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Equal sacrifice and fair burden-sharing in a public goods economy

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Abstract Applying a willingness-to-pay approach known from contingent valuation in environmental economics, we develop an ordinally based measure for the size of individual sacrifice that is connected with an agent's contribution to a public good. We construct a selection mechanism that picks the unique efficient solution among all allocations that have an equal sacrifice as defined in this way. We show that the solution thus obtained corresponds to Moulin's egalitarian equivalent allocation, conforms to both the ability-to-pay and the benefit principles, and has much in common with the Lindahl equilibrium.

Keywords Public goods · Cooperative solutions · Fairness · Egalitarian-equivalent solutions

JEL Classification H41

1 Introduction

Since the theory of public finance began in the nineteenth century, three famous principles of just taxation based on different normative ideas have been formulated (see,

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e.g., Musgrave 1959, for a historical review). The *equal sacrifice* principle requires that taxation should lead to the same (absolute or relative) loss of utility for everyone. In this way, a symmetrical, and thus fair, treatment of all citizens is ensured. Taxation according to *ability to pay,* on the other hand, requires that personal tax liability should be positively correlated with the taxpayer's income or wealth, and thus provides some kind of vertical equity for people with different financial capacities. In contrast to both equal sacrifice and ability-to-pay, the *benefit principle* also takes into account how the tax revenue is spent. It postulates that the individual tax burden should be related to the utility gain an agent derives from the governmental expenditures that are financed with her taxes. The benefit principle, therefore, reflects the quid pro quo fairness known from the market exchange of private goods.

Recently these basic concepts of just taxation have been attracting more attention in a field outside the framework of taxation theory in the ordinary sense. So it has become a major topic in the political debate and in economic research how to improve the supply of "global public goods." Climate protection has now become the most prominent of these goods (see, e.g., Kaul et al. 1999, 2003; Sandler 2004; Nordhaus 2005; Sandmo 2006, and Kaul and Conceição 2006). Provision of a public good, however, is inefficiently low when agents (or, in the case of an international public good, countries) do not cooperate, so that collective actions are required to overcome this underprovision problem (see, e.g., Sandler 1992, or Cornes and Sandler 1996, for a detailed explanation of this standard result in the theory of public goods).

In the international sphere cooperation often is, as in the case of the Kyoto protocol in climate policy, regulated by a convention which, in particular, stipulates how the contributions to the global public good are to be distributed among the participating countries (especially with climate protection see: Stern et al. 2006, pp. 450–467). The venerable principles of just taxation become relevant once again for designing the fundamental structure of such burden sharing arrangements: Countries will only be ready to accept an agreement when their advantage is in line with their financial obligations, i.e., if the benefit principle is satisfied. At the same time, cooperation can only be expected to be successful if no nation feels overburdened compared to its partners, and a fair distribution of cooperative efforts is achieved (see Ringius et al. 2002, and Sandler 2004, pp. 77–79). This concern for an equitable treatment of all participants is reflected by the equal sacrifice principle. In the field of climate change policy there is, moreover, a broad consensus that richer countries have a greater obligation to finance greenhouse gas abatement, and this could be considered to be an application of the ability-to-pay principle.

Despite their importance, and the frequency with which they receive casual reference in informal discussions, the principles of just taxation as guidelines for fair burden sharing have not been incorporated systematically into the theory of public goods. The purpose of this paper is, therefore, to identify an approach by which one particular efficient public good allocation that conforms simultaneously to these three principles is selected. Taking the equal sacrifice postulate as the starting point, we proceed as follows: In Sect. 2, we first describe how the individual sacrifice connected with a certain individual public good contribution can be measured by adopting a willingness-to-pay technique familiar in environmental economics. (For another application of the willingness to pay approach to the standard public good model; see

Bergstrom 2006.) With this approach, individual contributions to the public good that were originally measured in units of the private good are converted into public-good equivalents so that the public good serves as the numéraire. Thus, in contrast to the classical equal sacrifice approach in the theory of taxation, a cardinal measure for individual utility is not required. In Sect. 3, we first establish some basic properties of this sacrifice measure that are used throughout the paper. In Sect. 4, the equity norm is then applied to determine the set of public good allocations for which the level of this sacrifice is identical for all agents. Imposing allocative efficiency, i.e., the Samuelson rule, as a further normative postulate in Sect. 5 then gives the desired choice mechanism for public-good allocations. In Sect. 6, it is shown that this mechanism corresponds to Moulin's egalitarian-equivalent solution concept (see Moulin 1987, 1995). Thus, an alternative justification for this selection mechanism is provided that is more closely related to standard ideas of equal treatment and to the standard concepts of public finance. In this way, it also becomes possible to draw a parallel between the egalitarian equivalent solution in a public good economy and the classical Lindahl equilibrium, which is also done in Sect. 6. In Sect. 7, we finally show that the equal sacrifice selection rule described in this paper also satisfies the benefit principle and the ability-to-pay criterion, so that it, indeed, incorporates the three fundamental principles for fair burden sharing.[1]

2 Measuring individual sacrifice of public good contribution

We consider a standard public-good economy consisting of n agents $i = 1, \dots, n$ (see the classical treatments in Bergstrom et al. 1986, and Cornes and Sandler 1996). Agent i is endowed with an amount y_i of the private good, her income. Total income of all agents is denoted by $Y = \sum_{i=1}^{n} y_i$. The utility function of agent i is $u_i(x_i, G)$ where x_i is agent i's level of private consumption and G is public-good supply. It is defined for all $(x_i, G) \in \mathbb{R}^2_+$ and continuous everywhere. For $(x_i, G) \in \mathbb{R}^2_{++}$, each $u_i(x_i, G)$ is even twice continuously differentiable and strictly monotone increasing in both variables, it is strictly quasi-concave and both the private and the public good are assumed to be noninferior. The marginal rate of substitution between the public good and the private good at some point $(x_i, G) \in \mathbb{R}^2_{++}$ is denoted by $\pi_i(x_i, G) = \frac{\partial u_i/\partial G}{\partial u_i/\partial x_i}$. To avoid corner solutions, we furthermore suppose that $u_i(x_i, 0) = u_i(0, G)$ holds for all $x_i \geq 0$ and all $G \geq 0$. Then all indifference curves that pass through a point in $\mathbb{R}^2_{++}$ do not hit the coordinate axis, and $\lim_{x_i \to 0} \pi_i(x_i, G) = 0$ holds for any $G > 0$ and $\lim_{G \to 0} \pi_i(x_i, G) = \infty$ for any $x_i > 0$.

The public good is produced by a constant returns to scale summation technology. If agent i contributes $g_i := y_i - x_i$ to the public good, the total supply of the public good is given by

$$G = \sum_{i=1}^{n} g_i. \tag{1}$$

[1] An empirical account of concepts for fair burden sharing in international environmental agreements is given by Lange et al. (2007).

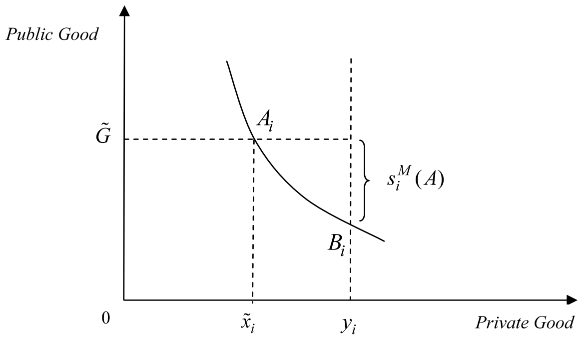


Fig. 1 Measuring individual sacrifice

Among all allocations that meet the budget constraint (1), we want to identify those in which the sacrifice for each agent is equal, and thus the equal sacrifice principle is satisfied. Applying this normative concept first of all requires the size of personal sacrifice to be measured in an adequate way. In this context, the simplest approach would be to identify agent i's sacrifice with the absolute level of her contribution g_i. But such a specification of sacrifice is only compatible with ethical intuition when all agents are completely identical, i.e., have the same income and the same preferences. Otherwise, one would expect a smaller income, or a lower preference for the public good, to increase agent i's subjective burden associated with some given contribution level g_i, since this contribution then is harder to bear.

The problem of finding an adequate measure of subjective individual sacrifice already showed up in the classical treatment of equal sacrifice of taxation where sacrifice was related to the loss of utility of income and not to income itself. In this approach, utility has to be cardinally measurable, which is in conflict with the usual assumption of purely ordinal preferences.[2] In the present paper, in which the agents' utility also depends on public good consumption, a measure of agent i's personal sacrifice is obtained by constructing a public-good equivalent to her contribution g_i. As we take the public good as the numéraire, the problem of having to make use of cardinal measurability of utility that arose in the classical equal sacrifice approach is avoided.[3]

Definition 1 Let $A = (\tilde{x}_1, \dots, \tilde{x}_n, \tilde{G})$ be some allocation. The individual sacrifice $s_i^M(A)$ that agent i makes in the allocation A is determined by

$$u_i\big(y_i, \tilde{G} - s_i^M(A)\big) = u_i(\tilde{x}_i, \tilde{G}). \tag{2}$$

The meaning of Definition 1 is visualized in Fig. 1.

By Definition 1, individual public-good contributions g_i are converted into equivalent public-good units, and thus are made comparable. This method for measuring

[2] For a modern treatment of the classical equal sacrifice approach see, e.g., Mitra and Ok (1996), or Moyes (2003).

[3] See Neill (2000) for an alternative approach to measuring individual sacrifice in a public-good economy that, as in the conventional treatments, refers to differences in cardinal utility.

personal sacrifice is analogous to the assessment of individual willingness to pay well known from contingent valuation studies in environmental economics (see, e.g., Ebert 1993, or Kolstad 2000, pp. 291–294). So, agent i's sacrifice $s_i^M(A)$ in a given allocation $A = (\tilde{x}_1, \dots, \tilde{x}_n, \tilde{G})$ is elicited as the answer to a willingness-to-pay question by which agent i is asked how much of the public good she would be ready to give up if—starting from her position $(\tilde{x}_i, \tilde{G})$—she could simultaneously reduce her public-good contribution to zero. Then agent i becomes indifferent between her position $A_i = (\tilde{x}_i, \tilde{G})$ attained in A and the position $B_i = (y_i, \tilde{G} - s_i^M(A))$ where private consumption is identical with the initially given income y_i and the public-good supply is reduced by the sacrifice level $s_i^M(A)$. In an alternative interpretation, $s_i^M(A)$ indicates agent i's willingness to pay (in units of the public good) for an increase of private consumption from $\tilde{x}_i$ to y_i.

For any utility level $\bar{u}_i$ of agent i, let $\varphi_i^h(x_i, \bar{u}_i)$ denote the inverse Hicksian demand function for the private good. It coincides with the marginal rate of substitution between the private good and the public good $1/\pi_i(x_i, G)$ when (x_i, G) varies along the given indifference curve which is identified with the utility level $\bar{u}_i$. Then $s_i^M(A)$ can be represented as an area below the inverse Hicksian demand function, i.e., as

$$s_i^M(A) = \int_{\tilde{x}_i}^{y_i} \varphi_i^h\big(x_i, u_i(\tilde{x}_i, \tilde{G})\big)\,\mathrm{d}x_i = \tilde{g}_i \int_{\tilde{x}_i}^{y_i} \frac{\varphi_i^h(x_i, u_i(\tilde{x}_i, \tilde{G}))}{\tilde{g}_i}\,\mathrm{d}x_i, \tag{3}$$

where $\tilde{g}_i := y_i - \tilde{x}_i$ is agent i's public-good contribution in the allocation A. Thus, the sacrifice of agent i in allocation A is obtained as this agent's contribution to the public good, weighted by the *average* marginal rate of substitution measured along the indifference curve $u_i(\tilde{x}_i, \tilde{G})$.

3 Properties of the sacrifice measure

In this section, we want to examine how the level of agent i's sacrifice depends on her position $(\tilde{x}_i, \tilde{G}) = (y_i - \tilde{g}_i, \tilde{G})$ attained in a certain allocation A and also on income y_i and preferences $u_i(x_i, G)$. In each of the following four steps of the analysis, we will vary only one of the parameters $\tilde{g}_i$, $\tilde{G}$ and y_i or the utility function $u_i(x_i, G)$, while the other three are kept constant. Part of the adjustment that is required by the transition from the original allocation A to a new feasible allocation called A' then has to be made by the other agents. The change of agent i's sacrifice, however, is not affected by the precise nature of the adjustments of the other agents so that they do not have to be described explicitly.

(i) If agent i's public good contribution is increased from $\tilde{g}_i$ to $\tilde{g}_i'$, her sacrifice obviously increases, since the new indifference curve $u_i(\tilde{x}_i', \tilde{G})$ is lower than the original indifference curve $u_i(\tilde{x}_i, \tilde{G})$.

(ii) If public-good supply grows from $\tilde{G}$ to $\tilde{G}'$, the argument is a little more complicated and crucially depends on the normality assumption. Letting $\tilde{u}_i := u_i(\tilde{x}_i, \tilde{G})$ and $\tilde{u}_i' := u_i(\tilde{x}_i, \tilde{G}')$, we consider the two inverse Hicksian demand functions $\varphi_i^h(x_i, \tilde{u}_i)$ and $\varphi_i^h(x_i, \tilde{u}_i')$ that correspond to these utility levels, respectively. From $\tilde{G}' > \tilde{G}$, we have $\tilde{u}_i' > \tilde{u}_i$. Then normality straightforwardly implies that $\varphi_i^h(x_i, \tilde{u}_i') > \varphi_i^h(x_i, \tilde{u}_i)$ holds for all x_i, so that the indifference curve through $(\tilde{x}_i, \tilde{G}')$ is, for any x_i, steeper than that through $(\tilde{x}_i, \tilde{G})$ (see Appendix A for details). Using our representation formula (3), this allows a comparison of the new sacrifice $s_i^M(A')$ (with the new position $(\tilde{x}_i, \tilde{G}')$ of agent i) and the original sacrifice level $s_i^M(A)$

$$s_i^M(A') = \int_{\tilde{x}_i}^{y_i} \varphi_i^h(x_i, \tilde{u}_i')\,\mathrm{d}x_i \geq \int_{\tilde{x}_i}^{y_i} \varphi_i^h(x_i, \tilde{u}_i)\,\mathrm{d}x_i = s_i^M(A). \tag{4}$$

(iii) If the income of agent i is increased from y_i to y_i' the effect on the sacrifice level again rests upon normality. Letting now $\tilde{u}_i' := u_i(y_i' - \tilde{g}_i, \tilde{G})$, we consider the inverse Hicksian demand function $\varphi_i^h(x_i, \tilde{u}_i')$. If a horizontal translation by $t := y_i' - y_i$ is made, normality implies $\varphi_i^h(x_i, \tilde{u}_i') \leq \varphi_i^h(x_i - t, \tilde{u}_i)$, i.e., moving to the right makes the indifference curves flatter (see again Appendix A). Denoting $\tilde{x}_i' = y_i' - \tilde{g}_i$, we then get the following estimate:

$$\begin{aligned} s_i^M(A') &= \int_{\tilde{x}_i'}^{y_i'} \varphi_i^h(x_i, \tilde{u}_i')\,\mathrm{d}x_i \leq \int_{\tilde{x}_i'}^{y_i'} \varphi_i^h(x_i - t, \tilde{u}_i)\,\mathrm{d}x_i \\ &= \int_{\tilde{x}_i}^{y_i} \varphi_i^h(x_i, \tilde{u}_i)\,\mathrm{d}x_i = s_i^M(A). \end{aligned} \tag{5}$$

This means that agent i's sacrifice becomes smaller if her income increases.

(iv) Finally, we suppose that agent i is substituted by another type of agent with a utility function $u_i'(x_i, G)$ which represents a stronger preference for the public good than the original utility function $u_i(x_i, G)$. This intensification of preferences for the public good is described by the assumption that the new utility function everywhere exhibits a higher marginal willingness to pay for the public good, i.e., that

$$\frac{\partial u_i'/\partial G}{\partial u_i'/\partial x_i} > \frac{\partial u_i/\partial G}{\partial u_i/\partial x_i} \tag{6}$$

holds for all consumption bundles (x_i, G). This condition in particular means that the indifference curve $\tilde{u}_i' := u_i'(\tilde{x}_i, \tilde{G})$ is flatter in the point $(\tilde{x}_i, \tilde{G})$ than the original indifference curve $\tilde{u}_i := u_i(\tilde{x}_i, \tilde{G})$. The two indifference curves $\tilde{u}_i$ and $\tilde{u}_i'$ cannot cross twice because this would violate assumption (6). So, the indifference curve $\tilde{u}_i'$ must lie above the indifference curve $\tilde{u}_i$ right to $\tilde{x}_i$ which clearly implies that agent i's sacrifice is reduced.

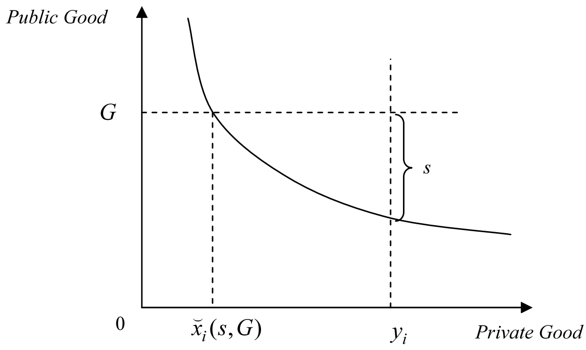
We summarize these findings as follows:

Proposition 1 *The individual sacrifice of an agent becomes higher if*

(i) *the public-good contribution, or*
(ii) *total public-good supply increases.*

The individual sacrifice of an agent is the lower

Fig. 2 Private consumption given s and G



(iii) *the higher an agent's income*, *or*
(iv) *the stronger an agent's preferences for the public good.*

4 Equal sacrifice allocations

Having developed a concept for the measurement of sacrifice it is now straightforward to characterize *equal* sacrifice allocations. This is made precise by the next definition.

Definition 2 Let an income distribution $(y_1, \ldots, y_n)$ and preferences $(u_1, \ldots, u_n)$ be given. An allocation $A = (\tilde{x}_1, \ldots, \tilde{x}_n, \tilde{G})$ is called an *equal sacrifice allocation* when there is some sacrifice level $s > 0$ such that

$$s_i^M(A) = s \quad \text{for all } i = 1, \ldots, n. \tag{7}$$

In order to show that such equal sacrifice solutions exist, we use the following construction in which we start with some public good level $G \in]0, Y[$. We then define for any sacrifice level $s \in [0, G[$ a private consumption level $\breve{x}_i(s, G)$ of agent i by letting

$$u_i\big(\breve{x}_i(s, G), G\big) = u_i(y_i, G - s). \tag{8}$$

Thus, as depicted in Fig. 2, $\breve{x}_i(s, G)$ is agent i's private consumption level when public-good supply is G and this agent should bear some sacrifice s. Since given our assumptions on preferences, all indifference curves are strictly decreasing and do not hit the G-axis, a unique private-good consumption level $\breve{x}_i(s, G) > 0$ exists for all $G \in]0, Y[$ and all $s \in [0, G[$. Obviously, $\breve{x}_i(s, G)$ is strictly decreasing in s for a given public-good level G and $\lim_{s \to G} \breve{x}_i(s, G) = 0$ holds. Moreover, for a fixed sacrifice level s, $\breve{x}_i(s, G)$ is increasing in G, since it follows from parts (i) and (ii) of Proposition 1 that otherwise the sacrifice level would increase.

Having established these properties of the function $\breve{x}_i(s, G)$, we now consider the function

$$H(s, G) := \sum_{i=1}^{n} \breve{x}_i(s, G) + G. \tag{9}$$

The function $H(s, G)$ defined by (9) describes how much aggregate income would be required if public-good supply were G and all agents $i = 1, \dots, n$ had the equal sacrifice level s. The function $H(s, G)$ is differentiable in both variables and strictly decreasing in s, and from the properties of the functions $\breve{x}_i(s, G)$, it follows that $H(0, G) = Y + G > Y$ and $\lim_{s \to G} H(s, G) = G < Y$. Thus, by continuity and monotonicity of $H(s, G)$ in G, the intermediate value theorem implies that there is a unique value of sacrifice $s^M(G)$, such that

$$H\big(s^M(G), G\big) = Y. \tag{10}$$

Hence, there exists a unique equal sacrifice allocation $(\breve{x}_1(s^M(G), G), \dots, \breve{x}_n(s^M(G), G), G)$ that fulfills the budget constraint (1) with public good supply G.

By (10), the function $s^M(G)$ is implicitly defined for all public good levels in $]0, Y[$, and since $\lim_{G \to 0} s^M(G) = 0$ and $\lim_{G \to Y} s^M(G) = Y$, this function takes on any value in this interval. Furthermore, from totally differentiating (9), we obtain

$$\frac{\partial s^M(G)}{\partial G} = -\frac{\partial H/\partial G}{\partial H/\partial s} > 0. \tag{11}$$

This inequality follows from $\partial H/\partial s < 0$ and $\partial H/\partial G > 0$, which holds since each $\breve{x}_i(s, G)$ is decreasing in s and increasing in G. As the function $s^M(G)$ is thus strictly increasing, it can be inverted. The inverse function of $s^M(G)$, which is called $G^M(s)$, then is defined on $]0, Y[$ and it is strictly increasing, also. This yields the following result.

Proposition 2 *For each $s \in [0, Y[$, there is a unique feasible allocation in which all agents have the equal individual sacrifice s.*

Proof Given s, let public-good supply be $G^M(s)$ and private consumption of agent i be $x_i^M(s) := \breve{x}_i(s, G^M(s))$. Then by the construction above, $(x_1^M(s), \dots, x_n^M(s), G^M(s))$ is a feasible allocation in which all agents have the same sacrifice level s. As $G^M(s)$ is a strictly increasing function, the sacrifice level must be different from s in any other feasible equal sacrifice allocation which shows uniqueness. □

5 The choice mechanism

Through Proposition 1 it becomes clear that there are infinitely many equal sacrifice solutions that could, depending on the sacrifice level s, be described by an "equal-sacrifice curve" $(x_1^M(s), \dots, x_n^M(s), G^M(s))$ in the $\mathbb{R}_+^{n+1}$-space.[4] We now want to show that on this curve there is one single point that gives a Pareto-optimal allocation.

For a proof, consider the marginal rates of substitution between the public good and the private good along the equal sacrifice curve, i.e., we denote

[4]See, e.g., Schlesinger and Sullivan (1986) for a similar construction in the Kolm-triangle for the two-person case.

$\pi_i^M(s) := \pi_i(x_i^M(s), G^M(s))$ for each agent $i = 1, \dots, n$ and each sacrifice level $s \in]0, Y[$. From our assumptions on preferences, we have $\lim_{s \to Y} \pi_i^M(s) = 0$ (as $\lim_{s \to Y} x_i^M(s) = 0$ and $\lim_{s \to Y} G^M(s) = Y$) and $\lim_{s \to 0} \pi_i^M(s) = \infty$ (as $\lim_{s \to 0} x_i^M(s) = y_i$ and $\lim_{s \to 0} G^M(s) = 0$). In order to apply the Samuelson rule, we now denote $\Pi^M(s) := \sum_{i=1}^n \pi_i^M(s)$ as the sum of these marginal rates of substitution. Then $\lim_{s \to Y} \Pi^M(s) = 0$ and $\lim_{s \to 0} \Pi^M(s) = \infty$ so that by the intermediate-value theorem, there is some $s^* \in]0, Y[$ for which $\Pi^M(s^*) = 1$. The feasible equal sacrifice allocation $(x_1^M(s^*), \dots, x_n^M(s^*), G^M(s^*))$ fulfills the Samuelson condition, and thus is Pareto-optimal.

In order to show that $(x_1^M(s^*), \dots, x_n^M(s^*), G^M(s^*))$ is the *unique* efficient allocation in the economy under consideration, we need a separate argument. For that, we first note that in an equal sacrifice allocation the utility levels different agents attain can never move in an opposite direction when the sacrifice level s is changed. This is obvious since—according to (2) and (7) and the definition of $G^M(s)$—the utility of each agent must change in the same direction as $G^M(s) - s$.

Now suppose that there are two different sacrifice levels s^* and s^{**} for which $\Pi^M(s^{**}) = \Pi^M(s^*) = 1$ holds, such that two Pareto-optimal allocations would exist. It is a direct consequence of our observation concerning the parallel change of all agent's utilities that in this case $u_i(x_i^M(s^*), G^M(s^*)) = u_i(x_i^M(s^{**}), G^M(s^{**}))$ for all $i = 1, \dots, n$, i.e., all agents have the same utility in both equal sacrifice solutions. Otherwise, a contradiction to the supposed Pareto optimality of the two equal sacrifice allocations would result.

Without loss of generality, $s^{**} > s^*$ may be assumed so that from strict monotonicity of the function $G^M(s)$, we get $G^M(s^{**}) > G^M(s^*)$. Having the same utility levels in both allocations thus requires $x_i^M(s^{**}) < x_i^M(s^*)$ for all agents $i = 1, \dots, n$. From the assumed normality of preferences, we then get $\pi_i^M(s^{**}) = \pi_i(x_i^M(s^{**}), G^M(s^{**})) < \pi_i(x_i^M(s^*), G^M(s^*)) = \pi_i^M(s^*)$ for all agents $i = 1, \dots, n$. This gives $1 = \Pi^M(s^{**}) = \sum_{i=1}^n \pi_i^M(s^{**}) < \sum_{i=1}^n \pi_i^M(s^*) = \Pi^M(s^*) = 1$ which is a contradiction. So we can conclude:

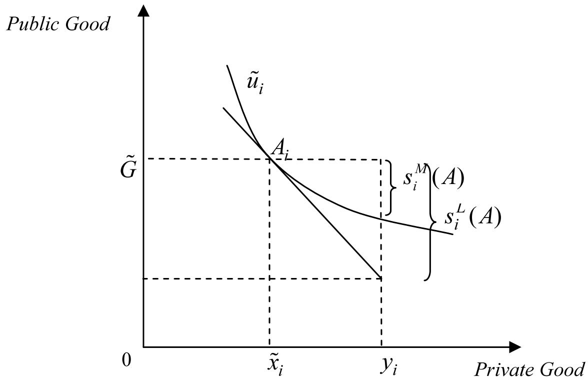
Proposition 3 *There is a unique sacrifice level s^* such that the equal sacrifice allocation $(x_1^M(s^*), \dots, x_n^M(s^*), G^M(s^*))$ is Pareto optimal.*

Using Proposition 3, the mechanism that picks an equal sacrifice solution is now characterized as follows:

Definition 3 Let a public-good economy be given by the income distribution $(y_1, \dots, y_n)$ and preferences $(u_1, \dots, u_n)$. Then the *equal sacrifice solution* for this public-good economy is defined as $(\hat{x}_1^M, \dots, \hat{x}_n^M, \hat{G}^M) = (x_1^M(\hat{s}^M), \dots, x_n^M(\hat{s}^M), G^M(\hat{s}^M))$, where the sacrifice level $\hat{s}^M := s^*$ is determined according to Proposition 3.

Given normality, the equal sacrifice solution as characterized by Definition 3 is well defined and unique.

Fig. 3 Compaaring the Moulin and the Lindahl sacrifice



6 Comparison with the literature

It is now straightforward that the equal sacrifice solution according to Definition 3 coincides with the egalitarian-equivalent allocation of the given economy (see Moulin 1987, 1995). Given an income distribution $(y_1, \ldots, y_n)$ and preferences $(u_1, \ldots, u_n)$ define $\overline{G}^M := \hat{G}^M - \hat{s}^M$. From condition (2), we then have

$$u_i\left(\hat{x}_i^M, \hat{G}^M\right) = u_i\left(y_i, \overline{G}^M\right) \tag{12}$$

so that $\overline{G}^M$ is the egalitarian-equivalent public-good supply in the sense of Moulin. Using a line of the argument different from that of Moulin, we have thus been able to link the egalitarian-equivalent solution concept to the equal sacrifice principle.[5] In this way, it becomes also possible to compare Moulin's approach with the much older Lindahl solution. To this end, we define—quite analogous to (2)—for any given feasible allocation $A = (\tilde{x}_1, \ldots, \tilde{x}_n, \tilde{G})$, an alternative *Lindahl-sacrifice* by letting

$$s_i^L(A) := \frac{\tilde{g}_i}{\pi_i(\tilde{x}_i, \tilde{G})}. \tag{13}$$

where $\tilde{g}_i = y_i - \tilde{x}_i$ again denotes agent i's contribution to the public good in A.

The sacrifice level according to (13) is measured by using the *marginal* valuation of the private good in units of the public good in position $(\tilde{x}_i, \tilde{G})$ instead of the *total* willingness to pay (see, e.g., Ebert 2003, and Ebert and Tillmann 2007, for a general discussion of the marginal valuation approach in a public goods economy). This Lindalian measurement device is visualized in Fig. 3. It is obvious from Fig. 3 that with strictly convex indifference curves $s_i^L(A) > s_i^M(A)$ is automatically implied, i.e., the level of the Lindahl sacrifice always exceeds the level of the sacrifice expounded in this paper.

It again follows from normality that the Lindahl sacrifice has the same properties as stated in Proposition 1. We can also identify equal sacrifice allocations that

[5] For justifications of the Moulin solution see—besides Moulin (1987) himself—Maniquet and Sprumont (2004).

are based on the Lindahlian sacrifice concept. The corresponding choice mechanism then picks an allocation $(\hat{x}_1^L, \ldots, \hat{x}_n^L, \hat{G}^L)$ that implies an equal Lindahl sacrifice for all agents, and as well is efficient. It has been shown in Buchholz and Peters (2007) that the Lindahl equal sacrifice allocation $(\hat{x}_1^L, \ldots, \hat{x}_n^L, \hat{G}^L)$ is identical with the standard Lindahl equilibrium that would be chosen if the agents $i = 1, \ldots, n$ acted as price-takers and agent i were confronted with the personalized Lindahl price $\hat{p}_i := \pi_i(\hat{x}_i^L, \hat{G}^L)$.[6]

Even though the sacrifice measures s_i^M and s_i^L are conceptually different, they may yield the same efficient equal sacrifice solutions under specific circumstances. This is, e.g., the case if all agents have identical Cobb–Douglas preferences. Then an application of both concepts implies that in the corresponding equal sacrifice solutions, the public-good contributions of all agents $i = 1, \ldots, n$ are proportional to their income levels y_i (see Appendix B for a detailed analysis of the Cobb–Douglas case).

7 Properties of the equal sacrifice solutions

In this section, we show that the equal sacrifice solution as characterized in this paper satisfies both the ability-to-pay principle and the benefit principle.[7] To make this precise, we first have to define exactly what these principles are to mean.

Concerning ability-to-pay, we assume that two agents j and k have identical preferences but differ with respect to their income, so that without loss of generality, $y_k > y_j$ holds. If some arbitrary choice mechanism E picks an allocation $(\hat{x}_1^E, \ldots, \hat{x}_n^E, \hat{G}^E)$ with individual public-good contributions $\hat{g}_i^E := y_i - \hat{x}_i^E$, this mechanism is said to satisfy the (weak) ability-to-pay principle if $\hat{g}_k^E \geq \hat{g}_j^E$ holds, i.e., if the richer agent k does not make a smaller contribution to the public good than the poorer agent j does.

Analogously, the (weak) benefit principle requires that given the same income level, an agent k with a stronger preference for the public good, as defined by reference to marginal willingness to pay in condition (6), should not make a smaller contribution to the public good than an agent j with a weaker preference.[8] If this condition is met, and if additionally, $y_j = y_k$ holds, the benefit principle is satisfied for a mechanism E if and only if $\hat{g}_k^E \geq \hat{g}_j^E$.

It is now a straightforward consequence of Proposition 1 that both principles are satisfied for equal sacrifice solutions: Assume that the public-good contribution of agent k would be smaller than that of agent j if the income of agent k were higher than that of agent j, or agent k's preferences for the public good were stronger than that of agent j. Combining the results of Proposition 1(i) with those in Proposition 1(iii) or (iv), respectively, implies that agent k would have to bear a smaller sacrifice than agent j which contradicts the equal sacrifice assumption.

[6] For other distributional features of the Lindahl solution see Buchholz et al. (2006).

[7] Concerning the empirical relevance of the two principles in the case of global public goods, see Barrett (2006, pp. 365–366).

[8] For a general discussion of the benefit principle in the public good context see Hines (2000) and, with a focus on the relationship between the benefit principle and ability-to-pay, Abbasian and Myles (2006).

8 Conclusion

This paper has shown how, in a standard public good economy, the venerable equal sacrifice principle can be applied to make a selection among efficient allocations. Unlike the traditional literature, however, we did not make use of cardinal measures like a sacrifice which is defined by a loss in utility. Instead, we obtained a sacrifice measure by transforming the individual expenses for the public good into public-good equivalents. The method by which this transformation was made was borrowed from the willingness-to-pay assessment well known from the contingent valuation techniques used in environmental economics. Those public-good allocations that show an equal sacrifice defined in this way and are Pareto-optimal turn out to be identical with the egalitarian-equivalent solutions as conceived by Moulin (1987). Moreover, they satisfy the ability-to-pay and the benefit principles properly defined.

The novel justification of the egalitarian-equivalent solution concept provided in this paper also makes it possible to recognize its similarity with the classical Lindahl equilibrium, since the Lindahl mechanism can also be interpreted as an application of the equal sacrifice principle. The difference, however, is that assessment of the sacrifice as made in our approach is based on *total* willingness-to-pay, or in an alternative interpretation, *average* valuation of the public good, whereas the Lindahl solution rests upon the valuation of the public-good contribution according to *marginal* willingness-to-pay. In special cases, both equal sacrifice solutions may coincide, but generally they will be different.

Measuring individual sacrifice by total valuations as in the present paper takes into account more information about individual preferences than the Lindahl approach, in which assessment of sacrifice is only based on marginal willingness-to-pay at a single point. In a world of full information, as assumed here (and also in the standard treatments of the Lindahl and the egalitarian-equivalent solution concepts), the solution described in this paper is, therefore, based on a more accurate valuation of individual sacrifice than is its Lindahl counterpart.

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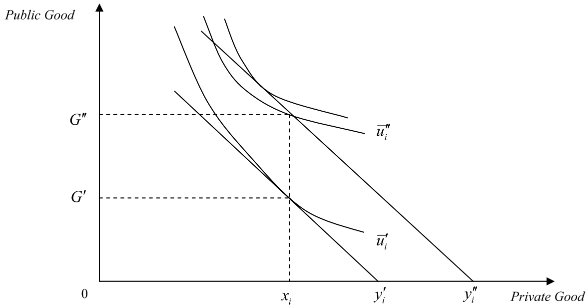
Appendix A: Steepness of indifference curves

Consider agent i and fix some level x_i of her private consumption. Let, as in the main text, $\bar{u}_i'$ and $\bar{u}_i''$ be two utility levels of agent i with $\bar{u}_i'' > \bar{u}_i'$. By G' and G'', we then denote the levels of public-good supply for which $u_i(x_i, G') = \bar{u}_i'$ and $\bar{u}_i'(x_i, G'') = \bar{u}_i''$ holds. Now assume $\varphi_i^h(x_i, \bar{u}_i'') < \varphi_i^h(x_i, \bar{u}_i')$, i.e., that the indifference curve $\bar{u}_i''$ at (x_i, G'') is flatter than the indifference curve $\bar{u}_i'$ at (x_i, G'). Thus, as depicted in Fig. 4, agent i endowed with the income $y_i'' := x_i + p_i'G''$ and confronted with the public-good price $p_i' := \varphi_i^h(x_i, \bar{u}_i')$ would demand less of the private good than x_i.

Fig. 4 Comparing public good demand



Since x_i is agent i's private good demand, given the income $y_i' := x_i + p_i' G'$ and the public good price p_i', and clearly $y_i' < y_i''$ holds, this would contradict the assumption that the private good is normal.

Since indifference curves are convex, it is a straightforward implication of this argument that indifference curves become steeper if public good consumption grows and private good consumption falls simultaneously.

Appendix B: The equal sacrifice solution in the Cobb–Douglas case

Let n agents $i = 1, \ldots, n$ be given by their income levels $y_1, \ldots, y_n$ and their Cobb–Douglas utility functions $u_i(x_i, G) = x_i G^{\rho_i}$. In the equal sacrifice solution $(\hat{x}_1^M, \ldots, \hat{x}_n^M, \hat{G}^M)$, the common equal sacrifice level $\hat{s}^M$ must satisfy $\hat{x}_i^M \cdot (\hat{G}^M)^{\rho_i} = y_i \cdot (\hat{G}^M - \hat{s}^M)^{\rho_i}$ for all agents $i = 1, \ldots, n$ which yields $\hat{s}^M = \hat{G}^M \cdot (1 - (\frac{\hat{x}_i^M}{y_i})^{\frac{1}{\rho_i}})$ for the common sacrifice level. For individual private consumption, we obtain $\hat{x}_i^M = A^{\rho_i} \cdot y_i$, where $A := \frac{\hat{G}^M - \hat{s}^M}{\hat{G}^M} < 1$ is a constant for the given public-good economy. The individual public-good contributions then are $\hat{g}_i^M = (1 - A^{\rho_i}) \cdot y_i$.

This expression clearly confirms the results of Proposition 4 in the main text for the Cobb–Douglas case. If two agents j and k have the same preferences, i.e., $\rho_j = \rho_k$ holds, but $y_k > y_j$, then $\hat{g}_k^M > \hat{g}_j^M$ so that the agent with the higher income makes a higher contribution to the public good and ability-to-pay is fulfilled. If, on the other hand, two agents j and k have the same income $y_j = y_k$, but $\rho_k > \rho_j$, we have again $\hat{g}_k^M > \hat{g}_j^M$, i.e., the agent with the stronger preference for the public good makes a higher contribution, which gives the benefit principle.

We now compare our equal sacrifice solution with the Lindahl equilibrium $(\hat{x}_1^L, \ldots, \hat{x}_n^L, \hat{G}^L)$ that results in the same situation. Here, individual public-good contributions are $\hat{g}_i^L = \frac{\rho_i}{1+\rho_i} y_i$ for agents $i = 1, \ldots, n$. When all agents have the same preferences, so that $\rho = \rho_i$ for $i = 1, \ldots, n$, it directly follows from our formulas that public-good contributions must be proportional to the individual income levels in both solutions. Since efficiency of the outcomes requires $\hat{G}^M = \hat{G}^L = \frac{\rho}{1+\rho} Y$ where $Y = \sum_{i=1}^n y_i$ is total income, both equal sacrifice solutions coincide when all agents have the same Cobb–Douglas preferences.

However, when agents have different Cobb–Douglas preferences, the equal sacrifice solution $(\hat{x}_1^M, \ldots, \hat{x}_n^M, \hat{G}^M)$ may differ from $(\hat{x}_1^L, \ldots, \hat{x}_n^L, \hat{G}^L)$. This is demonstrated by the following simple example: Let $n = 2$, $y_1 = y_2 = 1$ and $\rho_1 = 1$ and $\rho_2 = 2$. Then in Lindahl equilibrium clearly $\hat{x}_1^L = 0.5$, $\hat{x}_2^L = 0.33$, $\hat{g}_1^L = 0.5$, $\hat{g}_2^L = 0.67$, and $\hat{G}^L = 1.17$. The utility levels of the two agents are $\hat{u}_1^L = 0.58$ and $\hat{u}_1^L = 0.45$. On the other hand, in the equal sacrifice solution, $\hat{x}_2^M = (\hat{x}_1^M)^2$ holds which—combined with the feasibility condition and the Samuelson rule for efficiency—leads to the quadratic equation $3(\hat{x}_1^M)^2 + 2\hat{x}_1^M - 2 = 0$. Solving this equation for $\hat{x}_1^M$ yields $\hat{x}_1^M = 0.55$, which then implies $\hat{x}_2^M = 0.3$, $\hat{g}_1^M = 0.45$, $\hat{g}_2^M = 0.7$, and thus $\hat{G}^M = 1.15$. This shows that both solutions need not be identical. For the utility levels, we obtain $\hat{u}_1^M = 0.63$ and $\hat{u}_2^M = 0.4$ so that agent 1 with the lower preference for the public good is worse off in the Lindahl solution where for agent 2 the reverse result holds.

In the general case with nonhomogeneous Cobb–Douglas preferences, an explicit comparison between the two solutions is difficult to make since no closed form expression for the Moulin outcome exists in this case.

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