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TWO-CONTINUUM MECHANICS OF DIELECTRICS AS THE BASIS OF THE THEORY OF PIEZOELECTRICITY AND ELECTROSTRICTION

L. P. Khoroshun

A new principle for constructing the theory of coupled dynamic electroelasticity of dielectrics that have piezoelectric and electrostrictive effects is expounded. This theory is based on the purely mechanical two-continuum description of the deformation of dielectrics as a mixture of positive and negative charges coupled into pairs neutral molecules or elementary cells provided that an elastic potential exists and the partial stresses are in linear quadratic relationship with the difference of displacements of charges. Based on the definition of the vector of polarization of an elementary dielectric macrovolume and the electric field generated by it, the equations of two-continuum mechanics are transformed into coupled dynamic equations for the macrodisplacements of neutral molecules and electric-field strength that describe the piezoelectric and electrostrictive effects. Maxwell's equations follow from these equations as a particular case.

Keywords: dielectrics, two-continuum mechanics, piezoelectrics, electrostriction, polarization vector, Maxwell's equations

Introduction. In modern mechanics, static and dynamic problems of magnetoelasticity [15, 18] and electroelasticity [1, 2, 7, 9–12, 16, 19–21] are important. They are related to the piezoelectric effect that manifests itself as direct or inverse linear relationship between strains and electric-field strength. The piezoelectric effect is observed in solid dielectrics whose lattice has no center of symmetry. Also, solid, liquid, and gaseous dielectrics can display electrostriction [17] manifested as quadratic relationship between strains and electric-field strength, irrespective of the structure and symmetry of substance. In this case, there is no inverse electromechanical effect, i.e., homogeneous deformation does not lead to polarization and occurrence of electric field, unlike piezoelectric materials. In an oscillating electric field, mechanical vibrations caused by electrostriction occur at a frequency twice higher than the frequency of the field. The electrostrictive and piezoelectric effects are used in various electromechanical transducers at macro- and microscales.

The theory of electroelasticity [1, 2] is based on the equations of the statics or dynamics of an elastic body, the equation of electrostatics (acoustic approximation), and the constitutive equations relating the stress tensor and the electric-flux density to the strain tensor and an electric-field strength. The constitutive equations are derived from the condition of the existence of internal energy that is a function of strains and electric-flux density. The piezoelectric effect is described by quadratic dependence of internal energy on the corresponding parameters, and electrostriction by cubic dependence providing linear dependence of stresses on strains and quadratic dependence on electric-field strength.

The basic shortcoming of the acoustic approximation is the failure to account for the dynamics of the electromagnetic field, which does not allow describing coupled acoustic and electromagnetic oscillations. Moreover, the range of applicability of the acoustic approximation is limited by the frequency range in which permittivity remains constant. To eliminate these shortcomings, it is necessary to develop new principles of constructing the theory of coupled dynamic electroelasticity. Such a principle based on the pure mechanical two-continuum description of the behavior of dielectrics as mixtures of positive and

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S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine 03057, e-mail: stochac@inmech.kiev.ua. Translated from Prikladnaya Mekhanika, Vol. 54, No. 2, pp. 29–41, March–April, 2018. Original article submitted December 20, 2016.

negative charges is outlined in [13] and used to derive coupled dynamic equations of electroelasticity of piezoelectric materials and, in particular, equations of electrodynamics more general than Maxwell's equations [8, 14]. Here we generalize this theory to nonlinear electroelasticity associated with the electrostrictive effect.

1. Nonlinear Equations of the Two-Continuum Mechanics of Dielectrics. Consider an elementary macrovolume of a dielectric that consists of interacting neutral molecules or unit cells [6], each consisting of coupled (carriers of) positive and negative charges. In the initial state, the densities of positive, n_{10}, and negative, n_{20}, charges coincide and equal the number of molecules or unit cells in a unit macrovolume: $n_{10} = n_{20} = N$. The current values n_1 and n_2 satisfy the balance equations

$$\frac{\partial n_1}{\partial t} + (n_1 \dot{u}_i^1)_{,i} = 0, \quad \frac{\partial n_2}{\partial t} + (n_2 \dot{u}_i^2)_{,i} = 0, \tag{1.1}$$

where $\dot{u}_i^1$ and $\dot{u}_i^2$ are the vectors of velocities of positive and negative charges in an elementary macrovolume; overdot denotes substantial differentiation with respect to time,

$$\dot{u}_i^1 \equiv \frac{d_1 u_i^1}{dt} = \frac{\partial u_i^1}{\partial t} + u_{i,n}^1 \dot{u}_n^1, \quad \dot{u}_i^2 \equiv \frac{d_2 u_i^2}{dt} = \frac{\partial u_i^2}{\partial t} + u_{i,n}^2 \dot{u}_n^2. \tag{1.2}$$

Multiplying Eqs. (1.1) on the masses of positive and negative charges, m_1, m_2, respectively, we obtain the equations of conservation of mass

$$\frac{\partial \rho_1}{\partial t} + (\rho_1 \dot{u}_i^1)_{,i} = 0, \quad \frac{\partial \rho_2}{\partial t} + (\rho_2 \dot{u}_i^2)_{,i} = 0, \tag{1.3}$$

where $\rho_1 = n_1 m_1, \rho_2 = n_2 m_2$ are the mass densities of positive and negative charges, respectively.

Following the analogy with the theory of mechanical mixtures [6], we introduce partial stresses $\sigma_{ij}^1, \sigma_{ij}^2$ as components of the resultant of the forces acting on positive and negative charges of a dielectric area element divided by its area. Then the equations of conservation of momentum of positive and negative charges in an elementary macrovolume of dielectric can be represented as

$$\rho_1 \ddot{u}_i^1 = \sigma_{ij,j}^1 + R_i + F_i^1,$$

$$\rho_2 \ddot{u}_i^2 = \sigma_{ij,j}^2 - R_i + F_i^2, \tag{1.4}$$

where R_i is the resultant force of interaction between the positive and negative charges per elementary macrovolume; F_i^1, F_i^2 are the external volume forces acting on the respective charges; $\ddot{u}_i^1, \ddot{u}_i^2$ are substantial derivatives of velocities with respect to time,

$$\ddot{u}_i^1 \equiv \frac{d_1 \dot{u}_i^1}{dt} = \frac{\partial \dot{u}_i^1}{\partial t} + \dot{u}_{i,n}^1 \dot{u}_n^1, \quad \ddot{u}_i^2 \equiv \frac{d_2 \dot{u}_i^2}{dt} = \frac{\partial \dot{u}_i^2}{\partial t} + \dot{u}_{i,n}^2 \dot{u}_n^2. \tag{1.5}$$

The balance equations (1.3), (1.4) should be supplemented with the constitutive equations relating the dynamic and kinematic parameters. Let the dielectric be perfectly elastic. Following the analogy with the classical theory of elasticity [3], we multiply Eqs. (1.4) by $\dot{u}_i^1$ and $\dot{u}_i^2$, respectively, sum them, and integrate over some volume V of the dielectric bounded by a surface S. Doing so results in the law of conservation of energy

$$\int_V (\dot{T} + \dot{U})\, dV = \int_V (F_i^1 \dot{u}_i^1 + F_i^2 \dot{u}_i^2)\, dV + \int_S (\sigma_{ij}^1 n_j \dot{u}_i^1 + \sigma_{ij}^2 n_j \dot{u}_i^2)\, dS, \tag{1.6}$$

where n_j are the direction cosines of the normal to the surface S; T and $\dot{U}$ are the kinetic energy and the rate of increase in the internal energy,

$$T = \frac{1}{2}(\rho_1 \dot{u}_i^1 \dot{u}_i^1 + \rho_2 \dot{u}_i^2 \dot{u}_i^2), \quad \dot{U} = \sigma_{ij}^1 \dot{\varepsilon}_{ij}^1 + \sigma_{ij}^2 \dot{\varepsilon}_{ij}^2 - R_i (\dot{u}_i^1 - \dot{u}_i^2),$$

$$\varepsilon_{ij}^k = \frac{1}{2}(u_{i,j}^k + u_{j,i}^k) \quad (k = 1, 2). \tag{1.7}$$

It follows from (1.7) that the internal energy U is a function of the kinematic parameters $\varepsilon_{ij}^1, \varepsilon_{ij}^2, u_i^1 - u_i^2$ and the dynamic parameters $\sigma_{ij}^1, \sigma_{ij}^2, R_i$ are determined by the derivatives

$$\sigma_{ij}^1 = \frac{\partial U}{\partial \varepsilon_{ij}^1}, \quad \sigma_{ij}^2 = \frac{\partial U}{\partial \varepsilon_{ij}^2}, \quad R_i = -\frac{\partial U}{\partial (u_i^1 - u_i^2)}. \tag{1.8}$$

If the internal energy depends on the kinematic parameters as

$$U = \frac{1}{2}\lambda_{ijmn}^{11} \varepsilon_{ij}^1 \varepsilon_{mn}^1 + \lambda_{ijmn}^{12} \varepsilon_{ij}^1 \varepsilon_{mn}^2 + \frac{1}{2}\lambda_{ijmn}^{22} \varepsilon_{ij}^2 \varepsilon_{mn}^2 + h_{mij}^1 \varepsilon_{ij}^1 (u_m^1 - u_m^2)$$

$$+ h_{mij}^2 \varepsilon_{ij}^2 (u_m^1 - u_m^2) + \frac{1}{2}\kappa_{ij}(u_i^1 - u_i^2)(u_j^1 - u_j^2) - \frac{1}{2} h_{mnij}^1 \varepsilon_{ij}^1 (u_m^1 - u_m^2)(u_n^1 - u_n^2)$$

$$- \frac{1}{2} h_{mnij}^2 \varepsilon_{ij}^2 (u_m^1 - u_m^2)(u_n^1 - u_n^2), \tag{1.9}$$

then, using (1.8), we arrive at the nonlinear constitutive equations

$$\sigma_{ij}^1 = \lambda_{ijmn}^{11} \varepsilon_{mn}^1 + \lambda_{ijmn}^{12} \varepsilon_{mn}^2 + h_{mij}^1 (u_m^1 - u_m^2) - \frac{1}{2} h_{mnij}^1 (u_m^1 - u_m^2)(u_n^1 - u_n^2),$$

$$\sigma_{ij}^2 = \lambda_{ijmn}^{21} \varepsilon_{mn}^1 + \lambda_{ijmn}^{22} \varepsilon_{mn}^2 + h_{mij}^2 (u_m^1 - u_m^2) - \frac{1}{2} h_{mnij}^2 (u_m^1 - u_m^2)(u_n^1 - u_n^2),$$

$$R_i = -\kappa_{ij}(u_j^1 - u_j^2) - h_{imn}^1 \varepsilon_{mn}^1 - h_{imn}^2 \varepsilon_{mn}^2 + h_{ijmn}^1 (u_j^1 - u_j^2)\varepsilon_{mn}^1 + h_{ijmn}^2 (u_j^1 - u_j^2)\varepsilon_{mn}^2$$

$$(\lambda_{ijmn}^{\nu k} = \lambda_{mnij}^{k\nu} = \lambda_{jimn}^{\nu k} = \lambda_{ijnm}^{\nu k}, \ h_{ijmn}^k = h_{jimn}^k = h_{ijnm}^k, \ h_{imn}^k = h_{inm}^k, \ \kappa_{ij} = \kappa_{ji}). \tag{1.10}$$

Let

$$u_i^1 = u_i + u_i', \quad u_i^2 = u_i - u_i', \quad \sigma_{ij} = \sigma_{ij}^1 + \sigma_{ij}^2, \quad \sigma_{ij}' = \sigma_{ij}^1 - \sigma_{ij}^2. \tag{1.11}$$

Then Eqs. (1.3), (1.4) and (1.7), (1.9), (1.10) transform into

$$\frac{\partial \rho}{\partial t} + (\rho \dot{u}_i + \rho' \dot{u}_i')_{,i} = 0, \quad \frac{\partial \rho'}{\partial t} + (\rho' \dot{u}_i + \rho \dot{u}_i')_{,i} = 0, \tag{1.12}$$

$$\rho \ddot{u}_i + \rho' \ddot{u}_i' = \sigma_{ij,j} + F_i, \quad \rho' \ddot{u}_i + \rho \ddot{u}_i' = \sigma_{ij,j}' + 2R_i + F_i', \tag{1.13}$$

$$T = \frac{1}{2}\rho(\dot{u}_i \dot{u}_i + \dot{u}_i' \dot{u}_i') + \rho' \dot{u}_i \dot{u}_i', \quad U = \frac{1}{2}\lambda_{ijmn}^* \varepsilon_{ij} \varepsilon_{mn} + \overline{\lambda}_{ijmn} \varepsilon_{ij} \varepsilon_{mn}' + \frac{1}{2}\lambda_{ijmn} \varepsilon_{ij}' \varepsilon_{mn}'$$

$$+ h_{mij}^* \varepsilon_{ij} u_m' + h_{mij}' \varepsilon_{ij}' u_m' + 2\kappa_{ij} u_i' u_j' - h_{mnij}^* \varepsilon_{ij} u_m' u_n' - h_{mnij}' \varepsilon_{ij}' u_m' u_n', \tag{1.14}$$

$$\sigma_{ij} = \frac{\partial U}{\partial \varepsilon_{ij}} = \lambda_{ijmn}^* \varepsilon_{mn} + \overline{\lambda}_{ijmn} \varepsilon_{mn}' + h_{mij}^* u_m' - h_{mnij}^* u_m' u_n',$$

$$\sigma'_{ij} = \frac{\partial U}{\partial \varepsilon'_{ij}} = \bar{\lambda}_{mnij}\varepsilon_{mn} + \lambda_{ijmn}\varepsilon'_{mn} + h'_{mij}u'_m - h'_{mnij}u'_m u'_n, \tag{1.15}$$

$$2R_i = -\frac{\partial U}{\partial u'_i} = -4\kappa_{ij}u'_j - h^*_{imn}\varepsilon_{mn} - h'_{imn}\varepsilon'_{mn} + 2h^*_{ijmn}u'_j\varepsilon_{mn} + 2h'_{ijmn}u'_j\varepsilon'_{mn},$$

where

$$\ddot{u}_i = \frac{\partial \dot{u}_i}{\partial t} + \dot{u}_{i,n}\dot{u}_n + \dot{u}'_{i,n}\dot{u}'_n, \qquad \ddot{u}'_i = \frac{\partial \dot{u}'_i}{\partial t} + \dot{u}_{i,n}\dot{u}'_n + \dot{u}'_{i,n}\dot{u}_n,$$

$$\dot{u}_i = \frac{\partial u_i}{\partial t} + u_{i,n}\dot{u}_n + u'_{i,n}\dot{u}'_n, \qquad \dot{u}'_i = \frac{\partial u'_i}{\partial t} + u_{i,n}\dot{u}'_n + u'_{i,n}\dot{u}_n,$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \qquad \varepsilon'_{ij} = \frac{1}{2}(u'_{i,j} + u'_{j,i}), \qquad F_i = F_i^1 + F_i^2, \qquad F'_i = F_i^1 - F_i^2,$$

$$\lambda^*_{ijmn} = \lambda^{11}_{ijmn} + \lambda^{21}_{ijmn} + \lambda^{12}_{ijmn} + \lambda^{22}_{ijmn}, \qquad \bar{\lambda}_{ijmn} = \lambda^{11}_{ijmn} + \lambda^{21}_{ijmn} - \lambda^{12}_{ijmn} - \lambda^{22}_{ijmn},$$

$$\lambda_{ijmn} = \lambda^{11}_{ijmn} + \lambda^{22}_{ijmn} - \lambda^{12}_{ijmn} - \lambda^{21}_{ijmn}, \qquad h^*_{imn} = 2(h^1_{imn} + h^2_{imn}),$$

$$h'_{imn} = 2(h^1_{imn} - h^2_{imn}), \qquad h^*_{ijmn} = 2(h^1_{ijmn} + h^2_{ijmn}), \qquad h'_{ijmn} = 2(h^1_{ijmn} - h^2_{ijmn}),$$

$$\rho = \rho_1 + \rho_2, \qquad \rho' = \rho_1 - \rho_2. \tag{1.16}$$

Substituting (1.15) into (1.13), we arrive at a system of coupled dynamic equations for displacements of neutral molecules $u_i = (u_i^1 + u_i^2)/2$ and half mutual displacements $u'_i = (u_i^1 - u_i^2)/2$ of positive and negative charges:

$$\rho\ddot{u}_i + \rho'\ddot{u}'_i = \lambda^*_{ijmn}u_{m,nj} + \bar{\lambda}_{ijmn}u'_{m,nj} + h^*_{mnij}u'_{m,j} - h^*_{mnij}(u'_m u'_n)_{,j} + F_i,$$

$$\rho'\ddot{u}_i + \rho\ddot{u}'_i = \bar{\lambda}_{mnij}u_{m,nj} + \lambda_{ijmn}u'_{m,nj} - h^*_{imn}u_{m,n} + h_{mij}u'_{m,j} - 4\kappa_{ij}u'_j + 2h^*_{ijmn}u'_j u_{m,n} + 2h_{ijmn}u'_j u'_{m,n} + F'_i$$

$$(h_{mij} = h'_{mij} - h'_{imj}, \qquad h_{ijmn} = h'_{ijmn} - h'_{jmin}). \tag{1.17}$$

For isotropic dielectrics, the material tensors appearing in (1.17) are represented by the formulas

$$\lambda^*_{ijmn} = \lambda^*\delta_{ij}\delta_{mn} + 2\mu^* I_{ijmn}, \qquad \bar{\lambda}_{ijmn} = \bar{\lambda}\delta_{ij}\delta_{mn} + 2\bar{\mu}I_{ijmn}, \qquad \lambda_{ijmn} = \lambda\delta_{ij}\delta_{mn} + 2\mu I_{ijmn},$$

$$h^*_{ijmn} = h^*\delta_{ij}\delta_{mn} + 2l^* I_{ijmn}, \qquad h'_{ijmn} = h'\delta_{ij}\delta_{mn} + 2l' I_{ijmn}, \qquad \kappa_{ij} = \kappa\delta_{ij}, \qquad h^*_{imn} = h_{imn} = 0, \tag{1.18}$$

where $\lambda^*, \mu^*, \bar{\lambda}, \bar{\mu}, \lambda, \mu, h^*, l^*, h, l, \kappa$ are material constants; δ_{ij}, I_{ijmn} are unit tensors. Then Eqs. (1.17) become

$$\rho\ddot{u}_i + \rho'\ddot{u}'_i = \mu^* u_{i,rr} + (\lambda^* + \mu^*)u_{r,ri} + \bar{\mu}u'_{i,rr} + (\bar{\lambda} + \bar{\mu})u'_{r,ri} - h^*(u'_n u'_n)_{,i} - 2l^*(u'_i u'_n)_{,n} + F_i,$$

$$\rho'\ddot{u}_i + \rho\ddot{u}'_i = \bar{\mu}u_{i,rr} + (\bar{\lambda} + \bar{\mu})u_{r,ri} + \mu u'_{i,rr} + (\lambda + \mu)u'_{r,ri} - 4\kappa u'_i + 2h^* u'_i u_{n,n} + 2l^* u'_n(u_{i,n} + u_{n,i})$$

$$+ 2(h' - l')(u'_i u'_{n,n} - u'_n u'_{n,i}) + F'_i. \tag{1.19}$$

These equations can be simplified by assuming that charges are symmetric rather than detailing the structure of atoms, molecules, or unit cells; i.e., the material tensors are invariant under permutation of charges. In this case, it is necessary to set $\lambda^{11}_{ijmn} = \lambda^{22}_{ijmn}, \lambda^{12}_{ijmn} = \lambda^{21}_{ijmn}, h^1_{ijmn} = h^2_{ijmn}, h^1_{imn} = h^2_{imn}$ which leads to $\bar{\lambda}_{ijmn} = 0, h'_{ijmn} = 0, h'_{imn} = 0$. As a result, the constitutive equations (1.15) and the dynamic equations (1.17), (1.19) become simpler:

$$\sigma_{ij} = \lambda^*_{ijmn}\varepsilon_{mn} + h^*_{mij}u'_m - h^*_{mnij}u'_m u'_n, \qquad \sigma'_{ij} = \lambda_{ijmn}\varepsilon'_{mn},$$

$$2R_i = -4\kappa_{ij}u'_j - h^*_{imn}\varepsilon_{mn} + 2h^*_{ijmn}u'_j\varepsilon_{mn}, \tag{1.20}$$

$$\rho\ddot{u}_i + \rho'\ddot{u}'_i = \lambda^*_{ijmn}u_{m,nj} + h^*_{mij}u'_{m,j} - h^*_{mnij}(u'_m u'_n)_{,j} + F_i,$$

$$\rho'\ddot{u}_i + \rho\ddot{u}'_i = \lambda_{ijmn}u'_{m,nj} - h^*_{imn}u_{m,n} - 4\kappa_{ij}u'_j + 2h^*_{ijmn}u'_j u_{m,n} + F'_i, \tag{1.21}$$

$$\rho\ddot{u}_i + \rho'\ddot{u}'_i = \mu^* u_{i,rr} + (\lambda^* + \mu^*)u_{r,ri} - h^*(u'_n u'_n)_{,i} - 2l^*(u'_i u'_n)_{,n} + F_i,$$

$$\rho'\ddot{u}_i + \rho\ddot{u}'_i = \mu u'_{i,rr} + (\lambda + \mu)u'_{r,ri} - 4\kappa u'_i + 2h^* u'_i u_{n,n} + 2l^* u'_n(u_{i,n} + u_{n,i}) + F'_i. \tag{1.22}$$

The equations for perfectly liquid or gaseous dielectric follow, as a limiting case, from Eqs. (1.22) for isotropic elastic dielectric if $\mu^* = 0, l^* = 0$:

$$\rho\ddot{u}_i + \rho'\ddot{u}'_i = -p_{,i} + F_i \qquad (p = p_0 - \lambda^* u_{r,r} + h^* u'_n u'_n),$$

$$\rho'\ddot{u}_i + \rho\ddot{u}'_i = \mu u'_{i,rr} + (\lambda + \mu)u'_{r,ri} - 4\kappa u'_i + 2h^* u'_i u_{n,n} + F'_i, \tag{1.23}$$

where p and p_0 are the pressure in disturbed and resting liquid dielectric, respectively.

Equations (1.17), (1.19), (1.21)–(1.23) are invariant under Galilean transformation. If the nonlinear inertial terms are neglected, i.e., $\ddot{u}_i = \partial^2 u_i / \partial t^2$, $\ddot{u}'_i = \partial^2 u'_i / \partial t^2$ in (1.16) on the left-hand sides of (1.17), (1.19), (1.21)–(1.23), then they are no longer invariant under Galilean transformations.

2. Coupled Electromechanical Equations. The above coupled equations of nonlinear two-continuum mechanics of dielectrics include purely mechanical parameters $\sigma_{ij}, \varepsilon_{ij}, u_i, \rho$ and $\sigma'_{ij}, \varepsilon'_{ij}, u'_i, \rho', R_i$ describing the stress–strain state of the dielectric as a system of neutral molecules or cells and differences of parameters related to coupled charges that describe the polarization of the dielectric. For example, the displacement difference vector u'_i multiplied by the charge density of the dielectric is the polarization vector. Therefore, the task is to transform them into the coupled equations of mechanics of neutral molecules or cells and equations of electricity of coupled charges. To this end, we multiply the equations of balance of charge densities (1.1) by charges q and $-q$, respectively, and sum them. Taking (1.11) into account, we obtain the law of conservation of electric charge:

$$\frac{\partial\rho_e}{\partial t} + I_{i,i} = 0, \tag{2.1}$$

where

$$\rho_e = (n_1 - n_2)q, \quad I_i = I_i^{\text{con}} + I_i^{\text{pol}}, \quad I_i^{\text{con}} = \rho_e\dot{u}_i, \quad I_i^{\text{pol}} = (n_1 + n_2)q\dot{u}'_i, \tag{2.2}$$

where ρ_e is the density of polarization or coupled charges, which is the total density because of the absence of free charges; I_i^{con} is the convection current caused by the displacement of polarization charges; I_i^{pol} is the polarization current or rate.

If the charge densities coincide ($n_{10} = n_{20} = N$) at initial time, then integrating Eq. (2.1) over time, we obtain an equation in linear approximation:

$$\rho_e + P_{i,i} = 0, \tag{2.3}$$

where P_i is the polarization vector,

$$P_i = 2Nqu'_i. \tag{2.4}$$

The density of polarization charges ρ_e generates an electric field E_i that, according to the Gauss theorem [4, 5], is defined by

$$E_{i,i} = 4\pi k\rho_e, \tag{2.5}$$

where $k = 1$ and $k = 1/4\pi\varepsilon_0$ in the CGS and SI systems, respectively. Substituting (2.3) into (2.5), we arrive at the equation

$$(E_i + 4\pi kP_i)_{,i} = 0, \tag{2.6}$$

whose general solution is

$$E_i = -4\pi kP_i, \tag{2.7}$$

where the electric-field strength is represented in terms of scalar and vector potentials $E_i = \xi_{,i} + e_{imn}\eta_{n,m}$ satisfying the equations $\xi_{,rr} = -4\pi kP_{r,r}$, $\eta_{i,rr} = 4\pi ke_{imn}P_{n,m}$, where e_{imn} is the Levy–Chivita unit antisymmetric tensor.

The polarization caused by the mutual displacement of coupled charges can be caused by various factors such as an external electric field or a field of free charges, inertial forces, deformation of dielectric, change of temperature. If the dielectric is in an external static electric field E_i^{ex} associated with some density of free electric charges ρ_e^{ex}, according to the Gauss theorem,

$$E_{i,i}^{\text{ex}} = 4\pi k\rho_e^{\text{ex}}, \tag{2.8}$$

and the other factors causing polarization are absent, then, according to the experimental law [4, 5], the polarization vector of an unit volume is defined by

$$P_i = \beta E_i^*, \quad P_i = \varepsilon_0\beta E_i^*$$

$$(E_i^* = E_i^{\text{ex}} + E_i) \tag{2.9}$$

in the CGS and SI systems, respectively, where β is the polarizability or electric susceptibility of an unit volume of dielectric. Then the expression for electric-flux density D_i follows from (2.7)–(2.9) and has the following forms in the CGC and SI systems, respectively:

$$D_{i,i} = 4\pi\rho_e^{\text{ex}} \quad (D_i = E_i^* + 4\pi P_i = \chi E_i^*, \quad \chi = 1 + 4\pi\beta), \tag{2.10}$$

$$D_{i,i} = \rho_e^{\text{ex}} \quad (D_i = \varepsilon_0 E_i^* + P_i = \varepsilon_0\chi E_i^*, \quad \chi = 1 + \beta), \tag{2.11}$$

where χ is permittivity.

It follows from (2.7)–(2.11) that

$$D_i = E_i^{\text{ex}}, \quad D_i = \varepsilon_0 E_i^{\text{ex}}, \tag{2.12}$$

in the SGS and the SI systems, respectively, i.e. the electric-flux density [4] is just the strength of external electric field of free charges or a quantity proportional to it.

If dynamic processes occur in dielectric, the coupled charges are acted upon by inertial forces, making relations (2.9)–(2.11) invalid. In this case, to determine the polarization vector P_i or the electric field E_i generated by it, according to (2.7), it is necessary to derive dynamic equations. Therefore, the definition of displacement current in electrodynamics as time derivative of electric-flux density $\frac{1}{4\pi}\frac{\partial D}{\partial t}$ or $\frac{\partial D}{\partial t}$ in the SGS and SI systems, respectively, is incorrect because the concept of electric-flux density loses sense for dynamic processes. Moreover, according to (2.12) and the definition, the displacement current is only determined by the external electric field E_i^{ex} and is equal to zero if it is absent.

This conclusion also follows from the well-known justification of displacement current [5] based on the law of conservation of electric charge. Indeed, the law of conservation of the electric charge (2.1) with (2.5) can be represented as

$$(I_i^{\text{con}} + I_i^{\text{d}})_{,i} = 0, \tag{2.13}$$

where $I_i^{\text{d}} = \dfrac{1}{4\pi k}\dfrac{\partial E_i}{\partial t} + I_i^{\text{con}}$ is the displacement current for dielectric. Taking (2.3), (2.4), (2.7) into accoiunt, we conclude that the displacement current in dielectric is equal to zero.

Using formulas (2.4) and (2.7), we obtain

$$u_i' = \frac{1}{2Nq}P_i = -\nu E_i \qquad \left(\nu = \frac{1}{8\pi kNq}\right). \tag{2.14}$$

The volume forces F_i^1 and F_i^2 are represented as sums of purely mechanical and ponderomotive components caused by electric field:

$$F_i^1 = \overline{F}_i^1 + \overline{\overline{F}}_i^1, \quad F_i^2 = \overline{F}_i^2 + \overline{\overline{F}}_i^2, \quad \overline{\overline{F}}_i^1 = -\overline{\overline{F}}_i^2 = Nqk(E_i + E_i^{\text{ex}}), \tag{2.15}$$

where E_i^{ex} is the given strength of the external electric field. Using $\overline{F}_i^1 = \overline{F}_i^2$ and (1.16), we get

$$F_i = \overline{F}_i^1 + \overline{F}_i^2, \quad F_i' = \overline{\overline{F}}_i' = 2Nqk(E_i + E_i^{\text{ex}}). \tag{2.16}$$

Substituting (2.1)–(2.16) into (1.14), (1.20)–(1.23), we get the expressions for kinetic and internal energy:

$$T = \frac{1}{2}\rho(\dot{u}_i\dot{u}_i + \nu^2\dot{E}_i\dot{E}_i) - \nu\rho'\dot{u}_i\dot{E}_i, \quad U = \frac{1}{2}\lambda^*_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + \frac{1}{2}\nu^2\lambda_{ijmn}E_{ij}E_{mn}$$

$$-\nu h^*_{mij}\varepsilon_{ij}E_m + 2\nu^2\kappa_{ij}E_iE_j - \nu^2 h^*_{mnij}\varepsilon_{ij}E_mE_n \quad \left(E_{ij} = \frac{1}{2}(E_{i,j} + E_{j,i})\right), \tag{2.17}$$

the constitutive equations

$$\sigma_{ij} = \lambda^*_{ijmn}\varepsilon_{mn} - \nu h^*_{mij}E_m - \nu^2 h^*_{mnij}E_mE_n, \quad \sigma'_{ij} = -\nu\lambda_{ijmn}E_{mn},$$

$$2R_i = 4\nu\kappa_{ij}E_j - h^*_{imn}\varepsilon_{mn} - 2\nu h^*_{ijmn}E_j\varepsilon_{mn}, \tag{2.18}$$

the dynamic nonlinear equations of electroelasticity for anisotropic and isotropic dielectrics:

$$\rho\ddot{u}_i - \nu\rho'\ddot{E}_i = \lambda^*_{ijmn}u_{m,nj} - \nu h^*_{mij}E_{m,j} - \nu^2 h^*_{mnij}(E_mE_n)_{,j} + F_i,$$

$$\rho'\ddot{u}_i - \nu\rho\ddot{E}_i = -h^*_{imn}u_{m,n} - \nu(\lambda_{ijmn}E_{m,nj} - 4\kappa^*_{ij}E_j + 2h^*_{ijmn}E_ju_{m,n}) + 2NqkE_i^{\text{ex}}, \tag{2.19}$$

$$\rho\ddot{u}_i - \nu\rho'\ddot{E}_i = \mu^* u_{i,rr} + (\lambda^* + \mu^*)u_{r,ri} - \nu^2\left[h^*(E_nE_n)_{,i} + 2l^*(E_iE_n)_{,n}\right] + F_i, \tag{2.20}$$

$$\rho'\ddot{u}_i - \nu\rho\ddot{E}_i = -\nu\left[\mu E_{i,rr} + (\lambda + \mu)E_{r,ri} - 4\kappa^* E_i + 2h^* E_iu_{n,n} + 2l^* E_n(u_{i,n} + u_{n,i})\right] + 2NqkE_i^{\text{ex}},$$

and the equations of electrohydromechanics for a perfectly liquid dielectric:

$$\rho\ddot{u}_i - \nu\rho'\ddot{E}_i = -p_{,i} + F_i \quad (p = p_0 - \lambda^* u_{r,r} + \nu^2 h^* E_nE_n),$$

$$\rho'\ddot{u}_i - \nu\rho\ddot{E}_i = -\nu\left[\mu E_{i,rr} + (\lambda + \mu)E_{r,ri} - 4\kappa^* E_i + 2h^* E_iu_{n,n}\right] + 2NqkE_i^{\text{ex}}, \tag{2.21}$$

where

$$\ddot{u}_i = \frac{\partial \dot{u}_i}{\partial t} + \dot{u}_{i,n}\dot{u}_n + \nu^2 \dot{E}_{i,n}\dot{E}_n, \quad \ddot{E}_i = \frac{\partial \dot{E}_i}{\partial t} + \dot{u}_{i,n}\dot{E}_n + \dot{E}_{i,n}\dot{u}_n,$$

$$\dot{u}_i = \frac{\partial u_i}{\partial t} + u_{i,n}\dot{u}_n + \nu^2 E_{i,n}\dot{E}_n, \quad \dot{E}_i = \frac{\partial E_i}{\partial t} + u_{i,n}\dot{E}_n + E_{i,n}\dot{u}_n,$$

$$\kappa^*_{ij} = \kappa_{ij} + 4\pi k N^2 q^2 \delta_{ij}, \quad \kappa^* = \kappa + 4\pi k N^2 q^2. \tag{2.22}$$

Equations (2.19)–(2.22) should be supplemented with the law of conservation of mass, follow from (1.3), (1.11), (2.14):

$$\frac{\partial \rho}{\partial t} + (\rho \dot{u}_i - \nu \rho' \dot{E}_i)_{,i} = 0, \quad \frac{\partial \rho'}{\partial t} + (\rho' \dot{u}_i - \nu \rho \dot{E}_i)_{,i} = 0. \tag{2.23}$$

Equations (2.19)–(2.23) are invariant under Galilean transformation.

Multiplying Eqs. (1.13) by $\dot{u}_i, -\nu \dot{E}_i$, summing them, and integrating over some domain V bounded by a surface S, we obtain the law of conservation of energy

$$\int_V (\dot{T} + \dot{U}^*)\, dV = \int_V (F_i \dot{u}_i - 2\nu N q E^{\text{ex}}_i \dot{E}_i)\, dV + \int_S (\sigma_{ij} n_j \dot{u}_i - \nu \sigma'_{ij} n_j \dot{E}_i)\, dS, \tag{2.24}$$

where the parameters $T, \sigma_{ij}, \sigma'_{ij}, \dot{u}_i, \dot{E}_i$ are defined by (2.17), (2.18), (2.22), and U^* is expressed as

$$U^* = U + \nu N q E_i E_i = \frac{1}{2}\lambda^*_{ijmn}\varepsilon_{ij}\varepsilon_{mn} + \frac{1}{2}\nu^2 \lambda_{ijmn} E_{ij} E_{mn}$$

$$-\nu h^*_{mij}\varepsilon_{ij} E_m + 2\nu^2 \kappa^*_{ij} E_i E_j - \nu^2 h^*_{mnij}\varepsilon_{ij} E_m E_n. \tag{2.25}$$

The energy equation (2.24) also follows from (1.6) with (1.7)–(1.9), (1.11), (1.16), (2.14)–(2.16), (2.22).

The differential equations (2.19)–(2.21) describe the coupled dynamic processes of mechanical displacements of neutral particles of dielectric and changes of the electric field caused by polarization. The linear terms with coefficients h^*_{mij} describe the piezoelectric effects, while the nonlinear terms with coefficients h^*_{mnij}, h^*, l^* describe the electrostrictive effects. The differential equations must be supplemented with boundary conditions. Since the high-order derivatives with respect to the coordinates appearing on the right-hand sides of Eqs. (1.13), (2.19), (2.20) are the same as in the classical theory of elasticity, it is sufficient to set one of the conditions $u_i|_S$, $\sigma_{ij} n_j|_S$ and $E_i|_S$, $\sigma'_{ij} n_j|_S$ on the boundary S.

If the dielectric is perfectly liquid, then Eq. (2.21) should be supplemented with boundary conditions for the mechanical parameters $\dot{u}_i$ or p similar to those in classical hydromechanics and with boundary conditions for the electric parameters E_i, σ'_{ij} similar to those in the classical theory of elasticity; i.e., it is necessary to set one of the conditions $E_i|_S$, $\sigma'_{ij} n_j|_S$.

If two dissimilar dielectrics are in perfect mechanical and electric contact, which follows from the continuity of displacements of charges u^1_i, u^2_i and the normal and tangential stresses $\sigma^1_{ij} n_j, \sigma^2_{ij} n_j$, the interface conditions are as follows:

$$u^{(1)}_i|_S = u^{(2)}_i|_S, \quad \sigma^{(1)}_{ij} n_j|_S = \sigma^{(1)}_{ij} n_j|_S, \quad E^{(1)}_i|_S = E^{(2)}_i|_S, \quad \sigma'^{(1)}_{ij} n_j|_S = \sigma'^{(2)}_{ij} n_j|_S, \tag{2.26}$$

where the indices in brackets refer to the materials in contact. If there is densification, slippage, or accumulation of charges of like sign at the interface, the interface condition (2.26) ceases to be valid. In this case, it is necessary to construct an appropriate model instead of conditions (2.26).

The piezoelectric effect described by the terms with coefficients h^*_{imn} in Eqs. (2.18), (2.19) manifests itself only in solid dielectrics whose lattice has no center of symmetry. Piezoelectric transducers used in engineering are often made of preliminary polarized piezoceramics having transversely isotropic electroelastic symmetry. If the x_3-axis is directed along the axis of symmetry of such a material, the constitutive equations (2.18) will have the form

$$\sigma_{ij} = (\lambda^*_{11} - \lambda^*_{12})\varepsilon_{ij} + (\lambda^*_{12}\varepsilon_{kk} + \lambda^*_{13}\varepsilon_{33} - \nu \tilde{h}^*_{31} E_3)\delta_{ij}$$

$$-\nu^2\left[(h_{11}^*-h_{12}^*)E_iE_j+(h_{12}^*E_kE_k+h_{31}^*E_3^2)\delta_{ij}\right],$$

$$\sigma_{33}=\lambda_{13}^*\varepsilon_{kk}+\lambda_{33}^*\varepsilon_{33}-\nu\tilde{h}_{33}^*E_3-\nu^2(h_{31}^*E_kE_k+h_{33}^*E_3^2),$$

$$\sigma_{i3}=2\lambda_{44}^*\varepsilon_{i3}-\nu\tilde{h}_{15}^*E_i-2\nu^2h_{55}^*E_iE_3,$$

$$\sigma'_{ij}=-\nu\left[(\lambda_{11}-\lambda_{12})E_{ij}+(\lambda_{12}E_{kk}+\lambda_{13}E_{33})\delta_{ij}\right],$$

$$\sigma'_{33}=-\nu(\lambda_{13}E_{kk}+\lambda_{33}E_{33}),\quad \sigma'_{i3}=-2\nu\lambda_{44}E_{i3},$$

$$2R_i=4\nu\kappa_{11}E_i-2\tilde{h}_{15}^*\varepsilon_{i3}-2\nu\left[(h_{11}^*-h_{12}^*)\varepsilon_{ij}E_j+(h_{12}^*\varepsilon_{kk}+h_{13}^*\varepsilon_{33})E_i\right],$$

$$2R_3=4\nu\kappa_{33}E_3-\tilde{h}_{31}^*\varepsilon_{kk}-\tilde{h}_{33}^*\varepsilon_{33}-2\nu(2h_{55}^*\varepsilon_{i3}E_i+h_{31}^*\varepsilon_{kk}E_3+h_{33}^*\varepsilon_{33}E_3)\quad (i,j,k=1,2), \tag{2.27}$$

where matrix notation [7] is used for the electroelastic constants λ_{ijmn}^*, λ_{ijmn}, h_{ijmn}^*, h_{imn}^*, and $h_{imn}^*\to\tilde{h}_{ik}^*$.

Substituting (2.15), (2.16), and (2.27) into Eq. (1.12) and taking (2.22) into account, we obtain

$$\rho\ddot{u}_i-\nu\rho'\ddot{E}_i=\frac{1}{2}(\lambda_{11}^*-\lambda_{12}^*)u_{i,kk}+\frac{1}{2}(\lambda_{11}^*+\lambda_{12}^*)u_{k,ki}+(\lambda_{13}^*+\lambda_{44}^*)u_{3,3i}+\lambda_{44}^*u_{i,33}$$

$$-\nu(\tilde{h}_{31}^*E_{3,i}+\tilde{h}_{15}^*E_{i,3})-\nu^2\left[(h_{11}^*-h_{12}^*)(E_iE_j)_{,j}+h_{12}^*(E_kE_k)_{,i}+h_{31}^*(E_3^2)_{,i}+2h_{55}^*(E_iE_3)_{,3}\right]+F_i,$$

$$\rho\ddot{u}_3-\nu\rho'\ddot{E}_3=\lambda_{44}^*u_{3,kk}+(\lambda_{13}^*+\lambda_{44}^*)u_{k,k3}+\lambda_{33}^*u_{3,33}-\nu(\tilde{h}_{15}^*E_{k,k}+\tilde{h}_{33}^*E_{3,3})$$

$$-\nu^2\left[2h_{55}^*(E_{k,k}E_3+E_iE_{3,i})+h_{31}^*(E_kE_k)_{,3}+h_{33}^*(E_3^2)_{,3}\right]+F_3,$$

$$\rho'\ddot{u}_i-\nu\rho\ddot{E}_i=-\tilde{h}_{15}^*(u_{i,3}+u_{3,i})-\nu\left[\frac{1}{2}(\lambda_{11}-\lambda_{12})E_{i,kk}+\frac{1}{2}(\lambda_{11}+\lambda_{12})E_{k,ki}\right.$$

$$+(\lambda_{13}+\lambda_{44})E_{3,3i}+\lambda_{44}E_{i,33}-4\kappa_{11}^*E_i$$

$$\left.+(h_{11}^*-h_{12}^*)(u_{i,j}+u_{j,i})E_j+2(h_{12}^*u_{k,k}+h_{13}^*u_{3,3})E_i\right]+2NqkE_i^{\text{ex}},$$

$$\rho'\ddot{u}_3-\nu\rho\ddot{E}_3=-\tilde{h}_{31}^*u_{k,k}-\tilde{h}_{33}^*u_{3,3}-\nu\left[\lambda_{44}E_{3,kk}+(\lambda_{13}+\lambda_{44})E_{k,k3}\right.$$

$$\left.+\lambda_{33}E_{3,33}-4\kappa_{33}^*E_3+2h_{55}^*(u_{i,3}+u_{3,i})E_i+2h_{31}^*u_{k,k}E_3+2h_{33}^*u_{3,3}E_3\right]+2NqkE_3^{\text{ex}}. \tag{2.28}$$

Equations (2.28) are invariant under Galilean transformation. If the nonlinear inertial terms are neglected, i.e., $\ddot{u}_i=\partial^2u_i/\partial t^2, \ddot{E}_i=\partial^2E_i/\partial t^2$ in (2.22) on the left-hand sides of (2.28), then they are no longer invariant under Galilean transformations.

3. Equations of Electricity. In the equations of electroelasticity (2.19), (2.20), (2.28) and electrohydromechanics (2.21), the coupling of the mechanical and electric processes follows from the coupling of the constitutive equations (2.18) and the inertial interaction on the left-hand sides of the equations. Then, even purely mechanical or electric loads generate coupled electromechanical processes in dielectric. It is of interest to derive, at least theoretically, the equations of electricity to compare them with Maxwell's equations derived from the experimental laws of electrodynamics and the artificial concept of displacement current [8, 14]. To this end, we assume that neutral particles of dielectric move with a constant velocity, i.e., $\dot{u}_i=U_i=\text{const}$, and $F_i=0$. Then Eqs. (2.19) become

$$\rho'\left(\frac{\partial^2 E_i}{\partial t^2}+2U_n\frac{\partial E_{i,n}}{\partial t}+U_nU_pE_{i,np}\right)-\nu\rho\dot{E}_{i,n}\dot{E}_n=h^*_{mij}E_{m,j}+\nu h^*_{mnij}(E_mE_n)_{,j},$$

$$\rho\left(\frac{\partial^2 E_i}{\partial t^2}+2U_n\frac{\partial E_{i,n}}{\partial t}+U_nU_pE_{i,np}\right)-\nu\rho'\dot{E}_{i,n}\dot{E}_n=\lambda_{ijmn}E_{m,nj}-4\kappa^*_{ij}E_j-2\frac{Nqk}{\nu}E_i^{\text{ex}}. \quad (3.1)$$

Eliminating the terms $\dot{E}_{i,n}\dot{E}_n$ from (3.1), we obtain the equation of electrodynamics for uniformly moving elastic anisotropic dielectric, taking into account the piezoelectric and electrostrictive effects:

$$\frac{4\rho_1\rho_2}{\rho}\left(\frac{\partial^2 E_i}{\partial t^2}+2U_n\frac{\partial E_{i,n}}{\partial t}+U_nU_pE_{i,np}\right)$$

$$=\lambda_{ijmn}E_{m,nj}-\frac{\rho'}{\rho}\left[h^*_{mij}E_{m,j}+h^*_{mnij}(E_mE_n)_{,j}\right]-4\kappa^*_{ij}E_j-2\frac{Nq}{\nu}E_i^{\text{ex}}. \quad (3.2)$$

However, whether the piezoelectric and electrostrictive effects can exist while the dielectric uniformly moves without deformation should be studied separately.

If isotropic elastic and perfectly liquid dielectrics move uniformly and the electrostrictive effect is neglected, the electrodynamic equation below follows from (2.20), (2.21):

$$\frac{\partial^2 E_i}{\partial t^2}+2U_n\frac{\partial E_{i,n}}{\partial t}+U_nU_pE_{i,np}=c_2^2E_{i,rr}+(c_1^2-c_2^2)E_{r,ri}-s^2E_i-f_i, \quad (3.3)$$

where

$$c_1^2=\frac{(\lambda+2\mu)\rho}{4\rho_1\rho_2},\quad c_2^2=\frac{\mu\rho}{4\rho_1\rho_2},\quad s^2=\frac{\kappa^*\rho}{\rho_1\rho_2},\quad f_i=\frac{4\pi kN^2q^2\rho}{\rho_1\rho_2}E_i^{\text{ex}}. \quad (3.4)$$

Equations (3.2), (3.3) are invariant under Galilean transformation.

If the neutral particles of the dielectric are at rest ($U_i=0$), then Eqs. (3.1), (3.3) are no longer invariant under Galilean transformations. In particular, (3.3) yields

$$\frac{\partial^2 E_i}{\partial t^2}=c_2^2E_{i,rr}+(c_1^2-c_2^2)E_{r,ri}-s^2E_i-f_i. \quad (3.5)$$

If

$$\text{rot}\,E_i=-\frac{1}{c_2}\frac{\partial B_i}{\partial t}\quad(\text{rot}\,E_i=e_{imn}E_{n,m}), \quad (3.6)$$

we arrive at the second Maxwell's equation, which is a differential form of the experimental Faraday's law of induction, where B_i is the magnetic-flux density; e_{imn} is a unit antisymmetric tensor. Then, using the expression

$$E_{i,rr}=E_{r,ri}-e_{ipq}e_{qmn}E_{n,mp} \quad (3.7)$$

and (3.5), (3.6), we obtain

$$\frac{\partial^2 E_i}{\partial t^2}=c_2\frac{\partial}{\partial t}\text{rot}B_i+c_1^2E_{r,ri}-s^2E_i-f_i. \quad (3.8)$$

Setting $E_i^{\text{ex}}=0$, dropping the terms $c_1^2E_{r,ri}, s^2E_i$, and integrating Eqs. (3.8) over time subject to zero initial conditions, we obtain the first Maxwell's equation for dielectrics:

$$\mathrm{rot}\, B_i = \frac{1}{c_2} \frac{\partial E_i}{\partial t}. \tag{3.9}$$

However, according to [5, 8], the right-hand side of Eq. (3.9) was introduced by Maxwell into the initial equation to describe displacement current, although neither he nor his followers provided rigorous substantiation of the existence of such currents. Moreover, as shown in Sec. 2, the displacement current in dielectric is equal to zero. Actually, the right-hand side of Eq. (3.9) results from the integration of the inertial term of Eq. (3.8) characterizing the inertia of the polarization of an elementary volume of the dielectric.

Thus, Maxwell's equations follow, as a special case, from the electrodynamic equation (3.5) for fixed dielectric or its modification (3.6), (3.8). Equations (3.5), (3.6), (3.8), unlike Maxwell's equations, describe the dispersion of transverse electromagnetic waves and longitudinal electric waves associated with the propagation of polarization charge density. The term with E_i^{ex} allows describing forced electrodynamic processes in dielectrics.

Conclusions. Application of various electromechanical transducers based on the piezoelectric and electrostrictive effects requires further in-depth study and description of electromechanical interaction. The theory of electroelasticity is based on the equations of the static or dynamic equations of an elastic body, the equations of electrostatics (acoustic approximation), and the constitutive equations relating the stress tensor and the electric-flux density to the strain tensor and electric-field strength. It is assumed that internal energy is a function of strains and electric-flux density.

Shortcomings of the acoustic approximation based on the electrostatic equations are the impossibility of describing coupled acoustic and electromagnetic dynamic processes that can be observed as excitation of electromagnetic oscillations by acoustic oscillations. Moreover, there is no sufficient proof for the dependence of internal energy on electric-flux density, which, according to [4], represents only the portion of the electric field generated by free charges, i.e., is an external electric field irrespective of whether dielectric is present in it or not.

The new principle of constructing the theory of linear and nonlinear electroelasticity that is based on the equations of two-continuum mechanics of dielectrics as mixtures of positive and negative charges coupled into neutral molecules or cells allows eliminating the shortcomings. The internal energy is assumed a function of strains of mixture components and the difference of their displacements that is proportional to the polarization vector and the electric field generated by it. The electric-flux density equal to the external electric field appears in the equations as a component of the volume ponderomotive force generated by the total electric field. The derived coupled dynamic equations of electromagnetomechanics of dielectrics describe the piezoelectric and electrostrictive effects. Maxwell's equations follow from them when only the mutual displacements of charges in fixed isotropic dielectric are nonzero.

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