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NONSTATIONARY VIBRATIONS OF ELLIPTIC CYLINDRICAL SANDWICH SHELLS REINFORCED WITH DISCRETE STRINGERS

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The equations of the vibration of elliptic cylindrical sandwich shells with equally spaced stringers under nonstationary loading are derived. Models based on the Timoshenko-type shell theory are used. The dynamic equations are solved numerically by the integro-interpolation finite-differencing method for equations with discontinuous coefficients. The dynamic problem for an elliptic cylindrical sandwich shell reinforced with stringers is solved

Keywords: elliptic cylindrical sandwich shell, Timoshenko-type theory, forced vibrations, numerical solution

Introduction. Determining the stress–strain state of stringer-reinforced cylindrical sandwich shells is a challenge that can be resolved using theoretical models of laminated plates and shells. The numerical implementation of these models requires developing and improving efficient numerical methods for design of such structural members. The complexity of mechanical models of laminated shells and the use of essentially different kinematic and static hypotheses are the reason why there are many different design schemes and equations [1–22]. There are two approaches to the modeling of laminated shells: application of the hypotheses to the sandwich as a whole [3, 5, 7–11, 14–22] and application of hypotheses that take into account kinematic and static features of each layer [1, 6]. The models and theories of the second approach are called discrete-structural in [3, 6]. The above approaches can also be applied to sandwich shells: structurally orthotropic model and a model with discrete stringers. The second approach was used in [11, 12] to study the axisymmetric and nonaxisymmetric vibrations of discretely reinforced sandwich shells under nonstationary loading.

We will consider elliptical cylindrical sandwich shells reinforced with discrete stringers. It is obvious that the second approach can be used to model discretely reinforced multilayer shells. To derive the equations of vibration, we will use a geometrically linear theory and Timoshenko's hypotheses for shells and rods. To solve specific boundary-value problems, we will use an explicit finite-difference scheme for integration of equations. We will solve, as a numerical example, the problem of the forced nonaxisymmetric vibrations of an inhomogeneous shell under a distributed nonstationary load.

1. Problem Statement. Basic Equations. Let us consider a stringer-reinforced sandwich elliptic cylindrical shell under internal distributed nonstationary loading. The sandwich structure consists of two (inner and outer) coaxial cylindrical one-layer shells rigidly coupled by stringers. The structure is schematized in Fig. 1.

The coefficients of the first quadratic form and the curvatures of the coordinate surface of the shell are given by

$$A_1 = 1, \qquad k_2 = 0, \qquad A_2 = (a_k^2 \cos^2 \alpha_2 + b_k^2 \sin^2 \alpha_2)^{1/2},$$

$$k_2 = a_k b_k (a_k^2 \cos^2 \alpha_2 + b_k^2 \sin^2 \alpha_2)^{-3/2}, \tag{1.1}$$

where a_k and b_k are the semiaxes of the elliptical cross-section of the cylindrical shell.

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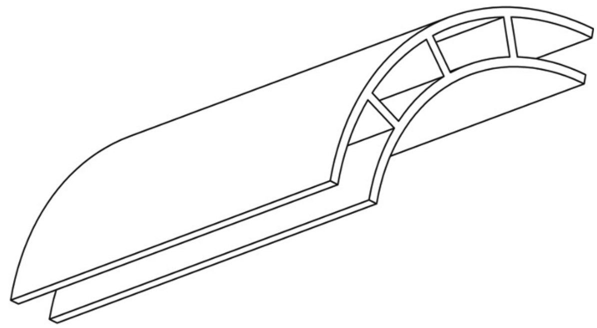

Fig. 1

It is assumed that the strain state of the inner and outer shells (indices "1" and "2", respectively) can be defined in terms of the generalized displacement vectors of the mid-surfaces $\overline{U}_1 = (u_1^1, u_2^1, u_3^1, \varphi_1^1, \varphi_2^1)^T$ and $\overline{U}_2 = (u_1^2, u_2^2, u_3^2, \varphi_1^2, \varphi_2^2)^T$. It is also assumed that the strain state of the ith stringer can be defined by the generalized displacement vector $\overline{U}_i = (u_{1i}, u_{2i}, u_{3i}, \varphi_{1i}, \varphi_{2i})^T$.

To derive the equations of the vibrations of the discretely reinforced sandwich structure, we will use the Hamilton–Ostrogradskii variational principle:

$$\delta \int_{t_1}^{t_2} (K - \Pi + A)\, dt = 0, \tag{1.2}$$

where K is the total kinetic energy of the elastic system; Π is its total potential energy; A is the work done by the external forces;

$$\delta\Pi = \delta\sum_{k=1}^{2} \Pi^k + \delta\sum_{i=1}^{I} \Pi_i, \qquad \delta K = \delta\sum_{k=1}^{2} K^k + \delta\sum_{i=1}^{I} K_i, \tag{1.3}$$

where Π^k and Π_i are the potential energies of the kth shell and ith stringer; K^k and K_i are the respective kinetic energies.

Carrying out standard transformations in (1.2) and considering the expressions for the potential and kinetic energies for the shells and stringers, we arrive at two groups of equations, according to [3]. The equations of vibration of the shell are

$$\frac{\partial T_{11}^k}{\partial s_1} + \frac{\partial S^k}{\partial s_2} = \rho_k h_k \frac{\partial^2 u_1^k}{\partial t^2}, \qquad \frac{\partial S^k}{\partial s_1} + \frac{\partial T_{22}^k}{\partial s_2} + k_2 T_{23}^k = \rho_k h_k \frac{\partial^2 u_2^k}{\partial t^2},$$

$$\frac{\partial T_{13}^k}{\partial s_1} + \frac{\partial T_{23}^k}{\partial s_2} - k_2 T_{22}^k + P_3^k(s_1, s_2, t) = \rho_k h_k \frac{\partial^2 u_3^k}{\partial t^2},$$

$$\frac{\partial M_{11}^k}{\partial s_1} + \frac{\partial H^k}{\partial s_2} - T_{13}^k = \rho_k \frac{h_k^3}{12} \frac{\partial^2 \varphi_1^k}{\partial t^2},$$

$$\frac{\partial H^k}{\partial s_1} + \frac{\partial M_{22}^k}{\partial s_2} - T_{23}^k = \rho_k \frac{h_k^3}{12} \frac{\partial^2 \varphi_2^k}{\partial t^2}, \qquad k = 1, 2, \tag{1.4}$$

for the inner and outer shells

$$[S]_i + \frac{\partial T_{11i}}{\partial s_1} = \rho_i F_i \frac{\partial^2 u_{1i}}{\partial t^2}, \qquad [T_{22}]_i + \frac{\partial T_{12i}}{\partial s_1} = \rho_i F_i \frac{\partial^2 u_{2i}}{\partial t^2},$$

$$[T_{23}]_i + \frac{\partial T_{13i}}{\partial s_1} = \rho_i F_i \frac{\partial^2 u_{3i}}{\partial t^2}, \quad [H]_i + \frac{\partial M_{11i}}{\partial s_1} - T_{13i} = \rho_i I_{1i} \frac{\partial^2 \varphi_{1i}}{\partial t^2},$$

$$[M_{22}]_i + \frac{\partial M_{12i}}{\partial s_1} = \rho_i I_{\mathrm{tw}\,i} \frac{\partial^2 \varphi_{2i}}{\partial t^2}, \quad i = \overline{1, I}, \tag{1.5}$$

for the ith stringer, where $[S]_i, [T_{22}]_i, [T_{23}]_i, [H]_i, [M_{22}]_i$ denote the total forces/moments of the inner and outer shells acting on the ith stringer.

The systems of equations (1.4), (1.5) are related by the kinematic contact conditions for the generalized displacement vectors of the mid-surface of the shells and of the cross-sectional centers of gravity of the ith stringer as follows:

$$u_{1i}(s_1) = u_1^k(s_1, s_{2i}) \pm H_i^* \varphi_1^k(s_1, s_{2i}), \quad u_{2i}(s_1) = u_2^k(s_1, s_{2i}) \pm H_i^* \varphi_2^k(s_1, s_{2i}),$$

$$u_{3i}(s_1) = u_3^k(s_1, s_{2i}), \quad \varphi_{1i}(s_1) = \varphi_1^k(s_1, s_{2i}), \quad \varphi_{2i}(s_1) = \varphi_2^k(s_1, s_{2i}),$$

$$H_i^*(s_1) = 0.5(h_k + h_i) \quad (k = 1, 2), \tag{1.6}$$

where ρ_k $(k = 1, 2)$ and ρ_i $(i = \overline{1, I})$ are the densities of the shell and stringer materials; h_k $(k = 1, 2)$ and h_i $(i = \overline{1, I})$ are the thicknesses and heights of the shells and stringers.

The forces/moments of the shells are determined in terms of the strains as

$$T_{11}^k = B_{11}^k(\varepsilon_{11}^k + \nu_{21}^k \varepsilon_{22}^k), \quad T_{22}^k = B_{22}^k(\varepsilon_{22}^k + \nu_{12}^k \varepsilon_{11}^k),$$

$$T_{13}^k = B_{13}^k \varepsilon_{13}^k, \quad S = B_{12}^k \varepsilon_{12}^k, \quad T_{13}^k = B_{13}^k \varepsilon_{13}^k, \quad S = B_{12}^k \varepsilon_{12}^k,$$

$$H = D_{12}^k \kappa_{12}^k, \quad M_{11}^k = D_{11}^k(\kappa_{11}^k + \nu_{21}^k \kappa_{22}^k), \quad M_{22}^k = D_{22}^k(\kappa_{22}^k + \nu_{12}^k \kappa_{11}^k), \tag{1.7}$$

while the strains are expressed in terms of the components of the generalized displacement vectors of the shell mid-surfaces as follows [2, 11, 12]:

$$\varepsilon_{11}^k = \frac{\partial u_1^k}{\partial s_1}, \quad \varepsilon_{22}^k = \frac{\partial u_2^k}{\partial s_2} + k_2 u_3^k, \quad \varepsilon_{12}^k = \frac{\partial u_2^k}{\partial s_1} + \frac{\partial u_1^k}{\partial s_2},$$

$$\varepsilon_{13}^k = \varphi_1^k + \theta_1^k, \quad \varepsilon_{23}^k = \varphi_2^k + \theta_2^k, \quad \theta_1^k = \frac{\partial u_3^k}{\partial s_1}, \quad \theta_2^k = \frac{\partial u_3^k}{\partial s_2} - k_2 u_2^k,$$

$$\kappa_{11}^k = \frac{\partial \varphi_1^k}{\partial s_1}, \quad \kappa_{22}^k = \frac{\partial \varphi_2^k}{\partial s_2}, \quad \kappa_{12}^k = \frac{\partial \varphi_2^k}{\partial s_1} + \frac{\partial \varphi_1^k}{\partial s_2}. \tag{1.8}$$

The relations between the forces/moments and the strains for the stringers are similar:

$$T_{11i} = E_i F_i \varepsilon_{11i}, \quad T_{12i} = G_i F_i \varepsilon_{12i}, \quad T_{13i} = G_i F_i \varepsilon_{13i},$$

$$M_{11i} = E_i I_{1i} \kappa_{11i}, \quad M_{21i} = G_i I_{\mathrm{tw}\,i} \kappa_{12i},$$

$$\varepsilon_{11i} = \frac{\partial u_{1i}}{\partial s_1}, \quad \varepsilon_{12i} = f_{2i} + \theta_{2i}, \quad \varepsilon_{13i} = f_{1i} + \theta_{1i},$$

$$\theta_{1i} = \frac{\partial u_{3i}}{\partial s_1}, \quad \theta_{2i} = \frac{\partial u_{2i}}{\partial s_1}, \quad \kappa_{11i} = \frac{\partial \varphi_{1i}}{\partial s_1}, \quad \kappa_{12i} = \frac{\partial \varphi_{2i}}{\partial s_1}, \tag{1.9}$$

where E_i and G_i are the material characteristics of the ith stringer; F_i, I_{1i}, and $I_{\text{tw}\,i}$ are the geometric parameters of the ith stringer.

The equations of vibration (1.4)–(1.9) are supplemented by appropriate boundary and initial conditions.

2. Numerical Problem-Solving Algorithm. The difference algorithm combines the integro-interpolation method for the construction of difference schemes with respect to the space coordinates s_1 and s_2 and an explicit finite-difference scheme with respect to the time coordinate t [2]. One feature of the problems of vibrations of inhomogeneous shells reinforced with discrete ribs is the presence of discontinuous coefficients in the original equations. Thus, the problems for elliptic cylindrical sandwich shells with stringers can be solved in two stages: (i) solution within the smooth domain and (ii) solution on the ith line of discontinuity along the OX-axis [11, 12].

Let us construct a difference algorithm for Eqs. (1.4) within the smooth domain:

$$\Omega_1 = \{s_{1l-1/2} \le s_1 \le s_{1l+1/2}, s_{2m-1/2} \le s_2 \le s_{2m+1/2}\} \quad \text{for} \quad t_{n-1/2} \le t \le t_{n+1/2}:$$

$$\int_t \iint_{\Omega_1} \left[\frac{\partial T_{11}^k}{\partial s_1} + \frac{\partial S^k}{\partial s_2} \right] d\Omega_1 dt = \int_t \iint_{\Omega_1} \left[\rho_k h_k \frac{\partial^2 u_1^k}{\partial t^2} \right] d\Omega_1 dt, \tag{2.1}$$

$$\int_t \iint_{\Omega_1} \left[\frac{\partial S^k}{\partial s_1} + \frac{\partial T_{22}^k}{\partial s_2} + k_2 T_{23}^k \right] d\Omega_1 dt = \int_t \iint_{\Omega_1} \left[\rho_k h_k \frac{\partial^2 u_2^k}{\partial t^2} \right] d\Omega_1 dt,$$

$$\ldots .$$

After standard transformations in (2.1), we obtain difference equations that approximate the original equations (1.4) in the smooth domain:

$$\frac{T_{11l+1/2,m}^{kn} - T_{11l-1/2,m}^{kn}}{\Delta s_1} + \frac{S_{l,m+1/2}^{kn} - S_{l,m-1/2}^{kn}}{\Delta s_2} = \rho h (U_{1lm}^{kn})_{\bar{t}t}, \tag{2.2}$$

$$\frac{S_{l+1/2,m}^{kn} - S_{l-1/2,m}^{kn}}{\Delta s_1} + \frac{T_{22l,m+1/2}^{kn} - T_{22l,m+1/2}^{kn}}{\Delta s_2} + \frac{1}{2} k_{2l,m} (T_{23l,m+1/2}^{kn} + T_{23l,m-1/2}^{kn}) = \rho h (U_{2lm}^{kn})_{\bar{t}t},$$

$$\ldots .$$

As a result, the components of the generalized vectors $\overline{U}_1$ and $\overline{U}_2$ are referred to the integer points of the difference scheme (l, m). The forces/moments used in differentiating with respect to s_1 are referred to the half-integer points $(l \pm 1/2, m)$, while the quantities used in differentiating with respect to s_2 are referred to the points $(l, m \pm 1/2)$.

To match the forces/moments in (2.2), we integrate Eqs. (1.7) over the domains

$$\Omega_2 = \{s_{1l-1} \le s_1 \le s_{1l},\ s_{2m-1/2} \le s_2 \le s_{2m+1/2}\},$$

$$\Omega_3 = \{s_{1l-1/2} \le s_1 \le s_{1l+1/2}, s_{2m-1} \le s_2 \le s_{2m}\}$$

$$\text{for} \quad t_{n-1/2} \le t \le t_{n+1/2}.$$

For example, in Ω_2 we have

$$\int_t \iint_{\Omega_2} [T_{11}^k] d\Omega_2 dt = \int_t \iint_{\Omega_2} [B_{11}^k (\varepsilon_{11}^k + \nu_{21}^k \varepsilon_{22}^k] d\Omega_2 dt, \tag{2.3}$$

$$\int_t \iint_{\Omega_2} [S] d\Omega_2 dt = \int_t \iint_{\Omega_2} [B_{12}^k \varepsilon_{12}^k] d\Omega_2 dt,$$

$$\ldots .$$

Integrating numerically, we obtain the following difference equations relating the forces/moments and strains:

$$T^{kn}_{11l\pm1/2,m} = B^k_{11}(\varepsilon^{kn}_{11l\pm1/2,m} + \nu^k_{21}\varepsilon^{kn}_{22l\pm1/2,m}),$$

$$S^{kn}_{l\pm1/2,m} = B^k_{12}\varepsilon^{kn}_{12l\pm1/2,m}, \tag{2.4}$$

$$\ldots .$$

The equations for the domain Ω_3 are similar.

The difference equations for the strains in (2.4) can be found by integrating Eq. (1.8) over the domains Ω_2 and Ω_3 for $t_{n-1/2} \le t \le t_{n+1/2}$:

$$\int_t \iint_{\Omega_2} [\varepsilon^k_{11}]\, d\Omega_2\, dt = \int_t \iint_{\Omega_2} \left[\frac{\partial u^k_1}{\partial s_1}\right] d\Omega_2\, dt,$$

$$\int_t \iint_{\Omega_2} [\varepsilon^k_{22}]\, d\Omega_2\, dt = \int_t \iint_{\Omega_2} \left[\frac{\partial u^k_2}{\partial s_2} + k_2 u^k_3\right] d\Omega_2\, dt, \tag{2.5}$$

$$\ldots .$$

After numerical integration in (2.5), we obtain the following difference equations relating the strains and the components of the generalized displacement vector:

$$\varepsilon^{kn}_{11l+1/2,m} = \frac{u^{kn}_{1l+1,m} - u^{kn}_{1l,m}}{\Delta s_1}, \tag{2.6}$$

$$\varepsilon^{kn}_{22l+1/2,m} = \frac{u^{kn}_{2l+1/2,m+1/2} - u^{kn}_{2l+1/2,m-1/2}}{\Delta s_1} + k_{2l+1/2,m}\frac{u^{kn}_{3l+1,m} + u^{kn}_{3l,m}}{2},$$

$$\ldots .$$

To find the numerical solution for the *i*th stringer, we consider the domains

$$\Omega_{1i} = \{s_{1l-1/2} \le s_1 \le s_{1l+1/2}\}, \quad \Omega_{2i} = \{s_{1l-1} \le s_1 \le s_{1l}\}, \quad \Omega_{3i} = \{s_{1l} \le s_1 \le s_{1l+1}\}$$

for $t_{n-1/2} \le t \le t_{n+1/2}$. The equations of vibration (1.5) for the *i*th stringer are integrated over the domain Ω_{1i}:

$$\int_t \int_{\Omega_{1i}} \left\{\frac{\partial T_{11i}}{\partial s_1} + [S]_i\right\} d\Omega_{1i}\, dt = \rho_i F_i \int_t \int_{\Omega_{1i}} \frac{\partial^2 u_{1i}}{\partial t^2}\, d\Omega_{1i}\, dt,$$

$$\int_t \int_{\Omega_{1i}} \left\{\frac{\partial \overline{T}_{12i}}{\partial s_1} + [T_{22}]_i\right\} d\Omega_{1i}\, dt = \rho_i F_i \int_t \int_{\Omega_{1i}} \frac{\partial^2 u_{2i}}{\partial t^2}\, d\Omega_{1i}\, dt,$$

$$\int_t \int_{\Omega_{1i}} \left\{\frac{\partial \overline{T}_{13i}}{\partial s_1} + [\overline{T}_{23}]_i\right\} d\Omega_{1i}\, dt = \rho_i F_i \int_t \int_{\Omega_{1i}} \frac{\partial^2 u_{3i}}{\partial t^2}\, d\Omega_{1i}\, dt,$$

$$\int_t \int_{\Omega_{1i}} \left\{\frac{\partial M_{11i}}{\partial s_1} - T_{13i} + [H]_i\right\} d\Omega_{1i}\, dt = \rho_i I_{1i} \int_t \int_{\Omega_{1i}} \frac{\partial^2 \varphi_{1i}}{\partial t^2}\, d\Omega_{1i}\, dt,$$

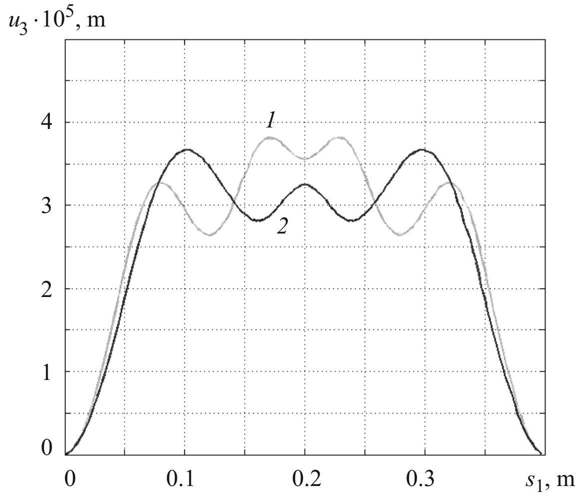

Fig. 2

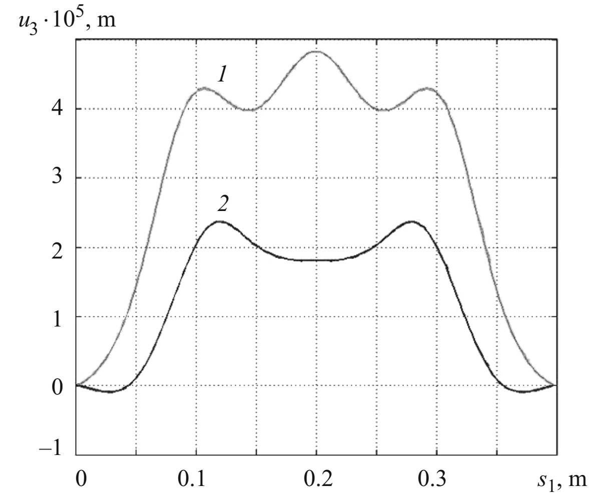

Fig. 3

$$\int_t \int_{\Omega_{1i}} \left\{ \frac{\partial M_{12i}}{\partial s_1} + [M_{22}]_i \right\} d\Omega_{1i} dt = \rho_i I_{\mathrm{tw}\,i} \int_t \int_{\Omega_{1i}} \frac{\partial^2 \varphi_{2i}}{\partial t^2} d\Omega_{1i} dt. \tag{2.7}$$

Integrating Eqs. (2.7) yields

$$\frac{T^n_{11il+1/2} - T^n_{11il-1/2}}{\Delta s_1} + [S]^n_l = \rho_i F_i (u^n_{1il})_{\bar{t}t},$$

$$\frac{T^n_{12il+1/2} - T^n_{12il-1/2}}{\Delta s_1} + [T_{22}]^n_l = \rho_i F_i (u^n_{2il})_{\bar{t}t},$$

$$\frac{T^n_{13il+1/2} - T^n_{13il-1/2}}{\Delta s_1} + [T_{23}]^n_l = \rho_i F_i (u^n_{3il})_{\bar{t}t},$$

$$\frac{M^n_{11il+1/2} - M^n_{11il-1/2}}{\Delta s_1} - \frac{1}{2}(T^n_{13il+1/2} + T^n_{13il-1/2}) + [H]^n_l = \rho_i I_{1i} (\varphi^n_{1il})_{\bar{t}t},$$

$$\frac{M^n_{12il+1/2} - M^n_{12il-1/2}}{\Delta s_1} + [M_{22}]^n_l = \rho_i I_{\mathrm{tw}\,i} (\varphi^n_{2il})_{\bar{t}t}. \tag{2.8}$$

The matched forces/moments for the ith stringer obtained for Eq. (2.8) are determined similarly to (2.3)–(2.6).

Thus, the above approach to constructing finite-difference equations for the shells and stringers ensures conservation of total mechanical energy at a difference level.

3. Numerical Example. Let us solve the problem of the forced vibrations of an elliptic cylindrical sandwich shell reinforced with stringers under an internal distributed impulsive load. The ends of the shell and stringers are clamped, i.e., the boundary conditions at $s_1 = 0$ and $s_1 = L$ are

$$u_1^k = u_2^k = u_3^k = \varphi_1^k = \varphi_2^k = 0 \quad (k = \overline{1,2}),$$

$$u_{1i} = u_{2i} = u_{3i} = \varphi_{1i} = \varphi_{2i} = 0 \quad (i = \overline{1,I}).$$

The initial conditions for system (1.1)–(1.6) at $t = 0$ are

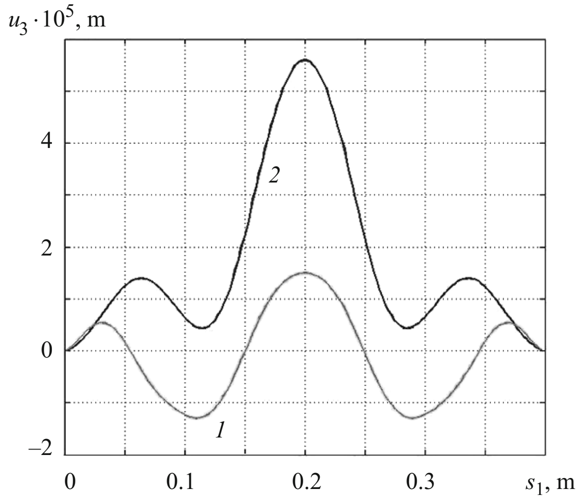

Fig. 4

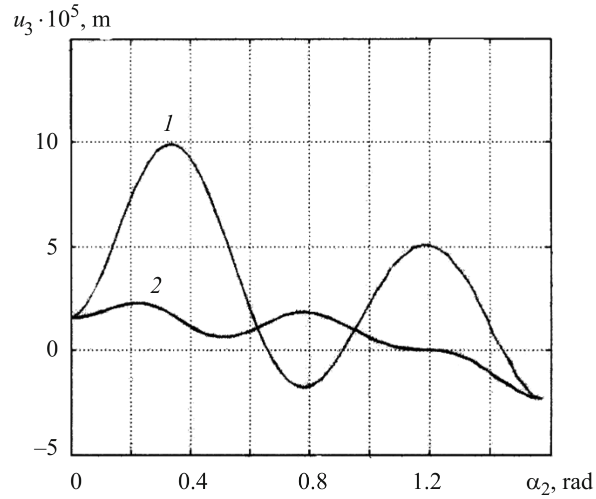

Fig. 5

$$u_1^k = u_2^k = u_3^k = \varphi_1^k = \varphi_2^k = 0,$$

$$\frac{\partial u_1^k}{\partial t} = \frac{\partial u_2^k}{\partial t} = \frac{\partial u_3^k}{\partial t} = \frac{\partial \varphi_1^k}{\partial t} = \frac{\partial \varphi_2^k}{\partial t} = 0 \qquad (k = 1, 2)$$

for the shells and

$$u_{1i} = u_{2i} = u_{3i} = \varphi_{1i} = \varphi_{2i} = 0,$$

$$\frac{\partial u_{1i}}{\partial t} = \frac{\partial u_{2i}}{\partial t} = \frac{\partial u_{3i}}{\partial t} = \frac{\partial \varphi_{1i}}{\partial t} = \frac{\partial \varphi_{2i}}{\partial t} = 0 \qquad (i = \overline{1, I})$$

for the stringers.

The geometrical and mechanical parameters of the structure: $L/h_1 = 80$, $h_1 = h_2$, $a_1/b_1 = 1.1$, $h_j = h_i$, $j = \overline{1, J}$, $E_1^1 = E_1^2 = E_i = E_j = 70$ GPa, $\nu_1^1 = \nu_1^2 = 0.3$, $\rho_1 = \rho_2 = \rho_i = \rho_j = 2.7 \cdot 10^3$ kg/m^3, where L is the length of the structure.

Consider the case where $I = 4$ and stringers are equally spaced between the inner and outer shells. The centers of gravity of stringer's cross-sections are projected onto the mid-surfaces of the shells along the lines $s_{2j} = (j-1)A_{2j}\alpha_{2j}$, $j = \overline{1, 4}$. The normal impulsive load is $P_3^1 = A \cdot [\eta(t) - \eta(t-T)]$, where $\eta(t)$ is the Heaviside function, A is the load amplitude, T is the load duration. Let $A = 10^6$ Pa, $T = 50$ µsec.

The numerical results are presented in Figs. 2–7, where curves *1* refer to the inner shell (strain, stress), and curves *2* refer to the outer shell. The stress–strain state and the dynamic behavior of stringer-reinforced elliptic cylindrical sandwich shells were analyzed on the time interval $t = 60T$.

Time points at which the kinematic and static quantities are maximum were considered as well.

Figure 2 shows u_3 as a function of s_1 in the sections $\alpha_2 = 0$ and $\alpha_2 = \pi/2$ (curves *1* and *2*, respectively) at the instant $t = 4T$ at which u_3 in the section $\alpha_2 = 0$ peaks.

Figure 3 shows u_3 as a function of s_1 in the section $\alpha_2 = \pi/4$. Curves *1* and *2* correspond to u_3 in the inner and outer shells, respectively, at the instant $t = 4.5T$ at which u_3 peaks. Similar curves for the instant $t = 9T$ at which u_3 in the outer shell peaks are shown in Fig. 4.

Figure 5 shows u_3 as a function of α_2 in the section $s_1 = L/2$ $(0 \le \alpha_2 \le \pi/2)$. Curves *1* and *2* correspond to u_3 in the inner and outer shells, respectively, at the instant $t = 2.5T$ at which u_3 peaks. Similar curves for the instant $t = 9T$ at which u_3 in the outer shell peaks are shown in Fig. 6.

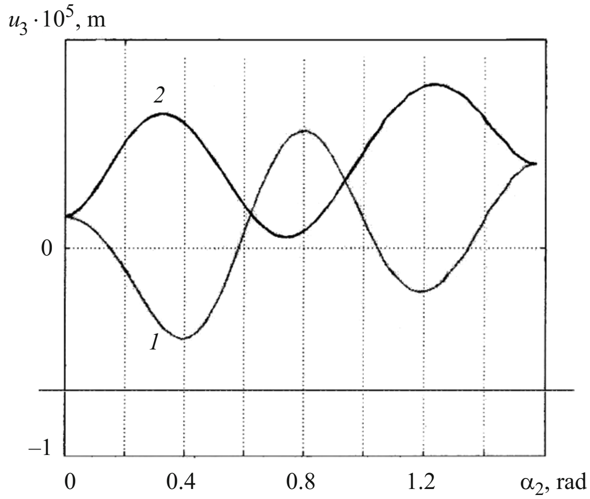

Fig. 6

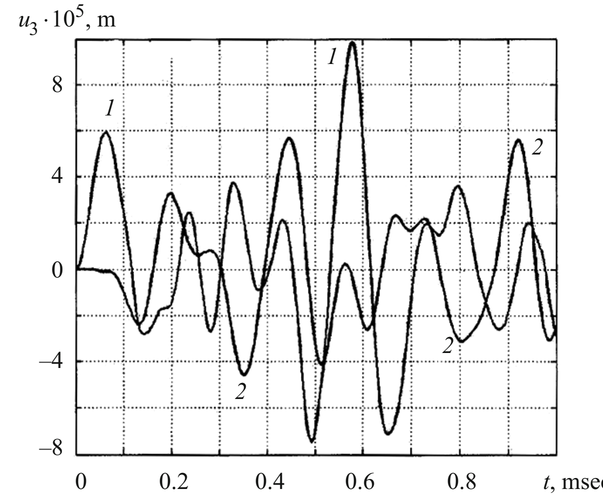

Fig. 7

Figure 7 shows u_3 at the point ($s_1 = L/2, \alpha_2 = \pi/4$). Curves *1* and *2* demonstrate the deflections of the inner and outer shells, respectively. It can be seen that the outer shells starts vibrating later than the inner shell. This lag is the time it takes a transverse wave to travel the distance from the projection of the center of gravity of stringer's cross-section onto the outer shell to the point of interest ($\Delta t \approx 3.25e^{-5}$).

Conclusions. We have derived, using the Hamilton–Ostrogradskii variational principle, the equations of vibration of elliptic cylindrical sandwich shells reinforced with stringers. Shell and rod models based on the Timoshenko hypotheses have been used. To solve the equations, an effective numerical method based on integro-interpolation equations in space coordinates and an explicit finite-difference scheme in time coordinate has been developed. The dynamic problem for a stringer-reinforced elliptic cylindrical sandwich shell under impulsive loading has been solved. The results obtained have been analyzed.

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