DOI 10.1007/s10778-018-0888-9
International Applied Mechanics, Vol. 54, No. 3, May, 2018

APPLICATION OF METHODS OF NUMERICAL ANALYSIS FOR STUDYING MECHANICAL PROCESSES IN BIOMECHANICS

A. Ya. Grigorenko[1], E. N. Pliska[2], G. V. Sorochenko[2], and N. N. Tormakhov[1]

Using the finite-element method, the stress–strain state of a tooth crown with a caries cavity acted upon by a vertical force is simulated. The tooth crown is modeled by a two-layer cylinder with outer enamel layer and inner dentinal layer. The effect of the location and geometry of the caries cavity and the mechanical properties of the enamel and dentin on the load-bearing capacity of the tooth crown is analyzed. It is shown that an increase in the length and depth of the cavity in the tooth crown leads to an increase in the stress intensity in the crown layers. The stress intensity peaks in the enamel layer near the walls of the caries cavities located near the tooth neck.

Keywords: stress–strain state, finite-element method, 3D theory, two-layer cylinder, caries

Introduction. A human tooth consists of a crown and root. The tooth crown is located beyond bone tissues of the upper and lower jaws and consists of outer enamel and inner dentinal layers. The enamel is brittle wear-proof material with a high elastic modulus, while the dentin has porous structure and elastic modulus is lower than that of the enamel. The central part of the crown contains soft tissues consisting of nerve fibers and blood vessels, which transmit information on loads on the tooth and promote metabolic processes in the tissues of the tooth crown [1].

One of the most widespread tooth diseases is caries caused by the microorganisms on the crown surface. Microorganisms produce acids and extracellular polysaccharides from simple sugars. The acids induce demineralization of the enamel and origin of cavities in it, while the extracellular polysaccharides generate biofilms that limit the access of oral fluid to caries [6, 8].

During nibbling and chewing, teeth are subject to high (up to 200 N) loads [1]. A caries cavity weakens the load-bearing capacity of the tooth crown, which may results in its fracture. Recently, new treatment techniques have been applied in stomatology. The efficiency of these techniques is evaluated by modeling the biomechanical processes in the tooth–jaw system rather than using the trial-and-error method [2–5, 10–13]. The modeling makes it possible to better study the processes in the tooth tissues and to develop more efficient treatment techniques.

A mathematic model of the stress–strain state of a tooth crown damaged by caries was proposed in [4]. It was assumed that the crown is a cylinder with a caries cavity described by cylindrical coordinates (height, depth, and circumference), while the tooth consists of elastic enamel and dentinal layers.

The mathematical modeling based on the finite-element method (FEM) has revealed a considerable increase in the stresses near the caries cavity which can give rise to microcracks and fracture of the tooth crown.

Below, we will study how the configuration of the caries cavity and elastic properties of the enamel and dentin influence the strength properties of the tooth crown. Particularly, we will investigate how the location and geometry of the caries cavity and mechanical properties of the enamel and dentin affect the load-bearing capacity of the tooth crown.

---

[1] S. P. Timoshenko Institute of Mechanics, Ukrainian National Academy of Sciences, 3 Nesterova St., Kyiv 03057, Ukraine; e-mail: metod@inmech.kiev.ua. [2] Bogomolets National Medical University, 34 Pobedy Av., Kyiv 03057, Ukraine; e-mail: plyska.e@gmail.com. Translated from Prikladnaya Mekhanika, Vol. 54, No. 3, pp. 136–144, May–June, 2018. Original article submitted on May 29, 2017.

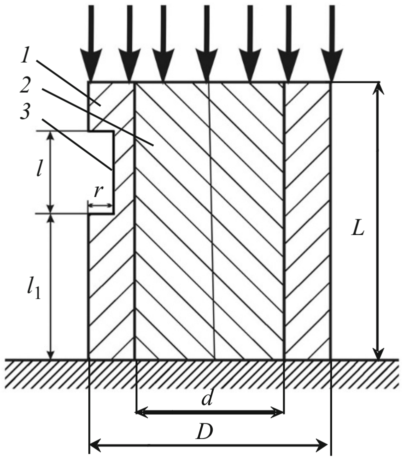

Fig. 1

1. Determining the Stress–Strain State. Let us analyze the SSS of a tooth crown with caries. Assume that the tooth crown has the form of a two-layer cylinder with a rectilinear caries cavity and one end clamped. The cylinder is shown in Fig. 1, where numbers *1*, *2*, and *3* denote enamel, dentin, and caries cavity, respectively. Let the outer layer be made of enamel, and the inner layer of dentin. The caries cavity is located at a distance of l_1 from the clamping point of crown and has height l and depth r. The crown has a symmetry plane, which allows us to restrict the consideration to half the crown cut off by the symmetry plane. Due to the presence of damage, we will use a spatial Cartesian coordinate system x, y, z.

The SSS of a piecewise homogeneous body that occupies a domain $\Omega \in \mathbf{R}^3$ $(\Omega = \Omega_1 \cup \Omega_2)$ with Lipschitz continuous boundary $\Gamma = \Gamma_\sigma \cup \Gamma_u$ is described by the equations of linear elasticity [4]. External surface forces represented by the components of the vector of surface load $\vec{\mathbf{p}} = (p_x, p_y, p_z)$ act on the boundary Γ_σ. The occlusion surface of the tooth crown under the vertical load p_y. A displacement vector $\vec{u}_\Gamma$ $(u_x^\Gamma, u_y^\Gamma, u_z^\Gamma)$ is specified on the boundary Γ_u.

The solution of the boundary-value problem must satisfy

the equilibrium equations

$$\frac{\partial \sigma^i_{xx}}{\partial x} + \frac{\partial \sigma^i_{yx}}{\partial y} + \frac{\partial \sigma^i_{zx}}{\partial z} = 0,$$

$$\frac{\partial \sigma^i_{xy}}{\partial x} + \frac{\partial \sigma^i_{yy}}{\partial y} + \frac{\partial \sigma^i_{zy}}{\partial z} = 0,$$

$$\frac{\partial \sigma^i_{xz}}{\partial x} + \frac{\partial \sigma^i_{yz}}{\partial y} + \frac{\partial \sigma^i_{zz}}{\partial z} = 0, \quad (x, y, z) \in \Omega_i, \ (i = 1, 2), \tag{1}$$

the kinematic equations

$$\vec{\varepsilon}^{\,i} = \mathbf{B}^i \vec{\mathbf{u}}^i \quad (x, y, z) \in \Omega_i, \ (i = 1, 2). \tag{2}$$

Hooke’s law

$$\vec{\sigma}^{\,i} = \mathbf{D}^i \vec{\varepsilon}^{\,i} \quad (x, y, z) \in \Omega_i, \ (i = 1, 2). \tag{3}$$

System (1)–(3) for all subdomains Ω_i is supplemented by the static boundary conditions

$$\sigma_{nx} = p_x, \quad \sigma_{ny} = p_y, \quad \sigma_{nz} = p_z \quad (x, y, z) \in \Gamma_\sigma, \tag{4}$$

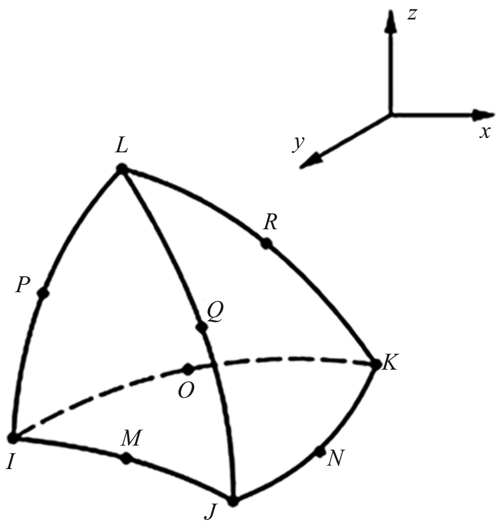

Fig. 2

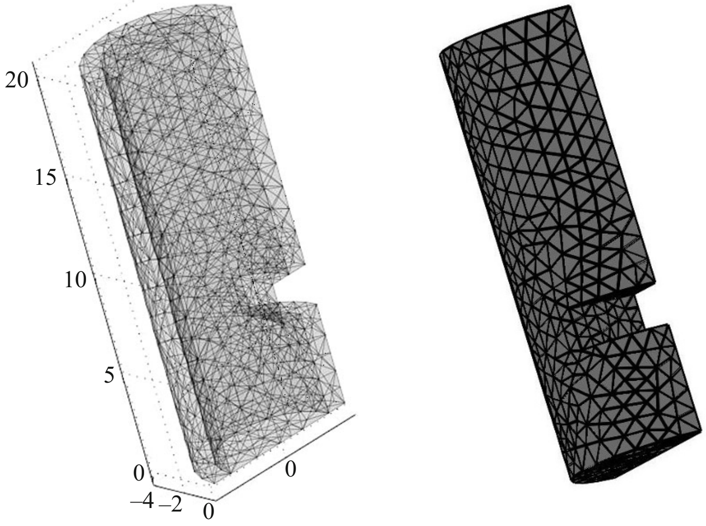

Fig. 3

the kinematic boundary conditions

$$u_x = u_x^\Gamma, \quad u_y = u_y^\Gamma, \quad u_z = u_z^\Gamma \quad (x, y, z) \in \Gamma_u \tag{5}$$

and the interface conditions between the subdomains Ω_i (perfect mechanical contact)

$$\sigma_{nx}^i = \sigma_{nx}^j, \quad \sigma_{ny}^i = \sigma_{ny}^j, \quad \sigma_{nz}^i = \sigma_{nz}^j \quad (i, j = 1, 2,\ i \neq j), \tag{6}$$

$$u_x^i = u_x^j, \quad u_y^i = u_y^j, \quad u_z^i = u_z^j \quad (i, j = 1, 2,\ i \neq j), \tag{7}$$

where

$$\sigma_{nx}^i = \sigma_{xx}^i \cos(\vec{\mathbf{n}}, x) + \sigma_{xy}^i \cos(\vec{\mathbf{n}}, y) + \sigma_{xz}^i \cos(\vec{\mathbf{n}}, z),$$

$$\sigma_{ny}^i = \sigma_{xy}^i \cos(\vec{\mathbf{n}}, x) + \sigma_{yy}^i \cos(\vec{\mathbf{n}}, y) + \sigma_{yz}^i \cos(\vec{\mathbf{n}}, z),$$

$$\sigma_{nz}^i = \sigma_{xz}^i \cos(\vec{\mathbf{n}}, x) + \sigma_{yz}^i \cos(\vec{\mathbf{n}}, y) + \sigma_{zz}^i \cos(\vec{\mathbf{n}}, z),$$

$\vec{\mathbf{n}}$ is the unit outward normal vector to the subdomain boundary Ω_i.

To solve the problem, we will use the finite-element method. According to the variational problem statement, we define a function $\vec{u} \in V \equiv \{\vec{v} \in W_2^{(1)}(\Omega) \mid \vec{v} = 0 \text{ on } \Gamma_u\}$ with which the Lagrange functional

$$I(u) = \frac{1}{2} \sum_{i=1}^{2} \int_{\Omega_i} (B^i u^i)^T D^i B^i u^i \, d\Omega_i - \sum_{i=1}^{2} \int_{\Omega_l} p^T u^i \, d\Omega_i \tag{8}$$

is minimized. The solution of system (1)–(7) or (8) is the fields of displacements, strains, and stresses. To evaluate the strength of structure elements, we use the Mises criterion

$$\sigma_M = \sqrt{0.5[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)]}. \tag{9}$$

To implement the FEM, we employ the Femlab (COMSOL Multiphysics 3.5) software package. To this end, the domain Ω was partitioned into tetrahedral finite elements (FE) shown in Fig. 2. The finite-element partition of the crown is demonstrated in Fig. 3.

TABLE 1

No.	Configuration	Crown dimensions, mm		
		l_1	l	r
1	K5205	5	2	0.5
2	K522	5	2	2
3	K0205	0	2	0.5
4	K2505	2	5	0.5
5	K0505	0	5	0.5

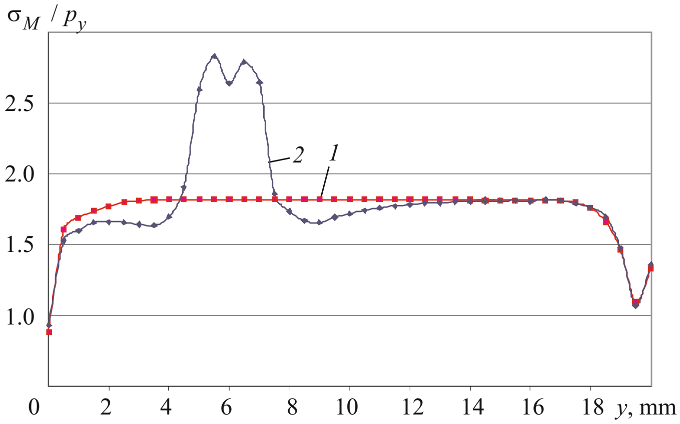

Fig. 4

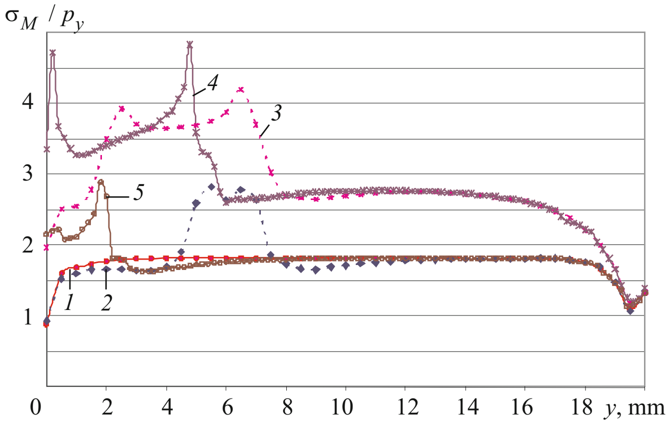

Fig. 5

2. Stress–Strain State of Tooth Crown Weakened by a Cavity. To evaluate the effect of the dimensions of the caries cavity and its location on the tooth crown, we considered five tooth-crown configurations (Table 1).

First, we determined the number of finite elements needed. The crown with caries cavity of K5205 configuration was partitioned into 3296, 13,184, and 52,736 finite elements. The stress intensities calculated with these numbers of finite elements are similar. Therefore, 3296 finite elements are sufficient to determine the stresses near the cavity. All the other calculations were performed with this number of finite elements. The stresses in the tooth crown containing a caries cavity were obtained in the case where the crown is made of enamel and has K5205 configuration. Figure 4 shows plots of relative stress intensity σ_M divided by the vertical load p_y on the tooth depending on the y-coordinate for the tooth crowns where numbers *1* and *2* refer to the cases where the cavity is absent or present, respectively.

The stress intensity in the crown of K5205 configuration is maximum near the cavity walls at the point with coordinates $x = -3.4$ mm, $y = 5$ mm, $z = 0.1$ mm. The maximum relative stress intensity in this configuration is equal to 2.78, which is double the stress intensity in the crown without caries cavity.

Figure 5 shows calculated relative stress intensity in the tooth crown of different configurations. Here numbers *1*–*5* correspond to the crown without cavity and to its configurations K5205, K2505, K0505, and K0205, respectively. As can be seen, the relative stress intensity in the K5205 and K0205 configurations where $l = 2$ mm is less than that in the configurations K2505 and K0505. Increasing the cavity length from 2 to 5 mm leads to an increase in the stress intensity by 59%. Comparing the stress intensities for the configurations K0205 and K0505 where the cavities are located at the crown clamping point near the tooth neck with those for the configurations K5205 and K2505 where the cavities are located in the middle of the crown, we see that the stress intensity in the former case is higher by 9.5% than in the latter case. We can now conclude that the risk of crown failure is higher when the caries cavity is located near the tooth neck.

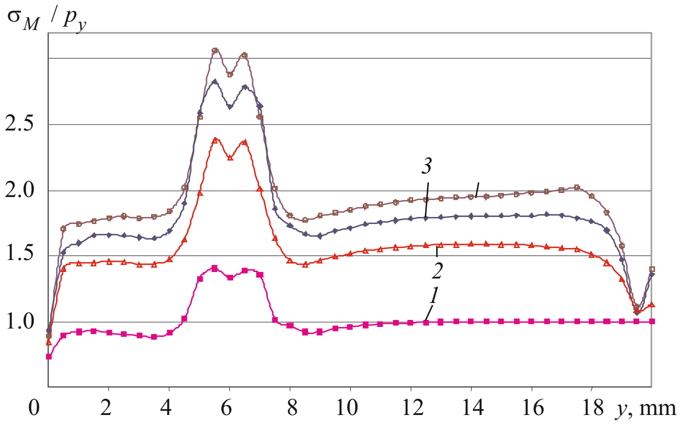

Fig. 6

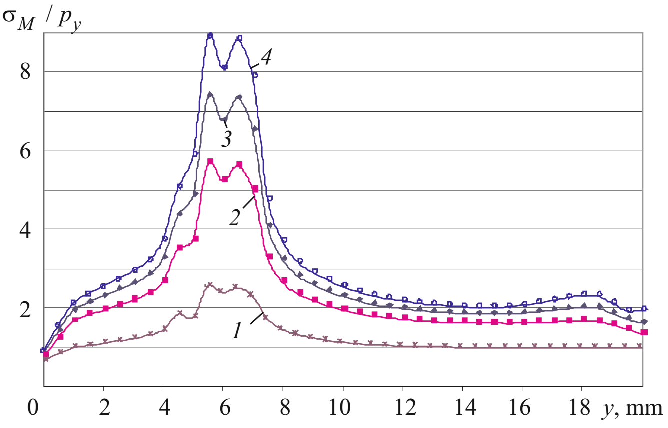

Fig. 7

The mechanical properties of the hard tooth tissues depend on such factors as the health condition, oral hygiene index, caries index, age, etc. The corrosive medium in the mouth causes demineralization of the enamel and dentin, which results in growth of micropores and degradation of the mechanical properties. Caries decreases the microhardness of the enamel by 30–40%. According to [7, 9], the elastic moduli of the enamel E_e and the dentine E_d may vary within 70–115 and 14–28 GPa, respectively.

In what follows, we will study how the ratio between the elastic moduli of the dentine and enamel affects the stress intensity near the caries cavity.

Figure 6 shows the stress intensity in the enamel layer divided by the surface load p_y for the K5205 crown configuration for the following values of E_e / E_d: 1, 3, 5, 8 (curves *1*, *2*, *3*, *4*, respectively). If the enamel and dentine moduli are equal, the stress intensity near the caries cavity exceeds the load on the occlusion surface of the crown by 49%. The stress intensity in the enamel increases with E_e / E_d. At $E_e / E_d = 8$, the maximum stress intensity near the caries cavity is higher than the surface load on the tooth by a factor of 3.

We also studied how the depth of the caries cavity affects the stress state of the tooth crown. Figure 7 shows the distributions of the relative stress intensity for the crown of K522 configuration for $r = 2$ mm and $E_e / E_d = 1, 3, 5, 8$ (curves *1*, *2*, *3*, *4*, respectively). Comparing Figs. 6 and 7 reveals that the stress intensity for the K5205 and K522 configurations increases with the depth of the caries cavity. If the elastic moduli of the enamel and dentin are equal, the stress intensity for the K522 configuration exceeds the surface load by 160%. If, however, $E_e / E_d = 8$, the stress intensity exceeds the surface load by a factor of 9.

The above results characterize the stress intensity in the tooth enamel. Figure 8 shows how the stress intensity in the dentine of the tooth crown pf the K522 configuration for $E_e / E_d = 1, 3, 5, 8$ (curves *1*, *2*, *3*, *4*, respectively). As is seen, the stress intensity in the tooth dentin is considerably lower than in the enamel, while in most of the crown, it is smaller than the external load on the occlusion surface of the crown. This indicates that the external enamel layer is the critical load-bearing element. The stress intensity exceeds the surface load p_y only in the vicinity of the caries cavity.

If $E_e / E_d = 1$, the stress intensity near the caries cavity is close to that in the enamel layer and exceeds p_y by 61%. If $E_e / E_d = 8$, the stress intensity in the dentin is less than in the enamel by a factor of 9 and exceeds the surface load only by 10%. Table 2 collects values of the relative stress intensities σ_M / p_y near the caries cavity divided by the surface load py for the enamel and dentin layers.

Conclusions. 1. Increase in the functional load and weakening, by a number of factors, of the structure and mechanical properties of the enamel and dentin in the tooth crown followed by development of the caries can induce such events as breaking off, microcracking, and fracture of the tooth crown.

2. Analysis of the stress state of the tooth crown with caries shows that an increase in the length and depth of the caries cavity in the tooth crown leads to a significant increase in the stress intensity both in the enamel and dentin layers. The stress intensity peaks near the walls of the caries cavity near the tooth neck.

3. The mechanical properties of the tooth hard tissues depend on such factors as the state of health and oral hygiene index. The stress distribution in the layers of the tooth crown depends on the ratio between the elastic moduli of the dentin and

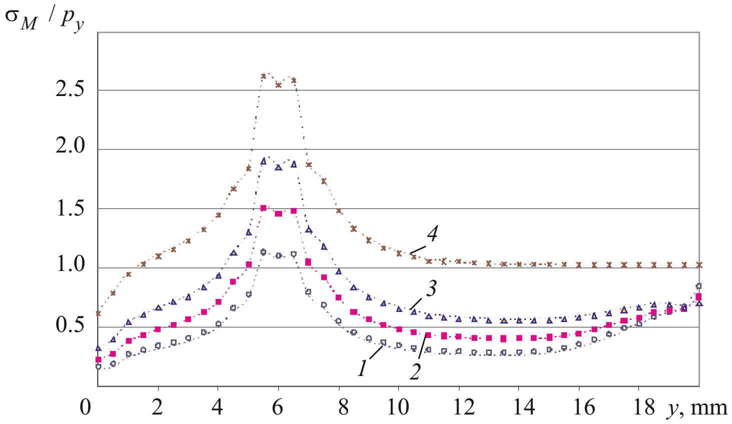

Fig. 8

TABLE 2

E_e / E_d	1	2	4	8
Enamel	2.65	5.91	7.91	9.55
Dentin	2.61	1.93	1.46	1.10

enamel. The enamel has a greater elastic modulus and bears the major portion of the external load. When the ratio is equal to eight, the stress intensity in the enamel layer (for a caries depth of 2 mm) exceeds the surface load on the tooth by a factor of 9. Moreover, the stress intensity in the dentin layer exceeds the surface load only by 10%. In the case of equal elastic moduli of the enamel and dentin, the stress intensities in the layers are similar.

4. The mathematical modeling based on the finite-element method and Femlab software package makes it possible to evaluate qualitatively how the location, geometry of the caries cavity, and the mechanical properties of the enamel and dentine influence the load-bearing capacity of the tooth crown.

5. The results obtained contribute to better insight into the effect of the dimensions and location of caries cavity on the stress–strain state of the tooth crown damaged by caries and allow developing more efficient treatment methods.

REFERENCES

1. A. I. Betel’man, *Orthopedic Stomatology* [in Russian], Meditsina, Moscow (1965).
2. O. Ya. Grigorenko, M. M. Dzharbus, S. I. Doroshenko, N. N. Tormakhov, and P. S. Flis, “Modeling the expansion of tooth arcs with an orthodontic device with an elastic member,” *Dop. NAN Ukrainy*, No. 2, 74–78 (1999).
3. O. Ya. Grigorenko, S. I. Doroshenko, N. I. Zhachko, N. N. Tormakhov, and P. S. Flis, “Modeling of the rotation of a tooth during instrumental treatment,” *Dop. NAN Ukrainy*, No. 7, 183–188 (2002).
4. Ya. M. Grigorenko, A. Ya. Grigorenko, M. F. Kopytko, A. N. Moskalenko, and L. A. Khomenko, “Mathematical modeling of functional load on hard tissues damaged by caries,” *Dop. NAN Ukrainy*, No. 8, 177–182 (2011).
5. Ya. M. Grigorenko, V. P. Nespryad’ko, V. V. Los’, and A. Ya. Grigorenko, “Prosthesis planning with intrabone implants based on the study of the stress–strain state of the bony tissue of jaws,” *Sovr. Stomatologiya*, No. 4, 122–129 (2004).
6. E. G. Zemnova, M. I. Zaslavskaya, E. V. Salina, and S. P. Rossanov, *Microflora of the Oral Cavity: Normal and Pathological*, NGMA, Nizhnii Novgorod (2004).

7. B. N. Zyryanov, P. A. Onloev, and A. P. Onloev, “Microhardness of tooth tissues in pathogenesis of teeth caries in inhabitants of Extreme North,” *Novoe v Stomatologii*, No. 10, 94–95 (2001).
8. V. K. Leont’ev and L. P. Kisel’nikova, *Child Therapeutic Stomatology. National Guide*, GEOTAR-MEDIA, Moscow (2010).
9. N. K. Loginova, A. G. Kolesnik, and V. S. Bartenev, “Physiology of enamel and dentin,” *Stomatologiya*, **85**, No. 4, 60–68 (2006).
10. K. L. Andersen, N. T. Mortensen, E. N. Pedersen, and B. Melsen, “Determination of stress levels and profiles in the periodontal ligaments by means of an improved three-dimensional finite element model for various types of orthodontic and natural force system,” *J. Biomedical Eng.*, **13**, No. 14, 293–303 (1991).
11. P. Flis, A. Grigorenko, N. Doroshenko, N. Tormakhov, and V. Filonenko, “Mechanical and mathematical modeling of class treatment with orthodontic appliances with a material ramp,” *EUREKA: Health Sciences*, 3(83), 60–68 (2016).
12. J. Middleton, M. L. Jones, and A. N. Wilson, “Three-dimensional analysis of orthodontic tooth movement,” *J. Biomedical Eng.*, **12** (4), 319–327 (1990).
13. C. G. Provatidis, “A comparative FEM–study of tooth mobility using isotropic and anisotropic models of the periodontal ligament,” *Medical Eng. Phys.*, **22**, 359–370 (2000).