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# THERMOVISCOELASTOPLASTIC DEFORMATION OF COMPOUND SHELLS OF REVOLUTION MADE OF A DAMAGEABLE MATERIAL

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**A technique for numerical analysis of the thermoviscoelastoplastic deformation of thin compound shells made of a damageable material in which a fracture front propagates is described. A procedure for automatic variation in the step of integration of the kinetic damage equation is developed. A two-layer cylindrical shell cooling by convection and subjected to internal pressure and tensile force is analyzed as an example. The numerical data are presented and analyzed**

**Keywords:** thermoviscoelastoplasticity, shell of revolution, fracture front

**Introduction.** The power engineering industry widely uses structures that are subjected to thermomechanical loading. Damage accumulation in and failure of such structures were studied in [1–3, 6–28, etc.]. While in service, structural members undergo elastic, plastic, and creep deformation. Inelastic deformation causes defects such as micropores and microcracks in the material, which may develop and cause failure of the structure. While the failure of a structure caused by plastic deformation occurs instantaneously and depends only on the external load, the damage of the material caused by creep develops with time, even under a constant load. In the process, the fracture front propagates, separating the material that has been fractured from the material that has not.

The idea of studying the damage front was set forth by Kachanov in [6] where the normal velocity of the fracture front was determined by integrating an equation for the damage (continuity) parameter. This idea was used in [7, 8, 15, etc.] to analytically solve problems for solids of canonical shapes: cylinders, disks, plates. This approach cannot be used for numerical solution of boundary-value problems for complex geometries because the stage of fracture propagation starts at discrete points, which substantially complicates the identification of the direction of the normal to the fracture front. Therefore, to identify the fracture front during the numerical solution of the problem in [1, 2, 7, 8, 16, 19, etc.], the elements of the body in which the damage parameter reached the critical value $\omega^*$ were excluded from consideration. Such an approach is naturally consistent with how a microcrack occurs in the body. Therefore, we will use this approach below to study the propagation of the fracture front in compound shells of revolution that cool by convection and undergo thermoviscoplastic deformation. The strength of the compound shell will be assessed using well-known failure criteria.

**1. Problem Formulation.** Consider a thin shell of revolution made by joining elements with differently shaped meridian. Within one element, the shell consists of $N$ isotropic layers of meridionally variable thickness and temperature-dependent material properties. Assume that the layers have been laid up without tension and do not separate and slip relative to each other. The position of an arbitrary point of the shell is defined by curvilinear orthogonal coordinates $s, \theta, \varsigma$, where $s$ ($s_a \le s \le s_b$) is the meridional coordinate, $\theta$ is the circumferential coordinate, and $\varsigma$ ($\varsigma_0 \le \varsigma \le \varsigma_N$) is the normal (to the coordinate surface $\varsigma = 0$) coordinate. The coordinates $\varsigma_0$ and $\varsigma_N$ correspond to the inside and outside surfaces of the shell, and the coordinates $\varsigma_k$ ($k = 1, \ldots, N-1$) to the interface between adjacent layers. The shell is assumed thin so that $\varsigma k_s$ and $\varsigma k_\theta$ (here $k_s$ and $k_\theta$ are the principal curvatures of the coordinate surface) can be neglected compared with unity. We also assume that the stresses $\sigma_{\varsigma\varsigma}$ can be neglected compared with the other normal stresses.

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Let the shell, being undeformed and at temperature $T = T_0$, begin to be subjected, at some reference time $t = 0$, to heating by convection and loading that both cause an axisymmetric stress–strain state (SSS), torsion, and plastic and creep strains. We will use a geometrically linear quasistatic problem formulation and the straight-line hypotheses [5]. The loading process is divided into short steps such that the loading history and the loading rate are described in the best possible way. At the *k*th step of loading, the problem of thermoviscoplasticity will be solved with the method of successive approximations. A meridional section of the shell is covered with a mesh. At its nodes, the temperature and the SSS are determined. To solve the problem, we will use the static and geometrical equations of the theory of thin layered shells. We will use the equations of deformation along paths of small curvature as constitutive equations. Such equations describing the thermoviscoelastoplastic deformation of shells of revolution made of a damageable material can be found in [3], where a procedure for solving the initial–boundary-value problem was outlined as well.

At constant temperature, the uniaxial creep rate is defined by the formula

$$\frac{d\varepsilon_{\mathrm{c}}}{dt} = \frac{Am\sigma_{\mathrm{e}}^{n} t^{m-1}}{(1-\omega)^{p}}, \tag{1.1}$$

where $\sigma_{\mathrm{e}}$ is the equivalent stress; $\omega$ is the damage parameter; $A, n, m, p$ are coefficients determined from experimental creep curves. Let $\omega = 0$ at $t = 0$, and $\omega = 1$ at the end $(t = t^{*})$ of the stage of latent fracture.

The parameter $\omega$ is a functional of the loading process [11]:

$$\omega = \omega[\sigma_{\mathrm{e}}(t), T(t), t] \tag{1.2}$$

and can be determined from Rabotnov's kinetic equation [10]:

$$\frac{d\omega}{dt} = C\left(\frac{\sigma_{\mathrm{e}}}{1-\omega}\right)^{Q}, \tag{1.3}$$

where $C$ and $Q$ are temperature-dependent coefficients.

It is assumed that creep damage is not accumulated in an element of the body when under compression $(\sigma_0 < 0)$. The equivalent stress can be found from a stress rupture criterion [11]:

$$\sigma_{\mathrm{e}} = \chi(\tau_{\mathrm{oct}} - \sigma_{\max}) + \sigma_{\max}, \tag{1.4}$$

where $\tau_{\mathrm{oct}}$ is the octahedral shear stress; $\sigma_{\max}$ is the maximum principal normal stress. The parameter $\chi$ is a function of the shear-stress intensity $S$ and temperature $T$. It is determined from the condition that the stress rupture curves for uniaxial tension and pure torsion coincide. If torsion test data are unavailable and the materials do not have rheological properties, the Sdobyrev criterion can be used to define the equivalent stress:

$$\sigma_{\mathrm{e}} = (S\sqrt{3} + \sigma_{\max})/2. \tag{1.5}$$

The parameter $\omega$ in each element of the body is determined during the solution of the boundary-value problem by numerically integrating Eq. (1.3). At the stage of latent fracture, the problem is solved up to the time point $t^{*}$ at which this parameter reaches the critical value $\omega^{*}$.

**2. Governing System of Equations.** In the chosen coordinate system, the constitutive equations relating stresses and strains have the form

$$\vec{\sigma} = [B]\vec{\varepsilon} - \vec{\sigma}^{(\mathrm{d})}, \qquad \vec{\sigma} = \{\sigma_{ss}, \sigma_{\theta\theta}, \sigma_{s\theta}, \sigma_{s\varsigma}, \sigma_{\theta\varsigma}\}^{T},$$

$$\vec{\varepsilon} = \{\varepsilon_{ss}, \varepsilon_{\theta\theta}, 2\varepsilon_{s\theta}, 2\varepsilon_{s\varsigma}, 2\varepsilon_{\theta\varsigma}\}^{T}, \qquad \vec{\sigma}^{(\mathrm{d})} = [B]\{\vec{\varepsilon}^{\,n} + (T - T_0)\vec{\alpha}_T\}, \tag{2.1}$$

where the superscript "T" denotes transposition; $[B]$, $\vec{\sigma}^{(\mathrm{d})}$, $\vec{\varepsilon}^{\,n}$, $\vec{\alpha}_T$ are, respectively, the stiffness matrix and the vectors of additional stresses, irreversible strains, and linear thermal expansion coefficients. The constitutive equations for forces and moments are presented in [3].

The system of static, kinematic, and constitutive equations allows us to reduce, at each step of loading, the thermoviscoplastic problem to a system of ordinary differential equations

$$\frac{d\vec{Y}}{ds} = P(s)\vec{Y} + \vec{f}(s) \tag{2.2}$$

for the unknown functions

$$\vec{Y} = \{\vec{N}, \vec{u}\}^T, \qquad \vec{N} = r\{N_s, N_{s\theta}, M_s, M_{s\theta}, Q_s\}^T, \qquad \vec{u} = \{u, v, \psi_s, \psi_\theta, w\}^T, \tag{2.3}$$

where $P(s)$ is the matrix of the system; $\vec{f}(s)$ is the vector of free terms; $N_s$, $Q_s$, and $M_s$ are the normal and transverse forces and bending moment acting in the section $s = \text{const}$; $N_{s\theta}$ and $M_{s\theta}$ are the shearing force and twisting moment acting in the same section; $u$ and $v$ are the displacements of particles of the coordinate surface in the $s$- and $\theta$-directions; $w$ is deflection; $\psi_s$ and $\psi_\theta$ are the complete angles of rotation of the straight element. The boundary conditions have the following brief form:

$$[G]\vec{Y} = \vec{g}, \tag{2.4}$$

where $[G]$ and $\vec{g}$ are the given matrix and vector of boundary conditions. In each approximation of a step of loading, the boundary-value problem (2.1)–(2.4) is reduced to Cauchy problems, which are integrated by the Runga–Kutta method in combination with Godunov's discrete orthogonalization.

**3. Problem-Solving Algorithm.** An algorithm for solving the thermoviscoplastic problem for a damageable material at the stage of latent fracture is outlined in [3]. The state of the shell is considered critical when the damage parameter takes on the critical value $\omega^*$ at, at least, one point of the shell. The value of $\omega$ at each point is found by numerical integration of the kinetic equation (1.3) over time using an explicit difference scheme.

Contrastingly, we will trace the propagation of the fracture front by excluding points of the shell at which the damage parameter has the critical value ($\omega \geq \omega^*$). To this end, we equate the elastic modulus at these points to zero. The strength of the shell can be assessed using criterion (1.4) or (1.5). The damage parameter can be determined using an implicit difference scheme. To this end, we represent the derivative in Eq. (1.3) as $(\omega - \tilde{\omega}) / \Delta t$, where $\omega = \omega(t + \Delta t)$, $\tilde{\omega} = \omega(t)$, $\Delta t$ is the duration of a step of loading. Let the right-hand side of Eq. (1.3) be referred to a time $t + \Delta t$. Then we obtain the following nonlinear equation for $\omega$:

$$\omega = \tilde{\omega} + \Delta t C \left( \frac{\sigma_e}{1 - \omega} \right)^Q. \tag{3.1}$$

In the $l$th approximation of the $m$th step of loading, this equation can be solved by fixed-point iteration:

$$\omega^{(k+1)} = \tilde{\omega} + \Delta t C \left( \frac{\sigma_e}{1 - \omega^{(k)}} \right)^Q \quad (k = 0, 1, 2, \ldots), \tag{3.2}$$

where $k$ is the iteration number. For definiteness, we will use the term "approximations" with reference to the solution of the physically nonlinear thermoviscoplastic problem and the term "iterations" with reference to formula (3.2). Let the value of $\omega$ found in the $(l-1)$th approximation be the starting iteration $\omega^{(0)}$ at the $l$th approximation. The iteration process is terminated once $|\omega^{(k+1)} - \omega^{(k)}| < \kappa\omega^{(k+1)}$, where $\kappa$ is a predefined small number.

The number of iterations needed for the iteration process to converge depends on the degree of nonlinearity. This circumstance allows us to formulate conditions for automatic change of the step of integration over time $\Delta t$: if at the $(m-1)$th step of loading, the maximum number $n$ of iterations over all elements of the shell appears less than some number $n_1$, then the step $\Delta t$ is doubled at the $m$th step. If at the $m$th step, the number $n$ of iterations appears greater than $n_2$, then it is necessary to halve the step $\Delta t$ and to repeat the calculation at the $(m-1)$th step. If at the $(m-1)$th step, the number of iterations $n_1 \leq n \leq n_2$, then the step is not changed at the $m$th step. The numbers $n_1$ and $n_2$ depend on the material properties and are found empirically. This procedure should necessarily be applied when the time point $t^*$ is approached and the parameter $\omega$ shows asymptotic behavior.

To promote convergence, we will also halve the step $\Delta t$ in the following cases:

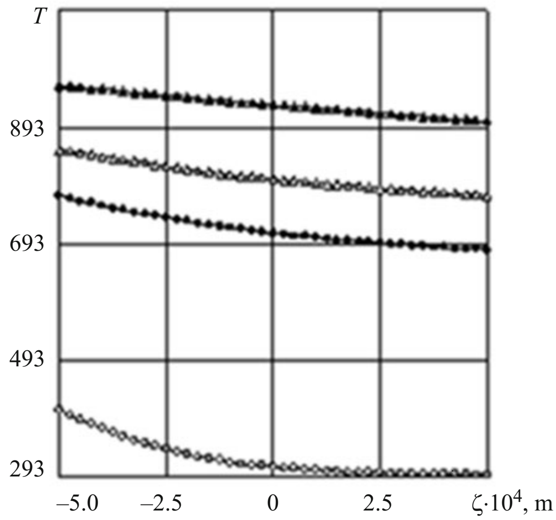

Fig. 1

TABLE 1

| $\varepsilon^*$, % | $\sigma$ at different temperatures $T$ | |
|---|---|---|
| | 273 | 2773 |
| 0 | 0 | 0 |
| 0.04 | 0.156 | 0.156 |
| 0.10 | 0.330 | 0.200 |
| 0.20 | 0.580 | 0.220 |
| 2.00 | 2.080 | 0.260 |

(a) if the number $N$ of successive approximations of the solution of the thermoviscoplastic problem at some step of loading stage exceeds a predefined number $N^*$;

(b) if the creep strain increment $\Delta\varepsilon_c$ over a step exceeds a predefined number $\Delta\varepsilon_c^*$;

(c) if the parameter $\omega$ exceeds a predefined critical value $\omega^*$.

If at some point of the shell, situation (c) occurs at two steps of loading in succession and $\omega^* \le \omega \le 1$, then we assume that the process of latent fracture is complete at this point. To ensure accuracy of the results, the step $\Delta t$ is bounded from above: $\Delta t \le \Delta t_{max}$.

If the loading process is nonisothermal, we need to know the distribution of temperature. The temperature field of a layered shell can be found by solving the nonstationary heat-conduction problem as described in [4].

**4. Example.** Let us analyze, as an example, the thermoviscoelastoplastic deformation of and fracture front propagation in an infinitely long two-layer cylindrical shell that cools by convection and is subjected to a surface load $q_\varsigma$, tensile force $N_s^*$. The radius of the mid-surface $R = 0.1$ and the thickness of the layers $h_1 = h_2 = 0.005$. Hereafter linear dimensions are measured in m, stresses in GPa, time in h, temperature in K, thermal diffusivity in m$^2$/h, thermal conductivity in W/(m·K), linear thermal expansion coefficient in K$^{-1}$, thermal transmittance in W/(m$^2$·K). The inside layer is made of a creep-resistant ceramic material (tantalum carbide). This material shows plastic behavior, but does not rheological behavior. The stress–strain curves of this

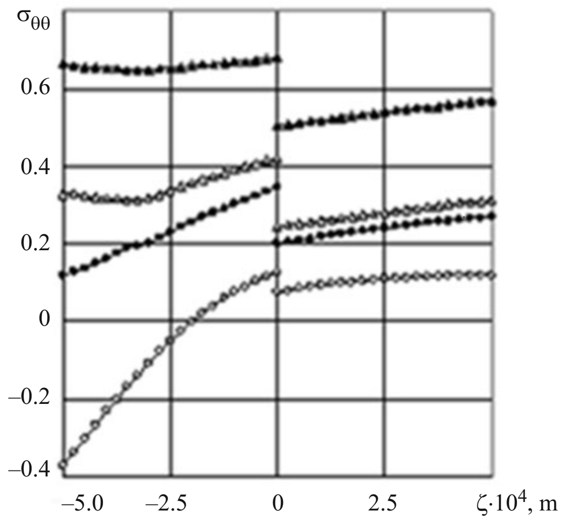


Fig. 2

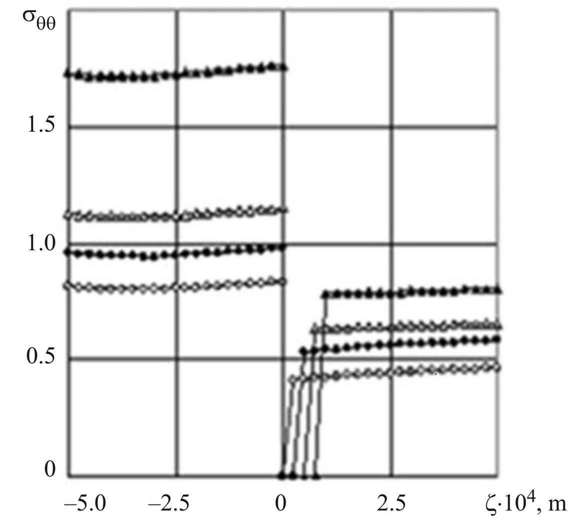


Fig. 3

material are given in Table 1. Poisson's ratio $\nu = 0.17$, linear thermal expansion coefficient $\alpha_T \cdot 10^6 = 12$, thermal diffusivity $a = 0.0158$, and thermal conductivity $\lambda = 19.5$. The outside layer is made of EI-437 alloy. Its mechanical properties are described in [3] where it is assumed that creep strains do not occur in the material in the temperature range from 293 to 573. Thermal characteristics of this material: $a = 0.031$, $\lambda = 22.2$.

During loading, $q_\varsigma$ and $N_s^*$ vary with time: $q_\varsigma = q_{\varsigma 0} f(t)$, $N_s^* = q_\varsigma R/2$, $f(t) = 1 - \exp(-50t)$, $q_{\varsigma 0} = 0.06$. The boundary conditions: $Q_s = 0$, $u = v = 0$, $\psi_s = \psi_\theta = 0$ for $s = s_a$ and $N_s = N_s^*$, $N_{\theta s} = 0$, $M_{\theta s} = 0$, $Q_s = 0$, $\psi_s = 0$ for $s = s_b$. The initial temperature of the shell $T_0 = 293$. Thermal transmittance $\alpha_1 = 1000$ and ambient temperature $\Theta_1 = 1100$ on the inside surface and $\alpha_2 = 200$ and $\Theta_2 = 293$ on the outside surface. The boundaries $s = s_a$ and $s = s_b$ are heat-insulated ($\alpha = 0$).

To solve the problem, we consider a section of the shell with directrix length $L = 0.0005$. The meridional section is covered with a uniform mesh consisting of 21 points throughout the thickness of each layer and three points along the circumference. Let $n_1 = 2$, $n_2 = 6$, $N^* = 100$, $\Delta\varepsilon_c^* = 0.0005$, $\omega^* = 0.6$, $\Delta t_{\max} = 1$. The initial time step $\Delta t_0 = 0.0001$, the error of the solutions of the thermoviscoplastic problem and the damage equation $\delta = \kappa = 0.001$. The above parameters ensure the convergence of the results.

The solution of the heat-conduction problem reveals that the temperature in the shell reaches its steady-state value at $t = 0.2$. The temperature on the inside, middle, and outside surfaces at this time point is equal to 965.1, 931.4, and 903.2, respectively. Figure 1 shows the distribution of temperature throughout the thickness of the shell at different time points. The curves are arranged in ascending order and correspond to times $t = 0.0005$, 0.01, 0.015, 0.2. Figure 2 shows the variation in the hoop stresses throughout the thickness of the shell at the same time points. It can be seen that, initially, high temperature gradients take place near the inside surface, giving rise to a plastic zone. This zone gradually grows and, beginning with $t = 0.0013$, it becomes an unloading zone. At $t = 0.002$, plastic strains also occur on the outside surface of the inside layer of the shell. Plastic strains in the outside layer occur first on the outside surface of the shell at $t = 0.0027$. Then the plastic zone gradually grows deep into the shell. At $t = 0.01$, plastic strains occur at each point throughout the thickness and the unloading zone near the inside surface of the shell starts contracting, disappearing at $t = 0.0165$. Creep strains occur first on the outside surface of the shell at $t = 0.0475$. Then the creep zone gradually grows deep into the outside layer, causing unloading in this layer. At the end of the stage of latent fracture, the plastic strains are maximum ($\Gamma_p^{\max} = 0.31\%$) at the point on the inside surface of the inside layer, and the creep strains are maximum ($\Gamma_c^{\max} = 0.24\%$) at the point on the inside surface of the outside layer. From this point, the fracture front begins to propagate at $t_1^* = 666.474$. As it advances deep into the outside layer, the cross-sectional area of the undamaged part of the shell decreases and the stresses in this part increase. The fracture front moves up until the time $t_2^* = 669.570$ at which fracture occurs at the point on the inside surface of the shell, according to the Sdobyrev criterion (1.5). At the time $t_2^*$, the damaged zone covers four points throughout the thickness of the outside layer.

The further analysis of the propagation of the fracture front is impossible because the calculated instantaneous strains $\varepsilon^*$ at the next step of loading exceed the maximum value given in the table.

The results indicate that the time of propagation of the fracture front is shorter than half a percent of the total time of loading. The kinetics of propagation of the fracture front is illustrated by Fig. 3. It shows the distribution of hoop stresses throughout the thickness of the shells at different times. The open circles, full circles, open triangles, and full triangles correspond to damaged zones that occur at one point, two points, three points, and four points, respectively. It should be noted that as the fracture front advances, the step of integration $\Delta t$ is considerably decreased (for example, $\Delta t = 0.15 \cdot 10^{-10}$ for $t = t_2^*$). This confirms the effectiveness of the above scheme of integration of Eq. (1.3).

For the other cases of load ($q_{\varsigma 0} = 0.05$ and $q_{\varsigma 0} = 0.07$), the fracture behavior appears the same. In the former case, $t_2^* =$ 12833.9, the time of propagation of the fracture front is 0.28% of the total time of loading, and creep fracture occurs at four points throughout the thickness. In the latter case, $t_2^* = 359.117$, the time of propagation of the fracture front is 0.54% of the total time, and fracture occurs at three points.

**Conclusions.** A technique for numerical analysis of the thermoviscoplastic state of compound thin shells of revolution in which a fracture front propagates has been proposed. The technique is based on well-known failure criteria. The results of analysis of a thin-walled two-layer shell subject to loads of different levels suggest that allowing for the second stage of fracture does not increase substantially the life of the shell, but makes it possible to identify the site of fracture more accurately.

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