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ANALYZING PROCESSES OF NONISOTHERMAL LOADING OF SHELLS OF REVOLUTION WITH ALLOWANCE FOR REPEATED PLASTIC STRAINS

M. E. Babeshko and V. G. Savchenko

An algorithm for numerical analysis of the elastoplastic stress–strain state of thin shells that deform along small-curvature paths under axisymmetric nonisothermal loading is developed. Repeated plastic strains are taken into account. The stress–strain state of a shell during heating and cooling is numerically analyzed.

Keywords: elastoplastic stress–strain state, repeated plastic strains

Introduction. Thin-walled structures may undergo plastic deformation under nonisothermal loading. During heating and cooling, the plastic zones may undergo unloading either elastically or plastically with repeated plastic strains decreasing the initial plastic strains. To evaluate the strength and efficiency of structural elements, it is necessary to numerically analyze their stress–strain state (SSS). Such an analysis is especially important when a structure is subject to repeated thermal and mechanical loading.

The thermoelastoplastic axisymmetric SSS of thin shells with allowance for loading history was numerically studied in [3–5, 9, etc.], where elastic unloading was mostly considered. The possible occurrence of secondary plastic strains was taken into account in [4, 7, 9], but further development of the deformation process was not considered.

In what follows, we will outline a technique for numerical analysis of thin shells of revolution that takes into account the origin, development, and change of plastic strains during axisymmetric nonisothermal loading. This technique continues the works [4, 7, 9] and is based on a more simple and efficient algorithm. It makes it possible to study thermal and mechanical loading processes whose direction changes more than once. The results of analysis of a specific process of variable nonisothermal loading of shells will be analyzed.

1. Problem Statement. Basic Relations. Consider a shell of revolution, which is initially unstressed and undeformed and at temperature $T = T_0$, subjected to mechanical loads and nonuniform heating. The shell is made of an isotropic material. Its meridian can consist of a finite number of elements with different geometry. To describe the shell, we will use curvilinear orthogonal coordinates s, θ, ς fixed to the undeformed continuous coordinate surface, where $s\,(s_a \le s \le s_b)$ is the meridional coordinate; s_a and s_b are the coordinates of shell ends; $\theta\,(0 \le \theta \le 2\pi)$ is the circumferential coordinate; $\varsigma\,(\varsigma_0 \le \varsigma \le \varsigma_k)$ is the normal (to the coordinate surface) coordinate. Here ς_0 and ς_k correspond to the inside and outside surfaces of the shell, respectively. The shell thickness $h = \varsigma_k - \varsigma_0$. The coordinate surface is either the midsurface or one of the surfaces of the shell.

We assume that the material of the shell is deformed either within or beyond the elasticity limits. In plastic zones, unloading with secondary plastic strains can occur, after which repeated loading with plastic strains can occur.

Suppose that the decrease in the yield stress in the zone of secondary plastic strains is equal to its increase during unloading under initial loading, i.e., the material demonstrates the ideal Bauschinger effect. The creep strains are assumed to be negligibly small compared with the elastic and plastic components. Assume that the temperature field of the shell is determined

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine 03057, e-mail: plast@inmech.kiev.ua. Translated from Prikladnaya Mekhanika, Vol. 53, No. 6, pp. 40–48, November–December, 2017. Original article submitted January 24, 2017.

either by solving the heat conduction problem or borrowed from other sources. We will solve the thermoplasticity problem in quasistatic statement using the Kirchhoff–Love hypotheses and the geometrically nonlinear shell theory [2]. To solve the problem, we divide the loading process into steps so that the points of division are as close as possible to the instants of change in the process direction.

To describe the behavior of the material, we use relations of the modified theory of deformation along small-curvature paths [4, 9], which are widely used to solve boundary-value problems of thermoplasticity [3–7, 9, 10]. In the case of active loading, these relations are identical to the equations of the flow theory [8, 11, etc.] associated with the Mises yield criterion.

Below we will use the theory of small-curvature processes linearized by the secondary-stress method [4, 9]. In accordance with this theory, the stresses σ_{ij} and strains ε_{ij} are related by Hooke's law with additional stresses in the general case of an orthogonal curvilinear coordinate system:

$$\sigma_{ij} = 2G\varepsilon_{ij} + 3\lambda\varepsilon_0\delta_{ij} - \sigma_{ij}^{(D)}, \tag{1}$$

where

$$\sigma_{ij}^{(D)} = 2G(e_{ij}^{(P)}) + K\varepsilon_T\delta_{ij}, \tag{2}$$

$$\lambda = \frac{2G\nu}{1-2\nu}, \quad \varepsilon_T = \alpha_T(T - T_0), \quad K = \frac{E}{1-2\nu}, \quad E = 2G(1+\nu), \quad \delta_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j, \end{cases} \tag{3}$$

where $e_{ij}^{(p)} = \varepsilon_{ij}^{(p)}$ are plastic strain components; E, G, ν, and α_T are the temperature-dependent elastic modulus, shear modulus, Poisson's ratio, and coefficient of linear thermal expansion of the material, respectively, T is temperature; $\varepsilon_0 = \varepsilon_{ii}/3$ is the first invariant of the strain tensor related to the first invariant of the stress tensor $\sigma_0 = \sigma_{ii}/3$ by

$$\sigma_0 = K(\varepsilon_0 - \varepsilon_T). \tag{4}$$

Since the shell is axisymmetrically loaded without torsion, its stress state is determined by the components σ_{ss} and $\sigma_{\theta\theta}$, while the strain state is determined by the components $\varepsilon_{ss}, \varepsilon_{\theta\theta}, \varepsilon_{\varsigma\varsigma}$. Then Eqs. (1) take the form

$$\sigma_{ss} = A_{11}\varepsilon_{ss} + A_{12}\varepsilon_{\theta\theta} - A_{1D},$$

$$\sigma_{\theta\theta} = A_{12}\varepsilon_{ss} + A_{22}\varepsilon_{\theta\theta} - A_{2D}, \tag{5}$$

where

$$A_{11} = A_{22} = \frac{E}{1-\nu^2}, \quad A_{12} = \nu A_{11},$$

$$A_{1D} = A_{11}(e_{ss}^{(p)} + \nu e_{\theta\theta}^{(p)}) + A_{11}(1+\nu)\varepsilon_T,$$

$$A_{2D} = A_{11}(e_{\theta\theta}^{(p)} + \nu e_{ss}^{(p)}) + A_{11}(1+\nu)\varepsilon_T, \tag{6}$$

where the plastic strains $e_{ss}^{(p)} = \varepsilon_{ss}^{(p)}$, $e_{\theta\theta}^{(p)} = \varepsilon_{\theta\theta}^{(p)}$, $\varepsilon_{\varsigma\varsigma}^{(p)} = -(\varepsilon_{ss}^{(p)} + \varepsilon_{\theta\theta}^{(p)})$ at an arbitrary Mth step are determined as the sum of increments Δ of these components:

$$\varepsilon_{ss}^{(p)} = \sum_{m=1}^{M}\Delta_m\varepsilon_{ss}^{(p)}, \quad \Delta_m\varepsilon_{ss}^{(p)} = \langle c_{ss}\rangle_m \Delta_m\Gamma_p, \quad \langle c_{ss}\rangle_m = \left\langle \frac{2\sigma_{ss} - \sigma_{\theta\theta}}{S} \right\rangle_m \quad (s, \theta), \tag{7}$$

where the angular brackets denote averaging, S is the intensity of tangential stresses,

$$S = \left[\frac{1}{3}(\sigma_{ss}^2 - \sigma_{ss}\sigma_{\theta\theta} + \sigma_{\theta\theta}^2)\right]^{1/2}, \tag{8}$$

Γ_p is the intensity of accumulated plastic shear strain,

$$\Gamma_p = \sum_{m=1}^{M-1} \Delta_m \Gamma_p + \Delta_M \Gamma_p. \qquad (9)$$

To determine $\Delta_M \Gamma_p$, we assume that the intensities of tangential stresses S and shear strains

$$\Gamma = \left[\frac{(\varepsilon_{ss} - \varepsilon_{\theta\theta})^2 + (\varepsilon_{\theta\theta} - \varepsilon_{\varsigma\varsigma})^2 + (\varepsilon_{\varsigma\varsigma} - \varepsilon_{ss})^2}{6} \right]^{1/2}$$

are related to the temperature T as follows:

$$S = \Phi(\Gamma, T). \qquad (10)$$

To individualize functions (10), we will use $\sigma \sim \varepsilon$ curves obtained in tests on cylindrical specimens subject to uniaxial tension at fixed temperature. Such curves at intermediate temperatures are plotted by linear interpolation. The stress σ and longitudinal strain ε in the specimen are related to the second invariants of the stress and strain deviators as follows [4]:

$$S = \frac{\sigma}{\sqrt{3}}, \quad \Gamma = \frac{S}{2G} + \Gamma_p, \quad \Gamma_P = \frac{\sqrt{3}}{2}\left(\varepsilon - \frac{\sigma}{E} \right). \qquad (11)$$

We suppose also that if unloading is elastic, then $\Gamma = S/(2G) + \Gamma_p^{(1)}$, where $\Gamma_p^{(1)}$ is the intensity of accumulated plastic shear strain (9) at the instant of unloading. If the unloading is accompanied by secondary plastic strains, then

$$S = \Phi_1(\Gamma, \Gamma_p^{(1)}, T). \qquad (12)$$

This function is plotted using (10), the value of $\Gamma_p^{(1)}$, and the associated value of $S^{(1)}$ at the instant of unloading. For repeated loading, we use

$$S = \Phi_2(\Gamma, \Gamma_p^{(2)}, T). \qquad (13)$$

This function is plotted using (10), the intensity of accumulated secondary plastic strain $\Gamma_p^{(2)}$, and associated value of $S^{(2)}$ at the instant of unloading in the zone of secondary plastic strains. In plotting (12) and (13), we suppose that

$$S^{(1)} + S_T^{(2)} = S^{(2)} + S_T^{(3)} = 2S_T^{(1)}, \qquad (14)$$

where $S_T^{(1)}$, $S_T^{(2)}$, and $S_T^{(3)}$ are the intensities of tangential stresses that correspond to the yield stresses in (10), (12), and (13). A method of plotting (12) and (13) is outlined in [4].

Thus, the increment $\Delta_M \Gamma_p$ at the current step of loading is determined using (10), (12), and (13) in the process of successive approximations. Relations (5) are used to establish the relationship among the forces N_s, N_θ, moments M_s, M_θ, strains, changes in curvature ε_s, ε_θ, κ_s, κ_θ of the coordinate surface of the shell:

$$N_s = C_{11}^{(0)}\varepsilon_s + C_{12}^{(0)}\varepsilon_\theta + C_{11}^{(1)}\kappa_s + C_{12}^{(1)}\kappa_\theta - N_{1D}^{(0)},$$

$$N_\theta = C_{12}^{(0)}\varepsilon_s + C_{22}^{(0)}\varepsilon_\theta + C_{12}^{(1)}\kappa_s + C_{22}^{(1)}\kappa_\theta - N_{2D}^{(0)},$$

$$M_s = C_{11}^{(1)}\varepsilon_s + C_{12}^{(1)}\varepsilon_\theta + C_{11}^{(2)}\kappa_s + C_{12}^{(2)}\kappa_\theta - N_{1D}^{(1)},$$

$$M_\theta = C_{12}^{(1)}\varepsilon_s + C_{22}^{(1)}\varepsilon_\theta + C_{12}^{(2)}\kappa_s + C_{22}^{(2)}\kappa_\theta - N_{2D}^{(1)}, \qquad (15)$$

where

$$C_{mn}^{(j)} = \int_{\varsigma_0}^{\varsigma_k} A_{mn} \varsigma^j d\varsigma, \quad N_{mD}^{(j)} = \int_{\varsigma_0}^{\varsigma_k} A_{mD} \varsigma^j d\varsigma \quad (m, n = 1, 2, \ j = 0, 1, 2). \tag{16}$$

Equations (15) and (16) in combination with the equilibrium and kinematic equations [2] constitute a system of 12 equations, which can be reduced to a system of six ordinary differential equations for the unknown functions $N_s, Q_s, M_s, u, w, \vartheta_s$, where Q_s is the shearing force; u and w are the meridional and normal displacements of points of the coordinate surface; ϑ_s is the angle of rotation of the normal to the coordinate surface. This system has the form

$$\frac{d\vec{Y}}{ds} = P(s)\vec{Y} + \vec{f}(s) \tag{17}$$

with the boundary conditions

$$B_1 \vec{Y}(s_a) = \vec{b}_1, \quad B_2 \vec{Y}(s_b) = \vec{b}_2, \tag{18}$$

where $\vec{Y}$ is the column vector of unknown functions, $\vec{Y} = \{N_s, Q_s, M_s, u, w, \vartheta_s\}$, $P(s)$ is the matrix of the system; $\vec{f}(s)$ is the column vector of additional terms; B_1 and B_2 are given matrices; $\vec{b}_1$ and $\vec{b}_2$ are given column vectors of boundary conditions.

The elements of the matrix $P(s)$ are calculated by the formulas

$$p_{11} = -\frac{\cos\varphi}{r}(1+\lambda_1), \quad p_{13} = -\frac{\cos\varphi}{r}\lambda_2, \quad p_{14} = \frac{\cos^2\varphi}{r^2}(\lambda_1 C_{12}^{(0)} + C_{22}^{(0)} + \lambda_2 C_{12}^{(1)}),$$

$$p_{15} = p_{14}\tan\varphi, \quad p_{16} = -\frac{\cos^2\varphi}{r^2}(\lambda_3 C_{12}^{(0)} + \lambda_4 C_{12}^{(1)} - C_{22}^{(1)}),$$

$$p_{21} = \frac{1}{R_s} - \frac{\lambda_1 \sin\varphi}{r}, \quad p_{22} = -\frac{\cos\varphi}{r}, \quad p_{23} = -\frac{\sin\varphi}{r}\lambda_2, \quad p_{24} = p_{15}, \quad p_{25} = p_{15}\tan\varphi,$$

$$p_{26} = p_{16}\tan\varphi, \quad p_{31} = -p_{22}\lambda_3, \quad p_{32} = -1, \quad p_{33} = -\frac{\cos\varphi}{r}(1-\lambda_4),$$

$$p_{34} = \frac{\cos^2\varphi}{r^2}(\lambda_1 C_{12}^{(1)} + C_{22}^{(1)} + \lambda_2 C_{12}^{(2)}), \quad p_{35} = p_{34}\tan\varphi,$$

$$p_{36} = -\frac{\cos^2\varphi}{r^2}(\lambda_3 C_{12}^{(1)} - C_{22}^{(2)} + \lambda_4 C_{12}^{(2)}), \quad p_{41} = \frac{C_{11}^{(2)}}{\delta_1}, \quad p_{42} = 0, \quad p_{43} = -\frac{C_{11}^{(1)}}{\delta_1},$$

$$p_{44} = -\lambda_1 p_{22}, \quad p_{45} = -p_{21}, \quad p_{46} = -p_{31}, \quad p_{51} = p_{52} = p_{53} = 0, \quad p_{54} = -p_{12},$$

$$p_{55} = 0, \quad p_{56} = -1, \quad p_{61} = p_{43}, \quad p_{62} = 0, \quad p_{63} = \frac{C_{11}^{(0)}}{\delta_1},$$

$$p_{64} = -p_{13}, \quad p_{65} = -p_{23}, \quad p_{66} = \lambda_4 p_{22}, \tag{19}$$

where $\lambda_1 = (C_{11}^{(1)} C_{12}^{(1)} - C_{12}^{(0)} C_{11}^{(2)})/\delta_1$, $\lambda_2 = (C_{11}^{(1)} C_{12}^{(0)} - C_{12}^{(1)} C_{11}^{(0)})/\delta_1$, $\lambda_3 = (C_{11}^{(2)} C_{12}^{(1)} - C_{12}^{(2)} C_{11}^{(1)})/\delta_1$, $\lambda_4 = (C_{11}^{(0)} C_{12}^{(2)} - C_{12}^{(1)} C_{11}^{(2)})/\delta_1$, $\delta_1 = C_{11}^{(0)} C_{11}^{(2)} - (C_{11}^{(1)})^2$.

The components of the vector $\vec{f}(s)$ are expressed as

TABLE 1

Step number	1	3	9	12	13	17	18	19	20	21	26
$N_s^* \cdot 10^{-3}$, N/m	120	160	284	20	–20	–250	–220	–20	20	140	284
q_ς, MPa	6	8	14.2	1	0	0	0	0	1	7	14.2
T, K	293	373	573	293	293	293	293	293	293	343	573

$$f_1 = -\frac{\cos\varphi}{r}[\lambda_1 N_{1D}^{(0)} + \lambda_2 N_{1D}^{(1)} + N_{2D}^{(0)}] - q_s, \quad f_2 = -\frac{\sin\varphi}{r}[\lambda_1 N_{1D}^{(0)} + \lambda_2 N_{1D}^{(1)} + N_{2D}^{(0)}] - q_\varsigma,$$

$$f_3 = \frac{\cos\varphi}{r}[\lambda_3 N_{1D}^{(0)} + \lambda_4 N_{1D}^{(1)} - N_{2D}^{(0)}], \quad f_4 = \frac{C_{11}^{(2)} N_{1D}^{(0)} - C_{11}^{(1)} N_{1D}^{(1)}}{\delta_1},$$

$$f_5 = 0, \quad f_6 = -\frac{C_{11}^{(1)} N_{1D}^{(0)} - C_{11}^{(0)} N_{1D}^{(1)}}{\delta_1}, \tag{20}$$

where q_s and q_ς are the components of the distributed load.

From (19) and (16) it follows that the elements of the matrix $P(s)$ depend on the geometry of the shell and the elastic properties of its material at the temperature of the current step, while components (20) of the vector $\vec{f}(s)$ additionally depend on the external loads and plastic strains, which must be adjusted during successive approximations.

The above formulas allow analyzing the SSS of the shell at any step of loading.

2. Problem-Solving Algorithm. Suppose that stress–strain curves, Poisson's ratios, and temperature-dependent coefficients of linear thermal expansion of the material are given. It is convenient to divide the process into steps so that the deformation would is elastic at the first step. Let us the plastic strains appearing in (6) be equal to zero in the first approximation at the first step of loading, i.e., we solve a thermoelastic problem. At the subsequent steps, the plastic strains (7) obtained at the previous step are used as the first approximation. In the subsequent approximations, the values found in the previous approximation are used. These values are used in calculating elements (20) of the column vector $\vec{f}(s)$, whereas the elements (19) of the matrix $P(s)$ are determined using the temperature-dependent material properties defined in the first approximatin and kept such in the subsequent approximations. After calculating the elements of the matrix $P(s)$ and the elements of the column vector $\vec{f}(s)$, we solve the boundary-value problem (17), (18) by reducing it to Cauchy problems, which are solved using the Runge–Kutta method and the discrete-orthogonalization method [1]. The functions found by solving the boundary-value problem are then used to determine the strains and stresses. Then we calculate

$$\Delta_M \Gamma_p = \sum_{i=1}^{L-1} \Delta_{Mi} \Gamma_p + \Delta_{ML} \Gamma_p,$$

$$\Delta_{ML} \Gamma_p = \frac{S - S^{(d)}}{2G}, \tag{21}$$

where L is the number of the current approximation at the Mth step. The value of S in (21) is calculated by formula (8), while the value $S^{(d)}$ is found from (10), (12), and (13), respectively, for initial loading, in the zone of secondary plastic strains, and for repeated loading. In the case of active initial loading, we use (10). The condition $\Delta\Gamma_p > 0$ is considered as the criterion of active loading, otherwise the shell element is unloaded, i.e., $\Delta\Gamma_p = 0$. In the case of unloading with sign reversal of the first invariant of

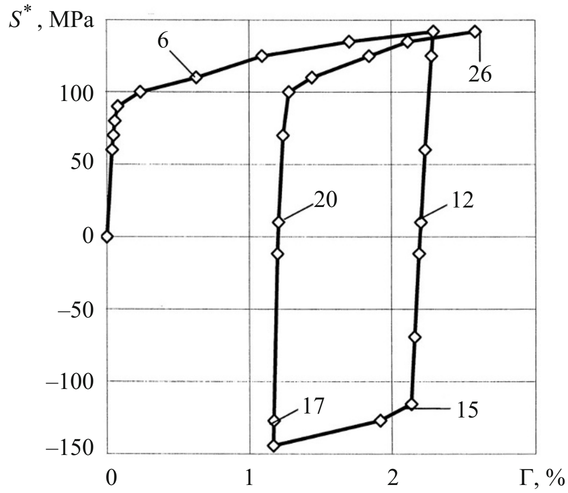

Fig. 1

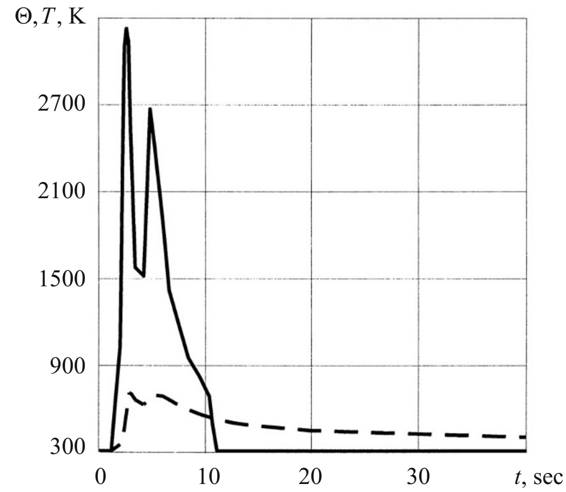

Fig. 2

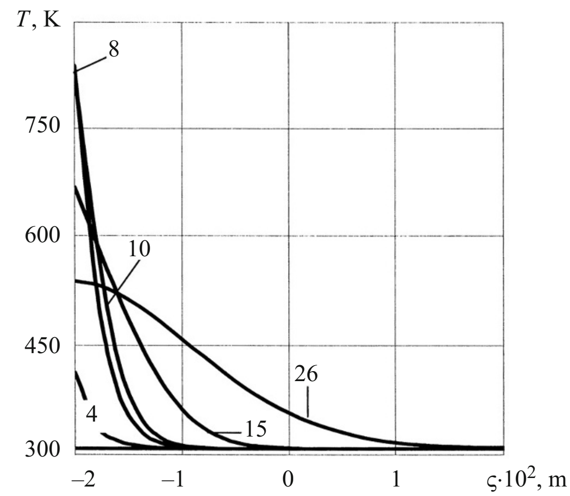

Fig. 3

the stress tensor $\sigma_0 = (\sigma_{ss} + \sigma_{\theta\theta})/3$, we use (12). Similarly, in the case of unloading in the zone of secondary plastic strains and sign reversal of σ_0, we use (13). The process of successive approximations is terminated once

$$|\Delta_{ML}\Gamma_p| \le \delta, \tag{22}$$

where δ is a predefined number.

3. Numerical Results. To validate the above algorithm, we determine the SSS of a cylindrical shell with mid-surface radius 0.1 m, thickness 0.01 m, and length 0.1 m. The shell made of Kh18N10T alloy is subjected axial force and internal pressure and is uniformly heated, the initial temperature $T_0 = 293$ K. The loads and temperature for 26 steps are summarized in Table 1.

The boundary conditions at all steps are the following:

$$Q_s = 0, \quad u = 0, \quad \vartheta_s = 0 \qquad \text{for } s = s_a;$$

$$N_s = N_s^*, \quad Q_s = 0, \quad \vartheta_s = 0 \qquad \text{for } s = s_b.$$

Under such loading conditions, the SSS of the shell is homogeneous. The problem is solved with the above technique by the method of successive approximations with accuracy (22) $\delta = 0.00001$. Since the problem is statically determinate, it can also

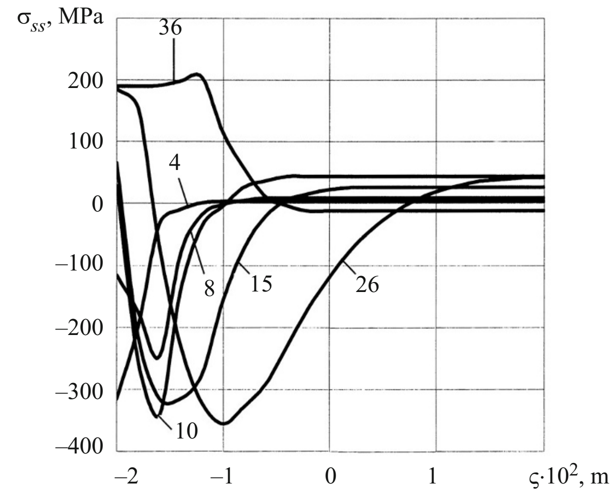

Fig. 4

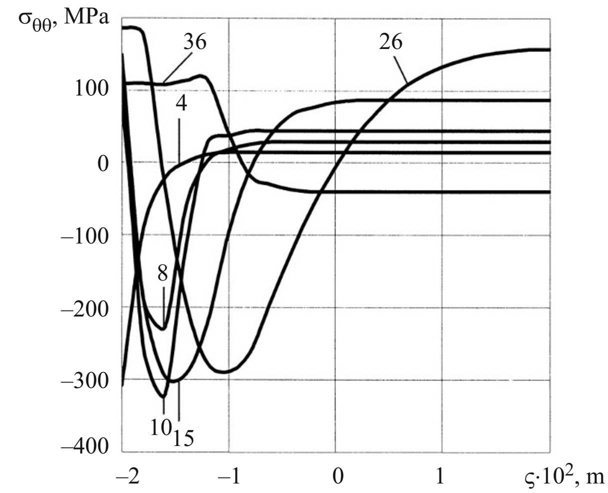

Fig. 5

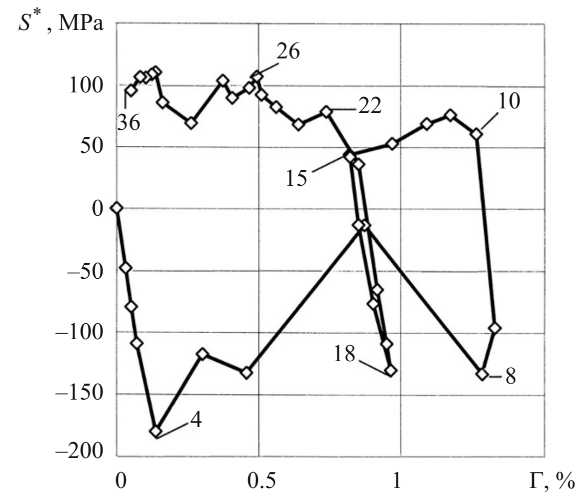

Fig. 6

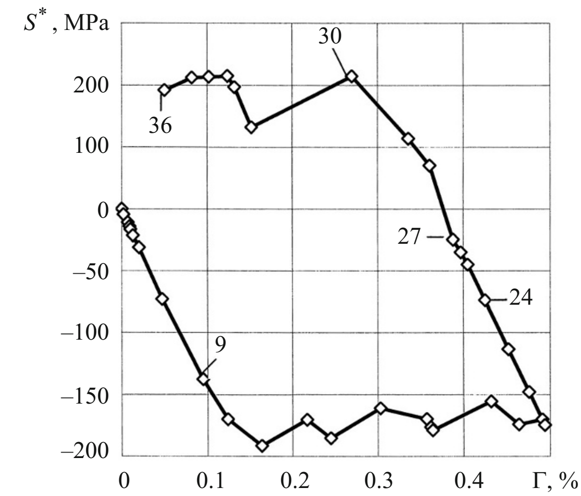

Fig. 7

be solved without successive approximations. Such a solution is in good agreement with that obtained with the technique proposed. This confirms the efficiency and accuracy of the algorithm. Figure 1 shows the S^*-versus-Γ diagram. Here $S^* = \text{sign}(\sigma_0)\cdot S$, the symbols indicate the ends of the steps, and the numbers are step numbers.

Using the above technique, we analyzed the thermostress state of an element of a launch pad [5]. The cross-section of the element is a square with 6 m side length and 0.04 m wall thickness. As shown in [5], the SSS of such structure can be determined by solving an axisymmetric thermoplasticity problem for a cylindrical shell of the same thickness whose diameter is equal to the square side.

In what follows, we will analyze the SSS of a long cylindrical shell with a mid-surface radius of 2.98 m and a thickness of 0.04 m. The shell is subject to nonstationary heating, the initial temperature $T_0 = 308$ K. After solving the heat-conduction problem with the technique [4, 9], we determine the nonstationary temperature field of the shell in the case of convective heat transfer to the environment and heat flow given on the inside surface.

The variation in the environment temperature near the inside surface during the first 40 seconds of the process is shown in Fig. 2 by a solid line. The dashed line represents the variation in temperature with time at the most heated point with coordinates $s = 0$, $\varsigma = -0.02$ m.

The thermoplasticity problem was solved with the above technique for a shell with a length of 0.1 m under no mechancial load. The symmetry conditions $Q_s = 0, u_s = 0, \vartheta_s = 0$ are prescribed on the boundaries $s = s_a$ and $s = s_b$.

The process of heating and cooling of the shell to the initial temperature $T = T_0 = 308$ K was divided into 36 steps. The temperature distribution throughout the shell thickness at some steps is presented in Fig. 3, where the numbers are step numbers. It was established that at the initial steps, compressive meridional and circumferential stresses occur near the inside surface, while the stresses in the remaining part of the shell are tensile. During heating and cooling, the shell undergoes stress redistribution. The zone of plastic strains begins on the inside surface and extends throughout the thickness ($-0.02 \le \varsigma \le -0.004$ m). In this zone, unloading occurs and secondary plastic strains occupy the zone $-0.02 \le \varsigma \le -0.008$ m throughout the thickness.

Some numerical results are presented in Figs. 4–7. The distribution of the meridional and circumferential stresses throughout the shell thickness is shown in Figs. 4 and 5, respectively. The numbers are step numbers. The plots for step 36 show the distribution of residual stresses throughout the thickness. How the loading and unloading proceed can be seen from the S^*-versus-Γ diagrams in the vicinity of the point ($s = 0, \varsigma = -0.02$ m) in Fig. 6 and of the point ($s = 0, \varsigma = -0.016$ m) in Fig. 7. The symbols correspond to the ends of steps, and the numbers are step numbers. From Fig. 6 it is seen that the first invariant of the stress tensor (4) changes its sign more than once, i.e., compression is changed to tension and vice versa. As is seen from Fig. 7, in the element being considered, which is heated less than the inside surface, plastic strains occur. Next unloading occurs and secondary plastic strains cause a strong decrease in the initial plastic strains. Figures 6 and 7 show that the residual values of the intensity of shear strains (and, hence, plastic strains) are insignificant and do not exceed 0.06%. This allows us to suppose that the SSS of the shell in an analogous loading process will be the same as in the process being studied.

Conclusions. We have developed an algorithm for numerical analysis of the thermoelastoplastic axisymmetric SSS of thin shells that deform along small-curvature paths and in which secondary plastic strains and repeated loading occur. The algorithm has been validated by analyzing a test example. The SSS of a cylindrical shell subject to nonstationary heating and cooling has been analyzed numerically.

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