compared to the linear version of description of viscoelastic properties.

Stalevich [2] determined the average relaxation and retardation times with the activating effects of deformation and load as parameters taken into account. The mathematical viscoelasticity model was simplified by considering nonlinearity in the integral relaxation and retardation kernels in the form of functions of the average relaxation and retardation times.

Modeling of viscoelastic processes of the indicated type is based on a description of generalized relaxation and creep curves by classes of normalized functions (probability integral, Kohlrausch function, normalized hyperbolic tangent, etc.) corresponding to some distributions of the relaxation and retardation times on a logarithmic reduced-time scale. However, the indicated normalized functions exhibit fast convergence to asymptotic values, which does not provide the required accuracy in predicting deformation processes for both long periods (months and years) and the small time intervals (seconds and fractions of a second) corresponding to the initial stage of a quasi-extension process.

Makarov [3] proposed a mathematical model for the viscoelastic properties of polymers based on the Cauchy probability distribution of relaxing and retarded particles:

$$E_{\varepsilon t} = E_0 - (E_0 - E_\infty)\varphi_{\varepsilon t}; \tag{1}$$

$$D_{\sigma t} = D_0 + (D_\infty - D_0)\varphi_{\sigma t}; \tag{2}$$

$$\varphi_{\varepsilon t} = \frac{1}{2} + \frac{1}{\pi}\ \arctan\left(\frac{1}{b_{n\varepsilon}}\ \ln\frac{t}{\tau_\varepsilon}\right); \tag{3}$$

$$\varphi_{\sigma t} = \frac{1}{2} + \frac{1}{\pi}\ \arctan\left(\frac{1}{b_{n\sigma}}\ \ln\frac{t}{\tau_\sigma}\right). \tag{4}$$

Here $E_{\varepsilon t} = \sigma/\varepsilon$ is the relaxation modulus, $D_{\sigma t} = \varepsilon/\sigma$ is the compliance, t is time, $1/b_{n\varepsilon}$ and $1/b_{n\sigma}$ are the intensity parameters for the relaxation and creep processes, respectively; τ_ε is the relaxation time (half the time required for relaxation), τ_σ is the retardation time (half the time required for creep), E_0 is the elastic modulus, E_∞ is the viscoelastic modulus D_0 is the initial compliance, D_∞ is the ultimate equilibrium compliance, ε is the strain, σ is the stress, $\varphi_{\varepsilon t}$ and $\varphi_{\sigma t}$ are the relaxation and creep functions, respectively, specified in the form of a normalized arctangent of the logarithm (NAL) of reduced time.

In contrast to the previously used normalized functions, the NAL function corresponding to the Cauchy probability distribution has a much lower rate of convergence to asymptotic values. The proposed version of mathematical modeling is expedient for predicting not only short- and long-term deformation processes in polymer materials but also for viscoelastic processes in polymer materials of complex macrostructure because the sum of random quantities distributed according to a normalized Cauchy law is known to be distributed according to the same law. In other words, if the relaxing and retarded particles in a polymer are assumed to be distributed in the characteristic relaxation and retardation times according to the Cauchy law, it can be assumed that macrorelaxing and macroretarded particles are distributed according to the same law [4].

An advantage of the mathematical model (1)–(4) is that it contains the minimum number of variables, which have a clear physical meaning. The parameters $E_0 = \lim\limits_{t\to 0} E_{\varepsilon t}$, $E_\infty = \lim\limits_{t\to\infty} E_{\varepsilon t}$, $D_0 = \lim\limits_{t\to 0} D_{\sigma t}$, and $D_\infty = \lim\limits_{t\to\infty} D_{\sigma t}$ are asymptotic values of the relaxation and compliance modulus. The structural parameter $b_{n\varepsilon}$ characterizes the relaxation rate and corresponds to the logarithm of half the reduced relaxation time for strain ε occurring in the time interval $t \in [t', t'']$; $\ln(t'/\tau_\varepsilon) = -b_{n\varepsilon}$ and $\ln(t''/\tau_\varepsilon) = b_{n\varepsilon}$. The structural parameter $b_{n\sigma}$ characterizes the creep rate and corresponds to the logarithm of half the reduced creep time for stress σ occurring in the time interval

$t \in [t', t'']$; $\ln(t'/\tau_\sigma) = -b_{n\sigma}$ and $\ln(t''/\tau_\sigma) = b_{n\sigma}$. The relaxation time $f_{\varepsilon_1\varepsilon} = \ln(t_1/\tau_\varepsilon)$ and retardation time $f_{\sigma_1\sigma} = \ln(t_1/\tau_\sigma)$ functions characterize the shifts of the relaxation and creep curves along the logarithmic time scale and are contained, respectively, in the structural strain–time functional argument

$$W_{\varepsilon t} = \frac{1}{b_{n\varepsilon}} \ln \frac{t}{\tau_\varepsilon} = \frac{1}{b_{n\varepsilon}} \Big(\ln \frac{t}{t_1} + \ln \frac{t_1}{\tau_\varepsilon} \Big) \tag{5}$$

and the structural-force time functional argument

$$W_{\sigma t} = \frac{1}{b_{n\sigma}} \ln \frac{t}{\tau_\sigma} = \frac{1}{b_{n\sigma}} \Big(\ln \frac{t}{t_1} + \ln \frac{t_1}{\tau_\sigma} \Big). \tag{6}$$

The slow convergence of the NAL function (for example, in comparison with the probability integral) to asymptotic values allows one to interpolate the relaxation modulus $E_{\varepsilon t}$ and compliance $D_{\sigma t}$ over a wide time range and, hence, to predict both short- and long-term deformation processes.

In using the NAL function for predicting deformation processes, we have noticed an interesting fact, which has not been observed in using various mathematical models of viscoelasticity: for some group of polymer materials (Mylar, etc.), the calculated asymptotic value of the viscoelastic modulus vanished ($E_\infty = \lim\limits_{t\to\infty} E_{\varepsilon t} = 0$) [3]. This implies that relaxation process is completed in an infinitely long time. This agrees with the physical model of polymers viscoelasticity and indicates that the proposed mathematical model of viscoelasticity (1)–(6) is valid.

It should be noted that the choice of a normalized function for the mathematical model of the viscoelastic properties of polymeric materials cannot be unequivocal and is complicated by the fact that no one functions can be preferred *a priori*. The basic criterion for the choice is experiment.

Studies of the viscoelastic characteristics of polymers have shown that values of the elastic modulus E_0 calculated using the proposed mathematical model (1)–(6) are larger than the corresponding values calculated using mathematical models based on different normalized functions and are close to the acoustic values E_a [4], which is also physically justified because the velocity of propagation of elastic interactions in polymers is close to the sound velocity. The viscoelastic modulus E_∞, which characterizes the lower asymptote of the relaxation modulus in long-term processes decreases, which actually leads to an increase in the relaxation time. This conclusion also refers to the creep process. This distinguishes the NAL function from the previously used normalized relaxation and retardation functions (for example, the probability integral, Kohlrausch function, hyperbolic tangent, etc.).

Thus, the use of the normalized NAL function as the basis of the mathematical model of viscoelasticity allows the deformation properties of polymers to be modeled with high accuracy, which, in turn, allows predictions of deformation processes over a wider range. Analytical specification of the NAL function and the fact that it belongs to the class of elementary functions simplify differential–integral transformations within framework of the mathematical model considered and facilitate the determination of viscoelastic characteristics.

Determining Mechanical Characteristics of Polymers. As an example of determining the mechanical characteristics of polymers, we consider processing of experimental data on the relaxation (Fig. 1) and creep (Fig. 2) of polyester fibers. In Fig. 1, it is evident that the relaxation curves obtained for various strain values can be made coincident with the generalized curve of the relaxation modulus $E_{\varepsilon t}$ approximated by the mathematical model (1), (3) by a parallel shift along the logarithmic time scale for the quantity $\ln(\tau_\varepsilon/t_1)$:

$$\ln(t/\tau_\varepsilon) = \ln(t/t_1) - \ln(\tau_\varepsilon/t_1).$$

This transformation of the relaxation curves is based on the so-called deformation–time analogy. From the values of the specified shifts, we determine the function $\ln(\tau_\varepsilon/t_1)$, which, in essence, specifies the relaxation times τ_ε (Fig. 3). The structural intensity parameter of relaxation $1/b_{n\varepsilon}$ is determined as the similarity coefficient of the generalized curve of the relaxation modulus and the normalized NAL function.

The asymptotic values of E_0 and E_∞ can be determined from the generalized dependence $E_{\varepsilon t}(\ln(t/t_1))$ (see Fig. 1). Thus, using the mathematical relaxation model (1), (3), from experimental relaxation curves (see Fig. 1) one obtains the main relaxation characteristics, which are then used to predict deformation processes. Similarly, from experimental creep curves (see Fig. 2), one obtains the creep parameters: the asymptotic values D_0 and D_∞, the intensity parameter $1/b_{n\sigma}$, and the retardation time function $\ln(\tau_\sigma/t_1)$ (Fig. 4).

The reliability of the obtained mechanical characteristics is verified by calculating the relaxation modulus and compliance from formulas (1) and (2) and by comparing the obtained values with experimental data (see Figs. 1 and 2).

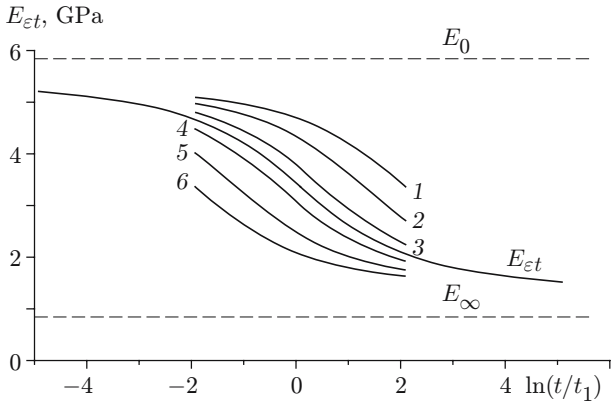

Fig. 1

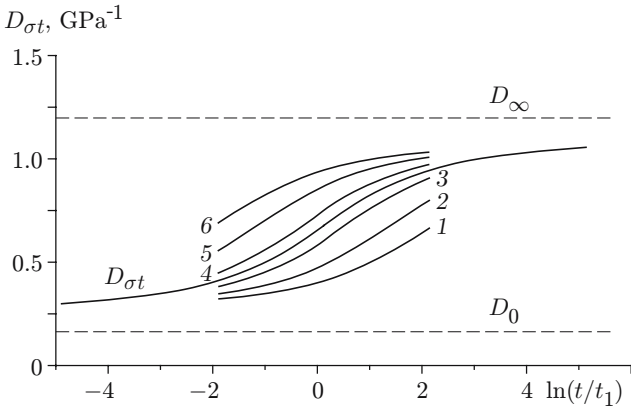

Fig. 2

Fig. 1. Stress relaxation curve (1–6) and generalized curve of the relaxation modulus $E_{\varepsilon t}$ for polyester fiber ($T = 20^\circ$C and $t_1 = 1$ min) for $\varepsilon = 1$ (1), 2 (2), 3 (3), 4 (4), 5 (5), and 6% (6).

Fig. 2. Creep curves (1–6) and a generalized curve of compliance $D_{\sigma t}$ for polyester fiber ($T = 20^\circ$C and $t_1 = 1$ min) for $\sigma = 82$ (1), 98 (2), 106 (3), 114 (4), 123 (5), and 136 MPa (6).

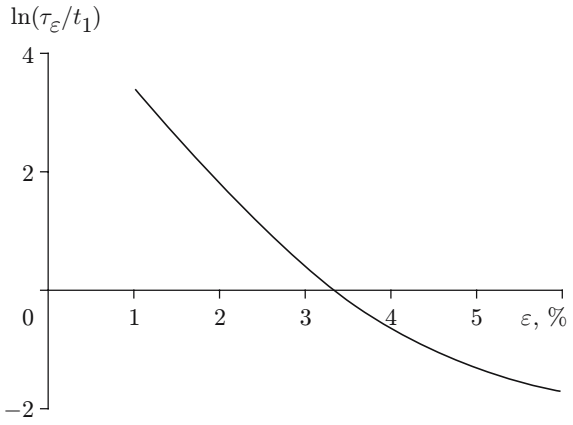

Fig. 3

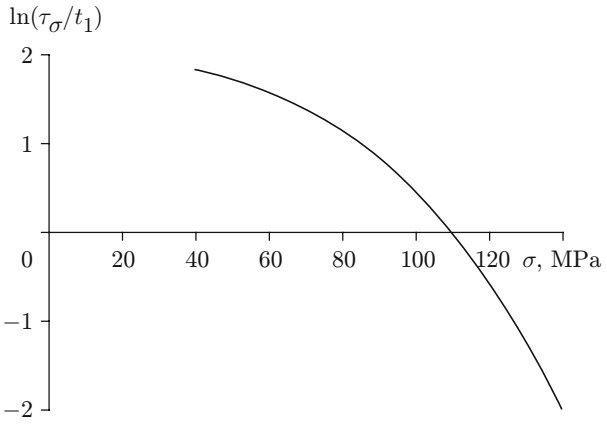

Fig. 4

Fig. 3. Relaxation time function for polyester fiber ($T = 20^\circ$C and $t_1 = 1$ min).

Fig. 4. Retardation time function for polyester fiber ($T = 20^\circ$C and $t_1 = 1$ min).

Predicting Deformation Processes. Polymer deformation processes are predicted using the well-known integral Boltzmann–Volterra relations

$$\sigma_t = E_0\varepsilon_t - (E_0 - E_\infty)\int\limits_0^t \varepsilon_\theta \varphi'_{\varepsilon,t-\theta}\,d\theta; \tag{7}$$

$$\varepsilon_t = D_0\sigma_t + (D_\infty - D_0)\int\limits_0^t \sigma_\theta \varphi'_{\sigma,t-\theta}\,d\theta \tag{8}$$

for nonlinear hereditary relaxation and nonlinear hereditary creep processes, respectively [2], with the integral relaxation and retardation kernels corresponding to the mathematical model (1)–(6):

$$\varphi'_{\varepsilon t} = \frac{\partial \varphi_{\varepsilon t}}{\partial t} = \frac{1}{\pi}\,\frac{1}{b_{n\varepsilon}}\,\frac{1}{1+W_{\varepsilon t}^2}\,\frac{1}{t}, \qquad \varphi'_{\sigma t} = \frac{\partial \varphi_{\sigma t}}{\partial t} = \frac{1}{\pi}\,\frac{1}{b_{n\sigma}}\,\frac{1}{1+W_{\sigma t}^2}\,\frac{1}{t}. \tag{9}$$

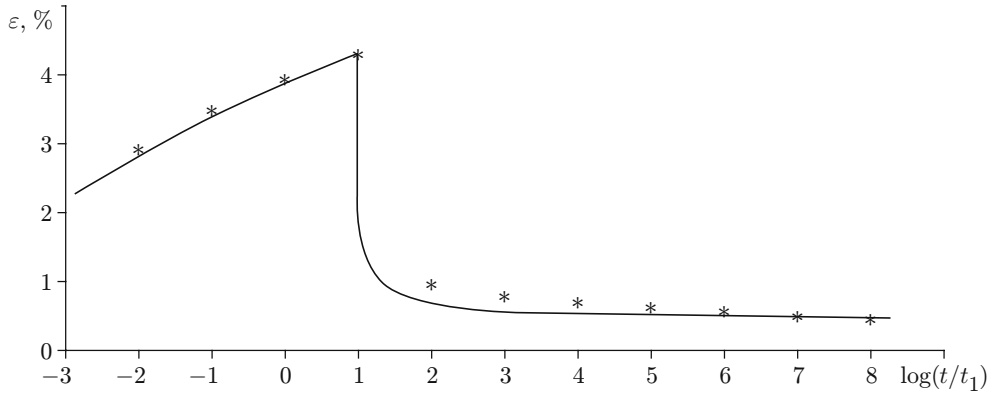

Fig. 5. Deformation–relaxation process for Mylar fiber with a linear density equal to $114 \cdot 10^{-6}$ kg/m ($T = 20^{\circ}$C, $\sigma = 182$ MPa, and $t_1 = 1$ min; load removal at $t = 10$ min): the curve refers to experimental data; the points are calculation results.

In modeling deformation processes, an advantage of using the integral kernels (9) and (10) as a consequence of the mathematical model (1)–(6) is that the range of confidential prediction can be extended toward the large and small times of the process with the prediction error decreased by reducing the effect of the quasi-instantaneous deformation factor at the beginning of the process.

In addition, the increase in the prediction accuracy is due to the use of methods for calculating the improper nonlinear hereditary integrals (7) and (8) based on nonuniform separation of the time scale with allowance for the specificity of the process considered [3]. For example, in predicting fast processes characterized by increasing strain rate, it is reasonable to separate the time scale in accordance with increasing geometric progression to take into account the effect of the quasi-instantaneous deformation factor at the beginning of the process. In predicting long-term processes characterized by decreasing strain rate, it is reasonable to separate the time scale in accordance with the decreasing geometric progression.

The methods developed to calculate the nonlinear hereditary viscoelasticity integral (7), (8) based on the mathematical model with the NAL function have been tested by calculations of various deformation–relaxation and reverse relaxation processes [4]. Calculation results for a long-term deformation–relaxation process of a Mylar fibers are presented in Fig. 5. It is evident that the calculation results are in good agreement with experimental data.

Separation of the Elastic and Viscoelastoplastic Components of the Mechanical Work of Deformation. The proposed methods for predicting deformation–relaxation processes can be used to develop new methods for dividing the specific (per unit volume) mechanical work of deformation [5]

$$a_t = \int_0^{\varepsilon_t} \sigma_t \, d\varepsilon = \frac{1}{2} E_0 \varepsilon_t^2 - \dot{\varepsilon}(E_0 - E_\infty) \int_0^t \int_0^t \varepsilon_{t-s} \varphi'_{\varepsilon s} \, ds \, dt$$

into the elastic ($a_{t0} = E_0^{-1} \sigma_t^2 / 2$) and viscoelastoplastic ($a_{tt} = a_t - a_{t0}$) components.

The fraction of the elastic component of the mechanical work a_{t0}/a_t can be determined from the formula for the extension process

$$\frac{a_{t0}}{a_t} = \Big[\varepsilon_t^2 + (1-c)^2 \Big(\int_0^t \varepsilon_{t-s} \varphi'_{\varepsilon s} \, ds \Big)^2 + 2\varepsilon_t (1-c) \int_0^t \varepsilon_{t-s} \varphi'_{\varepsilon s} \, ds \Big] \Big/ \Big(\varepsilon_t^2 + 2\dot{\varepsilon}(1-c) \int_0^t \int_0^t \varepsilon_{t-s} \varphi'_{\varepsilon s} \, ds \, dt \Big),$$

where $c = E_\infty / E_0$. The viscoelastoplastic component of the specific mechanical work or its fraction a_{tt}/a_t is determined similarly.

The elastic and viscoelastoplastic components of the mechanical work of deformation correspond to the elastic component ε_{t0} and viscoelastoplastic component ε_{tt} of the total deformation ε_t, which, in both the general

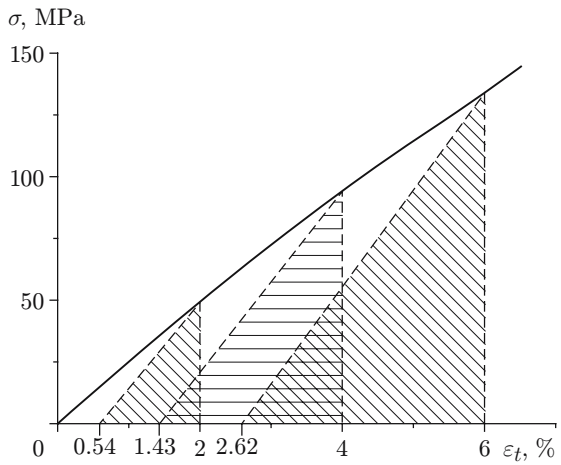

Fig. 6

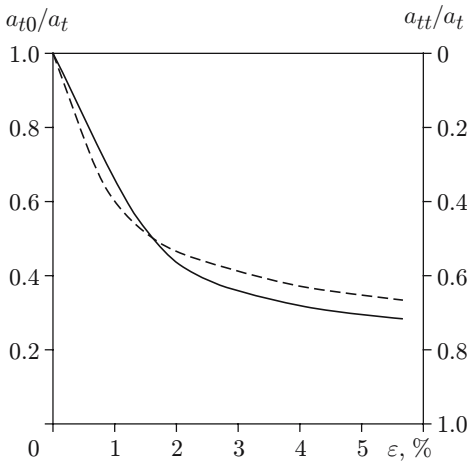

Fig. 7

Fig. 6. Decomposition of the total strain ε_t of polyester fiber in extension ($T = 20^\circ$C and $\dot{\varepsilon} = 0.083$ sec^{-1}) into the elastic component ε_{t0} and viscoelastoplastic component ε_{tt}.

Fig. 7. Contributions of the elastic component a_{t0}/a_t and viscoelastoplastic component a_{tt}/a_t to the mechanical work of deformation ($T = 20^\circ$C and $\dot{\varepsilon} = 0.083$ sec^{-1}): the solid curve refers to polyester fiber; the dashed curve to protective polyester fabric.

case for $\dot{\varepsilon} \neq$ const and in the particular case for $\dot{\varepsilon} =$ const, can be determined from the formulas

$$\varepsilon_{t0} = E_0^{-1}\sigma_t, \qquad \varepsilon_{tt} = \varepsilon_t - \varepsilon_{t0} = \Big(1 - \frac{E_\infty}{E_0}\Big) \int\limits_0^t \varepsilon_{t-s}\varphi'_{\varepsilon s}\, ds,$$

where the stress σ_t is calculated by the relation (8).

Figure 6 shows the decomposition of the total strain for uniform extension using as an example the experimental dependence $\sigma(\varepsilon_t)$ for polyester fiber for $\dot{\varepsilon} = 0.083$ sec^{-1}. The components $\varepsilon_{t0} = 1.46$ and $\varepsilon_{tt} = 0.54$ corresponds to the value $\varepsilon_{t0} = 1.46\%$; $\varepsilon_{t0} = 2.57$ and $\varepsilon_{tt} = 1.43$ to the value $\varepsilon_t = 4\%$; and $\varepsilon_{t0} = 3.38$ and $\varepsilon_{tt} = 2.62$ to the value $\varepsilon_t = 6\%$. The area under the extension diagram (solid curve) is equal to the total normalized (to unit volume) mechanical work of deformation, and the area of the corresponding dashed triangle is equal to the value of its elastic component. The components of the total mechanical work of deformation are calculated similarly. As the strain increases, the contribution of the elastic component to the mechanical work decreases and the contribution of the viscoelastoplastic component increases (Fig. 7).

The decomposition of the mechanical work of deformation into components is important in studies of the effect of external mechanical actions on polymer materials used as elements of protective structures [6]. For example, investigation of the components of the mechanical work of deformation allows one to analyze the deformation properties of polyester fabric employed in impact-resistant helmet liners and other polymer materials (see Fig. 3). At small strains ($\varepsilon = 2\%$), the elastic components of the strain and mechanical work are dominant. In this case, the contribution of the viscoelastoplastic component to the mechanical work is 30–40%. At the same time, an increase in the strain ($\varepsilon = 6\%$) leads to the dominance of the viscoelastoplastic components of the mechanical work of deformation, which characterize the viscoelastic and plastic deformation components. In this case, it is important to know how effectively the protective material can weaken the mechanical impact. The effectiveness is determined by the increase in the contribution of the viscoelastoplastic component of the mechanical strain energy with increasing strain. At the same time, at small strains, the dominance of the elastic components of the strain and strain energy implies that the material rapidly recovers its shape and is suitable for further use.

By introducing a correction for the accumulation of the irreversible strain component that does not dependent on the type of deformation process, one can calculate the total accumulated strain $(\varepsilon_t)_{\rm calc}$ by the formula [5]

$$(\varepsilon_t)_{\rm calc} = \varepsilon_{\rm res} + \varepsilon_t/\eta,$$

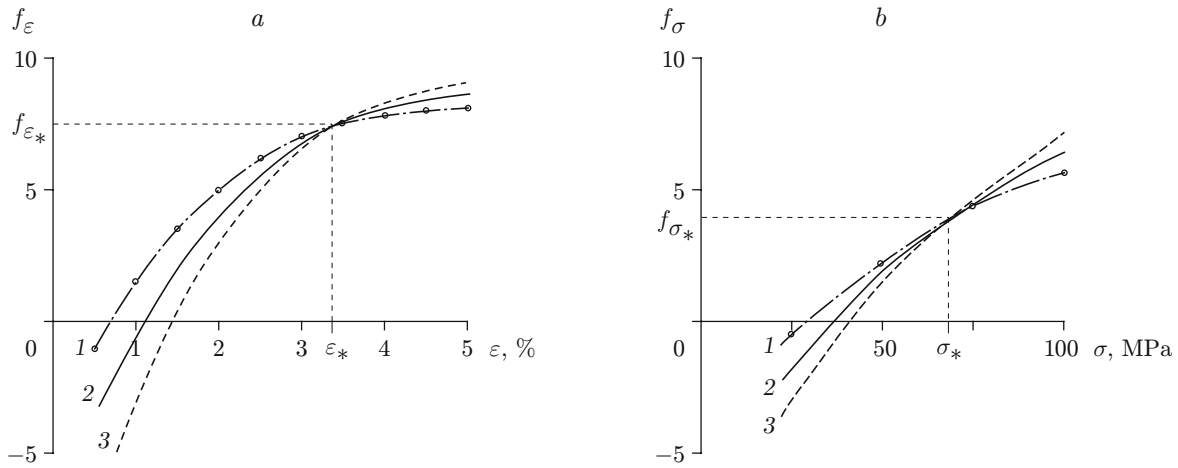

Fig. 8. Functions of the average relaxation time (a) and retardation time (b) for polyester fiber for $T = 20$ (1), 40 (2), and 60°C (3).

where ε_t is the strain calculated by formula (8), $\eta = (\varepsilon_* - \varepsilon_{\text{res}})\varepsilon_*$ is the strain reversibility coefficient determined experimentally, ε_* is the strain before load removal, and ε_{res} is the residual strain after load removal. The stress $(\sigma_t)_{\text{calc}}$ is calculated similarly, using a correction for strain irreversibility:

$$(\sigma_t)_{\text{calc}} = \eta\sigma_t + (1 - \eta)E_0\varepsilon_t.$$

Here σ_t is the stress calculated by formula (7).

Use of the indicated methods in calculations of complex deformation–relaxation (see Fig. 1) and reverse relaxation processes increases the prediction accuracy.

It should be noted that the separation of strain into components is rather conditional and can be performed using various methods. For example, during heating of a polymer to a certain temperature, irreversible plastic deformation becomes reversible.

Thus, the methods of separating the mechanical work of deformation and the corresponding strain into elastic and viscoelastoplastic components allow one to characterize the elastoplastic properties of materials, which is of great importance, for example, in sampling according to elastic and plastic criteria. Correction for strain irreversibility allows one to distinguish the plastic component in the viscoelastoplastic strain component, which also increases the accuracy in predicting both simple and complex nonlinear hereditary viscoelastic processes.

Predicting Thermoviscoelasticity. The indicated methods for determining the viscoelastic characteristics and predicting polymer deformation at variable temperature are based on the assumption of a linear temperature dependence of the statistical average relaxation and retardation time functions, which has been supported experimentally for the materials studied. Results of processing experimental curves of the relaxation and compliance moduli obtained at various temperatures are used to determine the linear transformations of the average relaxation and retardation time functions that correspond to rotations of the plots of the indicated functions through temperature-dependent angles, which provides values of the functions in specified temperature ranges (Fig. 8) [3]:

$$\varepsilon = \varepsilon_* + \tilde{\varepsilon}\cos\alpha - \tilde{f}_\varepsilon \sin\alpha, \qquad f_\varepsilon = f_{\varepsilon_*} + \tilde{\varepsilon}\sin\alpha + \tilde{f}_\varepsilon\cos\alpha,$$

$$\sigma = \sigma_* + \tilde{\sigma}\cos\beta - \tilde{f}_\sigma \sin\beta, \qquad f_\sigma = f_{\sigma_*} + \tilde{\sigma}\sin\beta + \tilde{f}_\sigma\cos\beta.$$

Here $\alpha = \alpha(T)$ and $\beta = \beta(T)$ are the angles of rotation the former coordinate systems $(\varepsilon, f_\varepsilon)$ and (σ, f_σ) around the points $(\varepsilon_*, f_{\varepsilon_*})$ and (σ_*, f_{σ_*}) relative to the new systems $(\tilde{\varepsilon}, \tilde{f}_\varepsilon)$, $(\tilde{\sigma}, \tilde{f}_\sigma)$, which depend on the temperature T; $f_\varepsilon = \ln(t_1/\tau_\varepsilon)$ and $f_\sigma = \ln(t_1/\tau_\sigma)$.

The other deformation characteristics (elastic and viscoelastic moduli, initial and ultimate equilibrium compliances, structural intensity parameters of the processes) are transformed similarly using linear dependences (Fig. 9).

Thus, the methods of linear transformation of viscoelastic characteristics in a specified temperature range allow one to predict deformation processes at any temperatures in the examined range, including the processes occurring at variable temperature.

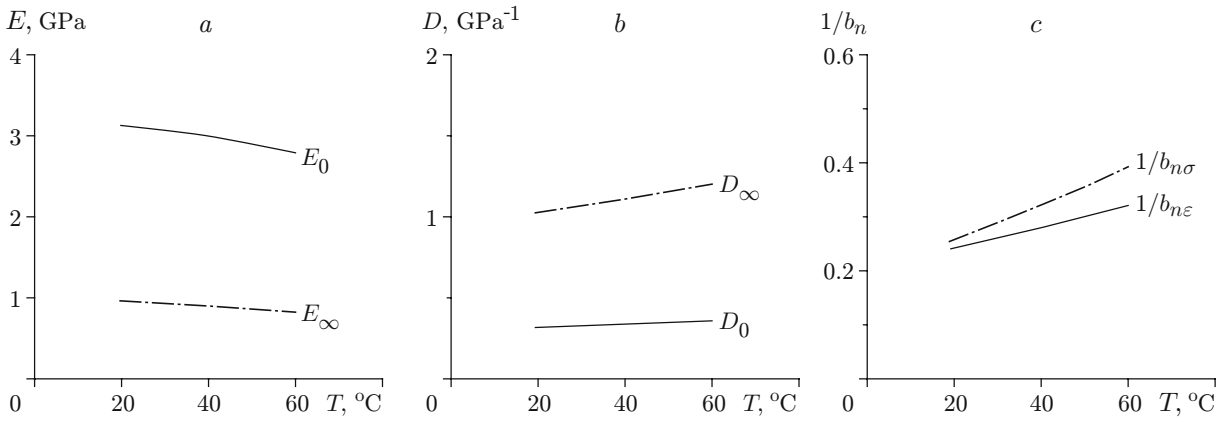

Fig. 9. Temperature dependences of the elastic modulus E_0 and viscoelastic modulus E_∞ (a), the initial compliance D_0 and ultimate equilibrium compliance D_∞ (b), and the intensity parameters of relaxation process $1/b_{n\varepsilon}$ and creep process $1/b_{n\sigma}$ (c) for polyester fiber.

Conclusions. Methods using a mathematical model with nonlinear hereditary integral relaxation and retardation kernels were proposed to analyze the deformation properties of polymers under nondestructive mechanical loads, which considerably increase the ranges of time, load, and strain in the analysis of viscoelastic processes.

Methods were developed to determine the viscoelastic characteristics which are parameters of the proposed mathematical model using results of short-term tests in simple relaxation and creep modes and to predict deformation–relaxation and reverse relaxation processes and other more complex deformation modes.

A method was proposed to separate the total mechanical work of deformation and its corresponding strain into the elastic and viscoelastoplastic components, which can be used to calculate polymer resistance in dynamic deformation modes, including an impact mode, and to estimate the ability of materials to resist mechanical actions and to recover the initial shape during operation.

A method for taking into account the pseudo-plastic component of irreversible strain in mathematical modeling of the viscoelastic properties of materials was developed which improves the reliability of predicting complex deformation modes.

Methods for accounting for the temperature dependence of viscoelastic characteristics in mathematical modeling of deformation properties of materials were proposed which allow one to predict deformation processes at variable temperature.

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