DOI 10.1007/s10778-015-0729-z
International Applied Mechanics, Vol. 51, No. 6, November, 2015

DYNAMIC ANALYSIS OF FLEXIBLE HOISTING ROPE WITH TIME-VARYING LENGTH*

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The governing equations of flexible hoisting rope are developed employing Hamilton's principle. Experiments are performed. It is found that the experimental data agree with the theoretical prediction very well. The results of simulation and experiment show that the flexible hoisting system dissipates energy during downward movement but gains energy during upward movement. Further, a passage through resonance in the hoisting system with periodic external excitation is analyzed. Due to the time-varying length of the hoisting rope, the natural frequencies of the system vary slowly, and transient resonance may occur when one of the frequencies coincides with the frequency of external excitation

Keywords: dynamic analysis; flexible hoisting rope; transient resonance

1. Introduction. While rope is employed in the hoisting industry such as mine hoists, elevators, cranes etc., it is subject to vibration due to its high flexibility and relatively low internal damping characteristics [1, 2]. Most often these systems are modeled as either an axially moving tensioned beam or a string with time-varying length and a rigid body at its lower end [3, 4]. It was shown that the vibration energy of the rope changes in general during elongation and shortening [5, 6]. When the rope length is being shortened, vibration energy increases exponentially with time, causing dynamic instability [7]. The study of rope vibration problems in flexible hoisting systems has attracted wide attention. Chi and Shu [8] calculated the natural frequencies associated with the vertical vibration of a stationary cable coupled with an elevator car. Terumichi and Ohtsuka et al. [9] assumed that the velocity of the string is constant and studied the transverse vibrations of a string with time-varying length and a mass–spring system at the lower end with theoretical and experimental methods. Fung and Lin [10] analyzed the transverse vibration of an elevator rope with time-varying length, and the time-varying mass and inertia of rotor were considered. A variable structure control scheme is proposed to suppress the transient amplitudes of vibrations. Kaczmarczyka and Ostachowiczb [11] studied the coupled vibration of a deep mine hoisting cable and built a distributed-parameter model. They found that the response of the catenary-vertical rope system may feature a number of resonance phenomena. Zhang and Agrawal [12] derived the governing equation of coupled vibration of a flexible cable transporter system with arbitrarily varying length. Zhu and Chen [13] investigated the control of an elevator cable with theoretical and experimental methods. A novel experimental method is developed to validate the uncontrolled and controlled lateral responses of a moving cable in a high-rise elevator and shows good agreement with the theoretical predictions. Zhang [14] presented a systematic procedure for deriving the model of a cable transporter system with arbitrarily varying cable length and proposed a Lyapunov controller to dissipate the vibratory

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* This research work was supported by the State Key Laboratory of Mechanical System and Vibration, Shanghai Jiaotong University (MSV-2010-06).

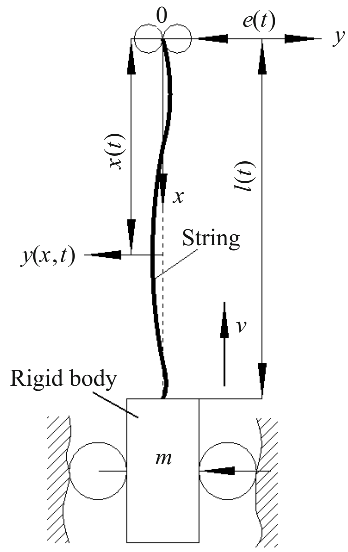


Fig. 1. Schematic of flexible hoisting string with time-varying length.

energy. Zhang and Zhu et al. [15] derived the governing equation and energy equation of longitudinal vibration of a flexible hoisting system with arbitrarily varying length.

While extensive studies focus individually on the vibration characteristics of a rope with time-varying length, the dynamic stability of a rope has also been studied by several researchers. Kumaniecka and Niziol [16] investigated the longitudinal-transverse vibration of a hoisting cable with slow variability of the parameters. The cable material non-linearity was taken into account and the unstable regions were identified by applying the harmonic balance method. The general stability characteristics of horizontally and vertically translating beams and strings with arbitrarily varying length and various boundary conditions were studied in Zhu and Ni [17]. While the amplitude of the displacement can behave in a different manner depending on the boundary conditions, the amplitude of the vibratory energy of a translating medium decrease and increase in general during extension and retraction, respectively. Lee [7] introduces a new technique to analyze the free vibration of a string with time-varying length by dealing with traveling waves. When the string length is being shortened, the free vibration energy increases exponentially with time, causing dynamic instability.

Extensive research efforts on a flexible hoisting rope with time-varying length have been done in the last few decades as aforementioned; however, most studies were restricted to cases with constant transport speed samples. The dynamic characteristics of a flexible hoisting rope with arbitrarily varying length are the subject of this investigation. The governing equations are developed employing the extended Hamilton's principle. The derived governing equations are shown to be nonlinear partial differential equations (PDEs) with variable coefficients. On choosing proper mode functions that satisfy the boundary conditions, the solution of the governing equations was obtained using Galerkin's method. In order to evaluate the mathematical model, an experimental set-up is built and some experiments are conducted. Comparing the experimental data to the simulation, a favorable result is obtained, which indicates that the proposed mathematical model is valid for a flexible hoisting rope. Further, the phenomenon of passage through resonance in a hoisting rope system is studied in this paper. Based on the proposed fundamental dynamic analyses, further vibration control can be adopted for such flexible hoisting systems in the near future.

2. Model of Flexible Hoisting System. A flexible hoisting system can be simplified as an axially moving string with time-varying length and a rigid body m at its lower end, as shown in Fig. 1. The rail and the suspension of the rail are assumed to be rigid. The string has Young's modulus E, diameter d, and the density per unit length ρ. The origin of coordinate is set at the top end of the string and the instantaneous length of string is $l(t)$ at time t. The instantaneous axial velocity, acceleration, and jerk of the string are $v(t) = \dot{l}(t)$, $a(t) = \dot{v}(t)$, and $j(t) = \dot{a}(t)$, respectively, where the overdot denotes time differentiation. At any instant t, the transverse displacement of the string is described by $y(x, t)$, at a spatial position x, where $0 \le x \le l(t)$. In an actual flexible hoisting system, the rotational unbalance of the traction motor or the abnormal off-track of the rope possibly causes vibration of the hoisting system. To reproduce this phenomenon, a transverse extrinsic disturbing excitation $e(t)$ is applied at the upper end of the string. In this paper, all the equations and derivations based on the following assumptions.

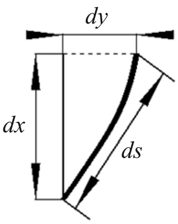


Fig. 2. A small element of the string in a deformed position.

1. Young's modulus E, diameter d, and density ρ of the string are always constants;

2. Only transverse vibration is considered here. The elastic distortion of the string arousing from the transverse vibration is much less than the length of the string;

3. The bending stiffness of the string, all the damp and friction, and the influence of air current are ignored.

2.1. Energy of Flexible Hoisting System. After the string is deformed, the position vector R of a point at x can be written as:

$$\boldsymbol{R} = x(t)i + y(x,t)j, \tag{1}$$

where i and j are the unit vectors along the x- and y-axes, respectively. The material derivative of R yields the velocity vector

$$\boldsymbol{V} = v(t)i + [y_t + vy_x]j, \tag{2}$$

where the subscript t denotes partial differentiation with respect to time, and the subscript x denotes partial differentiation with respect to space coordinate. Similarly, the position vector R_c and velocity vector V_c of the rigid body can be respectively written as:

$$\boldsymbol{R}_c = l(t)i + y(l(t),t)j, \tag{3}$$

$$\boldsymbol{V}_c = v(t)i + y_t(l(t),t)j. \tag{4}$$

Then, the kinetic energy of the flexible hoisting system is computed by

$$E_k(t) = \frac{1}{2}m\boldsymbol{V}_c \cdot \boldsymbol{V}_c\bigg|_{x=l(t)} + \frac{1}{2}\rho\int_0^{l(t)} \boldsymbol{V}\cdot\boldsymbol{V}dx. \tag{5}$$

The first term on the right-hand side of Eq. (5) represents the kinetic energy of the rigid body; the second term represents the kinetic energy of the string. The elastic strain energy of the string is

$$E_e(t) = \int_0^{l(t)} \left(P\varepsilon + \frac{1}{2}EA\varepsilon^2\right)dx, \tag{6}$$

where $P(x, t)$ is the quasi-static tension at spatial position x of the string at time t due to gravity. Since the string is acted upon by not only the weight of the concentrated mass at the lowest end, but also its own weight, the tension $P(x, t)$ is expressed as

$$P = [m + \rho(l(t) - x)]g. \tag{7}$$

And ε represents the strain measure at spatial position x of the string and can be expressed as

$$\varepsilon = (ds - dx)/dx. \tag{8}$$

As shown in Fig. 2, ds can be expressed as

$$ds \approx \sqrt{1 + (dy/dx)^2}\,dx \approx \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 - \frac{1}{8}\left(\frac{\partial y}{\partial x}\right)^4 + \ldots\right]dx \approx \left[1 + \frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right]dx. \tag{9}$$

Substituting Eq. (9) into Eq. (8) yields

$$\varepsilon = \frac{1}{2} y_x^2. \tag{10}$$

2.2. Free vibration equations. According to the characteristics of the top restriction of the string, the boundary conditions at $x(t) = 0$ are

$$y(0,t) = 0, \qquad y_t(0,t) = 0. \tag{11}$$

On substitution of Eqs (5) and (6) into Hamilton's principle,

$$\int_{t_1}^{t_2} (\delta E_\text{k}(t) - \delta E_\text{e}(t))dt = 0 \tag{12}$$

and apply the variational operation. Because the length $l(t)$ of the string changes with time, the standard procedure for integration by parts with respect to the temporal variable cannot apply. Applying Leibnitz's rule and part integration results in the following expressions:

$$\int_0^{l(t)} \rho(y_t + vy_x)\delta y_t dx = \rho \frac{\partial}{\partial t} \int_0^{l(t)} (y_t + vy_x)\delta y dx - \rho\left[v(y_t + vy_x)\delta y\right]_{l(t)} - \rho \int_0^{l(t)} \frac{\partial}{\partial t}(y_t + vy_x)\delta y dx. \tag{13}$$

Following the standard procedure for integration by parts with respect to the spatial variable and invoking Eq. (13), one obtains from Eq. (12),

$$-\int_{t_1}^{t_2} \left[m\frac{\partial}{\partial t} y_t(l,t) + Py_x + \frac{1}{2} EAy_x^3 \right] \delta y(l,t) dt$$

$$-\int_{t_1}^{t_2} \int_0^{l(t)} \left[\rho \frac{\partial}{\partial t}(y_t + vy_x) + \rho v \frac{\partial}{\partial x}(y_t + vy_x) - \frac{\partial}{\partial x}(Py_x) - EA\frac{\partial}{\partial x}\left(\frac{1}{2} y_x^3\right)\right] \delta y dx dt = 0. \tag{14}$$

Setting the coefficients of δw in Eq. (14) to zero yields the governing equations

$$\rho(y_{tt} + 2vy_{xt} + \dot{v}y_x + v^2 y_{xx}) - P_x y_x - Py_{xx} - \frac{3}{2} EAy_x^2 y_{xx} = 0, \quad 0 < x < l(t). \tag{15}$$

The first four terms in Eq. (15) correspond to the local, Coriolis, tangential, and centripetal acceleration, respectively. The resulting boundary conditions from Eq. (14) at $x = l(t)$ is

$$my_{tt} + Py_x + \frac{1}{2} EAy_x^3 = 0, \quad x = l(t). \tag{16}$$

The energy associated with the transverse vibration of the system is

$$E_v(t) = \frac{1}{2} my_t^2(l,t) + \frac{1}{2}\rho \int_0^{l(t)} [y_t(x,t) + vy_x(x,t)]^2 dx + \frac{1}{2}\int_0^{l(t)} \left(Py_x^2 + \frac{1}{4} EAy_x^4 \right) dx. \tag{17}$$

2.3. Forced Vibration Equations. When external excitation occurs at the upper end of the string, Eq. (15) must be adjusted. Compared with Eq. (12), the corresponding boundary conditions are changed into

$$y(0,t) = e(t), \quad y(l,t) = 0. \tag{18}$$

Obviously, the boundary conditions are non-homogeneous and difficult to be applied directly. Here, the procedure described in [5] is used to transform Eq. (15) with non-homogeneous boundary conditions into an equation of motion with homogeneous boundary conditions. The transverse displacement is expressed in the form

$$y(x,t) = w(x,t) + h(x,t), \tag{19}$$

where $w(x, t)$ is the part that satisfies the homogeneous boundary conditions and $h(x, t)$ is the part that does not satisfy the homogeneous boundary conditions. Substituting Eq. (19) into Eq. (15) yields

$$\rho(w_{tt} + 2vw_{xt} + \dot{v}w_x + v^2 w_{xx}) - P_x w_x - Pw_{xx} - \frac{3}{2}EAw_x^2 w_{xx} + \rho(h_{tt} + 2vh_{xt} + \dot{v}h_x + v^2 h_{xx}),$$

$$-P_x h_x - Ph_{xx} - EA(3w_x w_{xx} h_x + \frac{3}{2}w_{xx}h_x^2 + \frac{3}{2}w_x^2 h_{xx} + 3w_x h_x h_{xx} + \frac{3}{2}h_x^2 h_{xx}) = 0, \quad 0 < x\, l(t), \tag{20}$$

where $w(x, t)$ is the state variable. Equation (20) describes the transverse vibration of the flexible hoisting system under extrinsic disturbing excitation. The corresponding boundary condition is

$$mw_{tt} + Pw_x + \frac{1}{2}EAw_x^3 + mh_{tt} + Ph_x + \frac{1}{2}EA(3w_x^2 h_x + 3w_x h_x^2 + h_x^3) = 0, \quad x = l(t). \tag{21}$$

Set the function $h(x, t)$ to a first-order polynomial:

$$h(x,t) = a_0(t) + a_1(t)\frac{x}{l(t)}. \tag{22}$$

Then, when $x(t) = 0$ and $x(t) = l(t)$,

$$h(0,t) = e(t), \qquad h(l(t),t) = 0. \tag{23}$$

Substituting Eq. (23) into Eq. (22), we can obtain the coefficients $a_0(t)$ and $a_1(t)$:

$$a_0(t) = e(t), \qquad a_1(t) = -e(t). \tag{24}$$

Therefore,

$$h(x,t) = e(t) - e(t)\frac{x}{l(t)}. \tag{25}$$

Once $h(x, t)$ is known, the solutions for $w(x, t)$ can found from Eq. (20); $y(x, t)$ is obtained subsequently from Eq. (19). Equation (20) is a partial differential equation that describes the dynamics of the flexible hoisting string. The equation is defined over a time-dependent spatial domain rendering the problem non-stationary. Hence, the exact solution to this problem is not available, and recourse must be made to an approximate analysis. In what follows, numerical techniques are employed to obtain an approximate solution to the governing equation.

3. Discretization of the Governing Equation. Equation (20) is a partial differential equation with infinite dimensions, and many parameters are time-variant. It is impossible to obtain the exact analytical solution of Eq. (20). In this section, Galerkin's method is applied to truncate the infinite-dimensional partial differential equation into a nonlinear finite-dimensional ordinary differential equation with time-variant coefficients. Then, then they are solved with numerical methods. In order to map Eq. (20) onto a fixed domain, a new independent variable $\zeta = x/[l(t)]$ is introduced and the time-variant domain $[0, l(t)]$ for x is converted to a fixed domain [0, 1] for ζ. According to the characteristic of a taut translating string, the solution of the transverse vibration $w(x, t)$ is assumed to have the following form [11, 12]:

$$w(x,t) = \sum_{i=1}^{n} \varphi_i(\zeta) q_i(t) = \sum_{i=1}^{n} \varphi_i(x/l) q_i(t), \tag{26}$$

where $q_i(t)$ $(i = 1, 2, 3, \ldots, n)$ are the generalized coordinates with respect to $w(x, t)$; n is the number of included modes; $\varphi_i(\zeta)$ is a trial function [11, 12],

$$\varphi_i(\zeta) = \sqrt{2}\sin i\pi\zeta. \tag{27}$$

Consequently, expansion (26) results in expressions for partial derivatives of the transverse displacement function:

$$w_x(x,t) = \frac{1}{l}\sum_{i=1}^{n}\varphi_i'(\zeta)q_i(t), \qquad w_{xx}(x,t) = \frac{1}{l^2}\sum_{i=1}^{n}\varphi_i''(\zeta)q_i(t),$$

$$w_{xt}(x,t) = \sum_{i=1}^{n}\left[\frac{1}{l}\varphi_i'(\zeta)\dot{q}_i(t) - \frac{\zeta v}{l^2}\varphi_i''(\zeta)q_i(t) - \frac{v}{l^2}\varphi_i'(\zeta)q_i(t)\right],$$

$$w_{tt}(x,t) = \sum_{i=1}^{n}\varphi_i(\zeta)\ddot{q}_i(t) - \frac{2\zeta v}{l}\sum_{i=1}^{n}\varphi_i'(\zeta)\dot{q}_i(t)$$

$$+\left[\frac{2\zeta v^2}{l^2}\sum_{i=1}^{n}\varphi_i'(\zeta) - \frac{\zeta a}{l}\sum_{i=1}^{n}\varphi_i'(\zeta) + \frac{\zeta^2 v^2}{l^2}\sum_{i=1}^{n}\varphi_i''(\zeta)\right]q_i(t). \tag{28}$$

Substituting Eq. (28) into Eq. (20), multiplying the governing equation by $\varphi_j(\zeta)$ ($j = 1, 2, 3, \ldots, n$), integrating it from $\zeta = 0$ to 1, and using the boundary conditions and the orthonormality relation for $\varphi_i(\zeta)$ yield a discretized equation of transverse vibration for the flexible hoisting rope with time-variant coefficients:

$$\boldsymbol{M}\ddot{\boldsymbol{Q}} + \boldsymbol{C}\dot{\boldsymbol{Q}} + \boldsymbol{K}\boldsymbol{Q} + \boldsymbol{S}(\boldsymbol{Q}) = \boldsymbol{F}, \tag{29}$$

where $\boldsymbol{Q} = [q_1(t), q_2(t), \ldots, q_n(t)]^T$ is the vector of generalized coordinates; $\boldsymbol{M}$, $\boldsymbol{C}$, $\boldsymbol{K}$, and $\boldsymbol{F}$ are matrices of mass, damp, stiffness, and generalized force with respect to $\boldsymbol{Q}$, respectively; $\boldsymbol{S}(\boldsymbol{Q})$ is a higher-order term of generalized coordinate. The matrices are expressed as follows:

$$M_{ij} = \rho\delta_{ij}, \qquad C_{ij} = \int_0^1 \frac{2v}{l}(1-\zeta)\varphi_i'(\zeta)\varphi_j(\zeta)d\zeta,$$

$$K_{ij}(t) = \frac{\rho a}{l}\int_0^1 (1-\zeta)\varphi_i'(\zeta)\varphi_j(\zeta)d\zeta - \frac{\rho v^2}{l^2}\int_0^1 (1-\zeta)^2\varphi_i'(\zeta)\varphi_j'(\zeta)d\zeta$$

$$+\frac{\rho g}{l}\int_0^1 (1-\zeta)\varphi_i'(\zeta)\varphi_j'(\zeta)d\zeta - \frac{mg}{l^2}\int_0^1 \varphi_i''(\zeta)\varphi_j(\zeta)d\zeta - \frac{3EA}{2l^4}e^2(t)\int_0^1 \varphi_i''(\zeta)\varphi_j(\zeta)d\zeta,$$

$$S_j(\boldsymbol{Q}) = -\frac{3EA}{2l^4}\int_0^1\left(\sum_{i=1}^{n}\varphi_i'(\zeta)q_i(t)\right)^2\sum_{i=1}^{n}\varphi_i''(\zeta)q_i(t)\varphi\varphi_j(\zeta)d\zeta$$

$$-\frac{3EA}{l^4}e(t)\int_0^1 \sum_{i=1}^{n}\varphi_i'(\zeta)q_i(t)\sum_{i=1}^{n}\varphi_i''(\zeta)q_i(t)\varphi_j(\zeta)d\zeta,$$

$$\boldsymbol{F}_j = -\rho\left[\ddot{e}(t) + \frac{2v}{l}\dot{e}(t) + \frac{a}{l}e(t) - \frac{2v^2}{l^2}e(t)\right]\int_0^1 (1-\zeta)\varphi_j(\zeta)d\zeta - \frac{\rho g}{l}e(t)\int_0^1 \varphi_j(\zeta)d\zeta, \tag{30}$$

where the superscript " $'$ " denotes partial differentiation with respect to normalized variable ζ; δ_{ij} is the Kronecker delta defined by $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$ ($i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, n$). Solving the ordinary differential equation (29) with numerical methods may yield the instantaneous values of Q. Substituting these values into Eq. (26) may yield the instantaneous values of

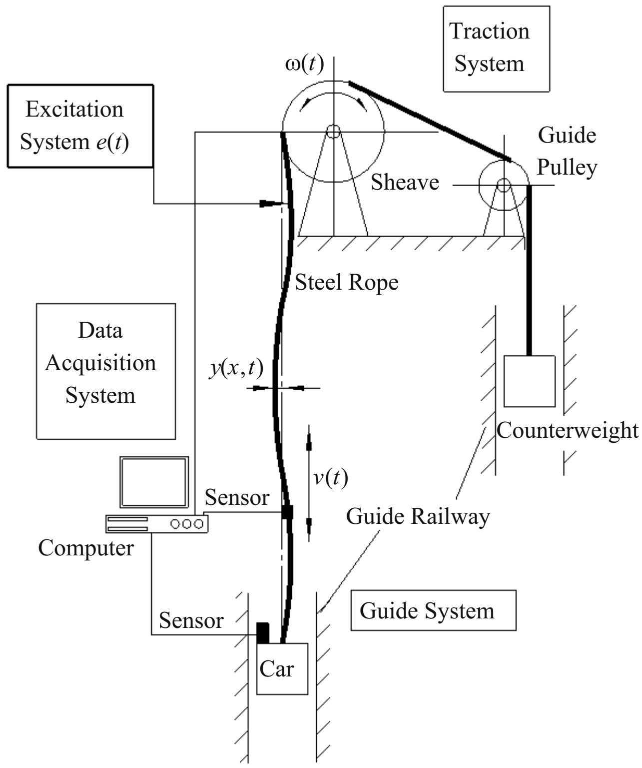


Fig. 3. Schematic diagram of experimental set-up for flexible hoisting system.

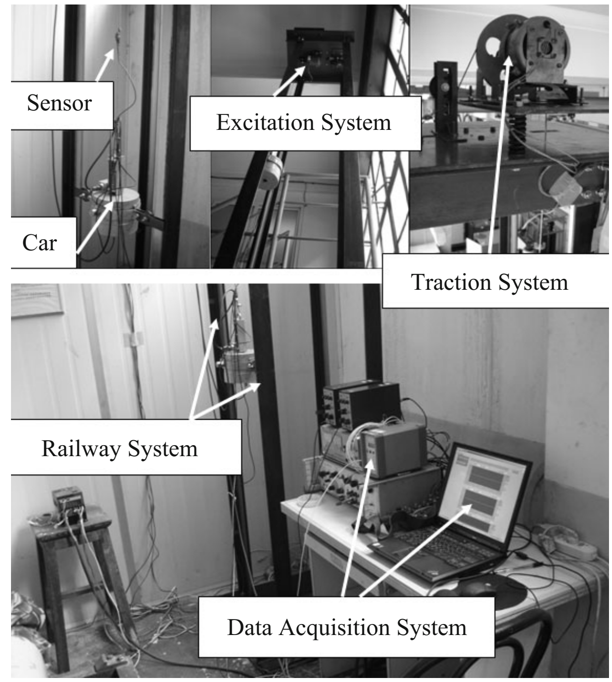


Fig. 4. Actual picture of the experimental set-up for flexible hoisting system.

the transverse vibration $w(x, t)$ of the string. The mathematical model defined by Eq. (29) illustrates the true dynamic nature of the flexible hoisting string, and can be used to predict and analyze the dynamic characteristics of a flexible hoisting string.

4. Experiment.

4.1. Experiment Set-up. To validate the mathematical model, an experimental set-up of flexible hoisting system was designed and built (Fig. 3). The set-up simulating the hoisting system of a traction elevator consists of a traction system, a guide system, an excitation system, and a data acquisition system. A frequency conversion motor is applied in the flexible hoisting system. The rotation speed of the motor may be controlled by adjusting the output of the transducer to obtain the anticipant motion curve of the hoisting system. A thin steel rope with a diameter of 3.2 mm is chosen as the hoisting rope. The model car and counterweight are made up of many weights. The mass of the car and counterweight is changeable by adding or reducing the number of weights.

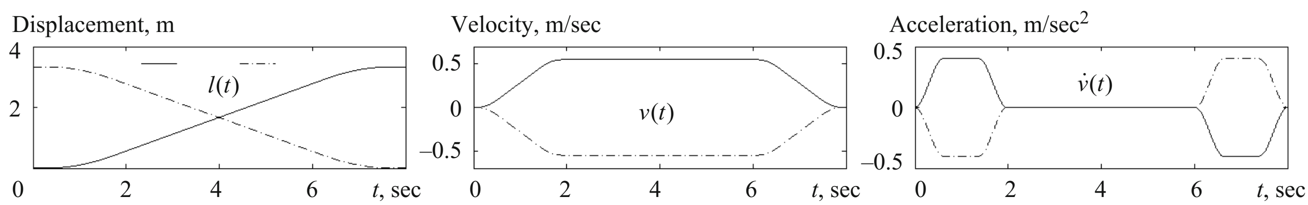
The hoisting rope at the car side is the main research object whose dynamic behavior will be studied in this set-up. To simulate the extrinsic disturbing excitation in an actual hoisting system, a transverse vibration exciter is applied at the top of the objective rope. The output of the exciter is decided by an adjustable signal generator. A micro-sensor with a mass of 4 g is attached at a certain position of the objective rope to acquire the transverse vibration acceleration of the rope. The signals from the micro-sensor are transmitted to a computer and saved. Figure 4 gives the actual picture of the experimental set-up. The main parameters of test are collected in Table 1.

4.2. Experiment Procedure. Now the transverse vibration of the flexible hoisting system will be calculated with a theoretical equation and tested with the experimental set-up, respectively. The results will then be compared. All the parameters used in the calculation and the test are the same. A downward or upward movement of the car is prescribed to be the input of the theoretical equations and experimental set-up. At the beginning, the car starts up at the top of the flexible hoisting system and goes down. When arriving at the bottom, the car pauses for a moment and turns back to the start. Figure 5 gives the prescribed displacement, velocity, acceleration curves of the flexible hoisting system, where the processes of acceleration, deceleration, and uniform speed downwards and upwards are included.

In an actual flexible hoisting system, the turning disbalance of the rotor or the abnormal off-track of the rope is the main origin of extrinsic disturbing excitation for the flexible hoisting system. It is supposed that a transverse extrinsic disturbing

TABLE 1. Parameters of experimental set-up of flexible hoisting system

Items	?	[ ]	Data values
Density per unit length	ρ	kg/m	0.042
Young's modulus	E	N/m^2	1×10^{12}
Rope diameter	d	m	3.2×10^{3}
Hoisting mass	m	kg	15
Excitation	$e(t)$	m	$5\times10^{4}\sin(\pi t)$
Minimum length of the string	$l_{min}(t)$	m	0.8
Maximum length of the string	$l_{max}(t)$	m	4.8
Maximum velocity	v_{max}	m/sec	0.55
Maximum acceleration	a_{max}	m/sec^2	0.4
Total travel time	t	sec	8
Number of transverse modes	n		4



a *b* *c*

Fig. 5. Movement profile of flexible hoisting system.

excitation $e(t)$ is applied at the top end of the flexible hoisting system to reproduce this phenomenon. The extrinsic disturbing excitation disturbs the dynamic behavior of the flexible hoisting rope only when the rope is moving. It is applied to the theoretical equations and experimental set-up.

During following calculation, the number n of included modes in $w(x, t)$ is set to 4, which was already proved to be a proper value with a great deal calculation results and comparisons. When $n = 4$, less calculating time and satisfying veracity of results may be simultaneously obtained.

4.3. Free Vibration Responses. The numerical simulations with the exact experiment parameters are conducted in order to compare with the experiments, and the experimental results are favorably compared with the simulations, which can be seen in Figs. 6 (downward movement) and 7 (upward movement). Comparing the results of test and calculation in Figs. 6 and 7 shows that the extent and trend of the vibration curves are similar. Therefore, it can be concluded that the theoretical equations, proposed in this paper, may be used to evaluate the vibration of a flexible hoisting rope.

Figure 6 displays reducing vibration amplitudes with increasing length of the rope during downward movement. This is because the energy of the flexible hoisting system transfers from the transverse vibration to the axial motion by bringing some mass into the domain of effective length, i.e., the axially hoisting rope is dissipative during downward movement, thus leading to a stabilized transverse dynamic response, as shown in Fig. 8*a*. A possible physical interpretation of the result is as follows: during downward movement, a negative external work is required to maintain the prescribed axial motion, which, in turn, brings

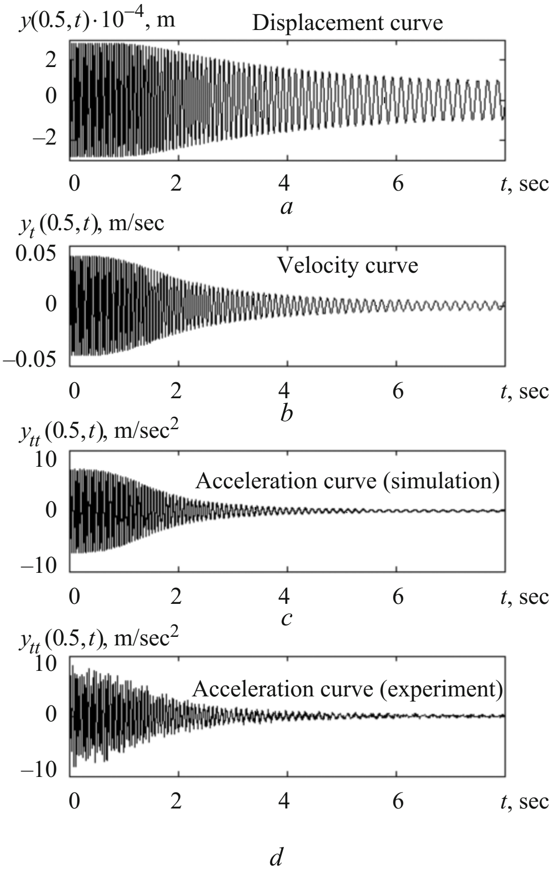


Fig. 6. Free vibration responses of the flexible hoisting rope at 0.5 m above the car during downward movement.

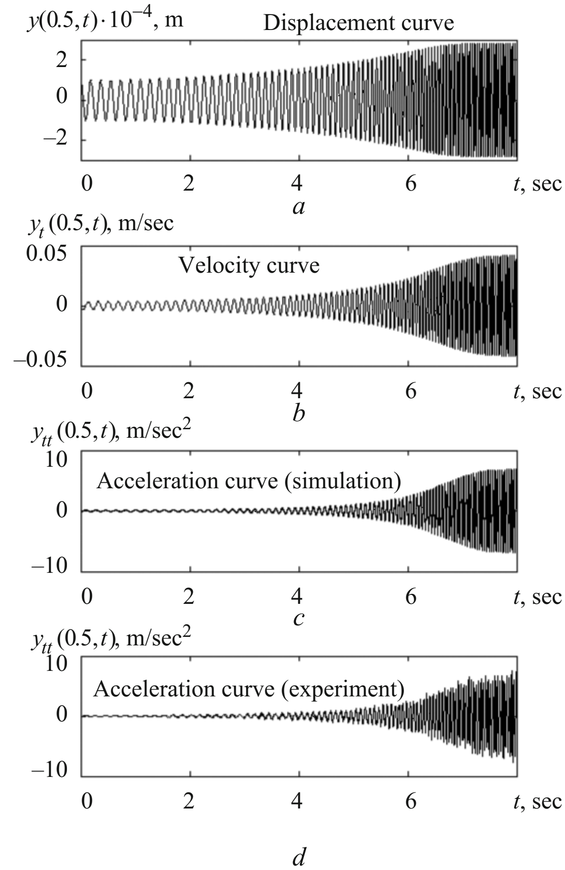


Fig. 7. Free vibration responses of the flexible hoisting rope at 0.5 m above the car during upward movement.

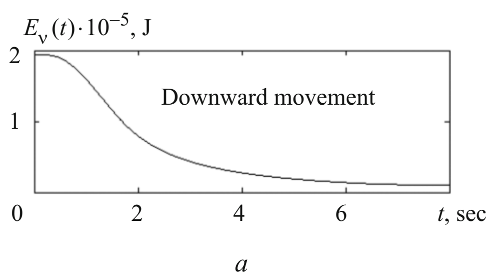


Fig. 8. Total energy associated with the transverse vibration of the system during movement.

about a convection of mass in the domain of effective length. At the same time, the frequencies of transverse vibration reduce with the increasing length of the rope. This is because the mass of the rope increases and the stiffness of the rope decreases, i.e., the rope becomes somewhat "softer."

By contrast, in Fig. 7, we observe that the vibration amplitudes of the rope increase with decreasing length of the rope during upward movement. This is because the energy of the flexible hoisting system transfers from the axial motion to the transverse vibration by leaving some mass out of the domain of effective length, i.e., the axially hoisting rope gains energy during upward movement, thus leading to an unstabilized transverse dynamic response, as shown in Fig. 8*b*.

A possible physical interpretation of the result is as follows: during upward movement, a positive external work is required to maintain the prescribed axial motion, which, in turn, brings about a convection of mass out of the domain of effective length. In the mean time, the frequencies of the transverse vibration increase with decreasing length of the rope. This is because the mass of the rope decreases and the stiffness of the rope increases, i.e., the rope becomes somewhat "stiffer."

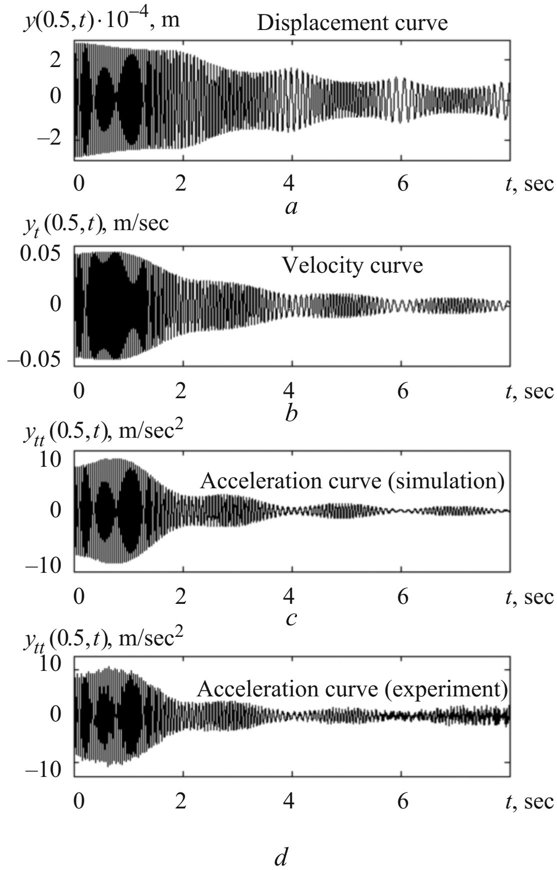


Fig. 9. Forced vibration responses of the flexible hoisting rope at 0.5 m above the car during downward movement.

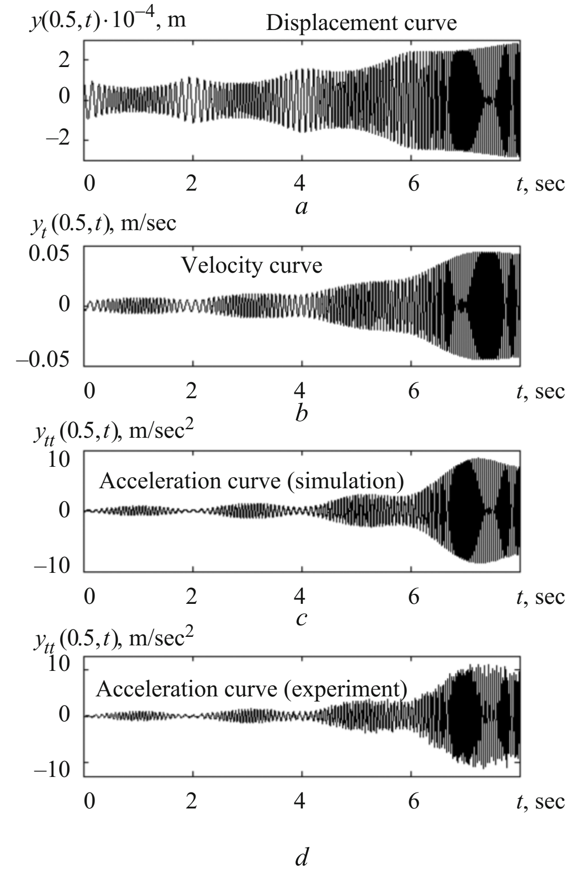


Fig. 10. Forced vibration responses of the flexible hoisting rope at 0.5 m above the car during downward movement.

4.4. Forced Vibration Responses. During movement of the hoisting system, the flexible hoisting system is subjected to vibration caused by various sources of excitation. They include excitations due to the irregularities of the guiding system and the rotational unbalance of the traction motor as well as environmental phenomena such as air current.

The system parameters are changing due to the time-varying length of the rope. The rate of variation of the length is, however, low, and the vibrations represent waves in a slowly varying domain. Hence, the hoisting rope is essentially a nonstationary vibration system with slowly varying frequencies. Therefore, a passage through resonance may occur when one of the slowly varying frequencies coincides with the frequency of the extrinsic disturbing excitation at some critical time instant.

Forced vibration responses for the hoisting rope with extrinsic disturbing excitation are illustrated in Figs. 9 (downward movement) and 10 (upward movement). From Figs. 9 and 10, it can be seen that a transient resonance occurs during movement of the hoisting system. The amplitudes exhibit oscillatory behavior before the resonance, and near the resonance the amplitudes increase rapidly and decline afterwards due to damping, developing damped beat phenomena. This is because one of the time-varying frequencies of the hoisting rope coincides with the frequency of the extrinsic disturbing excitation during movement of the hoisting system. It should be noted that the adverse dynamic response in the hoisting system promotes large oscillations in rope tension. The phenomenon cannot be ignored, as the high amplitude in the tension contributes directly to fatigue of the rope. Fatigue often results in hoisting ropes being discarded after lower working cycles. Therefore, suitable strategy can be sought to minimize the effects of adverse dynamic response of the system.

5. Conclusions. The nonlinear dynamic characteristics for a flexible hoist rope with time-varying length considering the coupling of axial movement and flexural deformation are analyzed in this paper. The flexible hoisting system is modeled as an axially moving string with time-varying length and a rigid body at its lower end. The governing equations are derived by using Leibnitz's rule and Hamilton's principle. Galerkin's method is used to truncate the infinite-dimensional partial differential equations into a set of nonlinear finite-dimensional ordinary differential equations with time-variant coefficients.

To validate the theoretical model, an experimental set-up of flexible hoisting system is built and some experiments are performed. By comparing the experimental results to the numerical simulation, a good agreement between the simulation and experiment is obtained, thus validating the mathematical model of flexible hoisting system. Based on the simulation and experiment, the following conclusions can be obtained:

1. A flexible hoisting rope with time-varying length experiences instability during upward movement, the natural frequencies are increasing because of the reducing mass and the increasing stiffness of the rope, and the energy transforms from the axial movement into the flexible deformation.

By contrast, it is stable during downward movement, the natural frequencies are decreasing because of the increasing mass and the reducing stiffness of the rope, and the energy coverts from the flexible deformation into the axial movement.

2. The flexible hoisting rope is a nonstationary oscillatory system with slowly varying frequencies. The transient resonance may occur when one of the time-varying frequencies of the hoisting rope coincides with the frequency of the extrinsic disturbing excitation.

3. The proposed theoretical model and analyses of the dynamic characteristics of the flexible hoisting system in this paper will be helpful for researchers to comprehend its dynamic behavior and develop a proper method to suppress the vibration in practice.

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