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STRESS STATE OF DISCRETELY STIFFENED ELLIPSOIDAL SHELLS UNDER A NONSTATIONARY NORMAL LOAD

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The problem of the forced non-axisymmetric vibration of stiffened ellipsoidal shells under nonstationary load is formulated. A numerical algorithm for solving the problem is developed, and the results obtained are analyzed.

Keywords: stiffened ellipsoidal shell, geometrically nonlinear theory, numerical method, nonstationary vibrations

Introduction. The problem of the forced vibration of stiffened shells is well understood. A literature review indicates that both axisymmetric and nonaxisymmetric harmonic vibrations of stiffened shells of simple geometry (cylindrical, conical, and spherical) have mainly been studied [1]. The forced vibrations of reinforced shells under impulsive loads were studied in [2] in detail. There are few studies on the dynamic behavior of stiffened shells of more complex geometry. Among such studies are [4, 16–18] that present results on the forced vibrations of complex geometry shells, namely stiffened ellipsoidal shells [4, 16]. The majority of studies on the dynamics of ellipsoidal shells deal with harmonic and free vibration as well as stability (the case of smooth-walled shell) [7–15, 19, 20]. It is of interest to study the nonaxisymmetric vibrations of rib-reinforced shells of more complex geometry under nonstationary loads.

In the present paper, the equations of nonaxisymmetric vibration of rib-reinforced ellipsoidal shells are derived. The refined model of shells and rods based on the Timoshenko hypotheses [2] is used to simulate the shell and ribs. The Hamilton–Ostrogradsky variational principle is used to derive the vibration equations. The numerical approach to solving the dynamic equations is based on the integro-interpolation finite-differencing technique for an equation with discontinuous coefficients. As an example, the problem of the nonaxisymmetric vibrations of a transversely reinforced ellipsoidal shell under a distributed internal load is solved.

1. Problem Statement. Let us consider an inhomogeneous elastic structure that is an ellipsoidal shell reinforced with transverse and longitudinal ribs. The equation of a smooth ellipsoid has the standard form

$$\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}=1, \tag{1.1}$$

where a and b are the ellipsoid semiaxes.

The parametric equations of the ellipsoid are as follows [3]:

$$x=a\sin\alpha_1\cos\alpha_2, \qquad y=a\sin\alpha_1\sin\alpha_2, \qquad z=b\cos\alpha_1, \tag{1.2}$$

where α_1 and α_2 are the Gaussian curvilinear coordinates corresponding to the meridional and circumferential directions, respectively.

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With (1.2), the expressions for the metrics and the shape of the shell's mid-surface [3] as well as the coefficients of the first quadratic form and the curvature of the mid-surface are derived in the following form:

$$A_1 = a(\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{1/2}, \quad A_2 = a \sin \alpha_1, \quad k = b / a,$$

$$k_1 = \frac{b}{a^2} (\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{-3/2}, \quad k_2 = \frac{b}{a^2} (\cos^2 \alpha_1 + k^2 \sin^2 \alpha_1)^{-1/2}. \tag{1.3}$$

To simulate the dynamic response of the structure mathematically, the geometrically nonlinear Timoshenko-type theory of shells based on the following assumption is used. With respect to the coordinate system (s_1, s_2, z), the displacement variation through the shell thickness is approximated as

$$u_1^z (s_1, s_2, z) = u_1 (s_1, s_2) + z\varphi_1 (s_1, s_2),$$

$$u_2^z (s_1, s_2, z) = u_2 (s_1, s_2) + z\varphi_2 (s_1, s_2),$$

$$u_3^z (s_1, s_2, z) = u_3 (s_1, s_2), \quad z \in [-h/2, h/2], \tag{1.4}$$

where $\overline{U} = (u_1, u_2, u_3, \varphi_1, \varphi_2)^T$ is the generalized displacement vector of the shell mid-surface; s_1 and s_2 are the lengths of meridional and circumferential arcs, respectively.

To develop the mathematical model describing the deformation of the *i*th rib directed along α_1-axis, the hypothesis of the rib cross-section indeformability is adopted in the frame of the geometrically nonlinear Timoshenko beam theory. The strain state of the *i*th rib is determined in terms of the components of the generalized displacement vector $\overline{U}_i = (u_{1i}, u_{2i}, u_{3i}, \varphi_{1i}, \varphi_{2i})^T$. The following approximation of the displacements over the cross-section of the *i*th rib is used:

$$u_{1i}^{yz} (s_1, y, z) = u_{1i} (s_1) + z\varphi_{1i} (s_1),$$

$$u_{2i}^{yz} (s_1, y, z) = u_{2i} (s_1) + z\varphi_{2i} (s_1),$$

$$u_{3i}^{yz} (s_1, y, z) = u_{3i} (s_1). \tag{1.5}$$

The interface conditions relating the components of the displacement vector of the cross-section center of mass of the *i*th rib directed along the α_1-axis and the components of the generalized displacement vector of the initial mid-surface are as follows [2]:

$$u_{1i} (s_1) = u_1 (s_1, s_{2i}) \pm h_{ci} \varphi_1 (s_1, s_{2i}),$$

$$u_{2i} (s_1) = u_2 (s_1, s_{2i}) \pm h_{ci} \varphi_2 (s_1, s_{2i}),$$

$$u_{3i} (s_1) = u_3 (s_1, s_{2i}),$$

$$\varphi_{1i} (s_1) = \varphi_1 (s_1, s_{2i}), \quad \varphi_{2i} (s_1) = \varphi_2 (s_1, s_{2i}), \tag{1.6}$$

where $h_{ci} = 0.5(h + h_i)$ is the distance between the mid-surface and centroidal line of the *i*th rib; h_i is the height of the *i*th rib directed along the α_1-axis; s_{2i} is the coordinate of the projection of the cross-sectional center of mass of the *i*th rib onto the coordinate mid-surface of the shell.

To develop the mathematical model describing the deformation of the *j*th rib directed along the α_2-axis, the hypothesis of the rib cross-section indeformability is adopted in the frame of the geometrically nonlinear Timoshenko beam theory as well. Once more, the strain state of the *j*th rib is determined in terms of the components of the generalized displacement vector $\overline{U}_j = (u_{1j}, u_{2j}, u_{3j}, \varphi_{1j}, \varphi_{2j})^T$. The approximation of the displacements over the cross-section of the *j*th rib is chosen in the form

$$u_{1j}^{xz} (x, s_2, z) = u_{1j} (s_2) + z\varphi_{1j} (s_2),$$

$$u_{2j}^{xz}(x, s_2, z) = u_{2j}(s_2) + z\varphi_{2j}(s_2),$$

$$u_{3j}^{xz}(x, s_2, z) = u_{3j}(s_2). \tag{1.7}$$

The interface conditions relating the components of the displacement vector of the cross-section center of mass of the jth rib directed along the α_2-axis and the components of the generalized displacement vector of the initial mid-surface are as follows [2]:

$$u_{1j}(s_2) = u_1(s_{1j}, s_2) \pm h_{cj}\varphi_2(s_{1j}, s_2),$$

$$u_{2j}(s_2) = u_2(s_{1j}, s_2) \pm h_{cj}\varphi_1(s_{1j}, s_2),$$

$$u_{3j}(s_2) = u_3(s_{1j}, s_2),$$

$$\varphi_{1j}(s_2) = \varphi_2(s_{1j}, s_2), \quad \varphi_{2j}(s_2) = \varphi_1(s_{1j}, s_2), \tag{1.8}$$

where $h_{cj} = 0.5(h + h_j)$ is the distance between the mid-surface and centroidal line of the jth rib; h_j is the height of jth rib directed along the α_2-axis; s_{1j} is the coordinate of the projection of cross-sectional center of mass of the jth rib onto the coordinate mid-surface of the shell skin.

In (1.6) and (1.8), the "+" or "–" sign corresponds to the case of a rib located on the outer or inner surface of the shell, respectively.

To derive the equations of motion of a discretely reinforced structure, the Hamilton–Ostrogradsky variational principle is used [2]:

$$\int_{t_1}^{t_2} \left[\delta(\Pi - K) + \delta A\right] dt = 0 \tag{1.9}$$

$$\left(\Pi = \Pi_0 + \sum_{i=1}^{n_1} \Pi_i + \sum_{j=1}^{n_2} \Pi_j, \quad K = K_0 + \sum_{i=1}^{n_1} K_i + \sum_{j=1}^{n_2} K_j\right), \tag{1.10}$$

where Π_0 and K_0 are the potential and kinetic energy of the shell, respectively; Π_i and K_i are the potential and kinetic energy of the ith rib; Π_j and K_j are the potential and kinetic energy of the jth rib; A is the work done by the external forces.

The expressions for δK and $\delta\Pi$ are of the form

$$\delta\Pi = \delta\Pi_0 + \sum_{i=1}^{n_1} \delta\Pi_i + \sum_{j=1}^{n_2} \delta\Pi_j, \quad \delta K = \delta K_0 + \sum_{i=1}^{n_1} \delta K_i + \sum_{j=1}^{n_2} \delta K_j,$$

$$\delta\Pi_0 = \iint_S \left[T_{11}\delta\varepsilon_{11} + T_{22}\delta\varepsilon_{22} + S\delta\varepsilon_{12} + T_{13}\delta\varepsilon_{13} + T_{23}\delta\varepsilon_{23} + M_{11}\delta\kappa_{11} + M_{22}\delta\kappa_{22} + H\delta(\tau_1 + \tau_2)\right] ds,$$

$$\delta\Pi_i = \int_{l_1} \left[T_{11i}\delta\varepsilon_{11i} + T_{12i}\delta\varepsilon_{12i} + T_{13i}\delta\varepsilon_{13i} + M_{11i}\delta\chi_{11i} + M_{12i}\delta\chi_{12i}\right] dl_1,$$

$$\delta\Pi_j = \int_{l_2} \left[T_{21j}\delta\varepsilon_{21j} + T_{22j}\delta\varepsilon_{22j} + T_{23j}\delta\varepsilon_{23j} + M_{21j}\delta\kappa_{21j} + M_{22j}\delta\kappa_{22j}\right] dl_2,$$

$$\delta K_0 = \rho h \iint_S \left[\frac{\partial u_1}{\partial t}\delta\frac{\partial u_1}{\partial t} + \frac{\partial u_2}{\partial t}\delta\frac{\partial u_2}{\partial t} + \frac{\partial u_3}{\partial t}\delta\frac{\partial u_3}{\partial t} + \frac{h^2}{12}\left(\frac{\partial\varphi_1}{\partial t}\delta\frac{\partial\varphi_1}{\partial t} + \frac{\partial\varphi_2}{\partial t}\delta\frac{\partial\varphi_2}{\partial t}\right) dS\right],$$

$$\delta K_i = \rho_i h_i \int_{l_1} \left[\frac{\partial u_{1i}}{\partial t}\delta\frac{\partial u_{1i}}{\partial t} + \frac{\partial u_{2i}}{\partial t}\delta\frac{\partial u_{2i}}{\partial t} + \frac{\partial u_{3i}}{\partial t}\delta\frac{\partial u_{3i}}{\partial t} + \frac{I_{1i}}{F_i}\frac{\partial\varphi_{1i}}{\partial t}\delta\frac{\partial\varphi_{1i}}{\partial t} + \frac{I_{cri}}{F_i}\frac{\partial\varphi_{2i}}{\partial t}\delta\frac{\partial\varphi_{2i}}{\partial t}\right] dl_1,$$

$$\delta K_j = \rho_j h_j \int_{l_2} \left[\frac{\partial u_{1j}}{\partial t} \delta \frac{\partial u_{1j}}{\partial t} + \frac{\partial u_{2j}}{\partial t} \delta \frac{\partial u_{2j}}{\partial t} + \frac{\partial u_{3j}}{\partial t} \delta \frac{\partial u_{3j}}{\partial t} + \frac{I_{crj}}{F_j} \frac{\partial \varphi_{1j}}{\partial t} \delta \frac{\partial \varphi_{1j}}{\partial t} + \frac{I_{2j}}{F_j} \frac{\partial \varphi_{2j}}{\partial t} \delta \frac{\partial \varphi_{2j}}{\partial t} \right] dl_2 . \tag{1.11}$$

After standard variation and integration using the shell–ith rib and shell–jth rib interface conditions (1.6) and (1.8) and the integral representation [2], functional (1.11) is represented as

$$\int_{t_1}^{t_2} \left\{ \iint_S \left\{ \left\{ \rho h \frac{\partial^2 u_1}{\partial t^2} - L_1(\overline{U}) + \sum_{i=1}^{n_1} \left[\rho_i F_i \frac{\partial^2 u_{1i}}{\partial t^2} - L_{1i}(\overline{U}_i) \right] \delta(\alpha_2 - \alpha_{2i}) \right. \right. \right.$$

$$\left. + \sum_{j=1}^{n_2} \left[\rho_j F_j \frac{\partial^2 u_{1j}}{\partial t^2} - L_{1j}(\overline{U}_j) \right] \delta(\alpha_2 - \alpha_{2j}) \right\} \delta u_1 + \left\{ \rho h \frac{\partial^2 u_2}{\partial t^2} - L_2(\overline{U}) \right.$$

$$\left. + \sum_{i=1}^{n_1} \left[\rho_i F_i \frac{\partial^2 u_{2i}}{\partial t^2} - L_{2i}(\overline{U}_i) \right] \delta(\alpha_2 - \alpha_{2i}) + \sum_{j=1}^{n_2} \left[\rho_j F_j \frac{\partial^2 u_{1j}}{\partial t^2} - L_{2j}(\overline{U}_j) \right] \delta(\alpha_2 - \alpha_{2j}) \right\} \delta u_2$$

$$+ \left\{ \rho h \frac{\partial^2 u_3}{\partial t^2} - L_3(\overline{U}) + \sum_{i=1}^{n_1} \left[\rho_i F_i \frac{\partial^2 u_{3i}}{\partial t^2} - L_{3i}(\overline{U}_i) \right] \delta(\alpha_2 - \alpha_{2i}) \right.$$

$$\left. + \sum_{j=1}^{n_2} \left[\rho_j F_j \frac{\partial^2 u_{1j}}{\partial t^2} - L_{3j}(\overline{U}_j) \right] \delta(\alpha_2 - \alpha_{2j}) \right\} \delta u_3 + \left\{ \rho \frac{h^3}{12} \frac{\partial^2 \varphi_1}{\partial t^2} - L_4(\overline{U}) \right.$$

$$+ \sum_{i=1}^{n_1} \left[\rho_i F_i \left(\frac{\partial^2 u_{1i}}{\partial t^2} + \frac{I_{1i}}{F_i} \frac{\partial^2 \varphi_{1i}}{\partial t^2} \right) - L_{4i}(\overline{U}_i) \right] \delta(\alpha_2 - \alpha_{2i})$$

$$\left. + \sum_{j=1}^{n_2} \left[\rho_j F_j \left(\frac{\partial^2 u_{1j}}{\partial t^2} + \frac{I_{crj}}{F_j} \frac{\partial^2 \varphi_{1j}}{\partial t^2} \right) - L_{4j}(\overline{U}_j) \right] \delta(\alpha_1 - \alpha_{1j}) \right\} \delta \varphi_1$$

$$+ \left\{ \rho \frac{h^3}{12} \frac{\partial^2 \varphi_2}{\partial t^2} - L_5(\overline{U}) + \sum_{i=1}^{n_1} \left[\rho_i F_i \left(\frac{\partial^2 u_{2i}}{\partial t^2} + \frac{I_{kri}}{F_i} \frac{\partial^2 \varphi_{2i}}{\partial t^2} \right) - L_{5i}(\overline{U}_i) \right] \delta(\alpha_2 - \alpha_{2i}) \right.$$

$$\left. \left. + \sum_{j=1}^{n_2} \left[\rho_j F_j \left(\frac{\partial^2 u_{2j}}{\partial t^2} + \frac{I_{2j}}{F_j} \frac{\partial^2 \varphi_{2j}}{\partial t^2} \right) - L_{5j}(\overline{U}_j) \right] \delta(\alpha_1 - \alpha_{1j}) \right\} \delta \varphi_2 \right\} dS$$

$$+ \int_{\Gamma_1} \left\{ \left[T_{11} + \sum_{i=1}^{n_1} T_{11i} \delta(\alpha_2 - \alpha_{2i}) \right] \delta u_1 + \left[(S + k_2 H) + \sum_{i=1}^{n_1} T_{12i} \delta(\alpha_2 - \alpha_{2i}) \right] \delta u_2 \right.$$

$$+ \left[T_{13} + \sum_{i=1}^{n_1} T_{13i} \delta(\alpha_2 - \alpha_{2i}) \right] \delta u_3 + \left[M_{11} + \sum_{i=1}^{n_1} (M_{11i} \pm h_i T_{11i}) \delta(\alpha_2 - \alpha_{2i}) \right] \delta \varphi_1$$

$$\left. + \left[H + \sum_{i=1}^{n_1} (M_{22i} \pm h_i T_{12i}) \delta(\alpha_2 - \alpha_{2i}) \right] \delta \varphi_2 \right\} dl_1$$

$$+\int_{\Gamma_2}\left\{\left[S+k_1H+\sum_{j=1}^{n_2}T_{21j}\delta(\alpha_1-\alpha_{1j})\right]\delta u_1+\left[T_{22}+\sum_{j=1}^{n_2}T_{22j}\delta(\alpha_1-\alpha_{1j})\right]\delta u_2\right.$$

$$+\left[T_{23}+\sum_{j=1}^{n_2}T_{23i}\delta(\alpha_1-\alpha_{1j})\right]\delta u_3+\left[H+\sum_{j=1}^{n_2}(M_{21j}\pm h_jT_{21j})\delta(\alpha_1-\alpha_{1j})\right]\delta\varphi_1$$

$$\left.+\left[M_{22}+\sum_{j=1}^{n_2}(M_{22j}\pm h_jT_{22j})\delta(\alpha_1-\alpha_{1j})\right]\delta\varphi_2\right\}dl_2\Bigg\}dt$$

$$-\iint_S\left[\rho h\left(\frac{\partial u_1}{\partial t}\delta u_1+\frac{\partial u_2}{\partial t}\delta u_2+\frac{\partial u_3}{\partial t}\delta u_3\right)+\rho\frac{h^3}{12}\left(\frac{\partial\varphi_1}{\partial t}\delta\varphi_1+\frac{\partial\varphi_2}{\partial t}\delta\varphi_2\right)\right]\Bigg|_{t_1}^{t_2}dS$$

$$-\int_{l_1}\left\{\sum_{i=1}^{n_1}\left[\rho_ih_i\left(\frac{\partial u_{1i}}{\partial t}\delta u_{1i}+\frac{\partial u_{2i}}{\partial t}\delta u_{2i}+\frac{\partial u_{3i}}{\partial t}\delta u_{3i}\right)\right.\right.$$

$$\left.\left.+\rho_i\left(I_{1i}\frac{\partial\varphi_{1i}}{\partial t}\delta\varphi_{1i}+I_{cri}\frac{\partial\varphi_{2i}}{\partial t}\delta\varphi_{2i}\right)\right]\delta(\alpha_2-\alpha_{2i})\Bigg|_{t_1}^{t_2}\right\}dl_1$$

$$-\int_{l_2}\left\{\sum_{j=1}^{n_2}\left[\rho_jh_j\left(\frac{\partial u_{1j}}{\partial t}\delta u_{1j}+\frac{\partial u_{2j}}{\partial t}\delta u_{2j}+\frac{\partial u_{3j}}{\partial t}\delta u_{3j}\right)\right.\right.$$

$$\left.\left.+\rho_j\left(I_{crj}\frac{\partial\varphi_{1j}}{\partial t}\delta\varphi_{1j}+I_{2j}\frac{\partial\varphi_{2j}}{\partial t}\delta\varphi_{2j}\right)\right]\delta(\alpha_1-\alpha_{1j})\Bigg|_{t_1}^{t_2}\right\}dl_2=0, \qquad (1.12)$$

herewith

$$\int_z\left(\varepsilon_{13}^z-\frac{\sigma_{13}^z}{G_{13}}\right)f_1(z)=0, \qquad \int_z\left(\varepsilon_{23}^z-\frac{\sigma_{23}^z}{G_{23}}\right)f_1(z)=0,$$

where

$$L_1(\overline{U})=\frac{1}{A_1A_2}\left\{\frac{\partial}{\partial\alpha_1}(A_2T_{11})-\frac{\partial A_2}{\partial\alpha_1}T_{22}+\frac{\partial}{\partial\alpha_2}[A_1(S+k_1H)]+\frac{\partial A_1}{\partial\alpha_2}(S+k_2H)\right\}+k_1T_{13},$$

$$L_2(\overline{U})=\frac{1}{A_1A_2}\left\{\frac{\partial}{\partial\alpha_1}\left[A_2(S+k_2H)\right]-\frac{\partial A_2}{\partial\alpha_1}(S+k_1H)+\frac{\partial}{\partial\alpha_2}(A_1T_{22})-\frac{\partial A_1}{\partial\alpha_2}T_{11}\right\}+k_2T_{23},$$

$$L_3(\overline{U})=\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(A_2T_{13})+\frac{\partial}{\partial\alpha_2}(A_1T_{13})\right]-k_1T_{11}-k_2T_{22}, \qquad (1.13)$$

$$L_4(\overline{U})=\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(A_2M_{11})-\frac{\partial A_2}{\partial\alpha_1}M_{22}+\frac{\partial}{\partial\alpha_2}(A_1H)+\frac{\partial A_1}{\partial\alpha_2}H\right]-T_{13},$$

$$L_5(\overline{U}) = \frac{1}{A_1 A_2}\left[\frac{\partial}{\partial \alpha_1}(A_2 H) - \frac{\partial A_2}{\partial \alpha_1} H + \frac{\partial}{\partial \alpha_2}(A_1 M_{22}) - \frac{\partial A_1}{\partial \alpha_2} M_{11}\right] - T_{23},$$

$$L_{1i}(\overline{U}_i) = \frac{1}{A_1}\frac{\partial T_{11i}}{\partial \alpha_1} + k_{1i} T_{13i}, \quad L_{2i}(\overline{U}_i) = \frac{1}{A_1}\frac{\partial T_{12i}}{\partial \alpha_1}, \quad L_{3i}(\overline{U}_i) = \frac{1}{A_1}\frac{\partial T_{13i}}{\partial \alpha_1} - k_{1i} T_{11i}, \tag{1.14}$$

$$L_{4i}(\overline{U}_i) = \frac{1}{A_1}\frac{\partial M_{11i}}{\partial \alpha_1} - T_{13i} \pm h_i\left(\frac{1}{A_1}\frac{\partial T_{11i}}{\partial \alpha_1} + k_{1i} T_{11i}\right), \quad L_{5i}(\overline{U}_i) = \frac{1}{A_1}\frac{\partial}{\partial \alpha_1}(M_{12i} \pm h_i T_{12i}),$$

$$L_{1j}(\overline{U}_j) = \frac{1}{A_2}\frac{\partial T_{21j}}{\partial \alpha_2}, \quad L_{2j}(\overline{U}_j) = \frac{1}{A_2}\frac{\partial T_{22j}}{\partial \alpha_2} + k_{2j} T_{23j},$$

$$L_{3j}(\overline{U}_j) = \frac{1}{A_2}\frac{\partial T_{23j}}{\partial \alpha_2} - k_{2j} T_{22j}, \quad L_{4j}(\overline{U}_j) = \frac{1}{A_2}\frac{\partial}{\partial \alpha_2}(M_{21j} \pm h_j T_{21j}),$$

$$L_{5j}(\overline{U}_j) = \frac{1}{A_2}\frac{\partial M_{22j}}{\partial \alpha_2} - T_{23j} \pm h_j\left(\frac{1}{A_2}\frac{\partial T_{22j}}{\partial \alpha_2} + k_{2j} T_{23j}\right). \tag{1.15}$$

In (1.11), $l_1 = A_1 d\alpha_1$, $l_2 = A_2 d\alpha_2$, Γ_1, and Γ_2 are the boundaries of the shell coinciding with the coordinate curves.

The standard calculations yield three groups of equations:

(i) the equations of vibrations of the shell between ribs:

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}(A_2 T_{11}) - \frac{\partial A_2}{\partial s_1} T_{22}\right] + k_1 \overline{T}_{13} + \frac{1}{A_1}\frac{\partial}{\partial s_2}(A_1 T_{21}) = \rho h \frac{\partial^2 u_1}{\partial t^2},$$

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}(A_2 T_{12}) - \frac{\partial A_2}{\partial s_1} T_{21}\right] + k_2 \overline{T}_{23} + \frac{1}{A_1}\frac{\partial}{\partial s_2}(A_1 T_{22}) = \rho h \frac{\partial^2 u_2}{\partial t^2},$$

$$\frac{1}{A_2}\frac{\partial}{\partial s_1}(A_2 \overline{T}_{13}) - k_1 T_{11} - k_2 T_{22} + P_3 + \frac{1}{A_1}\frac{\partial}{\partial s_2}(A_1 \overline{T}_{23}) = \rho h \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}(A_2 M_{11}) - \frac{\partial A_2}{\partial s_1} M_{22}\right] - T_{13} + \frac{1}{A_1}\frac{\partial}{\partial s_2}(A_1 M_{21}) = \rho \frac{h^3}{12}\frac{\partial^2 \varphi_1}{\partial t^2},$$

$$\frac{1}{A_2}\left[\frac{\partial}{\partial s_1}(A_2 M_{12}) + \frac{\partial A_2}{\partial s_1} M_{21}\right] + \frac{1}{A_1}\frac{\partial}{\partial s_2}(A_1 M_{22}) - T_{23} = \rho \frac{h^3}{12}\frac{\partial^2 \varphi_2}{\partial t^2}; \tag{1.16}$$

(ii) the equations of vibration of the ith rib directed along the α_1-axis:

$$\frac{\partial T_{11}}{\partial s_1} + k_{1i} T_{13i} + [S]_i = \rho_i F_i\left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{ci}\frac{\partial^2 \varphi_1}{\partial t^2}\right),$$

$$\frac{\partial T_{12i}}{\partial s_1} + [T_{22}]_i = \rho_i F_i\left(\frac{\partial^2 u_2}{\partial t^2} \pm h_{ci}\frac{\partial^2 \varphi_2}{\partial t^2}\right),$$

$$\frac{\partial T_{13i}}{\partial s_1} - k_{1i} T_{11i} + [T_{23}]_i = \rho_i F_i \frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{\partial M_{11i}}{\partial s_1} - T_{13i} \pm h_{ci}\left(\frac{\partial T_{11i}}{\partial s_1} + k_{1i}T_{13i}\right) + [H]_i = \rho_i F_i\left(\pm h_{ci}\frac{\partial^2 u_1}{\partial t^2} + \left(h_{ci}^2 + \frac{I_{1i}}{F_i}\right)\frac{\partial^2 \varphi_1}{\partial t^2}\right),$$

$$\frac{\partial M_{12i}}{\partial s_1} \pm h_{ci}\frac{\partial T_{12i}}{\partial s_1} + [M_{22}]_i = \rho_i F_i\left(\pm h_{ci}\frac{\partial^2 u_2}{\partial t^2} + \left(h_{ci}^2 + \frac{I_{cri}}{F_i}\right)\frac{\partial^2 \varphi_2}{\partial t^2}\right); \qquad (1.17)$$

(iii) the equations of vibration of the *j*th rib directed along the α_2-axis:

$$\frac{\partial \overline{T}_{21j}}{\partial s_2} + [T_{11}]_j = \rho_j F_j\left(\frac{\partial^2 u_1}{\partial t^2} \pm h_{cj}\frac{\partial^2 \varphi_1}{\partial t^2}\right),$$

$$\frac{\partial T_{22j}}{\partial s_2} + k_{2j}\overline{T}_{23j} + [S]_j = \rho_j F_j\left(\frac{\partial^2 u_2}{\partial t^2} \pm h_{cj}\frac{\partial^2 \varphi_2}{\partial t^2}\right),$$

$$\frac{\partial \overline{T}_{23j}}{\partial s_2} - k_{2j}T_{22j} + [\overline{T}_{13}]_j = \rho_j F_j\frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{\partial M_{21j}}{\partial s_2} \pm h_{cj}\frac{\partial \overline{T}_{21j}}{\partial s_2} + [M_{11}]_j = \rho_j F_j\left(\pm h_{cj}\frac{\partial^2 u_1}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{crj}}{F_j}\right)\frac{\partial^2 \varphi_1}{\partial t^2}\right),$$

$$\frac{\partial M_{22j}}{\partial s_2} - T_{23j} \pm h_{cj}\left(\frac{\partial T_{22j}}{\partial s_2} + k_{2j}\overline{T}_{23j}\right) + [H]_j = \rho_j F_j\left(\pm h_{cj}\frac{\partial^2 u_2}{\partial t^2} + \left(h_{cj}^2 + \frac{I_{2j}}{F_j}\right)\frac{\partial^2 \varphi_2}{\partial t^2}\right). \qquad (1.18)$$

The notation in (1.16)–(1.18) along with the corresponding expressions for the shell forces and moments as well as the expressions for the ribs are taken from [2].

The expressions for the shell forces and moments are

$$T_{11} = B_{11}\varepsilon_{11} + B_{12}\varepsilon_{22}, \quad T_{22} = B_{21}\varepsilon_{11} + B_{22}\varepsilon_{22}, \quad T_{12} = S + k_2 H, \quad T_{21} = S + k_1 H,$$

$$T_{13} = B_{13}\varepsilon_{13}, \quad T_{23} = B_{23}\varepsilon_{23}, \quad \overline{T}_{13} = T_{13} + T_{11}\theta_1 + S\theta_2, \quad \overline{T}_{23} = T_{23} + T_{22}\theta_2 + S\theta_1,$$

$$S = B_s\varepsilon_{12}, \quad M_{11} = D_{11}\chi_{11} + D_{12}\chi_{22}, \quad M_{22} = D_{21}\chi_{11} + D_{22}\chi_{22},$$

$$M_{12} = M_{21} = H, \quad H = D_s\chi_{12}. \qquad (1.19)$$

The expressions for the forces and moments for the ribs are:

$$T_{11i} = E_i F_i \varepsilon_{11i}, \quad T_{12i} = G_i F_i \varepsilon_{12i}, \quad T_{13i} = k_i^2 G_i F_i \varepsilon_{13i},$$

$$M_{11i} = E_i I_{1i}\chi_{11i}, \quad M_{12i} = G_i I_{cri}\chi_{12i}, \qquad (1.20)$$

$$T_{21j} = G_j F_j \varepsilon_{21j}, \quad T_{22j} = E_j F_j \varepsilon_{22j}, \quad T_{23j} = G_j F_j k_j^2 \varepsilon_{23j},$$

$$M_{21j} = G_j I_{crj}\chi_{21j}, \quad M_{22j} = E_j I_{2j}\chi_{22j}. \qquad (1.21)$$

The expressions for the quadratic approximation of the strains in the shell can be written as follows [2]:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial s_1} + k_1 u_3 + \frac{1}{2}\theta_1^2, \quad \varepsilon_{22} = \frac{\partial u_2}{\partial s_2} + \frac{1}{A_2}\frac{\partial A_2}{\partial s_1}u_1 + k_2 u_3 + \frac{1}{2}\theta_2^2,$$

$$\varepsilon_{12} = \omega + \theta_1\theta_2, \quad \varepsilon_{13} = \varphi_1 + \theta_1, \quad \varepsilon_{23} = \varphi_2 + \theta_2,$$

$$\omega = \omega_1 + \omega_2, \quad \omega_1 = \frac{\partial u_2}{\partial s_1}, \quad \omega_2 = \frac{\partial u_1}{\partial s_2} - \frac{1}{A_2}\frac{\partial A_2}{\partial s_1} u_2,$$

$$\theta_1 = \frac{\partial u_3}{\partial s_1} - k_1 u_1, \quad \theta_2 = \frac{\partial u_3}{\partial s_2} - k_2 u_2, \quad \chi_{11} = \frac{\partial \varphi_1}{\partial s_1}, \quad \chi_{22} = \frac{\partial \varphi_2}{\partial s_2} + \frac{1}{A_2}\frac{\partial A_2}{\partial s_1}\varphi_1,$$

$$\chi_{12} = \tau_1 + \tau_2 + k_1\omega_1 + k_2\omega_2, \quad \tau_1 = \frac{\partial \varphi_2}{\partial s_1}, \quad \tau_2 = \frac{\partial \varphi_1}{\partial s_2} - \frac{1}{A_2}\frac{\partial A_2}{\partial s_1}\varphi_2. \tag{1.22}$$

The strains for the ith rib can be expressed as follows:

$$\varepsilon_{11i} = \frac{\partial u_1}{\partial s_1} \pm h_{ci}\frac{\partial \varphi_1}{\partial s_1} + k_{1i}u_3 + \frac{1}{2}\theta_{1j}^2 + \frac{1}{2}\theta_{2j}^2, \quad \varepsilon_{12i} = \theta_{2i}, \quad \varepsilon_{13i} = \varphi_1 + \theta_{1i},$$

$$\theta_{1i} = \frac{\partial u_3}{\partial s_1} - k_{1i}(u_1 \pm h_{ci}\varphi_1), \quad \theta_{2i} = \frac{\partial u_2}{\partial s_1} \pm h_{ci}\frac{\partial \varphi_2}{\partial s_1}, \quad \chi_{11i} = \frac{\partial \varphi_1}{\partial s_1}, \quad \chi_{12i} = \frac{\partial \varphi_2}{\partial s_1}. \tag{1.23}$$

The expressions for the quadratic approximation of the strains for the transverse ribs are derived as

$$\varepsilon_{22j} = \frac{\partial u_2}{\partial s_2} \pm h_{cj}\frac{\partial \varphi_2}{\partial s_2} + k_{2j}u_3 + \frac{1}{2}\theta_{1j}^2 + \frac{1}{2}\theta_{2j}^2, \quad \varepsilon_{21j} = \theta_{2j}, \quad \varepsilon_{23j} = \varphi_2 + \theta_{1j},$$

$$\theta_{1j} = \frac{\partial u_3}{\partial s_2} - k_{2j}(u_2 \pm h_{cj}\varphi_2), \quad \theta_2 = \frac{\partial u_1}{\partial s_2} \pm h_{cj}\frac{\partial \varphi}{\partial s_2}, \quad \chi_{21j} = \frac{\partial \varphi_1}{\partial s_2}, \quad \chi_{22j} = \frac{\partial \varphi_2}{\partial s_2}. \tag{1.24}$$

The vibration equations (1.16)–(1.24) are supplemented with the natural boundary and initial conditions that follow from (1.12).

2. Numerical Algorithm. Equations (1.16)–(1.24) constitute a system of nonlinear partial differential equations for s_1, s_2, and t with spatial discontinuities in s_1 and s_2. The spatial discontinuities are the projection lines of the cross-sectional centers of mass of the longitudinal and transverse ribs onto the mid-surface of the ellipsoidal shell. Therefore, the numerical algorithm for solving the problem is constructed as follows: at the first step, the solution for the shell (1.16) between the ribs is derived. Then the solutions along the spatial discontinuity curves (1.17) and (1.18) are found [2]. The equations for both the shell between ribs and discontinuity curves are written and integrated. The difference algorithm is based on the integro-interpolation method of finite differencing with respect to the space coordinates and an explicit finite-difference scheme with respect to the time coordinate [2, 6]. The components of the generalized displacement vector are approximated at the integer nodes of the mesh, while the strain and forces are evaluated at half-integer nodes. This technique allows one to preserve the divergent difference representation of the differential equations and ensures the conservation of total mechanical energy at the difference level [5]. The continuous system is reduced to the finite-difference one in two steps. The first step is the finite-difference approximation of the divergent vibration equations in terms of forces and moments.

Integration of (1.16) using the explicit approximation with respect to the time coordinate yields the following difference equations for the ellipsoidal shell between ribs:

$$\frac{1}{A_{2\,l}}\left(\frac{A_{2\,l+1/2}T_{11\,l+1/2,m}^n - A_{2\,l-1/2}T_{11\,l-1/2,m}^n}{\Delta s_1}\right) - \frac{1}{A_{2\,l}}\frac{A_{2\,l+1/2} - A_{2\,l-1/2}}{\Delta s_1}T_{22\,l,m}^n$$

$$+\frac{1}{A_{1\,l}}\frac{A_{1\,l}T_{21\,l,m+1/2}^n - A_{1\,l}T_{21\,l,m-1/2}^n}{\Delta s_2} + k_{1\,l}T_{13\,l,m}^n = \rho h(u_{1\,l,m}^n)_{\bar{t}t},$$

$$\frac{1}{A_{2\,l}}\left(\frac{A_{2\,l+1/2}T_{12\,l+1/2,m}^n - A_{2\,l-1/2}T_{12\,l-1/2,m}^n}{\Delta s_1}\right) - \frac{1}{A_{2\,l}}\frac{A_{2\,l+1/2} - A_{2\,l-1/2}}{\Delta s_1}T_{21\,l,m}^n$$

$$+\frac{1}{A_{1\,l}}\left(\frac{A_{1\,l}T^{n}_{22\,l,m+1/2}-A_{1\,l}T^{n}_{22\,l,m-1/2}}{\Delta s_2}\right)+k_{2\,l}T^{n}_{23\,l,m}=\rho h(u^{n}_{2\,l,m})_{\bar{t}t},$$

$$\frac{1}{A_{2\,l}}\left(\frac{A_{2\,l+1/2}T^{n}_{13\,l+1/2,m}-A_{2\,l-1/2}T^{n}_{13\,l-1/2,m}}{\Delta s_1}\right)-k_{1\,l}T^{n}_{11\,l,m}$$

$$+\frac{1}{A_{1\,l}}\left(\frac{A_{1\,l}T^{n}_{23\,l,m+1/2}-A_{1\,l}T^{n}_{23\,l,m-1/2}}{\Delta s_2}\right)-k_{2\,l}T^{n}_{22\,l,m}+P^{n}_{3\,l,m}=\rho h(u^{n}_{3\,l,m})_{\bar{t}t},$$

$$\frac{1}{A_{2\,l}}\left(\frac{A_{2\,l+1/2}M^{n}_{11\,l+1/2,m}-A_{2\,l-1/2}M^{n}_{11\,l-1/2,m}}{\Delta s_1}\right)-\frac{1}{A_{2\,l}}\frac{A_{2\,l+1/2}-A_{2\,l-1/2}}{\Delta s_1}M^{n}_{22\,l,m}$$

$$+\frac{1}{A_{1\,l}}\left(\frac{A_{1\,l}M^{n}_{21\,l,m+1/2}-A_{1\,l}M^{n}_{21\,l,m-1/2}}{\Delta s_2}\right)-T^{n}_{13\,l,m}=\frac{\rho h^3}{12}(\varphi^{n}_{1\,l,m})_{\bar{t}t},$$

$$\frac{1}{A_{2\,l}}\left(\frac{A_{2\,l+1/2}M^{n}_{12\,l+1/2,m}-A_{2\,l-1/2}M^{n}_{12\,l-1/2,m}}{\Delta s_1}\right)-\frac{1}{A_{2\,l}}\frac{A_{2\,l+1/2}-A_{2\,l-1/2}}{\Delta s_1}M^{n}_{21\,l,m}$$

$$+\frac{1}{A_{1\,l}}\left(\frac{A_{1\,l}M^{n}_{22\,l,m+1/2}-A_{1\,l}M^{n}_{22\,l,m-1/2}}{\Delta s_2}\right)-T^{n}_{23\,l,m}=\frac{\rho h^3}{12}(\varphi^{n}_{2\,l,m})_{\bar{t}t}. \tag{2.1}$$

In the difference equations (2.1), the components of the generalized displacement vector $\overline{U}=(u_1,u_2,u_3,\varphi_1,\varphi_2)^T$ of the mid-surface of the ellipsoidal shell between the ribs are evaluated at the integer nodes of the difference mesh $\overline{U}_{l,m}=(u_{1\,l,m},u_{2\,l,m},u_{3\,l,m},\varphi_{1\,l,m},\varphi_{2\,l,m})^T$ with respect to the spatial coordinates.

The integration of (1.17) using the explicit approximation with respect to the time coordinate yields the following difference equations for the *i*th rib:

$$\frac{T^{n}_{11\,i\,l+1/2}-T^{n}_{11\,i\,l-1/2}}{\Delta s_1}+k_{1\,i\,l}T^{n}_{13\,i\,l}+[S]^{n}_{i}=\rho_i F_i\Big[(u^{n}_{1\,l})_{\bar{t}\,t}\pm h_{ci}(\varphi^{n}_{1\,l})_{\bar{t}\,t}\Big],$$

$$\frac{T^{n}_{12\,i\,l+1/2}-T^{n}_{12\,i\,l-1/2}}{\Delta s_1}+[T_{22}]^{n}_{i}=\rho_i F_i\Big[(u^{n}_{2\,l})_{\bar{t}\,t}\pm h_{ci}(\varphi^{n}_{2\,l})_{\bar{t}\,t}\Big],$$

$$\frac{T^{n}_{13\,i\,l+1/2}-T^{n}_{13i\,l-1/2}}{\Delta s_1}-k_{1\,i\,l}T^{n}_{11\,i\,l}+[T_{23}]^{n}_{i}=\rho_i F_i(u^{n}_{3\,l})_{\bar{t}\,t},$$

$$\frac{M^{n}_{11\,i\,l+1/2}-M^{n}_{21i\,l-1/2}}{\Delta s_1}-T^{n}_{13\,i\,l}\pm h_{ci}\left(\frac{T^{n}_{11\,i\,l+1/2}-T^{n}_{11i\,l-1/2}}{\Delta s_1}+k_{1\,i\,l}T^{n}_{13i\,l}\right)+[H]^{n}_{i}$$

$$=\rho_i F_i\left[\pm h_{ci}(u^{n}_{1\,l})_{\bar{t}\,t}+\left(h^2_{ci}+\frac{I_{1i}}{F_i}\right)(\varphi^{n}_{1\,l})_{\bar{t}\,t}\right],$$

$$\frac{M^n_{12\,i\,l+1/2}-M^n_{12i\,l-1/2}}{\Delta s_1}\pm h_{ci}\frac{T^n_{12ik+1/2}-T^n_{12ik-1/2}}{\Delta s_1}+[M_{22}]^n_i$$

$$=\rho_i F_i\left[\pm h_{ci}(u^n_{2\,l})_{\bar t\,t}+\left(h^2_{ci}+\frac{I_{cri}}{F_i}\right)(\varphi^n_{2\,l})_{\bar t\,t}\right]. \qquad (2.2)$$

In the difference equations (2.2), the components of the generalized displacement vector $\overline{U}_i=(u_{1i},u_{2i},u_{3i},\varphi_{1i},\varphi_{2i})^T$ of the cross-section center of mass of the ith rib are evaluated at the integer nodes of the difference mesh with respect to the spatial coordinates.

As previously, the integration of (1.17) using the explicit approximation with respect to the time coordinate yields the following difference equations for the jth rib:

$$\frac{T^n_{21\,j\,m+1/2}-T^n_{21\,j\,m-1/2}}{\Delta s_2}+[T_{11}]^n_j=\rho_j F_j\left[(u^n_{1\,m})_{\bar t\,t}\pm h_{ci}(\varphi^n_{1\,m})_{\bar t\,t}\right],$$

$$\frac{T^n_{22\,j\,m+1/2}-T^n_{22\,j\,m-1/2}}{\Delta s_2}+k_{2\,j\,m}T^n_{23\,j\,m}+[S]^n_j=\rho_j F_j\left[(u^n_{2\,m})_{\bar t\,t}\pm h_{ci}(\varphi^n_{2\,m})_{\bar t\,t}\right],$$

$$\frac{T^n_{23\,j\,m+1/2}-T^n_{23\,j\,m-1/2}}{\Delta s_2}-k_{2\,j\,m}T^n_{22\,j\,m}+[T_{13}]^n_j=\rho_j F_j(u^n_{3\,m})_{\bar t\,t},$$

$$\frac{M^n_{21\,j\,m+1/2}-M^n_{21j\,m-1/2}}{\Delta s_2}\pm h_{cj}\frac{T^n_{21\,j\,m+1/2}-T^n_{21j\,m-1/2}}{\Delta s_2}+[M_{11}]^n_j$$

$$=\rho_j F_j\left[\pm h_{cj}(u^n_{1\,m})_{\bar t\,t}+\left(h^2_{cj}+\frac{I_{crj}}{F_j}\right)(\varphi^n_{1\,m})_{\bar t\,t}\right],$$

$$\frac{M^n_{22\,j\,m+1/2}-M^n_{22j\,m-1/2}}{\Delta s_2}-T^n_{23\,j\,m}\pm h_{cj}\left(\frac{T^n_{22\,j\,m+1/2}-T^n_{22j\,m-1/2}}{\Delta s_2}+k_{2\,j\,m}T^n_{23\,j\,m}\right)+[H]^n_j$$

$$=\rho_j F_j\left[\pm h_{cj}(u^n_{2\,m})_{\bar t\,t}+\left(h^2_{cj}+\frac{I_{2j}}{F_j}\right)(\varphi^n_{2\,m})_{\bar t\,t}\right]. \qquad (2.3)$$

In the difference equations (2.3), the components of the generalized displacement vector $\overline{U}_j=(u_{1j},u_{2j},u_{3j},\varphi_{1j},\varphi_{2j})^T$ of the cross-section center of mass of the jth rib are evaluated at the integer nodes of the difference mesh with respect to the spatial coordinates.

The second step is the finite-difference approximation of the forces and moments as well as the correspondent strains in order to hold the finite-difference analog of the energy equation [5].

To analyze the stability of the linearized finite-difference equations, the necessary stability conditions in the form $\Delta t\le 2/\omega$ are used where $\omega=\max(\omega_0,\omega,\omega_j)$ are the maximum natural frequencies of the discrete-difference system of the shell and ith and jth ribs.

3. Numerical Example. As a numerical example, the problem of the forced vibration of an ellipsoidal shell reinforced with longitudinal and transverse ribs and rigidly fixed within the region $D=\{\alpha_{10}\le\alpha_1\le\alpha_{1N},\alpha_{20}\le\alpha_2\le\alpha_{2N}\}$ is solved. The shell is subjected to distributed normal loading $P_3(\alpha_1,\alpha_2,t)$. The boundary conditions are as follows: $\overline{U}(\alpha_{10},\alpha_2)=\overline{U}(\alpha_{1N},\alpha_2)=0$, $\overline{U}(\alpha_1,\alpha_{20})=\overline{U}(\alpha_1,\alpha_{2N})=0$. The zero initial conditions for all the components of the generalized displacement vector at t = 0 are assumed:

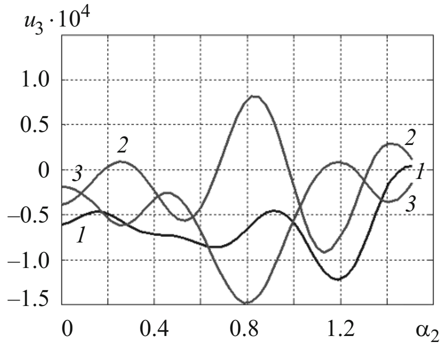

Fig. 1

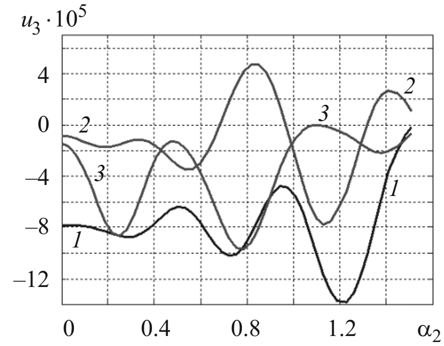

Fig. 2

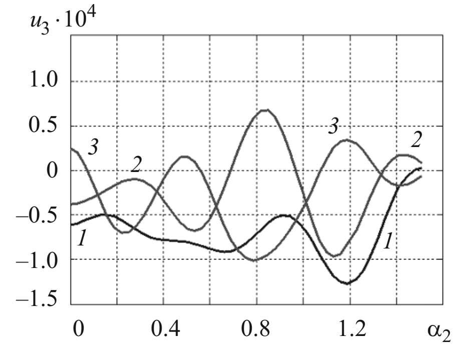

Fig. 3

$$u_1(\alpha_1,\alpha_2)=u_2(\alpha_1,\alpha_2)=u_3(\alpha_1,\alpha_2)=\varphi_1(\alpha_1,\alpha_2)=\varphi_2(\alpha_1,\alpha_2)=0,$$

$$\frac{\partial u_1(\alpha_1,\alpha_2)}{\partial t}=\frac{\partial u_2(\alpha_1,\alpha_2)}{\partial t}=\frac{\partial u_3(\alpha_1,\alpha_2)}{\partial t}=\frac{\partial \varphi_1(\alpha_1,\alpha_2)}{\partial t}=\frac{\partial \varphi_2(\alpha_1,\alpha_2)}{\partial t}=0.$$

The distributed normal load $P_3(\alpha_1,\alpha_2,t)$ is described by

$$P_3(\alpha_1,\alpha_2,t)=A\cdot\sin\frac{\pi t}{T}[\eta(t)-\eta(t-T)],$$

where A and T are the amplitude and duration of the load, respectively. Their values are chosen as $A=10^6$ Pa, $T=50$ μsec.

The problem is solved for the following geometrical and mechanical parameters of the reinforced structure (isotropic shell):

$$\alpha_{10}=\pi/12, \quad \alpha_{1N}=\pi-(\pi/12), \quad \alpha_{20}=-\pi/2, \quad \alpha_{2N}=\pi/2,$$

$$a/h=30, \quad k=1.5, \quad E=7\cdot10^{10}\ \text{Pa}, \quad \nu=0.33, \quad \rho=2.7\cdot10^3\ \text{kg/m}^3.$$

The geometrical and mechanical parameters of the transverse rib are: $h_j=4h, F_j=4h^2, E_j=E, G_j=G_{12}, \rho_j=\rho$.

The longitudinal rib is located in the cross-section $\alpha_2=0$ and directed along the α_1 axis. Its geometrical and mechanical parameters are: $h_i=4h, F_i=4h^2, E_i=E, G_i=G_{12}, \rho_i=\rho$.

The transverse rib is located in the cross-section $\alpha_1=\pi/2$ and directed along the α_2 axis.

The calculated results obtained on the time interval $t=20T$ are presented in Figs. 1–3, where the most typical curves for the generalized displacement u_3 are shown. They enable analyzing the stress state of the structure. Figures 1–3 illustrate the behavior of u_3 as a function of α_2 over the cross-section $\alpha_1=\pi/4$ (due to symmetry, the curves are shown for the region $0\le\alpha_2\le\pi/2$ along the α_2 axis).

Figure 1 corresponds to a longitudinal rib located in the cross-section $\alpha_2=0$ and directed along the axis α_1 at instants $t_1=5T$, $t_2=11T$, and $t_3=19T$. Analysis of the data shows that the maximum $\max\limits_{0\le t\le 20T}|u_3|=1.48\cdot10^{-4}$ m is reached at $t=19T$.

Figure 2 corresponds to a transverse rib located in the cross-section $\alpha_1=\pi/2$ and directed along the axis α_2 at the same instants $t_1=5T$, $t_2=11T$, and $t_3=19T$. The maximum $\max\limits_{0\le t\le 20T}|u_3|=1.38\cdot10^{-4}$ m is reached at $t=5T$.

Figure 3 corresponds to longitudinal and transverse ribs located in the cross-section $\alpha_1=\pi/2$ and directed along the axis α_2 at $t_1=5T$, $t_2=11T$, and $t_3=19T$. Analysis of the data shows that the maximum $\max\limits_{0\le t\le 20T}|u_3|=1.26\cdot10^{-4}$ m is reached at $t=5T$.

For the shell reinforced with longitudinal and transverse ribs, the maximum deflection is less by 10% than in the transversely reinforced shell and less by 17% than in the longitudinally reinforced structure.

In the figures, the positions of the ribs easily be identified.

Conclusions. The problem of the vibration of a discretely reinforced ellipsoidal shell has been formulated. Both the shell and ribs are simulated in the frame of the second-order curvilinear rod and shell theory using the Timoshenko model. For problems of this class, an effective numerical algorithm has been developed. It is based on a finite-difference scheme in the spatial coordinates and the explicit approximation with respect to the time coordinate. The results of numerical calculation as well as their analysis are presented as an example.

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