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DAMPING THE RADIAL VIBRATIONS AND SELF-HEATING OF VISCOELASTIC SHELL ELEMENTS WITH PIEZOELECTRIC SENSOR AND ACTUATOR

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The forced resonant axisymmetric vibrations and self-heating of a closed spherical shell, an infinitely long cylindrical shell, and a ring with piezoelectric sensor and actuator are studied. The effect of the temperature dependence of the complex characteristics of the passive material on the vibration amplitude and self-heating temperature is analyzed. The possibility of active damping of these vibrations with piezoelectric sensor and actuator is shown

Keywords: resonant axisymmetric vibrations, self-heating, closed spherical shell, infinitely long cylindrical shell, ring, piezoelectric sensor and actuator

Introduction. Thin-walled structural elements such as rings, spherical and cylindrical shells made of elastic and viscoelastic materials often undergo vibrations under harmonic loads. The vibrations of such elements can be damped with piezoelectric components acting as sensors and actuators, which is an active-damping method [9–13]. Harmonic loading causes self-heating of the material because of hysteresis losses, which should be taken into account in dynamic analysis of inelastic elements [3, 4].

The thermomechanical behavior of layered inelastic elements with piezoelectric layers under monoharmonic electromechanical loading was modeled in [2, 4, etc.]. Analytic and numerical results on the monoharmonic vibrations and self-heating of sandwich plates and cylindrical shells with piezolayers acting as sensors and actuators are reported in [5, 7, 8, etc.]. Mathematically, the vibrations of such objects are damped by using feedback to change the stiffness of the system.

We will solve the problem of damping the forced radial vibrations and self-heating of a thin-walled closed spherical shell, a thin-walled infinitely long cylindrical (plane strain) shell, and a ring (plane stress state) made of a passive (no piezoelectric effect) viscoelastic material with piezoelectric layers one of which is a sensor, and the other is an actuator. Damping is performed using one of the feedback mechanisms affecting the stiffness, dissipative, or inertial characteristics of the objects. The dependence of the viscoelastic properties of the passive material on the self-heating temperature is taken into account.

1. Problem Formulation and Solution. Consider a closed spherical shell, an infinitely long cylindrical shell, and a ring, each having three layers. Let the origin of coordinates $z=0$ be on the mid-surface of the middle viscoelastic layer of radius R and thickness h_0. The inner ($z \le -h_0/2$) and outer ($z \ge h_0/2$) layers of thicknesses h_1 and h_2, respectively, are made of elastic piezoceramics and are polarized across the thickness. There are continuous infinitely thin electrodes between the passive and piezoactive layers and on the outside surfaces of the piezolayers. The internal electrodes are kept at potentials ${}^m\varphi(\pm h_0/2)=0$ $(m=1,2)$. Let the layer of thickness h_1 be a sensor, and the layer of thickness h_2 be an actuator.

The shells are under centrally symmetric surface pressure $q_z = q\cos\omega t$ harmonically varying with time t with constant amplitude q and nearly resonant frequency ω. Moreover, a voltage ${}^2\varphi(h_0/2+h_2) - {}^2\varphi(h_0/2) = V_a$ of frequency equal to that of the mechanical load is applied to the actuator to balance it. Then a voltage ${}^1\varphi(-h_0/2-h_1) - {}^1\varphi(-h_0/2) = V_s$ of unknown amplitude is induced across the open-circuited electrodes of the sensor and the electric boundary condition $\iint_S {}^1D_z\, ds = 0$ is

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satisfied ($^1 D_z$ is the normal component of electric-flux density). Also, the shells transfer heat by convection to the environment of temperature T_0.

Due to the chosen geometrical and loading conditions, the Kirchhoff–Love hypotheses of the membrane theory of shells can be used to describe the electromechanical behavior of the whole sandwich and the electric-flux density in the piezolayers can be considered constant across the thickness ($D_z = \text{const}$) [4]. The viscoelastic behavior of the passive layer is described using the concept of temperature-dependent complex moduli [4]. Assuming that self-heating is a steady-state process, the self-heating temperature can be considered constant throughout the thickness of the whole sandwich.

Using the membrane theory of shells and the above assumptions, the equation of the harmonic vibrations of the objects can be written for amplitude variables as

$$\beta N_\theta - \rho_* R\omega^2 w = \tilde{q}, \tag{1}$$

where $\rho_* = \rho_1 h_1 + \rho_2 h_2 + \rho_0 h_0$, $\tilde{q} = R(h_0 + h_1 + h_2)q$, ρ_1, ρ_2, and ρ_0 are the specific densities of the piezoelectric and passive materials. $N_\theta = N'_\theta + iN''_\theta$ and $w = w' + iw''$ are the complex circumferential force and radial deflection.

The constitutive equations for the piezoceramic layers polarized along the z-axis are

$$^m\sigma_\theta = {^m c_{11}}\varepsilon_\theta - {^m b_{31}}\, {^m E_z}, \quad {^m D_z} = \beta\, {^m b_{31}}\varepsilon_\theta + {^m \tilde{\varepsilon}_{33}}\, {^m E_z}, \quad \varepsilon_\theta = \frac{w}{R}, \quad {^m E_z} = -\frac{d\, {^m\varphi}}{dz},$$

$$^m c_{11} = \beta_1 / {^m s^E_{11}}, \quad {^m\nu_E} = -{^m s^E_{12}} / {^m s^E_{11}}, \quad {^m b_{31}} = \beta_2\, {^m d_{31}} / {^m s^E_{11}}, \quad {^m\tilde{\varepsilon}_{33}} = {^m\varepsilon^T_{33}} - \beta_3\, {^m d^2_{31}} / {^m s^E_{11}}. \tag{2}$$

Hereafter $\beta = 2$, $\beta_1 = \beta_2 = 1/(1 - {^m\nu_E})$, $\beta_3 = 2\beta_2$ for the spherical shell; $\beta = 1$, $\beta_1 = 1/(1 - {^m\nu^2_E})$, $\beta_2 = 1/(1 - {^m\nu_E})\beta_3 = 2\beta_2$ for the cylindrical shell; $\beta = \beta_m = 1$ for the ring; ${^m s^E_{11}}, {^m s^E_{12}}, {^m d_{31}}, {^m\varepsilon^T_{33}}$ are the elastic compliances, piezoelectric modulus, and permittivity of the piezoceramics; ${^m E_z}$ is the normal component of electric-field strength.

For the viscoelastic passive layer h_0, the first and third equations in (2) hold, where ${^m c_{11}} = {^0 c_{11}} = \beta_1 E(T)$, ${^m b_{31}} = 0$; $E = E' + iE''$ is the temperature-dependent complex modulus of viscoelasticity; ${^m\nu_E} = \nu = \text{const}$ is Poisson's ratio.

With the goal of assessing the maximum effect of thermomechanical coupling, we will determine the steady-state self-heating temperature by solving the equation

$$-2\alpha_s\theta + \beta\beta_1 \frac{\omega}{2R^2} E''(\theta)|w|^2 = 0, \tag{3}$$

where $|w| = (w'^2 + w''^2)^{1/2}$, $\theta = T - T_0$; α_s is the heat-transfer coefficient to the environment.

Integrating Eqs. (2) over the thickness of the sandwich, we derive an expression for the circumferential force:

$$N_\theta = D_N \frac{w}{R} - {^1 b_{31}} V_s + {^2 b_{31}} V_a \quad (D_N = {^1 c_{11}} h_1 + {^2 c_{11}} h_2 + {^0 c_{11}} h_0). \tag{4}$$

The electric-flux density in the piezoelectric sensor is defined by the formula

$$^1 D_z = \beta\, {^1 b_{31}} \frac{w}{R} + {^1\tilde{\varepsilon}_{33}} \frac{V_s}{h_1}. \tag{5}$$

Subjecting expression (5) to the electric boundary condition on the electroded surfaces of the sensor, we determine the voltage amplitude

$$V_s = -\beta \frac{{^1 b_{31}} h_1}{{^1\tilde{\varepsilon}_{33}} R} w. \tag{6}$$

For the active damping of the mechanical vibrations of the shell elements with piezoactive sensor and actuator, we use feedback mechanisms mathematically implemented as a linear relationship between the actuator voltage V_a and the sensor voltage V_s and its first or second derivative with respect to time [4]:

$$V_a = -G_a V_s, \quad V_a = -i\omega G_{as} V_s, \quad V_a = \omega^2 \overline{G}_{as} V_s, \tag{7}$$

where $G_a, G_{as}, \overline{G}_{as}$ are parameters controlling the stiffness, dissipative, and inertial characteristics of the system, respectively; the sign "–" indicates that the actuator voltage V_a is antiphase to the mechanical load.

Substituting expression (4) into the vibration equation (1) and taking (6) and (7) into account, we get

$$|w| = \tilde{q} / (\Delta'^2 + \Delta''^2)^{1/2}, \tag{8}$$

where

$$\Delta' = \Delta_0 + \beta m_1 E'(\theta), \quad \Delta'' = \beta(m_1 E''(\theta) + \omega G_{as}\gamma_1),$$

$$\Delta_0 = \beta(D_N^0 + G_a\gamma_1) - \omega^2 (R\rho_* + \beta\overline{G}_{as}\gamma_1), \quad D_N^0 = ({}^1c_{11}h_1 + {}^2c_{11}h_2)/R + \gamma_2,$$

$$m_1 = \beta_1 h_0 / R, \quad \gamma_2 = {}^1b_{31}^2 h_1 / ({}^1\tilde{\varepsilon}_{33} R), \quad \gamma_1 = {}^2b_{31}\gamma_2 \, {}^1b_{31} h_1 / ({}^1\tilde{\varepsilon}_{33} R).$$

Combining relations (3) and (8) leads to a transcendental equation for the unknown temperature θ, given $E'(\theta)$ and $E''(\theta)$.

Let the temperature approximation of the components of the complex viscoelastic modulus be linear:

$$E'(\theta) = E_0' + E_1'\theta, \quad E''(\theta) = E_0'' + E_1''\theta. \tag{9}$$

Then the deflection amplitude (8) can be calculated by the formula

$$|w| = \tilde{q} / (n_0 + n_1\theta + n_2\theta^2)^{1/2}. \tag{10}$$

A cubic equation for determining the self-heating temperature follows from Eqs. (3), (9), and (10):

$$n_2\theta^3 + n_1\theta^2 + (n_0 - m_0 E_1''\tilde{q}^2)\theta - m_0 E_0''\tilde{q}^2 = 0, \tag{11}$$

where

$$n_0 = \Delta_1^2 + \Delta_2^2, \quad n_1 = 2(d_1\Delta_1 + d_2\Delta_2), \quad n_2 = d_1^2 + d_2^2, \quad d_{1,2} = \beta m_1 (E_1', E_1''),$$

$$\Delta_1 = \Delta_0 + \beta m_1 E_0', \quad \Delta_2 = \beta(m_1 E_0'' + \omega G_{as}\gamma_1), \quad m_0 = \beta\beta_1\omega/(4\alpha_s R^2).$$

For the temperature-independent components of the complex modulus, the deflection amplitude and self-heating can be found from (8) and (3) where it is necessary to set $E'(\theta) = E_0'$ and $E''(\theta) = E_0''$. The isothermal resonant frequency can be determined from the condition $\Delta' = 0$.

2. Analysis of the Calculated Results. Let us consider, as a numerical example, a ring with $R = 0.1$ m, $h_0 = 0.004$ m, $h_1 = h_2 = 0.5 \cdot 10^{-5}$ m damped using the feedback mechanism controlling the dissipative properties of the system ($G_a = 0$, $G_{as} \neq 0$, $\overline{G}_{as} = 0$). The passive layer is made of polymer [6] with the following coefficients of temperature approximation (9):

$$E_0' = 0.216594 \cdot 10^{10} \text{ Pa}, \quad E_1' = -0.236994 \cdot 10^8 \text{ Pa/°C},$$

$$E_0'' = 0.199358 \cdot 10^9 \text{ Pa}, \quad E_1'' = -0.190904 \cdot 10^7 \text{ Pa/°C},$$

$$\rho_0 = 929 \text{ kg/m}^3, \quad \nu = 0.3636, \quad T_0 = 20 \text{ °C}, \quad \alpha_s = 5 \text{ W/(m}^2\text{·°C)}.$$

The piezoelectric actuator and sensor are both made of TsTStBS-2 piezoceramics [1] with the following characteristis:

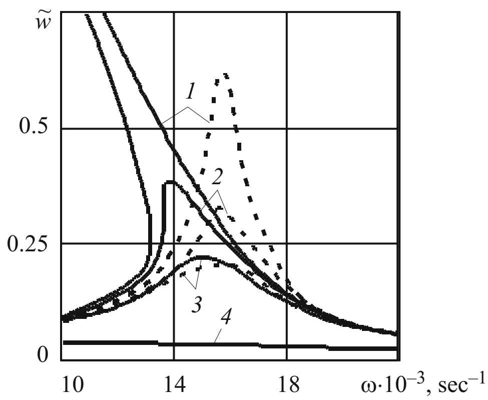

Fig. 1

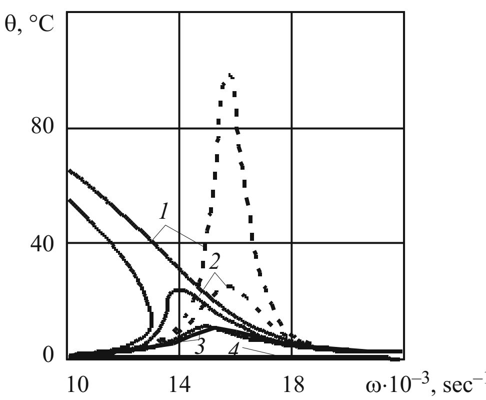

Fig. 2

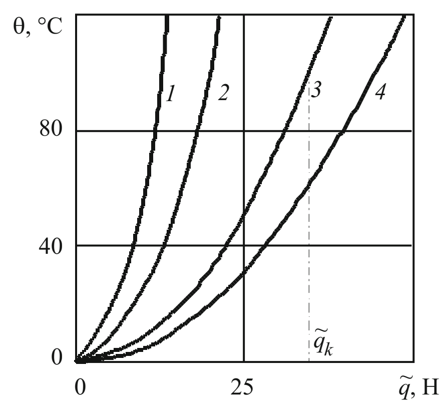

Fig. 3

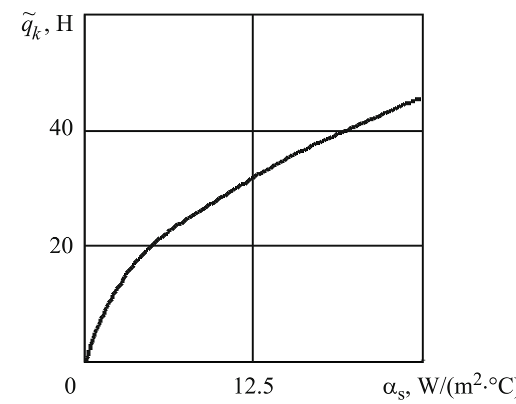

Fig. 4

$$s_{11}^E = 12.5\cdot 10^{-12}\ \text{m}^2/\text{N},\quad s_{12}^E = -4.62\cdot 10^{-12}\ \text{m}^2/\text{N},\quad d_{31} = -1.6\cdot 10^{-10}\ \text{C/m},$$

$$\varepsilon_{33}^T = 21\cdot 10^2\,\varepsilon_0,\quad \varepsilon_0 = 8.854\cdot 10^{-12}\ \text{F/m},\quad \rho_1 = \rho_2 = 7520\ \text{kg/m}^3.$$

Figures 1 and 2 show the frequency dependence of the dimensionless deflection amplitude $\tilde{w} = |w|\cdot 10^3 / h_0$ and self-heating temperature θ of the ring for $G_{as} = (0, 0.1, 0.2, 2.0)\cdot 10^{-2}$ (curves *1*, *2*, *3*, *4*, respectively) and $\tilde{q} = 20$ N. The dashed lines correspond to the isothermal complex viscoelastic modulus, while the solid lines to the temperature-dependent modulus. The temperature dependence makes the isothermal frequency characteristics softly nonlinear. Increasing the control G_{as} offsets the nonlinearity of both frequency characteristics and decreases the deflection and self-heating temperature up to the total absence of vibrations and heating.

There are harmonic loading and heat-transfer conditions under which the self-heating temperature of viscoelastic elements with passive and piezoactive components can reach the critical level θ_{cr} at which the system undergoes thermal failure because of the softening of the passive material or the depolarization of the piezoelectric material (Curie point). Figure 3 shows the dependence of the self-heating temperature θ of the undamped ring ($G_{as} = 0$) with temperature independent properties of the passive material on the mechanical load $\tilde{q}$ for $\omega = 15{,}800$ sec^{-1} and $\alpha_s = 2, 5, 15, 25$ W(m^2·°C) (curves *1*, *2*, *3*, *4*, respectively). The asterisk (*) on the ordinate axis indicates the temperature $\theta_{cr} = 100$ °C at which the polymer starts softening and which is lower than the Curie point of the piezoceramics. This temperature corresponds to the critical load amplitude $\tilde{q}_{cr}$ on the abscissa axis. The dependence of $\tilde{q}_{cr}$ on the heat-transfer coefficient α_s is shown in Fig. 4. It can be seen that the critical load $\tilde{q}_{cr}$ is zero if the system is perfectly thermally insulated ($\alpha_s \to 0$) and gradually increases tending to a constant level with increase in the heat-transfer coefficient.

Note that using the other feedback mechanisms defined by (7) for the active damping of the shell elements leads to qualitatively similar results.

Conclusions. The problem of the forced resonant radial vibrations and self-heating of viscoelastic elements (a closed spherical shell, an infinitely long cylindrical shell, and a ring) with a piezoelectric sensor and an actuator and passive layers with temperature-dependent properties has been formulated and solved. For the active damping of the mechanical vibrations of these elements, feedback mechanisms controlling the stiffness, dissipative, and inertial characteristics have been used. It has been shown that increasing the control parameter leads to a decrease in the amplitude of vibrations up to full damping. The effect of the temperature dependence of the viscoelastic properties of the material on the amplitude- and temperature–frequency characteristics and the heat-transfer coefficient has been analyzed. The critical amplitude of mechanical harmonic loading at which the self-heating temperature reaches a critical level and thermal failure occurs has been determined.

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