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INNOVATIVE DESIGN AND EQUIVALENT MECHANICAL PROPERTIES OF A NEW COMBINED HONEYCOMB STRUCTURE*

X. Li, R. He, X. Xu, R. Li, and Y. Wang

A new type of combined honeycomb structure produced by optimally organizing the hexagons and square unit cells is proposed. The related equivalent elastic parameters are studied based on the strain energy equivalent theory. A numerical simulation method is used to verify the correctness of the equivalent mechanical model. A good agreement between the simulation values and theoretical solutions is observed. The study of mechanical properties of the combined honeycomb structure shows the great advantages in mechanical properties as compared with the traditional hexagonal honeycomb structure. With the same equivalent density, the in-plane equivalent elastic modulus of the new combined honeycomb is improved to 22%, the shear modulus enhanced to 95%. The external equivalent elastic modulus and shear modulus are improved to 33% and 29%, respectively. The carried out study provides a theoretical basis for the combined honeycomb structure's further research and also gives a new way to improve the current honeycomb sandwich.

Keywords: combined honeycomb structure, equivalent elastic parameters, strain energy equivalent theory, numerical simulation

1. Introduction. As compared with traditional metal materials, sandwich structures show great advantages in weight, strength and stiffness, which make them have great value in engineering applications [1]. With continuous promotion and application in engineering field, people raise higher requirements of sandwich structure in their comprehensive mechanical performance, the traditional sandwich structures including honeycomb sandwich structures have gradually failed to meet the engineering demands, so there is an urgent need to develop the new ones. Currently, the Kagome and X-core structures have caused many researchers' interests and researches, these new honeycomb structures show their respective advantages to some extent [2], but they can't be widely promoted and applied soon because of their complicated core structures. For now, the honeycomb sandwich has been one of the most mature sandwich structures in structural design and processing technology, and the related researches are still going on [3–8]. Based on the background above as well as the perspective of processing technology and economy, there is still great engineering value to make improvements and innovation for the current mature honeycomb sandwich. According to the characteristic of square and hexagonal unit cell, this paper creatively combines these two type cells into a new combined unit cell and designs a new combined honeycomb structure. This combination makes the new honeycomb structure have great advantages in mechanical properties, and also provides more space for the engineering designers to make further optimization and design.

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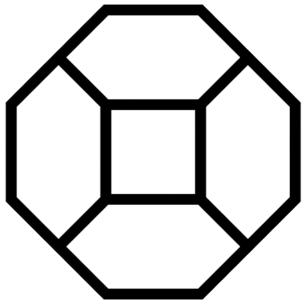

Fig. 1

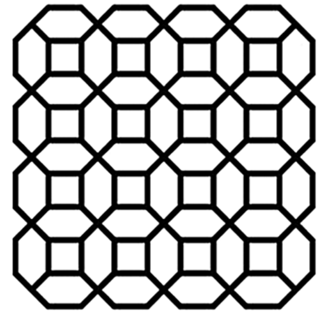

Fig. 2

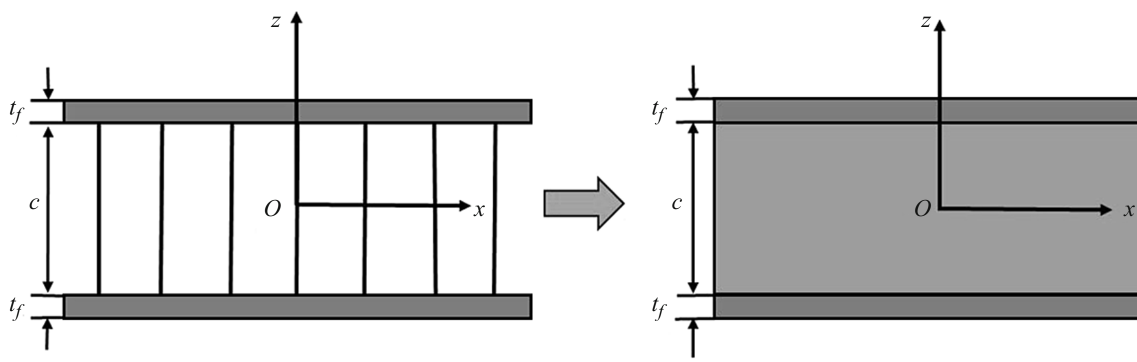

Fig. 3

2. Innovative Design of the New Combined Honeycomb Structure. Honeycomb core is the most important part in the honeycomb sandwich structure, a reasonable honeycomb core structure could greatly reduce the weight and enhance the sandwich structure's mechanical properties. With rapid development of processing technology, honeycomb structures are designed into a variety of forms to meet on the engineering demands. Based on the previous researches, the authors in this paper puts forward a new type of combined honeycomb structure from the perspective of bionics. The unit cell of the combined honeycomb structure is got by the optimizing arrangement of hexagonal and square unit cells, which is shown as Fig. 1, and the new combined honeycomb structure is designed as Fig. 2.

3. Mechanical Properties and Numerical Simulation of the New Combined Honeycomb Structure. In order to better study on the mechanical properties of the new honeycomb structure, it is necessary to establish an equivalent model which can reflect both the microscopic and macroscopic properties of the sandwich structure. So this paper introduces the concept of the equivalent mechanical model of sandwich structure. In this model, a homogeneous orthogonal layer was established and this layer has equivalent mechanical properties to the original honeycomb structure, the equivalent process is shown as Fig. 3.

At present, the researches on the mechanical equivalent model of sandwich structure are mainly focused on the honeycomb structure. Scholars represented by Gibson have done some work around the mechanical properties and established a variety of analysis models of honeycomb sandwich structures [9–21]. To determine the elastic constants of the mechanical equivalent model of the sandwich structure, Gibson proposed the classical cell theory. Now, most of the researches on the equivalent elastic constant of honeycomb structure are based on the classical cell theory. Based on the existing researches, this paper analyzes the mechanical equivalent model of the combined honeycomb structure through the energy equivalent theory, and the geometry size of unit cell is shown as Fig. 4.

In order to analyze the equivalent mechanical properties of the new combined honeycomb structure, we need to simplify the original unit cell, and the representative structural unit is finally got as Fig. 5.

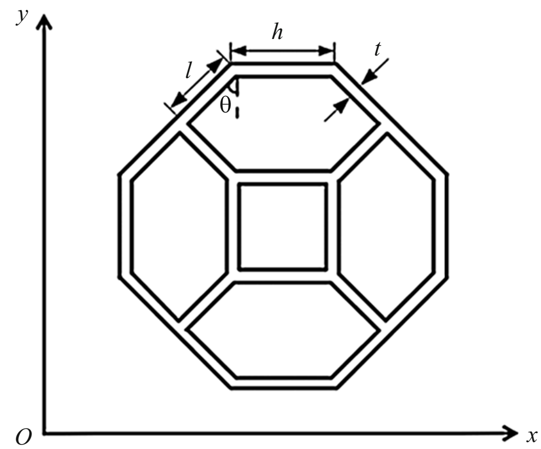

Fig. 4

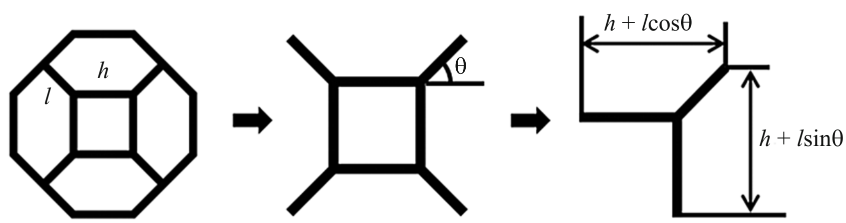

Fig. 5

3.1. Theoretical Solutions of the Combined Honeycomb Structure in Mechanical Properties.

3.1.1. The Equivalent Elastic Modulus in X Axis Direction. The related equivalent elastic parameters of the new combined honeycomb structure are deduced according to basic materials mechanics principles. Assuming that node *A* is in fixed condition, when the structure subjected to load in the *X* axis direction, the deformation mode can be shown as Fig. 6

$$\sum M_A = 0, \tag{1}$$

$$-M - M + P_x l \sin\theta = 0, \tag{2}$$

$$M = \frac{1}{2} P_x l \sin\theta, \tag{3}$$

$$P_x = \sigma_{cx} A_x = \sigma_{cx}(h + l\sin\theta)b, \tag{4}$$

where M is the bending moment of the cell node, P_x is the total force in the X axis direction, A_x is the equivalent area of the representative structural unit in the X axis direction, σ_{cx} is the equivalent stress in the X axis direction, b is the thickness of the honeycomb core.

The total equivalent strain energy in the representative structural unit can be expressed by the formula

$$\overline{U} = \frac{1}{2}\frac{\sigma_{cx}^2}{E_{cx}} V = \frac{1}{2}(\beta + \sin\theta)(\beta + \cos\theta)\frac{\sigma_{cx}^2}{E_{cx}} bl^2, \tag{5}$$

$$V = b(h + l\sin\theta)(h + l\cos\theta), \tag{6}$$

where E_{cx} is the equivalent elastic modulus of the combined honeycomb structure in the X axis direction, $\beta = h / l$.

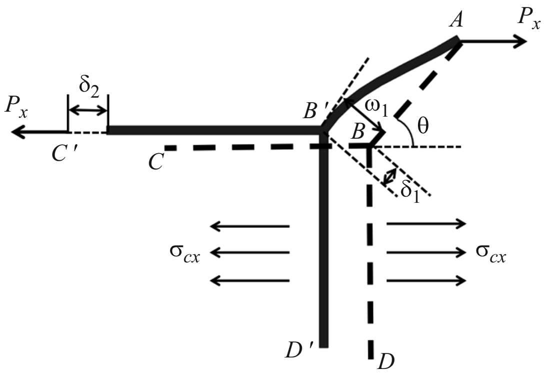

Fig. 6

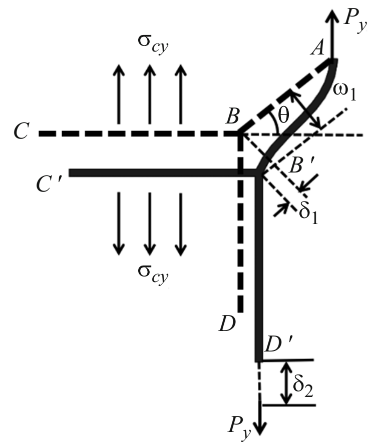

Fig. 7

The total strain energy in the representative structural unit is the sum of the strain energy of the cell wall *AB* and *BC*. According to Fig. 6, there exist tensile strain and bending strain in cell wall *AB*. The bending strain energy in the cell wall *AB* is

$$W_{AB1} = \int_0^l \frac{(P_x x \sin\theta - M)^2}{2E_s I} dx = \frac{P_x^2 \sin^2\theta}{24 E_s I} l^3. \tag{7}$$

The tensile strain energy in the cell wall *AB* is

$$W_{AB2} = \frac{N^2}{2E_s A_{AB}} l = \frac{P_x^2 \cos^2\theta}{2E_s A_{AB}} l. \tag{8}$$

The cell wall *BC* is only subjected to forces in the *X* axis direction, so that there only exists tensile strain energy in the cell wall *BC*, which can be expressed as

$$W_{BC} = \frac{N^2}{2E_s A} h = \frac{P_x^2}{2E_s A} h, \tag{9}$$

where $I = \frac{1}{12} bt^3$, $A = bt$, $\beta = h / l$. Then the total strain energy is

$$U = W_{AB1} + W_{AB2} + W_{BC} = \frac{P_x^2 l(\sin^2\theta l^2 + \cos^2\theta t^2 + \beta t^2)}{2E_s bt^3}. \tag{10}$$

According to the energy equivalent theory

$$\overline{U} = U, \tag{11}$$

$$\frac{1}{2}(\beta + \sin\theta)(\beta + \cos\theta)\frac{\sigma_{cx}^2}{E_{cx}} bl^2 = \frac{P_x^2 l(\sin^2\theta l^2 + \cos^2\theta t^2 + \beta t^2)}{2E_s bt^3}. \tag{12}$$

According to the equations above, the equivalent elastic modulus of the combined honeycomb structure in the *X* axis direction can be deduced as

$$E_{cx} = E_s \frac{t^3}{l^3} \frac{\beta + \cos\theta}{\beta + \sin\theta} \times \frac{1}{\sin^2\theta + \cos\theta \times t^2 / l^2 + \beta \times t^2 / l^2}. \tag{13}$$

3.1.2. The Equivalent Elastic Modulus in the Y Axis Direction. When the structure is only subjected to the load in the *Y* axis direction, the deformation mode is shown in Fig. 7. The total equivalent strain energy in the representative structural unit is

$$\overline{U} = \frac{1}{2}\frac{\sigma_{cy}^2}{E_{cy}}V = \frac{1}{2}(\beta + \sin\theta)(\beta + \cos\theta)\frac{\sigma_{cy}^2}{E_{cy}}bl^2. \tag{14}$$

The bending strain energy in the cell wall *AB* can be expressed as

$$W_{AB1} = \int_0^l \frac{(P_y y\cos\theta - M)^2}{2E_s I}dy = \frac{P_y^2\cos^2\theta}{24E_s I}l^3. \tag{15}$$

The tensile strain energy in the cell wall *AB* is

$$W_{AB2} = \frac{N^2}{2E_s A}l = \frac{P_y^2\sin^2\theta}{2E_s A}l. \tag{16}$$

The cell wall BD is only subjected to force in the *Y* axis direction, so that the tensile strain energy in the cell wall *BD* is

$$W_{BD} = \frac{N^2}{2E_s A}h = \frac{P_y^2}{2E_s A}h. \tag{17}$$

Therefore, the total strain energy is

$$U = W_{AB1} + W_{AB2} + W_{BD} = \frac{P_y^2 l(\cos^2\theta l^2 + \sin^2\theta t^2 + \beta t^2)}{2E_s bt^3}. \tag{18}$$

According to the energy equivalent theory, we have

$$\overline{U} = U, \tag{19}$$

$$\frac{1}{2}(\beta + \sin\theta)(\beta + \cos\theta)\frac{\sigma_{cy}^2}{E_{cy}}bl^2 = \frac{P_y^2 l(\cos^2\theta l^2 + \sin^2\theta t^2 + \beta t^2)}{2E_s bt^3}. \tag{20}$$

According to the equations above, the equivalent elastic modulus of the combined honeycomb structure in the *Y* axis direction can be expressed as

$$E_{cy} = E_s\frac{t^3}{l^3}\frac{\beta + \sin\theta}{\beta + \cos\theta} \times \frac{1}{\cos^2\theta + (\sin^2\theta + \beta)t^2/l^2}. \tag{21}$$

According to the theory put forward by Gibson [11], the in-plane equivalent shear modulus of the combined honeycomb structure can be expressed as

$$G_{cxy} = E_s\frac{t^3}{l^3}\frac{(\beta + \sin\theta)}{(2\beta + 1)\beta^2\cos\theta}, \tag{22}$$

$$G_{cyx} = E_s\frac{t^3}{l^3}\frac{(\beta + \cos\theta)}{(2\beta + 1)\beta^2\sin\theta}. \tag{23}$$

3.1.3. The External Equivalent Elastic Modulus. The representative structural unit subjected to load in the *Z* axis direction is schematized in Fig. 8.

If the pressure on the cell wall of this unit is *p*, then the total force on the cell wall is

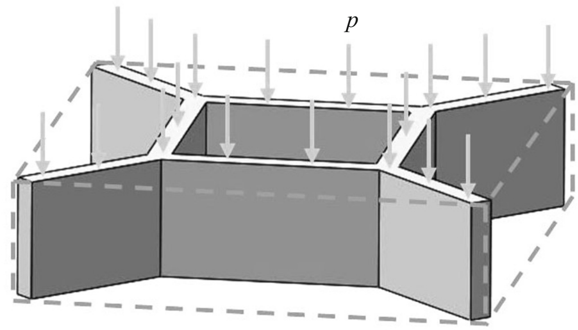

Fig. 8

$$F_c = 4pt(l+h). \tag{24}$$

The tensile strain of the cell wall is

$$\varepsilon_c = \frac{p}{E_s}, \tag{25}$$

where E_s is the elastic modulus of the basic material.

The equivalent pressure of the equivalent unit in the Z axis direction is

$$p_{cz} = \frac{F_c}{S} = \frac{4pt(l+h)}{(h+2l\sin\theta)(h+2l\cos\theta)}. \tag{26}$$

The equivalent strain of the equivalent unit in the Z axis direction is

$$\varepsilon_{cz} = \frac{p_{cz}}{E_{cz}}, \tag{27}$$

where E_{cz} is the equivalent elastic modulus of the combined honeycomb structure. Since the equivalent strain is equal to the actual strain, the following equation is easy to get:

$$\varepsilon_c = \varepsilon_{cz}. \tag{28}$$

From the equations above, the external equivalent elastic modulus of the combined honeycomb structure can be deduced as

$$E_{cz} = \frac{4(l+h)tE_s}{(h+2l\sin\theta)(h+2l\cos\theta)}. \tag{29}$$

3.1.4. The External Equivalent Shear Modulus. When the combined honeycomb structure is subjected to the external shear force, the shear flow in the structure can be shown as Fig. 9.

Taking the representative structural unit as the research subject, when the structure is subjected to the external shear force Q_{yz}, the shear flow in the structural unit is shown as Fig. 10.

The shear force of the representative structural unit is

$$Q_c = (2h+4l)\tau_c t, \tag{30}$$

where $\tau_c t$ is the shear flow in the representative structural unit.

The strain energy per volume of the cell wall is

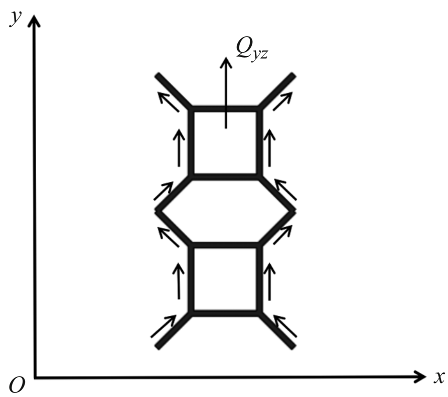

Fig. 9

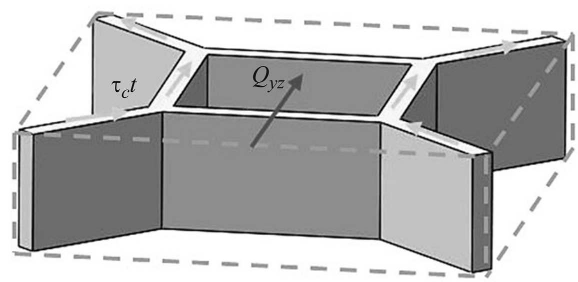

Fig. 10

$$u_c = \frac{1}{2}\tau_c \gamma_{yz} = \frac{\tau_c^2}{2G_c}, \tag{31}$$

where G_c is the shear modulus of the basic material.

Total strain energy of the representative structural unit can be expressed by the formula

$$U_C = u_c v_c = \frac{Q_c^2 b}{(4h + 8l)G_c t}. \tag{32}$$

Treating the representative structural unit as a homogeneous body and supposing that the unit is subjected to equivalent shear stress τ_{yz}, we obtain the external shear force Q_{yz}:

$$Q_{yz} = \tau_{yz}(h + 2l\sin\theta)(h + 2l\cos\theta). \tag{33}$$

If the external equivalent shear modulus of this equivalent unit is G_{cyz}, then the equivalent strain energy of the representative structural unit is

$$U_E = u_E \times v_E = \frac{\tau_{yz}^2}{2G_{cyz}} \times (h + 2l\cos\theta) \times (h + 2l\sin\theta) \times b = \frac{Q_{yz}^2 b}{2(h + 2l\sin\theta) \times (h + 2l\cos\theta) G_{cyz}}. \tag{34}$$

According to the energy equivalent theory,

$$U_E = U_C, \tag{35}$$

$$Q_{yz} = 2\tau_c th + 4\tau_c tl\sin\theta = 2\tau_c t(h + 2l\sin\theta). \tag{36}$$

The external equivalent shear modulus honeycomb structure G_{cyz} can be given by the following expression:

$$G_{cyz} = \frac{2G_c(h + 2l\sin\theta)t}{(2l + h)(h + 2l\cos\theta)}. \tag{37}$$

In the same way, the external equivalent shear modulus honeycomb structure G_{cxz} can be deduced as

$$G_{cxz} = \frac{2G_c(h + 2l\cos\theta)t}{(2l + h)(h + 2l\sin\theta)}. \tag{38}$$

3.1.5. Equivalent Density. The mass of the representative structural unit is

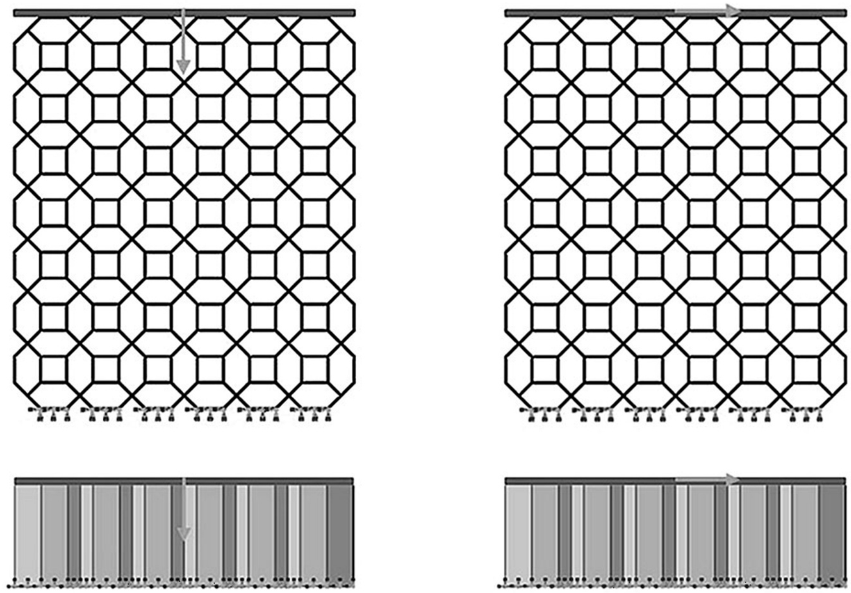

Fig. 11

$$m_1 = 4bt(h + l)\rho_s. \tag{39}$$

The equivalent volume of the representative structural unit is

$$V_e = b(h + 2l\sin\theta)(h + 2l\cos\theta). \tag{40}$$

The equivalent density of the combined honeycomb is

$$\rho_c = \frac{m_1}{V_e} = \frac{4\rho_s t(\beta + 1)}{l(\beta + 2\sin\theta)(\beta + 2\cos\theta)}. \tag{41}$$

In summary, the mechanical properties of the combined honeycomb structure can be expressed as

$$E_{cx} = E_s \frac{t^3}{l^3} \frac{\beta + \cos\theta}{\beta + \sin\theta} \times \frac{1}{\sin^2\theta + \cos^2\theta \times t^2 / l^2 + \beta \times t^2 / l^2}, \tag{42}$$

$$E_{cy} = E_s \frac{t^3}{l^3} \frac{\beta + \sin\theta}{\beta + \cos\theta} \times \frac{1}{\cos^2\theta + \sin^2\theta \times t^2 / l^2 + \beta \times t^2 / l^2}, \tag{43}$$

$$G_{cxy} = E_s \frac{t^3}{l^3} \frac{(\beta + \sin\theta)}{(2\beta + 1)\beta^2 \cos\theta}, \tag{44}$$

$$G_{cyx} = E_s \frac{t^3}{l^3} \frac{(\beta + \cos\theta)}{(2\beta + 1)\beta^2 \sin\theta}, \tag{45}$$

$$G_{\mathrm{cxz}} = \frac{2G_c(h + 2l\cos\theta)t}{(2l + h)(h + 2l\sin\theta)}, \tag{46}$$

$$G_{cyz} = \frac{2G_c(h + 2l\sin\theta)t}{(2l + h)(h + 2l\cos\theta)}, \tag{47}$$

TABLE

Results	E_{cx}, MPa	G_{cxy}, MPa	E_{cz}, MPa	G_{cxz}, MPa
Simulation values	52.4	12.1	6552	1193
Theoretical solution	56.6	11.3	6348	1158

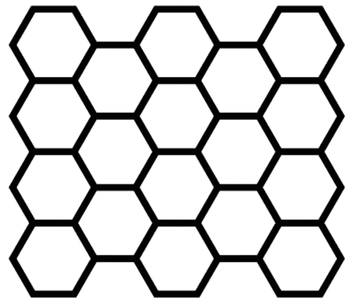

Fig. 12

$$E_{cz} = \frac{4(l+h)tE_s}{(h+2l\sin\theta)(h+2l\cos\theta)}, \tag{48}$$

$$\rho_c = \frac{4\rho_s(\beta+1)t}{l(\beta+2\sin\theta)(\beta+2\cos\theta)}. \tag{49}$$

3.2. Numerical Simulation Examples. The finite-element method was used to verify the theoretical solution. FE software ABAQUS was employed to carry out the simulations, and the finite-element model is shown as Fig. 11. In this case, t = 0.2 mm, $l = 2.7$ mm, $\beta = \sqrt{2}$, $\theta = 45°$, the basic material of the combined honeycomb structure is metal aluminum, whose mechanical properties are as follows: density $\rho_s = 2700$ kg/m^3; Young's modulus $E_s = 71$ GPa; Poisson's ratio $\upsilon_s = 0.33$. Applying in-plane and external load to the honeycomb structure to obtain the related elastic parameters, eventually the simulations values and theoretical solutions are shown in the table.

Obviously, the results got from the numerical simulations are in good agreement with theoretical solutions, which verifies the equivalent mechanical model of new combined honeycomb structure.

4. Comparison with Hexagonal Honeycomb Structure in Mechanical Properties. The hexagonal honeycomb structure is shown as Fig. 12, according to [11]; the equivalent mechanical parameters of the hexagonal honeycomb structure can be expressed as the following equations:

$$E_{cx}^h = E_{cy}^h = 2.31E_s\frac{t_h^3}{l_h^3}, \tag{50}$$

$$G_{cxy}^h = G_{cyx}^h = 0.58E_s\frac{t_h^3}{l_h^3}, \tag{51}$$

$$G_{cxz}^h = G_{cyz}^h = 0.58G_s\frac{t_h}{l_h}, \tag{52}$$

$$E_{cz}^h = 1.15E_s\frac{t_h}{l_h}, \tag{53}$$

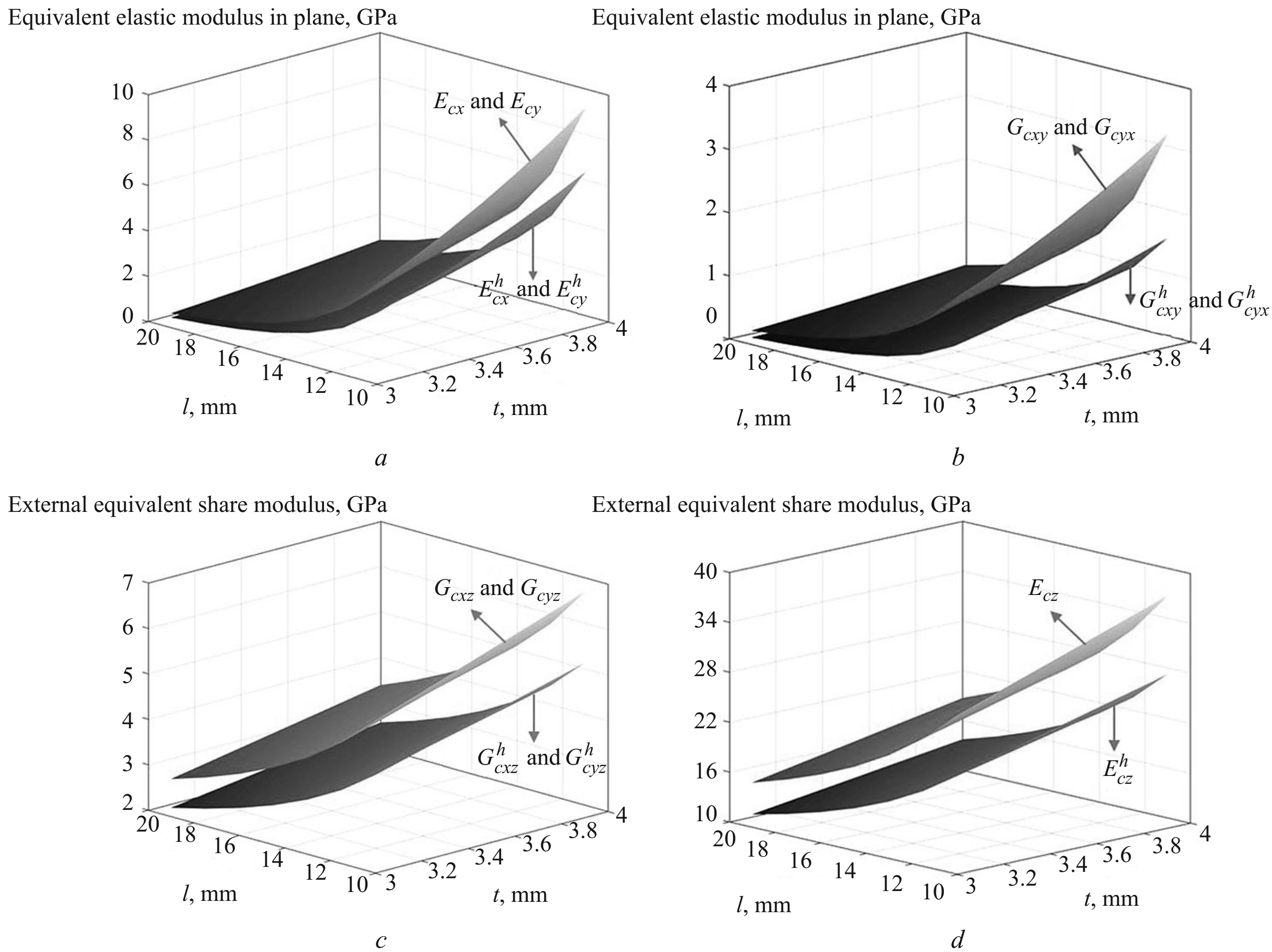

Fig. 13

$$\rho_c^h = 1.54\rho_s \frac{t_h}{l_h}, \tag{54}$$

where t_h is the thickness of the cell wall, l_h is the length the cell wall. When $\beta = 1$, $\theta = 45°$, the equivalent mechanical parameters of the combined honeycomb structure can be expressed as

$$E_{cx} = E_{cy} = 2E_s \frac{t^3}{l^3}, \tag{55}$$

$$G_{cxy} = G_{cyx} = 0.8E_s \frac{t^3}{l^3}, \tag{56}$$

$$G_{cxz} = G_{cyz} = 0.67G_s \frac{t}{l}, \tag{57}$$

$$E_{cz} = 1.37E_s \frac{t}{l}, \tag{58}$$

$$\rho_c = 1.37\rho_s \frac{t}{l}. \tag{59}$$

If $\rho_c = \rho'_c$ and $t = t_h$, then l and l_h are related by

$$l_h = 1.12l. \tag{60}$$

Then the equivalent mechanical parameters of the hexagonal honeycomb structure can be expressed as the following equations:

$$E'_{cx} = E'_{cy} = 1.64 E_s \frac{t^3}{l^3}, \tag{61}$$

$$G'_{cxy} = G'_{cyx} = 0.41 E_s \frac{t^3}{l^3}, \tag{62}$$

$$G'_{cxz} = G'_{cyz} = 0.52 G_s \frac{t}{l}, \tag{63}$$

$$E'_{cz} = 1.03 E_s \frac{t}{l}. \tag{64}$$

If t and l vary within a certain range, then the comparing surfaces reflecting the mechanical properties can be shown as Fig. 13.

The surfaces in Fig. 13 compare the mechanical properties of combined honeycomb and hexagonal honeycomb structure. It can be easily found that the new honeycomb structure have great advantages over the traditional hexagonal honeycomb when they are in the same equivalent density. From calculation, the in-plane equivalent elastic modulus of the combined honeycomb structure improved 22%, the shear modulus improved 95%, the external equivalent elastic modulus and shear modulus, respectively, improved 33% and 29%.

5. Conclusions. A new type of combined honeycomb structure has been created by optimizing the arrangement of square and hexagonal unit cells, the related equivalent mechanical parameters of which were deduced based on the energy equivalent theory and verified by finite-element simulations. From this paper, we can find that this new honeycomb structure has great advantages in mechanical properties over the traditional hexagonal honeycomb and also provides more functional design space for engineering designers to make further optimization. From the perspective of economics and processing technology, there is still great value to optimize and redesign the existing mature cellular honeycombs. The new combined honeycomb structure and design method in this paper are expected to provide some references for the further design and optimization of the honeycomb sandwich structures.

REFERENCES

1. L. J. Gibson, "Modelling the mechanical behavior of cellular materials," *Mater. Sci. Eng. A*, **110**, 1–36 (1989).
2. J. S. Park, J. H. Joo, B. C. Lee, et al., "Mechanical behaviour of tube-woven Kagome truss cores under compression," *Int. J. Mech. Sci.*, **53**, No. 1, 65–73 (2011).
3. T. Strek, H. Jopek, B. T. Maruszewski, et al., "Computational analysis of sandwich-structured composites with an auxetic phase," *Physica Status Solidi (b)*, **251**, No. 2, 354–366 (2014).
4. J. S. Kumar and K. Kalaichelvan, "Taguchi-grey multi-response optimization on structural parameters of honeycomb core sandwich structure for low velocity impact test," *Silicon*, **10**, No. 3, 879–889 (2018).
5. M. Montemurro, A. Catapano, and D. Doroszewski, "A multi-scale approach for the simultaneous shape and material optimisation of sandwich panels with cellular core," *Composites, Part B: Engineering*, **91,** 458–472 (2016).
6. J. Feng, J. Fu, C. Shang, et al., "Sandwich panel design and performance optimization based on triply periodic minimal surfaces," *Computer-Aided Design*, **115**, 307–322 (2019).
7. J. H. Sjolund, D. Peeters, and E. Lund, "Discrete material and thickness optimization of sandwich structures," *Compos. Struct.*, **217**, 75–88 (2019).
8. M. Gholami, R. A. Alashti, and A. Fathi, "Optimal design of a honeycomb core composite sandwich panel using evolutionary optimization algorithms," *Compos. Struct.*, **139**, 254–262 (2016).
9. J. R. Vinson, *The Behavior of Sandwich Structures of Isotropic and Composite Materials*, Routledge (2018).

10. R. Oftadeh, B. Haghpanah, J. Papadopoulos, et al., “Mechanics of anisotropic hierarchical honeycombs,” *Int. J. Mech. Sci.*, **81**, 126–136 (2014).
11. L. J. Gibson and M. F. Ashby, *The Mechanics of Honeycombs*, Cambridge University Press, Cambridge (1997).
12. W. S. Burton and A. K. Noor, “Assessment of continuum models for sandwich panel honeycomb cores,” *Comput. Meth. Appl. Mech. Eng.*, **145**, No. 3–4, 341–360 (1997).
13. F. Minghui and Y. Jiuren, “Equivalent elastic parameters of the honeycomb core,” *Acta Mech. Sinica.*, **31**, No. 1, 113–118 (1999).
14. W. E. Warren and A. M. Kraynik, “Foam mechanics: the linear elastic response of two-dimensional spatially periodic cellular materials,” *Mech. Mater.*, **6**, No. 1, 27–37 (1987).
15. Y. Chen, R. Das, and M. Battley, “Response of honeycombs subjected to in-plane shear,” *J. Appl. Mech.*, **83**, No. 6, 061004 (2016).
16. A. J. Wang and D. L. Mcdowell, “In-plane stiffness and yield strength of periodic metal honeycombs,” *J. Eng. Mater. Technol.*, **126**, No. 2, 137–156 (2004).
17. S. Balawi and J. L. Abot, “A refined model for the effective in-plane elastic moduli of hexagonal honeycombs,” *Compos. Struct.*, **84**, No. 2, 147–158 (2008).
18. S. Balawi and J. L. Abot, “The effect of honeycomb relative density on its effective in-plane elastic moduli: An experimental study,” *Compos. Struct.*, **84**, No. 4, 293–299 (2008).
19. D. Mousanezhad, H. Ebrahimi, B. Haghpanah, et al., “Spiderweb honeycombs,” *Int. J. Solids Struct.*, **66**, 218–227 (2015).
20. A. Ajdari, B. H. Jahromi, J. Papadopoulos, et al., “Hierarchical honeycombs with tailorable properties,” *Int. J. Solids Struct.*, **49,** No. 11–12, 1413–1419 (2012).
21. Y. Sun and N. M. Pugno, “In plane stiffness of multifunctional hierarchical honeycombs with negative Poisson’s ratio sub-structures,” *Compos. Struct.*, **106**, 681–689 (2013).