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# INFLUENCE OF THE STRESS MODE ON THE STRENGTH OF HIGH-PRESSURE VESSELS

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**A numerical method for the analysis of the stress–strain state and strength of a thin-walled structural member subject to increasing internal pressure is proposed. It is based on the constitutive equations describing the elastoplastic deformation of isotropic materials along small-curvature paths and taking into account the stress mode, the theory of thin shells of revolution, failure criterion, and a method to solve a boundary-value problem of plasticity. Numerical values of the critical load are found**

**Keywords:** elastoplastic deformation, shells of revolution, failure criterion, critical load

**Introduction.** Thin-walled high-pressure vessels are structural members of aircraft, vehicles, etc. The stress-strain state (SSS) and strength of such structural members were studied in [9–11, 13, 16–19, etc.]. The strength of high-pressure vessels depends on many factors, including geometry, material properties, loading conditions, etc. The material properties are known to strongly depend on the stress mode in some cases, which is manifested, for example, as a difference between the tensile and compressive stress–strain curves [20–22].

Here we will assess the effect of the stress mode on the critical load for a specific thin-walled member of a critical structure. To this end, we will solve, step by step, the axisymmetric problem of plasticity for a thin isotropic shell modeling the structural member under consideration, with and without regard to the stress mode. We will use the constitutive equations describing deformation along small-curvature paths and either incorporating [1, 7, 14, 15] or disregarding [5, 23] the stress mode. The calculated quantities describing the SSS of the shell will be used to validate the failure criterion formulated in [4] and to determine the critical internal pressure.

Note that the papers [6, 8, 12] describe methods for and the results of solving the axisymmetric problem of plasticity for thin isotropic shells using the theory of deformation along small-curvature paths and taking into account the stress mode, but do not evaluate the strength of these shells. Contrastingly, we will determine, step by step, the elastoplastic stress–strain state of a shell, taking into account the stress mode, and will evaluate its strength.

**1. Problem Statement and Basic Equations.** Consider a shell of revolution that is initially in stress/strain-free state at constant temperature $T = T_0 = 20$ °C and then is subjected to increasing uniform internal pressure. The shell is made of an isotropic material with properties dependent on the stress mode. The meridian of the shell can consist of a finite number of segments of different geometry. The shell is described in a curvilinear orthogonal coordinate system $s, \theta, \varsigma$ fixed to an undeformed continuous coordinate surface, where $s\,(s_a \le s \le s_b)$ is the meridional coordinate; $s_a$ and $s_b$ are the coordinates of the shell ends; $\theta\,(0 \le \theta \le 2\pi)$ is the circumferential coordinate; $\varsigma\,(\varsigma_0 \le \varsigma \le \varsigma_k)$ is the normal coordinate to the coordinate surface; $h = \varsigma_k - \varsigma_0$ is the thickness of the shell; $\varsigma_0$ corresponds to the inside surface of the shell and $\varsigma_k$ to the outside surface.

The coordinate surface is chosen to be the midsurface or one of the surfaces of the shell. The geometry of the coordinate meridian is completely described by functions $r(s)$ and $\varphi(s)$, where $r$ is the parallel radius; $\varphi$ is the angle between the normal to the coordinate surface and the negative $z$-axis. It is assumed that the load causes the shell to deform within and beyond the elastic range and the creep strains are negligible compared with the elastic and plastic strains. We will use a quasistatic problem

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formulation, the Kirchhoff–Love hypotheses, and the geometrically linear theory of shells [3]. To describe the deformation of isotropic materials, we will use the theory of deformation along small-curvature paths [7] that takes into account the stress mode. To characterize the stress mode, we will use the stress mode angle [2], which is expressed in terms of the second and third deviatoric stress invariants. This angle $\omega_\sigma$ indicates how the octahedral shear stress is oriented in the octahedral plane with respect to the negative projection (onto this plane) of the principal axis along which the minimum normal stress acts. In the general case, the constitutive equations of plasticity include two nonlinear functions that depend on the angle $\omega_\sigma$ and can be determined experimentally. One of the functions relates the first invariant of the stress tensor $\sigma_0$ and the first invariant of the strain tensor $\varepsilon_0$. The other function relates the second invariant of the stress deviator $S$ and the second invariant of the strain deviator $\Gamma$. These functions are individualized in reference tests on tubular specimens under proportional loading at different constant values of the angle $\omega_\sigma$. If the relationship between the first invariants of the stress and strain tensors is linear and the relationship between the second invariants of the respective deviators is independent of the stress mode, the constitutive equations [7, 14, 15] pass into the standard equations describing deformation along small-curvature paths [5]. Here we will use the version [7] of the constitutive equations [14, 15] where the relationship between the second invariants of the stress and strain tensors depends on the stress mode, while the relationship between the first invariants of the these tensors is linear.

To solve the problem, we divide the loading period into intervals (steps) with endpoints as close as possible to the onsets of possible unloading. At the $M$th step, we will use the differential equilibrium equations [3] for an element of an axisymmetrically loaded (without torsion) shell of revolution and the kinematic [3] and constitutive [7] equations. At an arbitrary point of the shell, these equations can be represented as a relationship between the stresses $\sigma_{ss}, \sigma_{\theta\theta}$ and the strains $\varepsilon_{ss}, \varepsilon_{\theta\theta}, \varepsilon_{\varsigma\varsigma}$ in the form of Hooke's law with additional stresses:

$$\sigma_{ss} = A_{11}\varepsilon_{ss} + A_{12}\varepsilon_{\theta\theta} - A_{1D}, \quad \sigma_{\theta\theta} = A_{12}\varepsilon_{ss} + A_{22}\varepsilon_{\theta\theta} - A_{2D}, \tag{1}$$

$$\varepsilon_{\varsigma\varsigma} = -\frac{\nu}{1-\nu}\left(\varepsilon_{ss} + \varepsilon_{\theta\theta}\right) - \frac{1-2\nu}{1-\nu}\left(e_{ss}^{(\mathrm{p})} + e_{\theta\theta}^{(\mathrm{p})}\right)$$

$$\left[A_{11} = A_{22} = \frac{E}{1-\nu^2}, \quad A_{12} = \nu A_{11}, \quad A_{1D} = A_{11}\left(e_{ss}^{(\mathrm{p})} + \nu e_{\theta\theta}^{(\mathrm{p})}\right), \quad A_{2D} = A_{11}\left(e_{\theta\theta}^{(\mathrm{p})} + \nu e_{ss}^{(\mathrm{p})}\right)\right], \tag{2}$$

where $E$ and $\nu$ are Young's modulus and Poisson's ratio of the material; $e_{ss}^{(\mathrm{p})}$ and $e_{\theta\theta}^{(\mathrm{p})}$ are the plastic components of the strain deviator. Thus, the first invariants of the stress and strain tensors are related by $\sigma_0 = K\varepsilon_0$, where $\sigma_0 = (\sigma_{ss} + \sigma_{\theta\theta})/3$, $\varepsilon_0 = (\varepsilon_{ss} + \varepsilon_{\theta\theta} + \varepsilon_{\varsigma\varsigma})/3$, $K = E/(1-2\nu)$ is the dilatation modulus, $G = E/2(1+\nu)$ is the shear modulus.

Then

$$e_{ss}^{(\mathrm{p})} = \varepsilon_{ss}^{(\mathrm{p})}, \quad e_{\theta\theta}^{(\mathrm{p})} = \varepsilon_{\theta\theta}^{(\mathrm{p})}, \quad e_{\varsigma\varsigma}^{(\mathrm{p})} = \varepsilon_{\varsigma\varsigma}^{(\mathrm{p})} = -(\varepsilon_{ss}^{(\mathrm{p})} + \varepsilon_{\theta\theta}^{(\mathrm{p})}).$$

The plastic strain components are found as the sum of increments of these components:

$$\varepsilon_{ss}^{(\mathrm{p})} = \sum_{m=1}^{M} \Delta_m \varepsilon_{ss}^{(\mathrm{p})}, \quad \Delta_m \varepsilon_{ss}^{(\mathrm{p})} = \langle c_{ss}\rangle_m \Delta_m \Gamma_{\mathrm{p}}^*, \quad \langle c_{ss}\rangle_m = \left\langle \frac{2\sigma_{ss} - \sigma_{\theta\theta}}{S} \right\rangle_m \quad (s, \theta), \tag{3}$$

where the angular brackets denote averaging over a step; $\Delta_M \Gamma_{\mathrm{p}}^*$ is the increment of plastic shear strain intensity $\Gamma_{\mathrm{p}}^*$,

$$\Gamma_{\mathrm{p}}^* = \sum_{m=1}^{M-1} \Delta_m \Gamma_{\mathrm{p}}^* + \Delta_M \Gamma_{\mathrm{p}}^*, \tag{4}$$

$S$ is the shear-stress intensity,

$$S = \left[\frac{1}{3}\left(\sigma_{ss}^2 - \sigma_{ss}\sigma_{\theta\theta} + \sigma_{\theta\theta}^2\right)\right]^{1/2}. \tag{5}$$

To determine $\Delta_k \Gamma_p^*$, we assume that the shear-stress intensity $S$, shear-strain intensity $\Gamma$, and the stress mode angle $\omega_\sigma$ are related as follows:

$$S = \Phi(\Gamma, \omega_\sigma), \tag{6}$$

where

$$\omega_\sigma = \frac{1}{3} \arccos\left[-\frac{3\sqrt{3}}{2} \frac{I_3(D_\sigma)}{S^3}\right] \quad \left(0 \le \omega_\sigma \le \frac{\pi}{3}\right), \tag{7}$$

$I_3(D_\sigma) = |s_{ij}|$ is the third invariant of the stress deviator $D_\sigma$, $s_{ij} = \sigma_{ij} - \sigma_0 \delta_{ij}$ are the components of the stress deviator [5].

To individualize function (6), we will use the following procedure from [7, 14, 15]: data of tests on proportionally loaded tubular specimens are used to set up the expression for the function $S(\Gamma)$ for several constant values of the angle $0 \le \omega_\sigma \le \pi/3$ and it is assuming that $\Gamma = S/2G + \Gamma_p^* = \Gamma_e + \Gamma_p^*$, $\Gamma_e$ is the elastic component of the shear-strain intensity. With (6), the increment $\Delta_M \Gamma_p^*$ over a step of loading is calculated by the method of successive approximations.

We use Eqs. (1) to derive the following relations among the forces $N_s$, $N_\theta$, moments $M_s, M_\theta$, strains $\varepsilon_s, \varepsilon_\theta$, and curvature changes $\kappa_s, \kappa_\theta$ of the coordinate surface:

$$N_s = C_{11}^{(0)} \varepsilon_s + C_{12}^{(0)} \varepsilon_\theta + C_{11}^{(1)} \kappa_s + C_{12}^{(1)} \kappa_\theta - N_{1D}^{(0)},$$

$$N_\theta = C_{12}^{(0)} \varepsilon_s + C_{22}^{(0)} \varepsilon_\theta + C_{12}^{(1)} \kappa_s + C_{22}^{(1)} \kappa_\theta - N_{2D}^{(0)},$$

$$M_s = C_{11}^{(1)} \varepsilon_s + C_{12}^{(1)} \varepsilon_\theta + C_{11}^{(2)} \kappa_s + C_{12}^{(2)} \kappa_\theta - N_{1D}^{(1)},$$

$$M_\theta = C_{12}^{(1)} \varepsilon_s + C_{22}^{(1)} \varepsilon_\theta + C_{12}^{(2)} \kappa_s + C_{22}^{(2)} \kappa_\theta - N_{2D}^{(1)} \tag{8}$$

$$\left[C_{mn}^{(j)} = \int_{\varsigma_0}^{\varsigma_k} A_{mn} \varsigma^j \, d\varsigma, \quad N_{mD}^{(j)} = \int_{\varsigma_0}^{\varsigma_k} A_{mD} \varsigma^j \, d\varsigma \quad (m, n = 1,2;\ j = 0,1,2)\right]. \tag{9}$$

Equations (8) together with the equilibrium and kinematic equations constitute a system of 12 equations. Let us reduce this system to a system of six ordinary differential equations for the unknown functions $N_s, Q_s, M_s, u, w, \vartheta_s$, where $Q_s$ is the shearing force; $u$ and $w$ are the meridional and normal displacements of particles of the coordinate surface; $\vartheta_s$ is the angle of rotation of the normal to the coordinate surface:

$$\frac{d\vec{Y}}{ds} = P(s)\vec{Y} + \vec{f}(s). \tag{10}$$

The boundary conditions are

$$B_1 \vec{Y}(s_a) = \vec{b}_1, \qquad B_2 \vec{Y}(s_b) = \vec{b}_2, \tag{11}$$

where $\vec{Y} = \{N_s, Q_s, M_s, u, w, \vartheta_s\}$ is the column vector of unknown functions; $P(s)$ is the matrix of the system; $\vec{f}(s)$ is the column vector of additional terms; $B_1$ and $B_2$ are given matrices; $\vec{b}_1$ and $\vec{b}_2$ are given column vectors of boundary conditions. The nonzero elements of $P(s)$ and $\vec{f}(s)$ are calculated by formulas from [5]. These formulas indicate that the elements of the matrix of the governing system depend on the geometry of the shell and the elastic properties of the material, while the components of the vector $\vec{f}(s)$ additionally depend on the external loads and plastic strains, which should be corrected during successive approximations.

The above formulas can be used to determine the SSS of the shell with allowance for the stress mode. The same formulas can be used when the stress mode is neglected. In that case, Eq. (6) is independent of the stress mode and is individualized based on simple tension test data.

Let us use the quantities describing the SSS of the shell to validate the chosen failure criterion with the purpose of determining the critical pressure. The load at which the following failure criterion is satisfied:

$$\sigma_e = \sigma_b, \tag{12}$$

where $\sigma_e$ is the equivalent stress; $\sigma_b$ is the ultimate strength of the material, is the critical load at which failure occurs. The equivalent stress is defined by Sdobyrev's formula [5]:

$$\sigma_e = \left(\sigma_i + \sigma_{max}\right)/2, \tag{13}$$

where $\sigma_{max}$ is the maximum principal normal stress; $\sigma_i$ is the stress intensity,

$$\sigma_i = S\sqrt{3}. \tag{14}$$

In the general case, the principal normal stresses are determined using formulas from [5]. In an axisymmetrically loaded thin shell, these stresses are $\sigma_{ss}$ and $\sigma_{\theta\theta}$, i.e., $\sigma_{max} = \max(\sigma_{ss}, \sigma_{\theta\theta})$ in (13).

**2. Problem-Solving Algorithm.** To determine the SSS of the shell with allowance for the stress mode, it is necessary to prescribe its geometry, boundary and loading conditions, and material properties. These material properties are characterized by Poisson's ratio and functions (6) specified in reference tests for $\omega_\sigma = 0, \pi/6, \pi/3$, indicating the values of $S_b = \sigma_b/\sqrt{3}$ for the same values of the stress mode angle. It is convenient that the partition into steps be done so that the shell deforms elastically at the first step. In the first approximation of the first step of loading, set $e_{ss}^{(p)} = e_{\theta\theta}^{(p)} = 0$ for each element of the shell and calculate the elements of $P(s)$ and $\vec{f}(s)$. Next, reduce the boundary-value problem (10), (11) to Cauchy's problems and use the Runge–Kutta and discrete-orthogonalization methods to solve them. The error of the solution is estimated, following Runge's rule, by halving the mesh of the partition. Use the calculated values of the unknown functions to find first the strain components and then the stress components (1) to determine the SSS of the shell at the first step. The process of successive approximations at the second or the *M*th stage is based on the SSS found at the previous $(M-1)$th step and the values of $(e_{ss}^{(p)})_{M-1}$, $(e_{\theta\theta}^{(p)})_{M-1}$, $(\Gamma_p^*)_{M-1}$. Next, calculate the shear-stress intensity (5) and the stress mode angle (7). By linear interpolation of functions (6) in the angle $\omega_\sigma$, identify the appropriate curve to calculate $S^{(d)}$ corresponding to the shear-strain intensity $\Gamma = (\Gamma_p^*)_{M-1} + S/2G$, where $S$ is defined by (5). Then $\Delta_M \Gamma_p^* = (S - S^{(d)})/2G$. Use this value to calculate the increments of the plastic strains (3). Then calculate the values of $A_{1D}$, $A_{2D}$ (2) and solve the boundary-value problem (10), (11) in the subsequent approximation. Generally, the increment of the plastic shear strain intensity in the *L*th approximation at the *M*th step is calculated by the formulas

$$\Delta_M \Gamma_p^* = \sum_{i=1}^{L-1} \Delta_{Mi} \Gamma_p^* + \Delta_{ML} \Gamma_p^*, \qquad \Delta_{ML} \Gamma_p^* = \frac{S - S^{(d)}}{2G}, \tag{15}$$

where $S^{(d)}$ is determined from (6) using the value of $\Gamma_p^*$ (4) found in the previous approximation. The process of successive approximations at a step is terminated once

$$\left|\Delta_{ML} \Gamma_p^*\right| \le \delta, \tag{16}$$

where $\delta$ is the predefined error of the solution of the plastic problem.

The above algorithm is applied only if the process is active loading. To identify whether the process is active loading or unloading in each element of the shell where plastic deformation occurs $(\Gamma_p^* > 0)$ at the current step of loading, it is necessary to test the condition $\Delta\Gamma_p^* > 0$ after finding the first approximation. If this condition is satisfied, then the process is active loading and

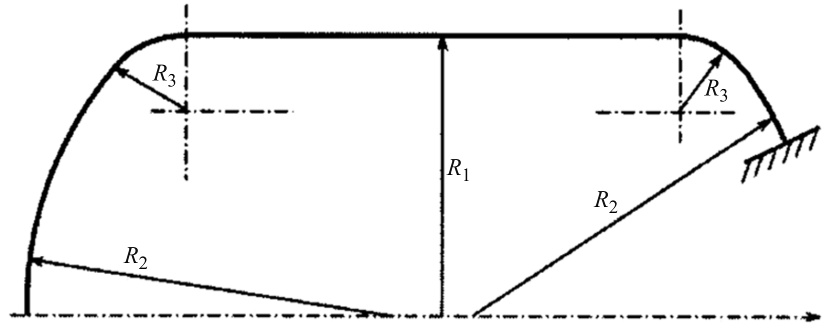


Fig. 1

the above algorithm is used. Otherwise, the process in the current element of the shell is unloading; then we should set $\Delta\Gamma_{\rm p}^{*} = 0$ for this element and use the values of the plastic strains corresponding to the end of the previous step.

To determine the critical load at each step of loading, it is necessary to test the failure criterion (12), (13). The computation is terminated once this criterion has been satisfied.

Whether the steps of loading are properly chosen is checked by halving the step and repeating computation. The steps are reduced until the values of the SSS obtained at the end of the process with different number of steps differ by less than a specified small amount.

**3. Numerical Results.** Let us determine the critical load for a shell modeling a motor housing subject to increasing internal pressure. The geometry of the shell is characterized by data on the meridian of its coordinate surface and the thickness. The inside surface of the shell is chosen to be the coordinate one. The meridian consists of the following concatenated segments: one spherical, one toroidal, four cylindrical, one toroidal, one spherical.

The following dimensions are indicated in Fig. 1: $R_1 = 6.63$, $R_2 = 8.5$, $R_3 = 2.5$ (hereinafter, linear dimensions are measured in centimeters, and angles in radians). The values of the meridional coordinate $s_i$ $(i = 1, \ldots, 8)$ at the end of the $i$th segment and the values of the angle $\varphi_i$ at the beginning of the segment are the following: $s_1 = 6.453106$ $(s_0 = s_a = 0)$, $s_2 = 8.482124$, $s_3 = 8.882124$, $s_4 = 9.482124$, $s_5 = 52.682124$, $s_6 = 53.82124$, $s_7 = 55.811141$, $s_8 = s_b = 57.163593$, $\varphi_1 = 0$, $\varphi_2 = 0.759189$, $\varphi_3 = \varphi_4 = \varphi_5 = \varphi_6 = \varphi_7 = \pi/2$, $\varphi_8 = 2.3824033$.

The thickness of the shell is constant and equal to 0.3 within segments 1 and 2, varies linearly from 0.3 to 0.15 along segment 3, and is constant and equal to 0.15 within segments 4–8.

The boundary conditions are: $u = 0, \vartheta_s = 0, Q_s = 0$ for $s_a = s_0$; $u = 0, w = 0, \vartheta_s = 0$ for $s_b = s_8$.

The shell is made of Kh18N10T alloy for which functions (6) for $\omega_\sigma = 0, \pi/6, \pi/3$ are given in [7, 14, 15]. The limiting values of the shear-stress intensity for the above values of the stress mode angles are equal to 324, 272, and 326 (MPa), respectively; Poisson's ratio $\nu = 0.27$.

The shell is subject to uniform internal pressure $q_\varsigma$ increasing from 4 MPa. The loading period is divided into steps so that the step size decreases as $S_{\rm b} = \sigma_{\rm b}/\sqrt{3}$ is approached. The computations demonstrate that plastic deformation occurs first within segment 7 of the shell under $q_\varsigma$ = 7 MPa and then, as the load is increased, within all the other segments, except for the vicinity of the pole of segment 1. The loading process is active, i.e., no unloading occurs. Initially, while the deformation is elastic, the meridional stresses that arise in the toroidal segment near the end $s = s_b$ are maximum, yet not critical. With further increase in the load and development of plastic deformation, the circumferential stresses in the cylindrical segment of the shell become maximum. This means that the shell will fail longitudinally. The chosen failure criterion indicates that failure will occur in the cylindrical segment of the housing on the outside surface of the shell in the neighborhood of the point $s = 51.7$ cm, where $\omega_\sigma = 0.42$. The corresponding breaking stress $\sigma_{\rm b} = 485$ MPa is reached at $q_\varsigma = 11.5$ MPa.

When the stress mode is disregarded (the function $S = F(\Gamma)$ is specified using simple tension test data $(\omega_\sigma = \pi/3)$), failure occurs in the same place, but the breaking stress $\sigma_{\rm b} = 565$ MPa when the critical pressure $q_\varsigma = 13.5$ MPa, which exceeds the value obtained with allowance for the stress mode by 17%.

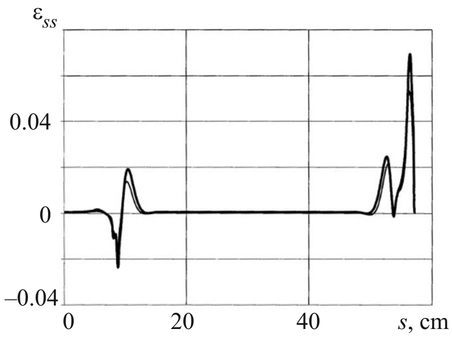


Fig. 2

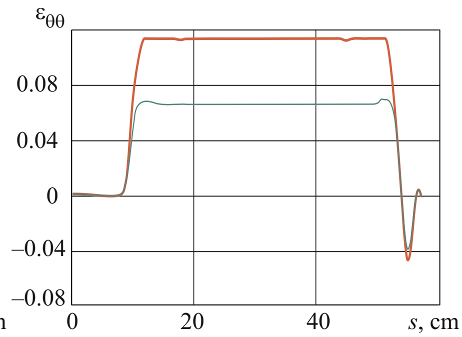


Fig. 3

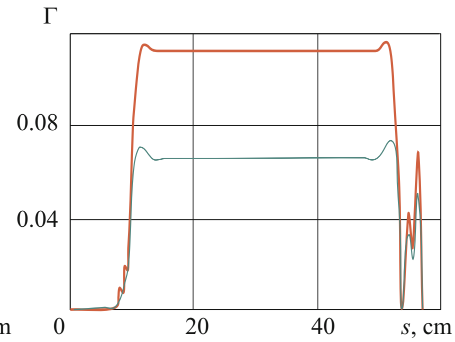


Fig. 3

The breaking stresses calculated with and without regard to the stress mode differ a little, whereas the strains differ substantially. The maximum values of the shear-strain intensity within the cylindrical segment of the shell calculated with and without regard to the stress mode differ by more than 60%.

Figures 2–4 demonstrate how the meridional (Fig. 2) and circumferential (Fig. 3) strains and the shear-strain intensity (Fig. 4) calculated with (heavy lines) and without (thin lines) regard to the stress mode vary along the meridian $s$ (in centimeters) of a shell with $\varsigma = h$.

**Conclusions.** A numerical method for determining the critical load for thin-walled structural members in the form of shells of revolution subject to increasing pressure has been proposed. The method is based on the geometrically linear theory of thin shells, the theory of deformation along small-curvature paths, and a failure criterion and takes into account the stress mode. For a shell made of Kh18N10T alloy as an example, it has been shown that the shear-strain intensities calculated with and without regard to the stress mode differ by 60%, while the values of the critical pressure differ by 17%.

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