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VIBRATIONS OF CYLINDRICAL SANDWICH SHELLS WITH ELASTIC CORE UNDER LOCAL LOADS

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The vibrations of a cylindrical sandwich shell with elastic core under local loads are studied. The Kirchhoff–Love hypotheses are applied to the isotropic base layers. For the thick core, the work of transverse shear and thickness reduction are taken into account, and the displacements are assumed to vary linearly with the transverse coordinate. The displacements are considered continuous at the interfaces. The Winkler hypothesis is applied to the elastic core. The analytical solutions to the problems of natural and forced vibrations are found. The variation in displacements in the shell under forces per unit length is studied

Keywords: vibrations, cylindrical sandwich shell, Winkler hypothesis, local loads

Introduction. Thin plates and shells are widely used in modern sectors of industry and construction, which necessitates the development of methods for dynamic or static design of such structural elements. The free vibrations of stepped cylindrical shells with cracks made of functionally graded materials were studied in [1, 8]. Dynamic problems for homogeneous and sandwich cylindrical shells on an elastic foundation are addressed in [7, 9, 10, 16]. The vibrations and dynamic characteristics of inhomogeneous and sandwich beams with or without elastic foundation are studied in [5, 6, 11, 12, 19, 21]. The dynamic behavior of various sandwich panels and plates, including those on an elastic foundation was analyzed experimentally and theoretically in [2–4, 13–15, 17, 18, 22] where the dependence of the natural frequencies on the geometrical and elastic characteristics of the layers and foundation was analyzed parametrically and the displacements under sudden, local, and resonant loads were studied. The low-amplitude vibrations of a circular cylindrical sandwich shell on an elastic reinforcement under local loads have not been studied.

1. Problem Formulation. The Kirchhoff–Love hypotheses are assumed to apply to the thin isotropic base layers of the shell. It is also assumed that the thick core undergoes transverse shear and thickness reduction, and the displacements vary linearly with the transverse coordinate and are continuous at the interfaces. Figure 1*a* shows a design model for the cylindrical shell. The thickness of the *k*th layer is denoted by h_k; $h_3 = 2$ (Fig. 1*b*).

Let the independent variables be the tangential displacements u_α^k and deflections w^k of the mid-surfaces of the base layers along the x_α- and *z*-axes of a right coordinate system aligned with the lines of principal curvature of the mid-surface of the core and the outward normal, respectively (Fig. 1*c*).

Thus, the displacements in the base layers ($c \le z \le c + h_1$, $-c - h_2 \le z \le -c$) are defined by

$$u_\alpha^{kz} = u_\alpha^k + (z \mp a_k)\psi_\alpha^k, \qquad a_k = c + 0.5h_k,$$

$$\psi_1^k = -w^k{}_{,1}, \qquad \psi_2^k = (R \pm a_k)^{-1}(u_2^k - w^k{}_{,2}). \tag{1.1}$$

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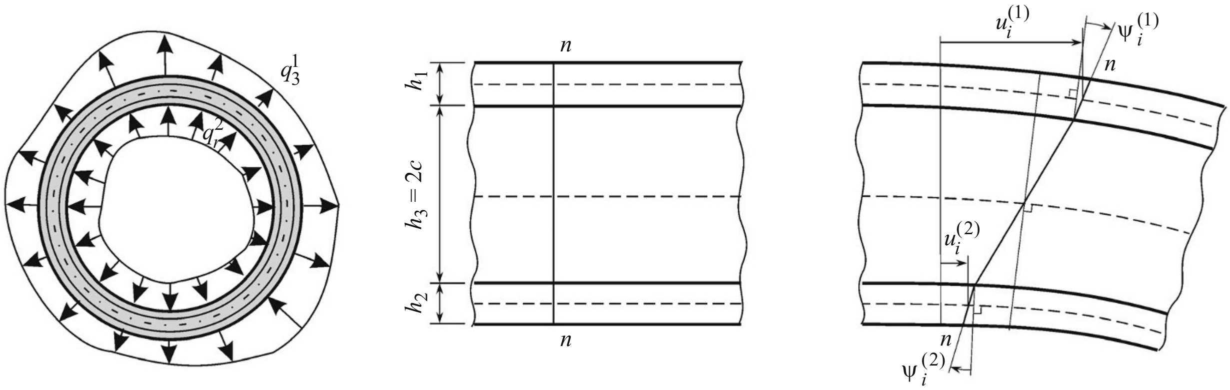

Fig. 1

Hereafter, the Greek indices take the values 1 and 2; the Latin indices take the values 1, 2, 3 (unless otherwise specified); the lower sign corresponds to $k = 2$ (layer number); ψ_α^k is the angle of rotation of the deformed normal in the kth base layer; the subscript after the comma denotes partial differentiation with respect to the coordinate.

Since the displacements are continuous at the interfaces between the layers, for the core ($-c \le z \le c$) we have

$$u_1^{3z} = 0.5\sum_{k=1}^{2}(1 \pm z/c)(u_1^k \pm 0.5h_k w^k{}_{,1}),$$

$$u_2^{3z} = \sum_{k=1}^{2}(1 \pm z/c)((0.5 \mp D_{k2})u_2^k \pm D_{k2} w^k{}_{,2}),$$

$$w^{3z} = 0.5\sum_{k=1}^{2}(1 \pm z/c)w^k,$$

where $D_{k1} = h_k/4$, $a_k = c + 0.5h_k$, $D_{k2} = 0.25h_k(1 \pm a_k/R)^{-1}R^{-1}$. Hereafter the brackets in the index "3" (core number) and other numerical indices will be omitted.

A distributed load q_l^1 and the reaction of the elastic inertialess Winkler medium is applied to the outside surfaces of the base layers (Fig. 1a)

$$q_{3r}^2 = -\kappa_0 w^2, \qquad q_{\alpha r}^k = 0, \tag{1.2}$$

where κ_0 is the modulus of subgrade reaction.

The equations of motion of the sandwich shell and the boundary conditions for forces are derived from the Lagrange variational principle taking into account the work of inertial forces and the core reaction (1.2):

$$\sum_{k=1}^{2}\Big[a_{m\alpha 1}^k u_\alpha^k{}_{,\alpha\alpha} + a_{m\alpha 2}^k u_\alpha^k{}_{,\beta\beta} + a_{m\alpha 3}^k u_\alpha^k + a_{m\alpha 4}^k u_\beta^k{}_{,\alpha\beta}$$

$$+ a_{m\alpha 5}^k w^k{}_{,\alpha} + a_{m\alpha 6}^k w^k{}_{,\alpha\alpha\alpha} + a_{m\alpha 7}^k w^k{}_{,\alpha\beta\beta}\Big] - b_\alpha^m \ddot{u}_\alpha^m = (\mp 0.5h_m c_2^m \delta_{\alpha 2} - R)m_m q_\alpha^m \delta_{m1},$$

$$\sum_{\gamma=1}^{2}\sum_{k=1}^{2}\Big[a_{m31}^{\gamma k} w^k{}_{,\gamma\gamma\gamma\gamma} + a_{m32}^k w^k{}_{,1122} + a_{m33}^{\gamma k} w^k{}_{,\gamma\gamma} + (a_{m34}^k - Rm_m \kappa_0^m \delta_{2m})w^k + a_{m35}^{\gamma k} u_\gamma^k{}_{,\gamma\gamma\gamma}$$

$$+ a_{m36}^{\gamma k} u_\gamma^k{}_{,\gamma} + a_{m37}^{\gamma k} u_\gamma^k{}_{,\gamma\beta\beta}\Big] - b_3^m \ddot{w}^m = -Rm_m\Big[q_3^m \pm 0.5h_m(q_1^m{}_{,1} + R^{-1}c_2^m q_2^m{}_{,2})\Big]\delta_{m1},$$

$$m, \alpha, \beta = 1, 2, \quad \alpha \neq \beta, \tag{1.3}$$

where δ_{mk} are Kronecker deltas; the coefficients are expressed in terms of the geometric characteristics of the layers and elastic moduli of their materials:

$$a_{k11}^{k} = K_k^+ h_k (1 \pm a_k) + K_3^+ c(2 \pm c)/3, \quad a_{\alpha 11}^{k} = K_3^+ c/3, \quad a_{k12}^{k} = G_k LN_k + G_3 S_k / 2,$$

$$a_{\alpha 12}^{k} = G_3 S_3 / 2, \quad a_{k13}^{k} = -G_3 / (2c), \quad a_{\alpha 13}^{k} = G_3 / (2c),$$

$$a_{k14}^{k} = h_k \left(K_k + \frac{1}{3} G_k \right) + \frac{4}{3} B_{k2} c \left(K_3 + \frac{1}{3} G_3 \right), \quad a_{\alpha 14}^{k} = \frac{2}{3} \left(K_3 + \frac{1}{3} G_3 \right) B_{k2} c,$$

$$a_{k15}^{k} = K_k^- h_k + K_3 (5c/6 \pm 0.5) - G_3 (\pm h_k / (4c) + 13c/8 \pm 5/6),$$

$$a_{\alpha 15}^{k} = K_3 (\mp 0.5 + c/6) + G_3 (\mp h_k / (4c) + c/18 \mp 1/6),$$

$$a_{k16}^{k} = -K_k^+ h_k^3 / 12 \pm K_3^+ h_k c(2 \pm c)/6, \quad a_{\alpha 16}^{k} = \mp \frac{1}{6} K_3^+ h_k c,$$

$$a_{k17}^{k} = -G_k (h_k \mp (a_k \pm 1) LN_k) \pm 4K_3 D_{k2} c/3 + G_3 (\pm 4D_{k2} c/9 \pm h_k S_k / 4),$$

$$a_{\alpha 17}^{k} = \mp \frac{2}{3} K_3 D_{k2} c \mp G_3 \left(\frac{2}{9} D_{k2} c + h_k S_3 / 4 \right),$$

$$a_{k21}^{k} = K_k^+ (2h_k c_2^k + A_k (c_2^k)^2 + (1 \mp 2c_2^k (a_k \pm 1)) LN_k) + 2K_3^+ B_{k2}^2 S_k,$$

$$a_{\alpha 21}^{k} = 2K_3^+ B_{12} B_{22} S_3, \quad a_{\alpha 22}^{k} = 4G_3 B_{12} B_{22} c/3,$$

$$a_{k22}^{k} = G_k h_k \left(h_k^2 c_2^k / 6 + (1 \pm a_k)(h_k^2 (c_2^k)^2 / 12 + 1) \right) + 4G_3 B_{k2}^2 c(2 \pm c)/3,$$

$$a_{k23}^{k} = -2G_3 B_{k2}^2 (1/c + S_k \mp 2), \quad a_{\alpha 23}^{k} = 2G_3 B_{12} B_{22} (1/c - S_3),$$

$$a_{k24}^{k} = \left(K_k + \frac{1}{3} G_k \right) h_k + 4 \left(K_3 + \frac{1}{3} G_3 \right) B_{k2} c/3, \quad a_{\alpha 24}^{k} = 2 \left(K_3 + \frac{1}{3} G_3 \right) B_{\alpha 2} c/3,$$

$$a_{k25}^{k} = K_k^+ \left(h_k c_2^k + (1 \mp c_2^k (a_k \pm 1)) LN_k \right) + K_3 B_{k2} (S_k \pm 1)$$

$$+ G_3 B_{k2} \left((7/3 \mp 2D_{k2}) S_k + 2D_{k2} (2 \mp 1/c) \mp 5/3 \right),$$

$$a_{\alpha 25}^{k} = K_3 B_{\alpha 2} (S_3 \mp 1) + G_3 B_{\alpha 2} \left((7/3 \pm 2D_{k2}) S_3 \mp 2D_{k2} / c \mp 1/3 \right),$$

$$a_{k26}^{k} = -K_k^+ c_2^k \left(h_k + A_k c_2^k \mp (a_k \pm 1) LN_k \right) \pm 2K_3^+ B_{k2} D_{k2} S_k, \quad a_{\alpha 26}^{k} = \mp 2K_3^+ B_{\alpha 2} D_{k2} S_3,$$

$$a_{k27}^{k} = -\frac{1}{12} h_k^3 c_2^k \left(K_k + G_k \left(\frac{4}{3} + c_2^k (1 \pm a_k) \right) \right) \pm \frac{2}{3} B_{k2} c \left(+K_3 h_k + G_3 \left(2D_{k2} (2 \pm c) + \frac{1}{3} h_k \right) \right),$$

$$a_{\alpha 27}^{k} = \mp \frac{1}{3} B_{\alpha 2} c \left(K_3 h_k + G_3 \left(4D_{k2} + \frac{1}{3} h_k \right) \right),$$

$$a_{k31}^{1k} = -h_k^3 K_k^+ (1 \pm a_k)/12 - K_3^+ h_k^2 c/12, \quad a_{k31}^{2k} = -K_k^+ (c_2^k)^2 A_k - 2K_3^+ D_{k2}^2 S_k,$$

$$a_{\alpha 31}^{1k} = K_3^+ h_1 h_2 c/12,$$

$$a_{k32}^{k} = -K_k h_k^3 c_2^k / 6 - G_k \left(h_k^3 c_2^k \left(2/3 + c_2^k (1 \pm a_k) \right) / 12 + A_k \right)$$

$$-4K_3 D_{k2} h_k c / 3 - G_3 \left[4D_{k2} c \left(D_{k2} (2 \pm c) + h_k / 3 \right) / 3 + h_k^2 S_k / 8 \right],$$

$$a_{\alpha 32}^{k} = K_3 \, c(D_{22} h_1 + D_{12} h_2) / 3 + G_3 \left(c(4D_{22} h_1 + D_{12} h_2 + 12 D_{12} D_{22}) / 9 + h_1 h_2 S_3 / 8 \right),$$

$$a_{k33}^{1k} = \mp K_3 h_k (5c/6 \pm 0.5) + G_3 \left(h_k (5/6 + h_k / (8c)) + c(2/3 \pm c/3 \pm 13 h_k / 18) \right),$$

$$a_{\alpha 33}^{1k} = K_3 \left(h_1 (1 - c/3)/4 + h_2 (1 + c/3)/4 \right)$$

$$+ G_3 \left[h_1 (1 - c/3)/12 + h_2 (1 + c/3)/12 + h_1 h_2 / (8c) + c/3 \right],$$

$$a_{k33}^{2k} = 2K_k^+ c_2^k \left(h_k \mp (a_k \pm 1) LN_k \right) \mp 2K_3 D_{k2} (S_k \pm 1)$$

$$+ G_3 \left(D_{k2} (10 \mp 14 S_k)/3 + 2D_{k2}^2 (1/c + S_k \mp 2) + 0.5 S_k \right),$$

$$a_{\alpha 33}^{2k} = K_3 \left(D_{12} (1 - S_3) + D_{22} (1 + S_3) \right) + G_3 \left(D_{22} \left(1 + 7S_3 \right) / 3 \right.$$

$$\left. + D_{12} (1 - 7S_3)/3 + 2D_{12} D_{22} (1/c - S_3) + 0.5 S_3 \right), \quad a_{\alpha 31}^{2k} = 2K_3^+ D_{12} D_{22} S_3,$$

$$a_{k34}^{k} = -K_k^+ LN_k - K_3 (0.5 S_k + 0.5/c \pm 1) - 2G_3 (S_k + 1/c \mp 1)/3,$$

$$a_{\alpha 34}^{k} = -K_3^+ (S_3 - 1/c)/2, \quad a_{k35}^{1k} = K_k^+ h_k^3 / 12 \mp K_3^+ h_k c(2 \pm c)/6,$$

$$a_{k35}^{2k} = K_k^+ c_2^k \left(h_k + c_2^k A_k \mp (a_k \pm 1) LN_k \right) \mp 2K_3^+ B_{k2} D_{k2} S_k,$$

$$a_{\alpha 35}^{1k} = \mp K_3^+ h_\alpha c / 6, \quad a_{\alpha 35}^{2k} = \mp 2K_3^+ B_{k2} D_{\alpha 2} S_3,$$

$$a_{k36}^{1k} = -K_k^- h_k - K_3 (5c/6 \pm 0.5) + G_3 (13c/18 \pm h_k / (4c) \pm 5/6),$$

$$a_{k36}^{2k} = -K_k^+ \left(h_k c_2^k + \left(1 \mp c_2^k (a_k \pm 1) \right) LN_k \right) - K_3 B_{k2} (S_k \pm 1)$$

$$\pm G_3 (2B_{k2} D_{k2} (1/c \mp 2 + S_k) \mp B_{k2} (7S_k \mp 5)/3),$$

$$a_{\alpha 36}^{1k} = -K_3 (c/6 \pm 0.5) - G_3 (c/18 \pm h_\alpha / (4c) \pm 1/6),$$

$$a_{\alpha 36}^{2k} = -K_3 B_{k2} (S_3 \pm 1) \mp G_3 \left(2B_{k2} D_{\alpha 2} (1/c - S_3) \pm B_{k2} (7S_3 \pm 1)/3 \right),$$

$$a_{k37}^{1k} = G_k \left(h_k \mp (a_k \pm 1) LN_k \right) \mp 4K_3 D_{k2} c / 3 \mp G_3 (4D_{k2} c / 9 + h_k S_k / 4),$$

$$a_{k37}^{2k} = h_k^3 c_2^k / 12 \left(K_k + G_k (4/3 + c_2^k (1 \pm a_k)) \right) + 2B_{k2} c / 3 \left(\mp K_3 h_k \mp G_3 (h_k / 3 + 2D_{k2} (2 \pm c)) \right),$$

$$a_{\alpha 37}^{1k} = \mp 2K_3 D_{\alpha 2} c / 3 \mp G_3 (2D_{\alpha 2} c / 9 + h_\alpha S_3 / 4),$$

$$a_{\alpha 37}^{2k} = \mp K_3 B_{k2} h_\alpha c / 3 \mp G_3 B_{k2} c (h_\alpha / 3 + 4D_{\alpha 2}) / 3,$$

$$LN_1 = \ln \left| (1 + c + h_1)/(1 + c) \right|, \quad LN_2 = \ln \left| (1 - c)/(1 - c - h_2) \right|, \quad LN_3 = \ln \left| (1 + c)/(1 - c) \right|,$$

$$S_\alpha = \pm (2 \mp 1/c) + 0.5 \left(1 \mp (2 \mp 1/c) 1/c \right) LN_3, \quad S_3 = 1/c + 0.5(1 - 1/c^2) LN_3,$$

$$A_k = \mp h_k(a_k \pm 1) + \left(a_k^2 \pm (2a_k \pm 1)\right) LN_k,$$

$$K_k^+ = K_k + 4G_k / 3, \quad K_k^- = K_k - 2G_k / 3 \quad (k, \alpha = 1, 2; k \neq \alpha),$$

$$m_m = 1 \pm (c + h_m) R^{-1}, \quad b_1^m = 2R[\rho_m I_1^m + 0.25 \rho_3 I_2^{\pm}], \quad b_3^m = b_1^m,$$

$$b_2^m = 2R\left[\rho_m (I_1^m + 2R^{-1}(1 \pm a_1 / R)^{-1} I_3^m + R^{-2}(1 \pm a_1 / R)^{-2} I_5^m) + \rho_3 (B_{m2})^2 I_2^{\pm}\right].$$

To use the Bubnov–Galerkin method, we expand the unknown displacements and load into series of basis functions:

$$u_\beta^k = \sum_{m,n} \psi_{\beta mn}^k (x, \varphi) T_{\beta mn}^k (t), \quad w_k = \sum_{m,n} \psi_{3mn}^k (x, \varphi) T_{3mn}^k (t),$$

$$q_l^k = \sum_{m,n} \psi_{qlmn}^k (x, \varphi) q_{lmn}^k (t). \tag{1.4}$$

The boundary conditions are satisfied by selecting the basis functions $\psi_{\beta mn}^k$ and ψ_{3mn}^k. Substituting expressions (1.4) into system (1.3), we obtain a system of ordinary differential equations for the function $T_{jmn}(t)$:

$$\sum_{j=1}^{6} P_{ljmn} T_{jmn} + q_{lmn}(t) = b_l \ddot{T}_{lmn} \quad (l = 1, ..., 6), \tag{1.5}$$

where P_{ljmn} are coefficients determined in satisfying the boundary conditions and dependent on the wave numbers m and n, which characterize the vibration mode and the number of nodal lines; q_{lmn} are the coefficients of the expansion of the components of the external load into series of basis functions.

2. Free Vibrations. The equations of the free vibrations of the cylindrical sandwich shell follows from (1.6) when $q_{lmn} = 0$. Assuming that all points of the shell oscillate with the same frequency, the equations can be derived after representing the functions $T_{jmn}(t)$ in the form

$$T_{lmn}(t) = A_{lmn} \sin(\omega_{mn} t + \alpha_{mn}), \tag{2.1}$$

where A_{lmn} and ω_{mn} are the amplitudes and frequencies of vibration; α_{mn} are the initial phases.

Omitting the indices m and n for brevity and substituting (2.1) into system (1.5), we arrive at a generalized eigenvalue problem:

$$[P]\{A\} = -\omega^2 [B]\{A\}, \tag{2.2}$$

where $[P]$ is a square matrix of the sixth order composed of the coefficients P_{ljmn}; $[B]$ is a diagonal matrix of b_l; $\{A\}$ is a vector composed of amplitudes A_1.

Denoting $\lambda = -\omega^2$ and inverting the matrix $[B]$ because it is not degenerate, we reduce (2.2) to a standard eigenvalue problem:

$$[R]\{A\} = \lambda\{A\}, \quad [R] = [B]^{-1}[P]. \tag{2.3}$$

The transition from (2.3) to (2.2) and the calculation of the eigenvalues λ can be conducted with the help of standard programs present on any computer. The calculated values of ω_i^2 ($i = 1, \ldots, 6$) can then be used to find the eigenvector $\{A_i\}$.

All the numerical results discussed below refer to a circular cylindrical sandwich shell simply supported at the ends by fixed rigid supports. The elastic medium inside the shell has stiffness $\kappa_0^2 = \kappa_0$, and there is no elastic medium outside the shell (unless otherwise specified). The thicknesses of the layers are divided by radius R: $h_1 = h_2 = 0.02$, $c = 0.025$. The base layers are made of D16T alloy; the core is made of fluoroplastic (see [20] for the mechanical characteristics of these materials). In the

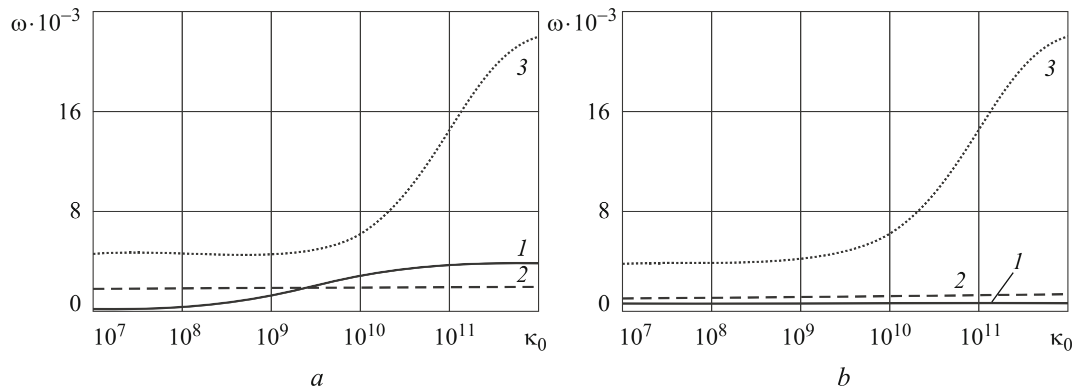

Fig. 2

figures and tables, the frequencies ω_{mnp} are measured in sec^{-1}, the stiffness coefficient κ_0 is measured in Pa/m, the shell length $L = 10R$, the radius $R = 1$.

Figure 2*a* shows the natural frequencies ω_{011}, ω_{012}, and ω_{013} (curves *1*, *2*, and *3*, respectively) versus the stiffness coefficient κ_0. Similar curves for the frequencies ω_{101}, ω_{102}, and ω_{103} are shown in Fig. 2*b*.

The elastic medium has a very weak effect on the frequency ω_{101}. If the stiffness of the medium is low ($\kappa_0 < 10^8$ Pa/m), this effect on the other frequencies is insignificant. The elastic medium of medium stiffness ($10^9 < \kappa_0 < 10^{11}$ Pa/m) increases the frequency ω_{103} by a factor of 5.

3. Forced Vibrations. Let us expand the unknown functions $T_{lmn}(t)$ (2.1) into a finite series of orthonormalized eigenfunctions:

$$T_{lmn} = \sum_{i=1}^{6} \delta_{lmni} \zeta_{mni}.$$

Substituting this expression into (1.5) and using the orthogonality of the natural modes, we arrive at six independent equations for the functions ζ_{mni} (for each m and n):

$$\ddot{\zeta}_{mni} + \omega_{mni}^2 \zeta_{mni} = \tilde{q}_{mni}(t),$$

$$\tilde{q}_{mni} = \sum_{l=1}^{6} q_{lmn} \delta_{lmni} \Big/ \sum_{l=1}^{6} b_l \delta_{lmni}^2, \tag{3.1}$$

where $\tilde{q}_{mni}$ are the components of the reduced load (either mechanical or thermal); δ_{lmni} are form factors.

Solving Eqs. (3.1), we obtain the required displacements as sums of the original coordinate functions (1.4) and products between ζ_{mni} and coefficients.

Let us consider special cases of local loading of a cylindrical shell filled with a material distributed over the area or the length (forces per unit length).

Uniformly Distributed Ring Load. Let a uniformly distributed ring load of intensity q_0 be applied to the outside surface of the shell (Fig. 3*a*):

$$q_3^1 = q_0(t)(H_0(x_1 - x) - H_0(x_0 - x)), \tag{3.2}$$

where x_0 and x_1 are the coordinates of the edges of the load ring.

The coefficients of the expansion of load (3.2) into series (1.4) are

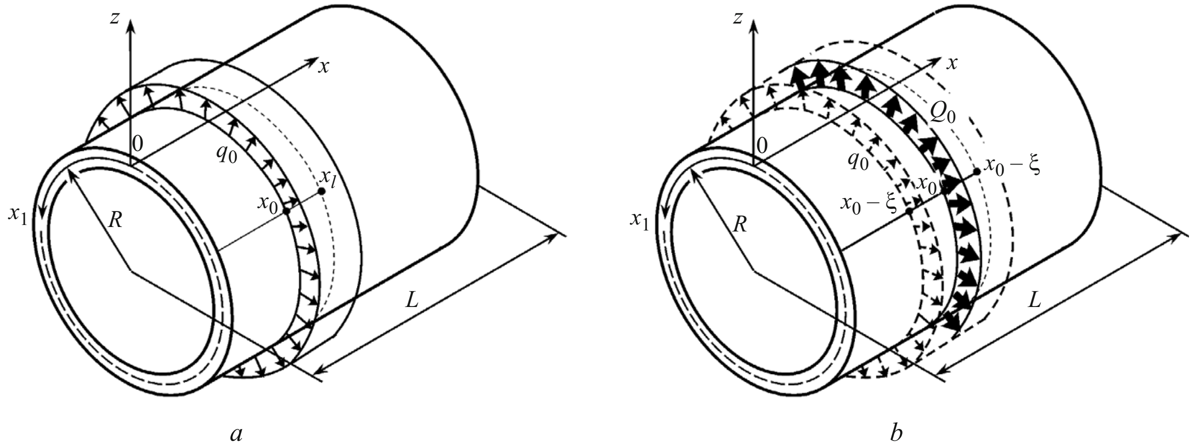

Fig. 3

$$q_{3m}^1 = \frac{2q_0(t)}{L}\int_{x_0}^{x_1}\sin\left(\frac{\pi m}{L}x\right)dx = \frac{2q_0(t)}{\pi m}\left(\cos\left(\frac{\pi m}{L}x_0\right)-\cos\left(\frac{\pi m}{L}x_1\right)\right). \quad (3.3)$$

The displacements in the base layers are defined by (1.1); the functions $\zeta_{mi}(t)$ follow from Eq. (3.1).

Force per Unit Length. Let an axisymmetric load per unit length of intensity $Q_0(t)$ distributed along a circle be applied to the outside surface of the shell (Fig. 3*b*): $Q(x,t)=Q_0(t)H_0(x-x_0)H_0(x_0-x)$, where x_0 is the coordinate of the section in which the load is applied.

To solve the problem, we use results (3.3) for a surface load distributed over the interval ($x_0-\xi \le x \le x_0+\xi$). Assuming that ξ is small, substituting $q_0 = Q_0/(2\xi)$ into (3.3), letting ξ tend to zero, and keeping Q_0 constant, we obtain

$$q_{3m}^1 = \lim_{\xi\to 0}\left[\frac{2Q_0(t)}{\pi m 2\xi}\left(\cos\left(\frac{\pi m}{L}(x_0-\xi)\right)-\cos\left(\frac{\pi m}{L}(x_0+\xi)\right)\right)\right] = \frac{2Q_0(t)}{L}\sin\left(\frac{\pi m}{L}x_0\right).$$

The numerical results discussed below refer to a shell of length $L = 2R$ under a compressive load per unit length of intensity $Q_0 = -10^6$ N/m. Figures 4 and 5 show the deflections of the base layers k = *1* (curve *1*) and $k = 2$ (curve *2*) in the middle of the shell (Figs. 4*a* and 5*a*) and the horizontal displacements at the right end of the shell (Figs. 4*b* and 5*b*) without ($\kappa_0 = 0$) and with ($\kappa_0 = 10^{11}$ Pa/m) elastic core.

Depending on the relative coordinate of the load circle, the deflections peak if the ring load is applied in the midlength of the shell. The deviation of the load by 10%*L* from the section decreases the vertical displacements to one third the value. With increase in the stiffness of the elastic medium, the region of local changes in the deflections near the section becomes narrower and their maximum decreases. If the elastic medium is of high stiffness, the deflections of the base layers differ substantially, i.e., the compression of the core is maximum.

The horizontal displacements at the right end are maximum when the load circle is in the section $x = (0.82–0.92)L$. As the stiffness of the elastic medium is increased, the maxima move to the right and the difference between the displacements in the layers decreases. The instants at which the displacements are maximum were chosen.

Conclusions. We have constructed a dynamic mechanical/mathematical model of a cylindrical sandwich shell with elastic core. The problems of free and forced vibrations have been solved. It has been established that with distance from the area of application of local loads, the displacements quickly decay with increase in the stiffness of the medium.

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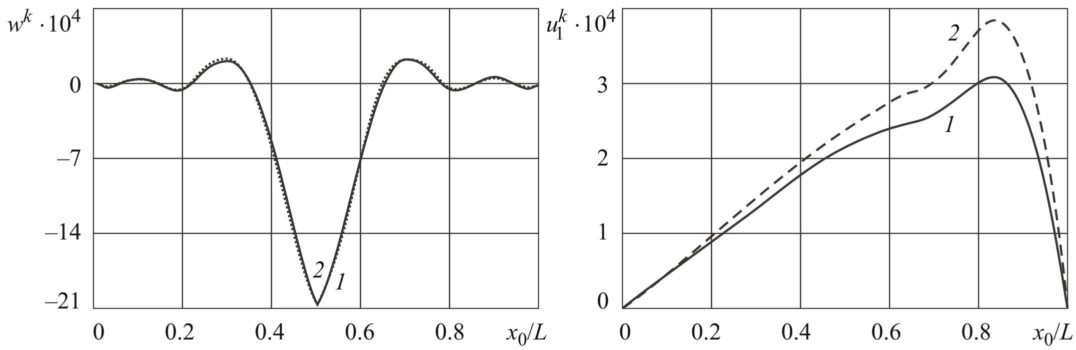
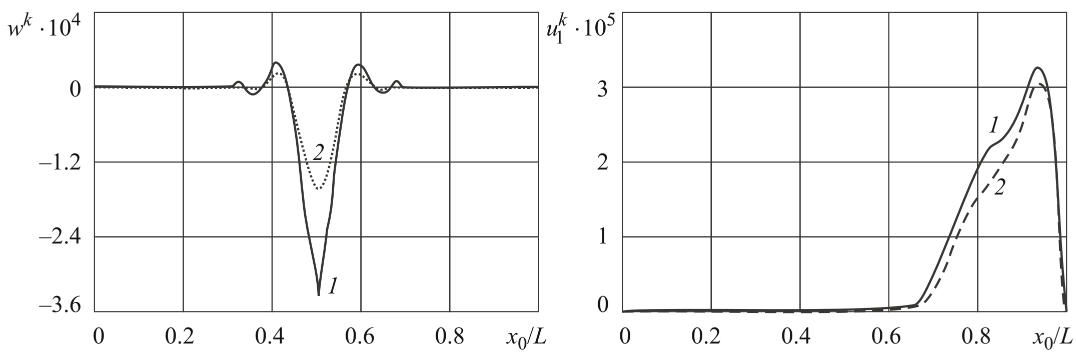


a *b*

Fig. 4



a *b*

Fig. 5

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