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NONSTATIONARY VIBRATIONS OF ELECTROELASTIC CYLINDRICAL SHELL IN ACOUSTIC LAYER

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The propagation of a pressure wave in a fluid bounded by two parallel plane boundaries generated by an infinitely long cylindrical electroelastic shell submerged into the fluid is considered. To describe the motion of the shell and the processes in the fluid, the equations of the linear theory of shells generalized to the case of electromechanics and the acoustic approximation are used. The problem-solving method is based on application of the image source method, the method of separation of variables, and Laplace integral transform. The method is used to reduce the problem to an infinite system of Volterra equations with delay arguments, which is numerically solved using the reduction method and regularizing procedures. The hydrodynamic pressure is calculated for the case where either step-wise or sinusoidal electric pulse load is applied to the continuous electrodes of the shell.

Keywords: acoustic layer, electroelastic cylindrical shell, Laplace integral transform, image source method

Introduction. The wide use of piezoelectric transducers in numerous hydroacoustic applications maintains interest in theoretical investigations in the field of hydroelectroelasticity. Till now, the processes in systems containing spherical and cylindrical piezotransducers located in an infinite acoustic medium or in the vicinity of a plane boundary have been studied in detail. The vast bibliography devoted to the field mentioned can be found in the review [9] and books [4, 6]. The necessity of elaborating effective solution methods for applied problems of nonstationary hydroelectroelasticity is dictated by the improvements in piezotransducer performance due to usage of pulse radiation or reception mode. For the class of problems mentioned, it is possible to investigate the evolution of dynamic process and transducer behavior under short-time electric or mechanical loads of complex shape with possible jumps in characteristics at the front.

Different aspects of radiation and diffraction of acoustic fields generated by electroelastic cylindrical/spherical shells in multiply connected domains with account of multiple reflections of waves from the boundaries demand further study.

The research area was addressed actively for the partial case of a point source of harmonic acoustic waves. A number of effective numerical methods have recently been proposed to study the acoustic field in a domain of complex geometry (boundary element methods, fundamental solutions method, and finite-difference method). Detailed information on the approaches can be found, for example, in the books [5, 12] and papers [11, 13]. The publications [7, 8] deserve to be mentioned among few papers on the radiation and diffraction problems for nonpoint sources. In [8], an approximate algorithm is proposed for the calculation of a plane acoustic field formed by the primary field of a circular acoustic radiator and the secondary field induced by a diffracted incident wave on the parallel boundaries of a waveguide as a result of a single reflection. The analytical solution for the plane stationary problem of acoustical wave diffraction by an elastic cylinder in a plane waveguide with rigid boundaries was found in [7] using the image source method [3, 14], which is experimentally proved to be highly effective.

In the present work, we propose a semi-analytical technique based on Laplace transform and image source method to solve the problem of nonstationary vibration of an electroelastic shell, the problem of pressure wave radiation and diffraction in an acoustic region bounded by two parallel boundaries.

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1. Problem Statement. The nonstationary processes in a liquid medium bounded by two parallel planes $y = l_1$ and $y = -l_2$ ($l_* > 0$) of the Cartesian coordinate system are under investigation. The medium is considered to be ideal compressible, and the acoustical model [2] is used to simulate its motion

$$\nabla^2 \Phi = \frac{1}{c_\mathrm{w}^2} \frac{\partial^2 \Phi}{\partial t^2}, \qquad p = -\rho_\mathrm{w} \frac{\partial \Phi}{\partial t}, \qquad \vec{v} = \vec{\nabla}\Phi, \tag{1}$$

where Φ is the velocity potential of the disturbed motion of the liquid; p is the hydrodynamic pressure; $\vec{v}$ is the velocity of the medium particles; c_w and ρ_w are the sound speed and the medium density, respectively; t is the time; ∇^2, $\vec{\nabla}$ are Laplace and Hamilton operators, respectively.

Assuming that upper boundary ($y = l_1$) is free while the medium interacts with the rigid wall at the lower boundary ($y = -l_2$), the boundary conditions for the potential function Φ have to yield zero pressure on the surface $y = l_1$ and zero displacements normal to the wall $y = -l_2$:

$$\Phi\big|_{y=l_1} = 0, \qquad \left.\frac{\partial \Phi}{\partial y}\right|_{y=-l_2} = 0, \tag{2}$$

which should be supplemented by the pressure wave damping condition at infinity:

$$\lim_{x^2+y^2+z^2\to\infty} \Phi = 0. \tag{3}$$

In a more general case, the "impedance" conditions [9] can be set on the plane boundaries:

$$\left.\left(\zeta_{1,j}\Phi + \zeta_{2,j}\frac{\partial \Phi}{\partial y}\right)\right|_{y=(-1)^{j+1} l_{2(1-\tilde{e}(j/2))}} = 0 \quad (j = 1, 2, 3, \ldots), \tag{4}$$

where the constants $\zeta_{1,j}$ and $\zeta_{2,j}$ characterize the properties of the layer boundaries ($\zeta_{1,j} = 0$ corresponds to a rigid wall, while $\zeta_{2,j} = 0$ to a free surface); $\tilde{e}(x)$ is a function returning the fractional part of x.

An infinitely long cylindrical electroelastic shell submerged into an acoustic medium is the source of nonstationary waves. The vibrations of the shell are caused by a specified mechanical loading $q_0(t)$ uniformly distributed over the inner surface of the shell and/or voltage V_0 applied to the continuous conducting coatings (electrodes). It is assumed that the shell consists of two layers rigidly connected with each other. The outer layer is of constant thickness h_m and is made of an elastic material, while the inner layer is of thickness h_p and made of thickness polarized piezoceramics with 6mm crystal symmetry class. Assuming that the shell axis coincides with the applicate axis Oz and deformation patterns are the same in the each plane perpendicular to Oz, the equations of motion of the shell with respect to the polar coordinate system (r, θ) are of the following form [17]:

$$-\frac{D_N}{R_0^2}\left(\frac{\partial u_0}{\partial \theta} + w\right) + \frac{D_M}{R_0^4}\left(\frac{\partial^3 u_0}{\partial \theta^3} - \frac{\partial^4 w}{\partial \theta^4}\right) + q + \frac{e_1}{R_0}V = m_h \frac{\partial^2 w}{\partial t^2},$$

$$\frac{D_N}{R_0^2}\left(\frac{\partial^2 u_0}{\partial \theta^2} + \frac{\partial w}{\partial \theta}\right) + \frac{D_M}{R_0^4}\left(\frac{\partial^2 u_0}{\partial \theta^2} - \frac{\partial^3 w}{\partial \theta^3}\right) = m_h \frac{\partial^2 u_0}{\partial t^2}. \tag{5}$$

Here $w(t, \theta)$ is the shell deflection; $u_0(t, \theta)$ is the tangential displacement of the datum surface [17]; $q(t, \theta)$ is the external load normal to the datum surface; $V(t, \theta) = V_0(t)$.

It is worth mentioning here that the system of equations (5) is derived using the Kirchhoff–Love hypotheses generalized to electromecanics. The notation used and the expressions for the coefficients are taken from [17] and are omitted here.

It is also assumed for system (5) that zero electric potential is applied to the internal electrode and the function q contains both the specified mechanical loading $q_0(t)$ and unknown hydrodynamic loading imposed by the ambient medium:

$$q = q_0 - p|_{r=R_0}. \tag{6'}$$

The normal components of the velocity of both the medium particles and shell points should be equal to each other on the shell surface ($r = R_0$) (continuous contact condition):

$$\frac{\partial w}{\partial t} = \left.\frac{\partial \Phi}{\partial r}\right|_{r=R_0}. \tag{6''}$$

The hydroelectroelastic system under consideration is assumed to be at rest before the electromechanical excitation starts ($t = 0$).

The system of equations (1) can be rewritten using dimensionless parameters and variables. All the linear dimensions and components of the displacement vector l_*, h_*, R_0, w, and u_0 are normalized by the radius R_0 of the shell datum surface; the velocity v is normalized by c_{w}, while time t, loads p and q_0, potential Φ, and electric signal V are divided by factors R_0 / c_{w}, $\rho_{\text{w}} c_{\text{w}}^2$, and $D_N R_0 / e_1$, respectively. Thus, the initial system of equations for the medium takes the form

$$\Delta\Phi = \frac{\partial^2 \Phi}{\partial t^2}, \qquad p = -\frac{\partial \Phi}{\partial t}, \qquad v_{\text{r}} = \frac{\partial \Phi}{\partial r}, \tag{7}$$

while the equations for the shell become

$$\frac{\partial u_0}{\partial \theta} - \delta \frac{\partial^3 u_0}{\partial \theta^3} + w + \delta \frac{\partial^4 w}{\partial \theta^4} + \alpha^2 \frac{\partial^2 w}{\partial t^2} = \beta q + V,$$

$$\alpha^2 \frac{\partial^2 u_0}{\partial t^2} - (1+\delta) \frac{\partial^2 u_0}{\partial \theta^2} - \frac{\partial w}{\partial \theta} + \delta \frac{\partial^3 w}{\partial \theta^3} = 0 \tag{8}$$

(here $\alpha^2 = m_h c_{\text{w}}^2 / D_N$, $\beta = \rho_{\text{w}} \alpha^2 / m_h$, $\delta = D_M / D_N$ are constant coefficients).

The boundary conditions (2)–(4), (6) obviously remain unchanged.

2. Problem-Solving Technique. The problem is solved using the image source method [3, 14]. The method suggest introducing additional sources that generate the acoustic waves multiply reflected from the layer boundaries $y = l_1$ and $y = -l_2$. Choice of the image source positions is dictated by the simplicity of the boundary conditions on the plane surfaces $y = l_1$ and $y = -l_2$. We assume that the first ($j = 1$) and second ($j = 2$) additional sources are located symmetrically to the electroelastic shell ($j = 0$) about the planes $y = l_1$ and $y = -l_2$, respectively. The next image sources are symmetrical reflections of the image sources with the preceding number. For example, the source $j = 3$ is assumed to be symmetrical to $j = 2$ about $y = l_1$ while $j = 4$ is symmetrical to $j = 1$ about $y = -l_2$. The numbering procedure is organized according to the algorithm described. The transient duration T and the wave travel time from the Jth source to the location of interest $(r_J - R)/c_{\text{w}} > T$; r_J is the distance from the Jth source center to the location of interest) specify the total number of the image sources needed. The arrangement of the real ($j = 0$) and image ($j \geq 1$) sources is shown in Fig 1.

Due to the nonlinearity of the problem, the acoustic field in the medium can be represented as a superposition of the fields induced by the shell and image sources:

$$\Phi = \sum_{j=0}^{J} \varphi_j, \tag{9}$$

where φ_j are the primary ($j = 0$) and reflected ($j \geq 1$) wave potentials.

Both the total velocity potential Φ and each of its components φ_j have to meet the wave equation (1). The general solution can be represented in terms of modified Bessel functions I_n and Macdonald functions K_n. Taking into account the damping condition at infinity (3), zero initial conditions and symmetry of the acoustic field in the medium about the Cartesian coordinate plane $x = 0$, the Laplace transformed solution of Eq. (1) [10] can be expanded into series:

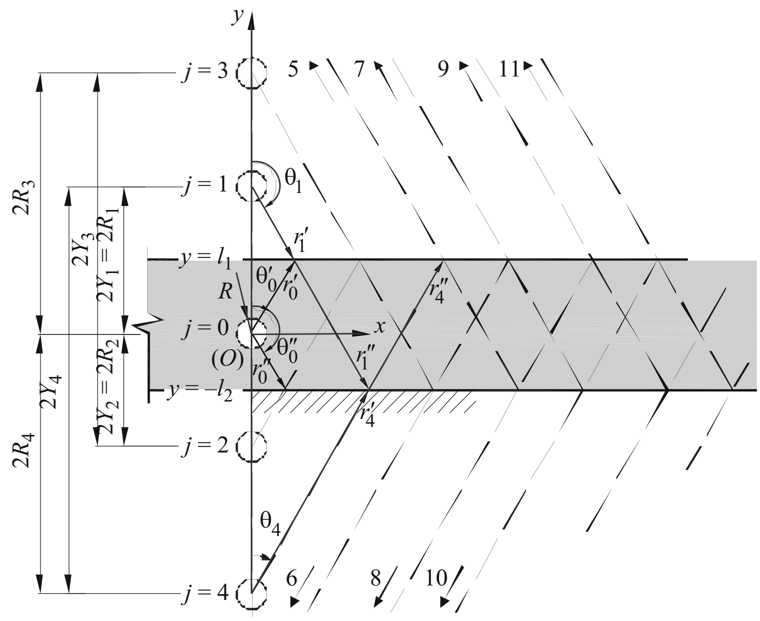


Fig. 1

$$\varphi_j^L(s,r_j,\theta_j)=\sum_{n=0}^{\infty}A_{j,n}^L(s)\frac{1}{s}e^{sR}K_n(sr_j)\cos(n\theta_j), \tag{10}$$

where Laplace-transformed functions are marked by superscript "L" ($f^L(s,z)=L\{f(t,z)\}$); s is the transform parameter; $A_{j,n}^L$ are arbitrary functions that should be determined from the boundary conditions; $R=1$ is the dimensionless radius of the shell; (r_j, θ_j) stands for the polar coordinate system whose origin coincides with the center of the jth source (θ_j is the angle between the y-axis and the line connecting the jth source with the point of interest). Figure 1 allows us to derive the relations between the coordinates. Some of them are listed below:

$$r_1'\sin\theta_1=r_0'\sin\theta_0',\quad r_1'\cos\theta_1=r_0'\cos\theta_0'-2Y_1,\quad 2Y_1=2l_1,$$

$$r_2'\sin\theta_2=r_0''\sin\theta_0'',\quad r_2'\cos\theta_2=r_0''\cos\theta_0''+2Y_2,\quad 2Y_2=2l_2,$$

$$r_3'\sin\theta_3=r_2''\sin\theta_2,\quad r_3'\cos\theta_3=r_2''\cos\theta_2-2Y_3,\quad 2Y_3=2Y_1+4l_2,$$

$$r_4'\sin\theta_4=r_1''\sin\theta_1,\quad r_4'\cos\theta_4=r_1''\cos\theta_1+2Y_4,\quad 2Y_4=2Y_2+4l_1,$$

$$r_4''\sin\theta_4=r_5'\sin\theta_5,\quad r_5'\cos\theta_5=r_4''\cos\theta_4-2Y_5,\quad 2Y_5=2Y_3+4l_1\ \text{etc.}$$

Then using the expression $\frac{\partial}{\partial y}=\frac{1}{\cos\theta_j}\frac{\partial}{\partial r_j}-\frac{1}{r_j\sin\theta_j}\frac{\partial}{\partial\theta_j}$ and the relations $\theta_1=\pi-\theta_0'$ and $r_1'=r_0'$; $\theta_2=\pi-\theta_0''$ and $r_2'=r_0''$; $\theta_3=\pi-\theta_2$ and $r_3'=r_2''$; $\theta_4=\pi-\theta_1$ and $r_4'=r_1''$; $\theta_5=\pi-\theta_4$ and $r_5'=r_4''$ etc., which are valid for $y=l_1$ and $y=-l_2$, expressions (9), (10), and (4) yields relations between coefficients $A_{j,n}^L(s)$ for $j\geq 1$:

$$A_{j,n}^L=(-1)^n\kappa_j A^L_{\hat{e}[j+4\tilde{e}(j/2)-3],n}, \tag{11}$$

where $\hat{e}(x)=x\cdot H(x)$; $H(x)$ is the Heaviside function; $\kappa_j=(\zeta_{2,j}-\zeta_{1,j})/(\zeta_{2,j}+\zeta_{1,j})$ ($\kappa_j=(-1)^j$ in the case of the classical boundary conditions (2)).

The theorems of summation of cylindrical functions [3] are used to transform between the polar coordinates (r_j, θ_j) ($j\geq 1$) and the coordinates (r_0, θ_0):

$$K_n(sr_j)e^{in\theta_j} = \sum_{m=-\infty}^{\infty}(-1)^m K_{n-m}(s2R_j)I_m(sr_0)e^{i[(n-m)\Theta_j+m\theta_0]} \quad \text{for} \quad r_0 < 2R_j,$$

$$K_n(sr_j)e^{in\theta_j} = \sum_{m=-\infty}^{\infty}(-1)^{n-m} I_{n-m}(s2R_j)K_m(sr_0)e^{i[(n-m)\Theta_j+m\theta_0]} \quad \text{for} \quad r_0 > 2R_j,$$

where $(2R_j, \Theta_j)$ are the coordinates of the pole of the polar system (r_0, θ_0) in the jth coordinate system.

If $R \le r_0 < 2R_j$, then

$$K_n(sr_j)\cos(n\theta_j) = \sum_{m=-\infty}^{\infty}(-1)^m K_{n-m}(s2R_j)I_m(sr_0)\cos((n-m)\Theta_j)\cos(m\theta_0)$$

$$= \sum_{m=0}^{\infty}\overline{\omega}_m(-1)^{m+(n-m)2\widetilde{e}(j/2)}\left[K_{n+m}(s2R_j)+K_{n-m}(s2R_j)\right]I_m(sr_0)\cos(m\theta_0),$$

$$2R_j = [2l_1+2l_2]\cdot\widetilde{E}\left(\frac{j-2}{4}\right)+2l_1\delta_{0,\widetilde{e}((j-1)/4)}+2l_2\delta_{0,\widetilde{e}((j-2)/4)}, \qquad \Theta_j = \pi\cdot 2\widetilde{e}\left(\frac{j}{2}\right), \tag{12}$$

where $\widetilde{E}(x) = x - \widetilde{e}(x)$, $\overline{\omega}_x = 1-0.5\delta_{0,x}$, $\delta_{0,x} = \begin{cases} 1, & x=0; \\ 0, & x\neq 0 \end{cases}$ is the Kronecker delta.

Using expressions (9), (10), and (12), the potential Φ can be transformed to the coordinates (r_0, θ_0) (the subscript "0" is omitted further on). The formula takes the form

$$\Phi^L = \sum_{n=0}^{\infty}A^L_{0,n}(s)\frac{1}{s}e^{sR}K_n(sr)\cos(n\theta)$$

$$+\sum_{j=1}^{J}\sum_{n=0}^{\infty}A^L_{j,n}(s)\frac{1}{s}e^{sR}\sum_{m=0}^{\infty}\overline{\omega}_m(-1)^{m+(n-m)2\widetilde{e}(j/2)}\left[K_{n+m}(s2R_j)+K_{n-m}(s2R_j)\right]I_m(sr)\cos(m\theta). \tag{13}$$

Changing the order of summation and using the notation

$$f^L_{n,m}(s,z) = \frac{1}{s^n}e^{-sz}I_m(sz), \qquad g^L_{n,m}(s,z) = \frac{1}{s^n}e^{sz}K_m(sz) \tag{14}$$

we get the expression

$$\Phi^L(s,r,\theta) = \sum_{n=0}^{\infty}\Phi^L_n(s,r)\cos(n\theta), \tag{15}$$

where

$$\Phi^L_n(r,\theta) = e^{-s(r-R)}A^L_{0,n}(s)g^L_{1,n}(s,r)$$

$$+\sum_{j=1}^{J}e^{-s(2R_j-R-r)}\sum_{m=0}^{\infty}A^L_{j,m}(s)\{\overline{\omega}_n(-1)^{n+(m-n)2\widetilde{e}(j/2)}[g^L_{1,m+n}(s,2R_j)+g^L_{1,m-n}(s,2R_j)]f^L_{0,n}(s,r)\}.$$

Recall that the recursive relations (11) hold for the functions $A^L_{j,m}(s)$.

Using relations (7), we obtain the following formulas for the hydrodynamic pressure p and radial velocity v_r of particles in the acoustic medium:

$$p^L(s,r,\theta) = -s\Phi^L = \sum_{n=0}^{\infty} p_n^L(s,r)\cos(n\theta), \tag{16}$$

$$v_{\mathrm{r}}^L(s,r,\theta) = \frac{\partial \Phi^L}{\partial r} = \sum_{n=0}^{\infty} v_n^L(s,r)\cos(n\theta), \tag{17}$$

where

$$p_n^L(s,r) = -\mathrm{e}^{-s(r-R)} A_{0,n}^L(s) g_{0,n}^L(s,r) - \sum_{j=1}^{J} \mathrm{e}^{-s(2R_j - R - r)} \sum_{m=0}^{\infty} A_{j,m}^L(s) \Big\{ \overline{\omega}_n (-1)^{n+(m-n)2\widetilde{e}(j/2)}$$

$$\times \Big[g_{0,m+n}^L(s,2R_j) + g_{0,m-n}^L(s,2R_j) \Big] f_{0,n}^L(s,r) \Big\},$$

$$v_n^L(s,r) = \mathrm{e}^{-s(r-R)} A_{0,n}^L(s) s G_n^L(s,r)$$

$$+ \sum_{j=1}^{J} \mathrm{e}^{-s(2R_j - R - r)} \sum_{m=0}^{\infty} A_{j,m}^L(s) \{ \overline{\omega}_n (-1)^{n+(m-n)2\widetilde{e}(j/2)} [g_{0,m+n}^L(s,2R_j) + g_{0,m-n}^L(s,2R_j)] s F_n^L(s,r) \},$$

where

$$G_0^L(s,z) = -g_{1,1}^L(s,z), \quad G_n^L(s,z) = -g_{1,n-1}^L(s,z) - \frac{n}{z} g_{2,n}^L(s,z) \quad \text{for} \quad n \geq 1,$$

$$F_0^L(s,z) = f_{1,1}^L(s,z), \quad F_n(s,z) = f_{1,n-1}^L(s,z) - \frac{n}{z} f_{2,n}^L(s,z) \quad \text{for} \quad n \geq 1.$$

The general solution of the Laplace-transformed system (8) is expanded into a series of normal modes of the the shell:

$$w^L = \sum_{n=0}^{\infty} a_n^L(s)\cos(n\theta), \quad u_0^L = \sum_{n=1}^{\infty} b_n^L(s)\sin(n\theta), \tag{18}$$

where $a_n^L(s)$, $b_n^L(s)$ are coefficients to be found.

Expressions similar to (18) can be derived for the functions V^L and $q^L = q_0^L - p^L|_{r=R}$ that describe the electric potential on the outer electrode and mechanical load, respectively,

$$q^L = \sum_{n=0}^{\infty} \Big(c_n^L(s) - p_n^L|_{r=R} \Big) \cos(n\theta), \quad V^L = \sum_{n=0}^{\infty} d_n^L(s)\cos(n\theta) \tag{19$'$}$$

with the coefficients

$$c_0^L = q_0^L, \quad d_0^L = V_0^L, \quad c_n^L = d_n^L = 0 \quad \text{for} \quad n \geq 1 \tag{19$''$}$$

if the assumptions $V(t,\theta) = V_0(t)$ and $q_0(t,\theta) = q_0(t)$ hold.

Substituting (18) and (19′) into Eqs. (8) yields an algebraic system of equations for the coefficients $a_n^L(s)$ and $b_n^L(s)$:

$$a_n^L(\xi_n^{(1)} + \alpha^2 s^2) + b_n^L \xi_n^{(2)} = \beta(c_n^L - p_n^L) + d_n^L,$$

$$a_n^L \xi_n^{(2)} + b_n^L(\xi_n^{(3)} + \alpha^2 s^2) = 0, \tag{20}$$

where $\xi_n^{(1)} = 1 + \delta n^4$, $\xi_n^{(2)} = n + \delta n^3$, $\xi_n^{(3)} = (1+\delta)n^2$.

With (19″), the solution of system (20) takes the form

$$a_0^L = \left(\beta\left(q_0^L - p_0^L|_{r=R}\right) + V_0^L\right)\bar{f}^L,$$

$$a_n^L = -\beta p_n^L \bar{g}^L, \quad b_n^L = \beta \frac{\xi_n^{(2)}}{D_n} p_n^L \quad \text{for} \quad n \geq 1. \tag{21}$$

Let

$$\bar{f}^L(s) = (1+\alpha^2 s^2)^{-1}, \quad \bar{g}^L(s) = (\xi_n^{(3)} + \alpha^2 s^2)/D_n,$$

$$D_n(s) = (\xi_n^{(3)} + \alpha^2 s^2)(\xi_n^{(1)} + \alpha^2 s^2) - \xi_n^{(2)2}.$$

It is obvious that the coefficients $a_n^L(s)$ and $b_n^L(s)$ (21) are related to $A_{j,n}^L(s)$ in (10) by the formula for p_n^L from (16). Therewith, the coefficients $A_{j,n}^L$ for $j \geq 1$ can be expressed in terms of $A_{0,n}^L$ (see (11)). To find $A_{0,n}^L$, it is necessary to substitute the results obtained into condition (6″):

$$sw^L = v_{\mathrm{r}}|_{r=R}.$$

As a result, we obtain an infinite algebraic system of equations. The system relates the *n*th mode of the displacement component $a_n^L(s)$ to the modes $A_{0,n}^L(s)$ of the potential function Φ^L ($a_n^L = s^{-1} \cdot v_n^L|_{r=R}$):

$$A_{0,0}^L(s)\left[G_0^L(s,R) - \beta g_{0,0}^L(s,R)\bar{f}^L(s)\right]$$

$$+\sum_{j=1}^{J} e^{-s(2R_j - 2R)} \sum_{m=0}^{\infty} A_{j,m}^L(s)(-1)^{m2\tilde{e}(j/2)} g_{1,m}^L(s, 2R_j)\left[F_0^L(s,R) - \beta f_{0,0}^L(s,R)\bar{f}^L(s)\right] = (\beta q_0^L + V_0^L)\bar{f}^L(s),$$

$$A_{0,n}^L(s)\left[G_n^L(s,R) - \beta g_{0,n}^L(s,R)\bar{g}^L(s)\right] + \sum_{j=1}^{J} e^{-s(2R_j - 2R)} \sum_{m=0}^{\infty} A_{j,m}^L(s)\Big\{\varpi_n (-1)^{n+(m-n)2\tilde{e}(j/2)}$$

$$\times\left[g_{0,m+n}^L(s,2R_j) + g_{0,m-n}^L(s,2R_j)\right] \times \left[F_n^L(s,R) - \beta f_{0,n}^L(s,R)\bar{g}^L(s)\right]\Big\} = 0 \quad (n \geq 1). \tag{22}$$

Note that the infinite system (22) is formulated in terms of the unknown functions $A_{0,n}^L$ of the transform parameter *s*. Reduction and direct solving of the system demands the expansion of functional determinants whose order corresponds to the number of normal modes retained. Application of the technique leads to complex and cumbersome formulas causing fundamental mathematical difficulties associated with the recovery of the original functions. To avoid this, the direct inversion of the system is performed by the technique proposed in [2] for nonstationary hydroelasticity problems. The boundary condition (6″) holds in the original domain in this case. It allows us to reduce the nonstationary hydroelectroelastic problem under consideration to an infinite system of Volterra equations of the first kind with delay arguments [1, 9]:

$$A_{0,0}(t) \overset{0}{*} \left[G_0(t,R) - \beta g_{0,0}(t,R) \overset{0}{*} \bar{f}(t)\right]$$

$$+\sum_{j=1}^{J}\sum_{m=0}^{\infty} A_{j,m}(t) \overset{2(R_j - R)}{*} \left[(-1)^{m2\tilde{e}(j/2)} g_{1,m}(t, 2R_j) \overset{0}{*} \left(F_0(t,R) - \beta f_{0,0}(t,R) \overset{0}{*} \bar{f}(t)\right)\right] = (\beta q_0 + V_0) \overset{0}{*} \bar{f}(t),$$

$$A_{0,n}(t) \overset{0}{*} \left[G_n(t,R) - \beta g_{0,n}(t,R) \overset{0}{*} \bar{g}(t) \right]$$

$$+\sum_{j=1}^{J}\sum_{m=0}^{\infty} A_{j,m}(t) \overset{2(R_j-R)}{*} \left\{ \left(\varpi_n (-1)^{n+(m-n)2\tilde{e}(j/2)} \left[g_{1,m+n}(t,2R_j) + g_{1,m-n}(t,2R_j) \right] \right) \right.$$

$$\left. \overset{0}{*} \left[F_n(t,R) - \beta f_{0,n}(t,R) \overset{0}{*} \bar{g}(t) \right] \right\} = 0 \qquad (n \geq 1). \tag{23}$$

Here operator "*" denotes the convolution of original functions ($A(t) \overset{t_0}{*} B(t) = H(t-t_0)\int_0^{t-t_0} A(\tau) B((t-t_0)-\tau)d\tau$).

It is worth mentioning here that convolution, delay, and similarity theorems [10] are applied for analytical inversion. The inversions of the functions $g_{0,m}(t,z)$, $f_{0,m}(t,z)$ from (23) are listed in a table of Laplace transform [16]:

$$g_{0,m}(t,z) = \frac{+\cosh(m \cdot \operatorname{arcosh}(1+t/z))}{z\sqrt{(1+t/z)^2-1}},$$

$$f_{0,m}(t,z) = H(t)H(t-2z)\frac{\cos(m \cdot \arccos(1-t/z))}{\pi z\sqrt{1-(1-t/z)^2}}.$$

The inversions $g_{n,m}(t,z)$ and $f_{n,m}(t,z)$ for $n > 0$ are obtained by integrating $g_{0,m}(t,z)$ and $f_{0,m}(t,z)$ n times (see [1]). The kernels $G_m(t,z)$ and $F_m(t,z)$ are linear combinations of the functions $g_{n,m}(t,z)$ and $f_{n,m}(t,z)$.

The system of integral equations (23) can be solved numerically using the reduction and quadrature methods. Reduction order is determined by the trial-and-error method. The process stops when the boundary condition is satisfied to the accuracy set.

The delay arguments allow step-by-step solution of the system at time intervals between the moments $T_j = 2(R_j - R)$ arranged in ascending order. Here T_j is the time it takes the acoustic wave to travel the distance from the jth image source to the shell (see Fig. 1). Thus, at the first step of the transient process, the functions $A_{0,n}(t)$ for $0 < t \leq \min(T_j)$ are determined using (23):

$$A_{0,0}(t) \overset{0}{*} \left[G_0(t,R) - \beta g_{0,0}(t,R) \overset{0}{*} \bar{f}(t) \right] = (\beta q_0 + V_0) \overset{0}{*} \bar{f}(t), \qquad A_{0,n}(t) = 0 \quad (n \geq 1).$$

Then the values obtained are transposed to the right-hand side and used in determining the functions $A_{0,n}(t)$ at the next time interval. The functions $A_{j,n}(t)$ $(j \geq 1)$ from the left-hand side are expressed in terms of $A_{0,n}(t)$ by applying the recursive relations (11).

With $A_{j,n}(t)$ $(j,n \geq 0)$, the hydrodynamic pressure p and the velocity v_r at an arbitrary point of the acoustic medium can be calculated. Thus, the summation of the integrals

$$p_n(t,r) = -A_{0,n}(t) \overset{r-R}{*} g_{0,n}(t,r) - \sum_j \bar{p}_{j,n}(t,r),$$

where

$$\bar{p}_{j,n}(t,r) = \sum_{m=0}^{\infty} A_{j,m} \overset{r-R}{*} \left\{ \varpi_n (-1)^{n+(m-n)2\tilde{e}(j/2)} \left[g_{0,m+n}(t,2R_j) + g_{0,m-n}(t,2R_j) \right] \overset{2R_j}{*} f_{0,n}(t,r) \right\}$$

yields the coefficients $p_n(t,r) = L^{-1}\{p_n^L(s,r)\}$ of the Fourier series (16).

The expression for $\bar{p}_{j,n}(t,r)$ is written for the case $r > 2R_j - R$. According to the addition theorem, the formula somewhat changes if $r > 2R_j + R$:

$$\bar{p}_{j,n}(t,r) = \sum_{m=0}^{\infty} A_{j,m} \overset{r-R}{*} \left\{ \bar{\omega}_n (-1)^{(m-n)[1+2\tilde{e}(j/2)]} \left[f_{0,m+n}(t,2R_j) + f_{0,m-n}(t,2R_j) \right] \overset{2R_j}{*} g_{0,n}(t,r) \right\}.$$

To study the nonstationary vibrations of the electroelastic shell interacting with the acoustic layer plane boundaries, the expression $a_n^L = s^{-1} \cdot v_n^L|_{r=R}$ that determines the coefficients $a_n(t)$ of the shell radial vibration $w(t,\theta)$ according to (18) should be inverted:

$$a_n(t) = A_{0,n}(t) \overset{0}{*} G_n(t,R)$$

$$+ \sum_{j=1}^{J} \sum_{m=0}^{\infty} A_{j,m}(t) \overset{2(R_j-R)}{*} \bar{\omega}_n (-1)^{n+(m-n)2\tilde{e}(j/2)} \left[g_{0,m+n}(t,2R_j) + g_{0,m-n}(t,2R_j) \right] \overset{0}{*} F_n(t,R).$$

In doing so, the expressions for all the other variables of interest can be derived.

3. Numerical Results. Water (c_{w} = 1500 m/sec, ρ_{w} = 1000 kg/m^3) is chosen as an acoustical medium for a layer of dimensionless thickness $l_1 + l_2 = H = 20$ for numerical experiments.

The upper boundary of the liquid layer is the pressure free surface ($\kappa_j = -1$, $j = 1, 3, 5, \ldots$) while the lower boundary is of "impedance" class with $\kappa_j = 0.9$ ($j = 2, 4, 6, \ldots$). The distance between the axis of the cylindrical electroelastic shell and the lower boundary is $l_2 = 0.3H$. The shell is considered to be composed of PZT-5 piezoceramic layer of thickness h_{p} = 0.04 and titanium alloy VT-6 (h_{m} = 0.02). The properties of the materials are listed in [17].

In the particular case under consideration, the shell can be modeled as a linear source to a first approximation if the datum surface radius $R = 1$ is significantly smaller than the distance between the plane surfaces. Therefore, the influence of the deflected waves on the shell vibrations can be neglected. This assumption simplifies system (23) substantially because the sum over the subscript j is equal to zero in this case. Moreover, $A_{0,n}(t) = 0$ for $n \geq 1$, while $A_{0,0}(t)$ is the solution of the Volterra equation of the first kind,

$$A_{0,0}(t) \overset{0}{*} \left[g_{1,1}(t,R) + \beta g_{0,0}(t,R) \overset{0}{*} \bar{f}(t) \right] = -(\beta q_0(t) + V_0(t)) \overset{0}{*} \bar{f}(t). \tag{24}$$

The assumption is validated by the numerical experiments performed in [1]. It was shown that the pressure waves induced by the first reflection influence the shell vibrations to some extent even for a small distance between the plane boundary and the shell. The secondary wave effect is much weaker and can be neglected to the accuracy demanded.

Having values of $A_{0,0}(t)$ and using relations (7), (9) and (10) (i.e., avoiding the use of the addition theorem), we can calculate the hydrodynamic pressure at an arbitrary point of the medium:

$$p^L = -\sum_{j=0}^{J} A_{j,0}^L(s) e^{-s(r_j - R)} g_{0,0}^L(s, r_j) \;\Rightarrow\; p = -\sum_{j=0}^{J} A_{j,0} \overset{r_j - R}{*} g_{0,0}(t, r_j), \tag{25}$$

where r_j is the distance between the pole of the jth coordinate system associated with the jth image source and the point of interest.

The pressure at points lying on the lower boundary of the layer with coordinates (H, $-l_2$) and ($3H$, $-l_2$) of the rectangular coordinate system with the origin on the shell axis is shown in Fig. 2. These results are marked with $r \approx H$ and $r \approx 3H$, respectively, in Fig. 2. The coordinates of the points in the polar coordinate system (r_0, θ_0) = (r, θ) are ($\sqrt{x^2 + y^2}$, $(\pi/2) - \arctan(y/x)$). The pressure profile on the shell radiating surface is illustrated by the curve $r = R$. It is assumed that the shell vibrations are induced by a step-wise electric signal $V_0(t) = H(t)$ applied to the electrodes while the shell is free of internal mechanical loading ($q_0(t) = 0$). If the electromechanical loading of the shell is $\beta q_0(t) + V_0(t) = H(t)$, the result will be similar.

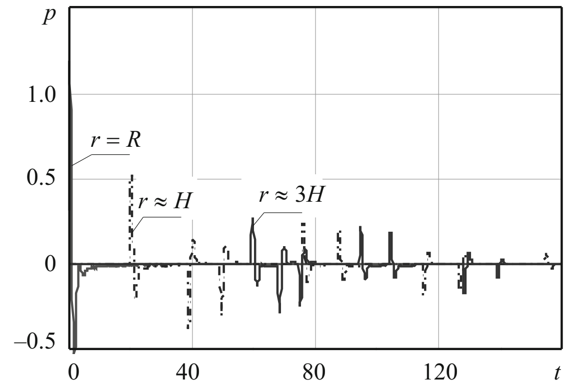


Fig. 2

Special regularization algorithms were used to solve the integral equation (24). But for the particular case of the loading $\beta q_0(t)+V_0(t)=H(t)$, the problem can be reduced to a system of algebraic equations and solved by conventional methods. Using the equality $L\{H(t)\}=1/s$, it is easy to derive the following equation in the s-domain:

$$A_{0,0}^L(s)\left[g_{0,1}^L(s,R)+\frac{\beta}{\alpha}g_{0,0}^L(s,R)\tilde{f}^L(s)\right]=-\bar{f}^L(s).$$

The inverse of the functions $\tilde{f}^L(s)=\alpha s\bar{f}^L(s)$ and $\bar{f}^L(s)$ from the previous equation can be derived using the operational calculus tables [16]:

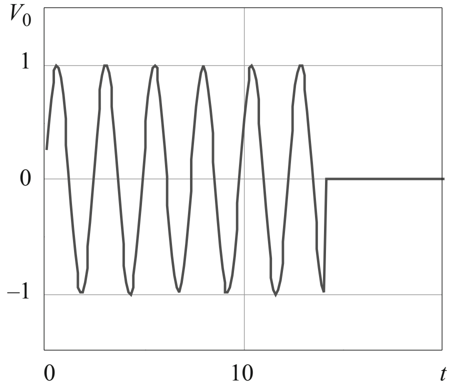
$$\bar{f}(t)=L^{-1}\{\bar{f}^L(s)\}=\frac{1}{\alpha}\sin\left(\frac{t}{\alpha}\right),$$

$$\tilde{f}(t)=L^{-1}\{\tilde{f}^L(s)\}=\frac{1}{\alpha}\cos\left(\frac{t}{\alpha}\right).$$

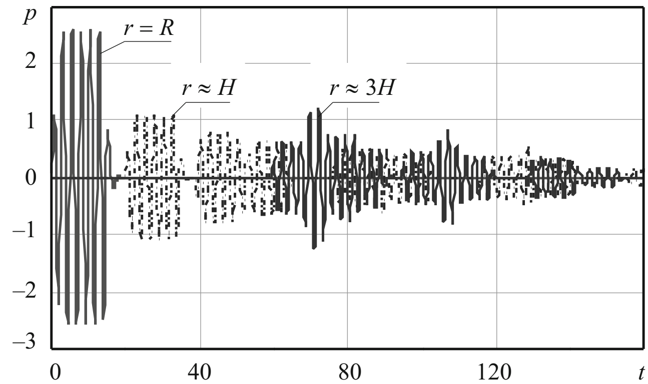
Provided the results for the case $\beta q_0(t)+V_0(t)=H(t)$ are calculated, the physical values for the nonstationary load of arbitrary profile $V_0(t)(q_0(t))$ can be easily obtained by using Duhamel's integral. Figure 3 shows the pressure curves at the same points of the layer for excitation by the electric single-frequency signal $V=\sin(\omega t)\cdot H(T_V-t)$ (Fig. 3*a*). Here $T_V=5.75\times(2\pi/\omega)$ is the signal duration; $\omega=1/\alpha$ is the frequency of pulse vibration of the shell in vacuum.

As follows from Fig. 2, the pressure on the shell surface increases sharply under step-wise loading followed by low amplitude oscillations that decay rapidly. The acoustic pulse profile formed propagates with velocity c_w in the medium but the wave amplitude decreases proportionally to square root of distance ($\sqrt{1/r}$) according to the well-known laws of acoustics. The superposition of the acoustic pulses from the actual source and image sources causes much more complex patterns of pressure profile at the points of interest. Similar effects are discussed in [1, 15] where different aspects of elastic and electroelastic shell vibrations in an infinite acoustic medium and acoustic half-space are studied. The steady-state vibration occurs quite rapidly under harmonic excitation of the shell by a signal of resonance frequency (Fig. 3*a*). In this regime, the amplitudes of the pressure waves radiated into the medium remain constant (Fig. 3*b*). Negligible vibrations of the shell due to inertia effects occur after the termination of loading. The maximum pressure decreases as the distance between the excitation source and the point of interest increases under a short pulse while an increase in the pulse duration causes a deviation from this pattern (Fig. 3*b*). For instance, the superposition of the primary and two reflected waves ($j=\overline{0,2}$) at the point $(3H,-l_2)$ causes 6% higher pressure then at the point $(H,-l_2)$ that lies approximately 3 times closer to the source (curves $r\approx H$ and $r\approx 3H$).

Conclusions. A numerical–analytic solution technique for the problem of nonstationary vibration of an infinitely long cylindrical electroelastic shell submerged into an acoustic layer of constant thickness has been developed. The problem has been formulated within the framework of acoustic approximation along with classical thin shell theory based on the Kirchhoff–Love hypotheses generalized to electromecanics. The impedance boundary conditions have been formulated on the plane surfaces of



a



b

Fig. 3

the layer enabling one to simulate different combinations of the surface properties. Base on the technique developed, the problem has been reduced to an infinite system of Volterra integral equations of the first kind solved numerically over known time intervals.

Numerical results have been obtained for the case where the distance between the shell center and the nearest boundary of the acoustic medium substantially exceeds the shell radius. For this geometrical pattern, the influence of reflected waves on the shell vibrations is negligible as was mentioned in different previous publications. The calculation algorithm can be simplified significantly, the fundamental results of the hydroelectroelasticity theory [1, 2] and superposition principle as well as image sources method [3] can be applied. It has been shown that for long acoustical signals, the hydrodynamic pressure at remote points can exceed the pressure at the points located closer to the medium excitation source.

The technique proposed can be extended to the case of nonparallel boundaries of the acoustic layer. The results obtained can be used for the class of applied problems referred to as inverse problems of hydroelectrolasticity. In particular, the identification of the disturbance source location in the layer or the pattern of the stationary or nonstationary acoustic signal generated by a source can be studied by the technique.

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