Irrigation and Drainage Systems (2006) 20: 57–68

Cropping pattern planning under water supply from multiple sources

G. TSAKIRIS & M. SPILIOTIS
Laboratory of Reclamation Works and Water Resources Management, National Technical University of Athens, 9 Iroon Polytechniou Street, Athens 15773, Greece;
E-mail: water@survey.ntua.gr

Abstract. Conventionally, irrigation development planning has been based on cropping pattern selection aiming at maximizing the revenue from irrigation activities. In the real world however, several complexities make the cropping pattern selection a more complicated mathematical problem. Of great interest is the case of water supply from multiple sources (e.g. surface and groundwater) in which a multi-criteria approach is most appropriate. Goal programming has been used in the past to solve cropping pattern selection problems, with criteria of a similar nature, the net benefit being included as a constraint. This paper presents a methodology, based on the fuzzy set theory, for enhancing the goal programming approach to solve similar problems under various sets of criteria of a different nature. In the proposed methodology the net benefit maximization is considered together with all other criteria. The methodology is illustrated using data from the Thessaly Plain in Greece.

Key words: cropping pattern, fuzzy sets, goal programming, irrigation development planning, multi-criteria approach

Introduction

Irrigation development planning was traditionally based on cropping pattern selection aiming at maximizing the revenue from irrigation activities. Due to a number of constraints and the desire to secure crop diversification, operational research techniques have been employed for finding optimal cropping patterns. Even before the 1970s, linear programming had been used as a powerful method of this selection. Soon after, due to complications in water availability pattern and the need to incorporate various criteria, multi-objective methods were proposed. Of particular interest is the approach of goal programming which incorporates a number of objectives and a number of constraints in a comprehensive manner (e.g. Tsakiris & Kiountouzis 1984; Vedula & Kummar 1996; Mohammadi 1998; Carvallo et al. 1998).

However, in most multi-criteria approaches such as in goal programming formulations there are two major shortcomings, which might result in an inability to apply these approaches. Namely the weights of the criteria should be known and agreed by all parties involved before the optimization procedure. Also, the criteria should be of the same or similar nature in order to decide upon the corresponding weights in a meaningful way.

This paper aims at overcoming these disadvantages by employing a goal programming formulation, extended by incorporating elements of the fuzzy sets theory.

The goal programming model

During recent decades, most of the single-objective problems of the past have been conceived as multi-objective problems. In accordance with this tendency, cropping pattern selection with supply from multiple water sources has been formulated as a typical goal programming problem. The aim of the goal programming methodology is to minimize the deviation from expected available water quantities, whereas the benefit from irrigation activities was incorporated in an indirect way as a constraint (Tsakiris & Kiountouzis 1984).

The standard form of the goal programming model has been presented by Charnes & Cooper (1977) and may be mathematically described as follows: Find vector $\underline{D}^{+},\ \underline{D}^{-}$ such that:

$$\text{minimize} \quad \underline{w}^{+}\underline{D}^{+} + \underline{w}^{-}\underline{D}^{-} \tag{1}$$

Subject to the constraints:

$$\begin{aligned} &A\underline{x} - I\underline{D}^{-} + I\underline{D}^{+} = \underline{g} \\ &\underline{D}^{+} \cdot \underline{D}^{-} = 0 \\ &\underline{D}^{+},\ \underline{D}^{-} \geq 0 \end{aligned} \tag{2}$$

In this formulation $\underline{w}^{+}$ and $\underline{w}^{-}$ are row vectors with non-negative constant elements representing the relative weights to be assigned to positive $\underline{D}^{+}$ and negative $\underline{D}^{-}$ column vectors of over-achievement and under-achievement of the goals, respectively. A is a matrix of coefficients, $\underline{x}$ is a column vector of the decision variables, *I* the identity matrix and $\underline{g}$ a column vector of desired "goals" to be met "as closely as possible".

According to the goal programming formulation for cropping pattern selection, the optimization problem can be expressed as follows:

$$\text{minimize} \quad \sum_{i=1}^{p} w_i \left| V_i - \sum_{j=1}^{n} R_{ij} x_j \right| \tag{3}$$

For simplification reason

$$\left.\begin{aligned} D_i^+ &= \begin{cases} \left| V_i - \sum\limits_{j=1}^{n} R_{ij} x_j \right|, & \text{when } V_i - \sum\limits_{j=1}^{n} R_{ij} x_j > 0 \\ 0, & \text{otherwise} \end{cases} \\ D_i^- &= \begin{cases} \left| V_i - \sum\limits_{j=1}^{n} R_{ij} x_j \right|, & \text{when } V_i - \sum\limits_{j=1}^{n} R_{ij} x_j < 0 \\ 0, & \text{otherwise} \end{cases} \end{aligned}\right\} \tag{4}$$

where n is the number of the crops to be irrigated, p the number of months covered by the model, x_j the land parcel allocated to the jth crop, R_{ij} the irrigation water requirements of the jth crop during the ith month, V_i the available volume of water from the main source (e.g. surface water) during the ith month, and w_i the relative weight to be assigned to positive and negative deviations of the irrigation water requirements from the water availability levels (V_i-goal). In physical terms the weight w_i can be interpreted as a penalty cost.

Taking into account that the negative penalty cost w_i^- is different from the positive penalty cost w_i^+, one can replace the aforementioned minimization problem with the following:
Find x_j such that

$$\text{minimise} \quad \sum_{i=1}^{p} (w_i^+ D_i^+ + w_i^- D_i^-) \tag{5}$$

Subject to the constraints:

$$\begin{aligned} & D_i^+, \quad D_i^- \geq 0 \\ & \sum_{j=1}^{n} (R_{ij} x_j + D_i^+ - D_i^-) = V_i \\ & x_j = 0, \quad j = 1(1)n \quad \text{and} \quad i = 1(1)p \end{aligned} \tag{6}$$

According to the goal programming methodology other crisp constraints (constraints with no uncertainty) should be added such as the land constraints.

From the above formulation it can be concluded that the economic aspect of the problem, may be expressed by incorporating another crisp constraint. However it does not seem wise to add another goal, which arises from the net benefit concept due to the following reasons.

(1) The two sets of goals are of an entirely different nature.
(2) The weights to be assigned to the two sets of goals are unknown, or cannot be rationally decided.

Apart from that, the economic aspect of the irrigation planning should be one of the main criteria on which the decision will be based, and therefore it is not sufficient to be included as an additional constraint. For the above reasons the goal programming approach will be enhanced by using elements of the fuzzy set theory.

Fuzzy set theory to enhance multi-objective analysis

In general, a multi-objective programming model is formulated for maximizing (or minimizing) several objectives simultaneously, subject to a set of constraints.

According to fuzzy set theory various types of membership functions can be used to support the fuzzy analytical framework, the most popular of which are those of linear type. Here, two types of fuzzy objectives may be separately formulated. For these objectives non-decreasing and non-increasing linear membership functions are assumed, respectively (Chang et al., 1997) (Figure 1).

The membership function for the maximization of the objective function Z_k should be achieved as follows:

$$\mu_k(x) = \begin{cases} 1 & \text{if} \quad Z_k(x) \geq U_k \\ 0 & \text{if} \quad Z_k(x) \leq L_k \\ \frac{Z_k(x)-L_k}{U_k-L_k} & \text{if} \quad L_k \leq Z_k(x) \leq U_k \end{cases} \tag{7}$$

where L_k is the lowest acceptable level and U_k the aspired level for the objective function Z_k, which should be maximized (Zimmermman 1984; Tsakiris & Spiliotis 2002; & Jairaj & Vedula 2000). The membership function for the minimization of the objective function is expressed in a similar way.

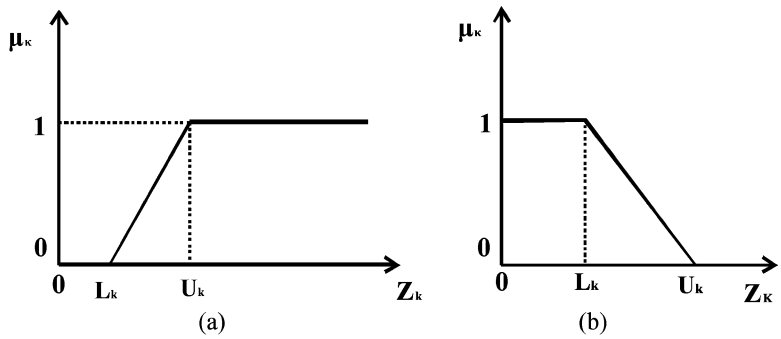

Figure 1. Diagrammatic representation of fuzzy linear membership function. The maximization (a) and minimization (b) linear membership functions.

In most practical applications, the lowest acceptable level and the aspired level can be determined from the ideal solutions of the multi-objective programming problem. At first, the multi-objective problem is solved for each objective separately. Let $(x^{(1)*},\ x^{(2)*}, \ldots, x^{(K)*})$ be the matrix of the optimal values achieved for each solution.

Then the values of all the K-objective functions can be calculated at all these K optimal solutions. Thus a pay-off matrix P is expressed as follows:

$$P = \begin{bmatrix} Z_1\big(x^{(1)*}\big), Z_1\big(x^{(2)*}\big), \ldots, Z_1\big(x^{(K)*}\big) \\ Z_2\big(x^{(1)*}\big), Z_2\big(x^{(2)*}\big), \ldots, Z_2\big(x^{(K)*}\big) \\ \vdots \\ Z_K\big(x^{(1)*}\big), Z_K\big(x^{(2)*}\big), \ldots, Z_K\big(x^{(K)*}\big) \end{bmatrix} \tag{8}$$

The lowest acceptable and the aspired levels are selected from the above matrix (Li & Lai 2000). The elements of the diagonal of the matrix represent the K aspired levels for each objective.

Having all the individual membership functions, the problem is to determine a global evaluation of x, through a fuzzy operator M_w with respect to all objectives (Li & Lai 2000). Thus the global membership function $\mu(x)$ becomes:

$$\mu(x) = M_w(\mu_1(x),\ \mu_2(x),\ \ldots,\ \mu_K(x)) \tag{9}$$

which shows to what degree each decision $x \in X$ satisfies all the objectives. For this purpose, various operators M_w can be used. In this study the min intersection is implemented.

In fuzzy set theory, the fuzzy mathematical programming aims at satisfying the fuzzy objectives and constraints, and a decision in a fuzzy environment is thus defined as the intersection of those membership functions corresponding to the fuzzy objectives and constraints.

With the use of membership functions and the min fuzzy intersection it holds:

$$\mu(x) = \min(\mu_1(x), \ldots, \mu_K(x)) \tag{10}$$

Since the decision maker should conclude in a crisp decision proposal, it seems appropriate that he should suggest the dividend with the highest degree of membership function in the fuzzy set decision. Therefore,

$$\max\ \mu = \max\{\min(\mu_1(x), \ldots, \mu_K(x))\} \tag{11}$$

Mathematically, using an auxiliary variable λ, the problem can be expressed as follows (Zimmermann 1984):

$$\max \quad \lambda \tag{12}$$

subject to $\mu_k(x) \geq \lambda$

$$\begin{aligned} k &= 1(1)K \\ \lambda &\in [0, 1] \\ x &\in X \end{aligned} \tag{13}$$

where λ expresses the common degree of satisfaction for all objectives. It is noted that the crisp constraints remain in the above formulation.

It can be concluded that by using the above procedure a compromise solution between several objectives can be found.

Two objective functions are incorporated in this study. Namely the maximization of net benefit and the minimization of deviations from the available water quantities (Equation (3)). Solving each problem separately for each objective, the pay-off matrix can be determined.

Application

Data from Thessaly Plain in Greece were used to demonstrate the proposed methodology. An area of 1000 ha is cultivated with the main crops being cotton, corn and sugar beet. Irrigation water is provided from two sources, that is, surface water without storage, and groundwater. Surface water availability is limited and is calculated after the subtraction of the minimum allowable discharge devoted to sustain the downstream ecosystem (Figure 2).

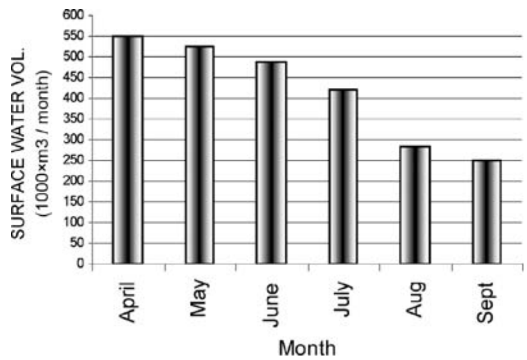

Figure 2. Monthly volume of surface water (main source) available (main source) (m^3/month).

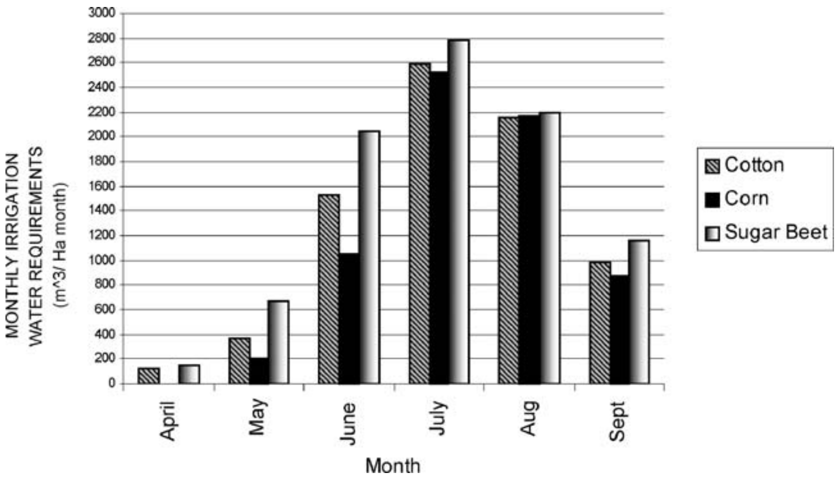

Figure 3. Monthly irrigation water requirements of the crops (m^3/ha).

Irrigation water requirements of the irrigated crops R_j are presented in Figure 3.

It should be mentioned that the cropping pattern will be selected, aiming at maximization of benefit from the irrigation activities, as well as the maximization of exploitation of available surface water quantities.

The problem is formulated following the methodology previously presented.

The decision variables x_j are the parcels of land allocated to each crop: cotton, corn and sugar beet respectively.

For every month the water quantity balance is calculated. If the main source does not meet the monthly irrigation water requirement, then pumping from the groundwater reservoir will cover the shortage. In this case, the monthly groundwater quantity which is used is equal to D_i^-. If the monthly requirements are less than the available surface quantity then a surplus quantity will appear, as D_i^+.

The goal constraints for the ith month for the three crops become:

$$\sum_{j=1}^{3} R_{ij}\, x_j + D_i^+ - D_i^- = V_i \tag{14}$$

In general case the net benefit which should be maximized is the net benefit from the irrigated crops minus the extra cost which is related to pumping from the groundwater reservoir.

$$\max \left(\sum_{j=1}^{3} (B_j x_j) - \sum_{i=1}^{6} (D_i^- \cdot w_i^-) \right) \tag{15}$$

Table 1. The net average revenue from irrigated crops in the area.

Crop	Revenue (€/ha)
Cotton	2824.7
Corn	1534.0
Sugar beet	989.0

where B_j is the net benefit from the jth crop (€/ha) under irrigation from the surface water (Table 1), and w_i^- the cost for pumping each unit volume of groundwater.

The cost per cubic metre of water pumped from the groundwater reservoir was calculated from local data as the average from a battery of boreholes in the area and was found to be

$$w_{\text{pump}} = 0.16\,€/\text{m}^3. \tag{16}$$

The maximum exploitation of available surface water, which may be also considered as an environmental goal, may be achieved by minimizing the deviation of used water from the surface water available each month.

The penalty cost w^- is chosen to be the cost of pumping of the unit volume of groundwater and is associated with the negative deviation. It is interesting to note that, the weight assigned to the weight of positive deviation is chosen to be equal to the opportunity cost. This is turn can be estimated as the weighted average of the net benefit from irrigation activities from the land equally allocated to three crops.

$$w_i^+ = r_i = \frac{\sum_{j=1}^{3} x_j r_{ij}}{x_0} \tag{17}$$

It should be mentioned that r_{ij} is estimated by disaggregation of the crop yield using a dated production function for each crop (Dooremdos & Kassam 1979; Tsakiris 1985). The estimated cost w_i^+ is presented in Table 2.

Based on the above, the objective of minimizing water quantity deviations becomes:

$$\min \sum_{i=1}^{6} (r_i D_i^+ + w_{\text{pump}} D_i^-) \tag{18}$$

Other crisp constraints may be also formulated as follows:

Table 2. Estimated monthly cost w^+ of each m^3 of water from the main source.

Month	€/m^3
April	1.84
May	1.09
June	0.25
July	0.14
August	0.16
September	0.16

Land availability constraint

The sum of the land parcels allocated to various crops should be equal to the total available area for irrigation, namely

$$\sum_{j=1}^{3} x_j = x_0 = 1000\,(\text{ha}) \tag{19}$$

Land allocation constraints

Management and crop diversity requirement considerations set a maximum and a minimum irrigated acreage allocated to each crop:

$$\delta_j x_0 \le x_j \le \mu_j x_0, \quad j = 1,\ 2,\ 3 \tag{20}$$

where δ_j and μ_j are fractions of irrigated area allocated to the *j*th crop.

For the purpose of this study δ_j was taken as 0.1 and $\mu_j = 0.7$.

The problem formulated above, can now be solved according to the following steps.

I. Step 1: The problem is solved separately for each objective.
II. Step 2: The membership function for each goal is determined with the use of a pay-off table produced in step 1 (Table 3).

Table 3. The pay-off table.

Goal	Net benefit (NB)	Environmental penalty (EP)
Max NB	1,459,102€ ($\mu = 1$)	1,919,817€ ($\mu = 0$)
Min EP	644,875€ ($\mu = 0$)	1,779,773€ ($\mu = 1$)

Therefore, the membership function of net benefit takes the following form:

$$\mu_1(\mathrm{NB}) = \begin{cases} 0 & \text{if} \quad \mathrm{NB} \leq 644,875 \\ 1 & \text{if} \quad \mathrm{NB} \geq 1,459,102 \\ \frac{\mathrm{NB}-644,875}{1,459,102-644,875} & \text{if} \quad 644,875 \leq \mathrm{NB} \leq 1,459,102 \end{cases} \tag{21}$$

Also the membership function of the deviations from available surface water (environmental penalty minimization) takes the following form:

$$\mu_2(\mathrm{EP}) = \begin{cases} 1 & \text{if} \quad \mathrm{EP} \leq 1,779,773 \\ 0 & \text{if} \quad \mathrm{EP} \geq 1,919,817 \\ 1 - \frac{\mathrm{EP}-1,779,773}{1,919,817-1,779,773} & \text{if} \quad 1,779,773 \leq \mathrm{EP} \leq 1,919,817 \end{cases} \tag{22}$$

III. Step 3: Having selected the min-section aggregator and using Equation (12), the problem can now be formulated as:

$$\max \quad \lambda. \tag{23}$$

Subject to the following constraints.

- Constraints based on the fuzzy multi-criteria analysis:

$$\mu_1(NB) \geq \lambda, \quad \mu_2(EP) \geq \lambda. \tag{24}$$

- Crisp constraints for the land parcels:

$$x_1 + x_2 + x_3 = 1000, \quad 0.1\, x_0 \leq x_j \leq 0.7\, x_0, \quad j = 1(1)3. \tag{25}$$

- Goal constraints on water deviations (Equation (14)).

The solution achieved is:

$$\lambda = 0.592, \quad x_1 = 572.9\,\text{ha}, \quad x_2 = 100\,\text{ha}, \quad x_3 = 327.1\,\text{ha},$$
$$\mathrm{NB} = 1,127,219€, \quad \mathrm{EP} = 1,836,849€.$$

Also the positive and the negative deviations realized are in m^3:

$$D_1^+ = 429,204, \quad D_2^+ = 72,597, \quad D_3^+ = D_4^+ = D_5^+ = D_6^+ = 0,$$
$$D_1^- = D_2^- = 0, \quad D_3^- = 1,161,522, \quad D_4^- = 2,223,818,$$
$$D_5^- = 1,882,869, \quad D_6^- = 781,686.3$$

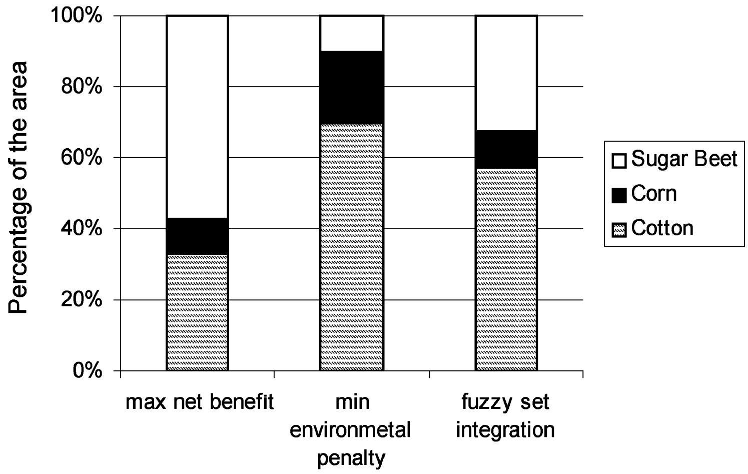

Figure 4. Comparison of solutions.

It is interesting to see that the results obtained seem more balanced between the two different approaches, which give emphasis to both the net benefit maximization from irrigation activities and the rational use of the available water from the two sources. This can be seen when the results are compared with the pay-off table (Table 3).

Figure 4 illustrates the cropping pattern as calculated based on:

(a) max net benefit;
(b) min deviations from available surface water;
(c) fuzzy set integration.

Concluding remarks

Cropping pattern planning in irrigated agriculture has been traditionally based on the concept of maximization of net benefit. However due to the involvement of various other factors in the decision making, multi-objective methods as the Goal Programming approach have been proposed in the past, mainly in case with multiple water sources. In these attempts criteria of similar nature were considered and the net benefit was included as a constraint rather than as a part of the objective function.

In this paper the integration of the above two approaches was demonstrated by using elements of the fuzzy set theory. The main advantage of the proposed methodology is that it avoids the subjectivity of assigning weights to the criteria of different nature. The methodology follows a well-defined procedure and reaches meaningful results.

A useful extension of the methodology can be devised by using other multi-objective methods and by incorporating other fuzzy aggregators from those used in this study.

References

Carvallo H., Holzapfel E., Lopez M. & Marino M. 1998. Irrigated cropping optimization. *Journal of Irrigation and Drainage Engineering* 124: 67–72.

Charnes A. & Cooper W. 1977. Goal programming and multiple objective function. *European Journal of Operational Research* 1: 39–54.

Dooremdos J. & Kassam A.S. 1979. Yield response to water. FAO Irrigation and Drainage Paper, No. 33, Rome.

Kindler J. & Tyszewski S. 1995. On the value of fuzzy concepts in hydrology and water resources management. In Z. Kundzewicz (Ed) *New Uncertainty Concepts in Hydrology and Water Resources*, Cambridge University Press, pp. 126–132.

Klir G. & Yuan B.T. 1995. *Fuzzy Sets and Fuzzy Logic Theory and its Applications*. Prentice Hall.

Li L. & Lai K.K. 2000. A fuzzy approach to the multiobjective transportation problem. *Computer and Operation Research* 27: 43–57.

Chang N. B., Wen C.G. & Chen Y.L. 1997. A fuzzy multi-objective programming approach for optimal management of the reservoir watershed. *European Journal of Operational Research* 99: 289–302.

Mohammadi E.M. 1998. Irrigation planning integrated approach. *Journal of Water Resources Planning and Management* 124: 272–279.

Papadopoulos B. & Sirpi M. 1999. Similarities in fuzzy regression models. *Journal of Optimization Theory and Applications* 102(2): 373–383.

Tsakiris G. 1985. Evaluating the effect of non-uniform and deficient irrigation. Part 1. *Advances in Water Resources* 8: 77–81.

Tsakiris G. & Kiountouzis E. 1984. Optimizing the interbasin water transfer for irrigation development. In G. Tsakiris (Ed) *Proceeding of the Fifth International Conference on Water Resources Planning and Management`Water in the Year 2000*', pp. 4.13–4.24.

Tsakiris G. & Spiliotis M. 2002. *A methodology for solving water resources management problems using fuzzy linear programming*. Research Monography, Laboratory of Reclamation Works and Water Resources Management, NTUA, pp. 1–24 (in Greek).

Tsakiris G. & Spiliotis M. 2002. Water allocation based on fuzzy objective function and fuzzy constraints. In G. Tsakiris (Ed) *Proceedings of the Fifth International Conference of EWRA*, `*Water Resources Management in the Era of Transition*', pp. 252–267.

Vedula S. & Kumar N. 1996. An integrated model for optimal reservoir operation for irrigation of multiple crops. *Water Resources Research* 34: 1101–1108.

Jairaj P.G. & Vedula S. 2000. Multireservoir system optimization using fuzzy mathematical programming. *Water Resources Management* 14: 457–472.

Zimmermann H.J. 1984. *Fuzzy Set Theory and Decision Analysis*. Elsevier, Amsterdam.

Zimmermann H.J. 1991. *Fuzzy Set Theory and its Applications*. Kluwer Academic Publishers.