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EXPERIMENTAL STUDY OF THE VIBRATIONS OF A COMPOSITE CYLINDRICAL SHELL WITH A FILLER SUBJECT TO TWO-FREQUENCY EXCITATION

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Experimental results on the nonlinear dynamic deformation of the elastic wall of a glassfiber-reinforced plastic cylindrical shell (either "dry" or filled) during beating caused by kinematic two-frequency loading are discussed. It is revealed that the nonlinear deformation of a shell undergoing beating, especially at the two close frequencies of the modes $n = 3$ and $n = 5$, can be accompanied by the alteration of amplitude and deformation mode between one mode $n = 3$ and combined mode $(n = 3) + (n = 5)$ and the alternation of one mode $n = 3$ between a traveling wave and a standing wave

Keywords: glass-reinforced plastic cylindrical shell, filler, coupled bending modes, two-frequency excitation

Introduction. The deformation of elastic cylindrical shells is strongly dependent on major factors (external periodic loading) and other factors. For example, the presence of a filler (fluid or loose material) in thin-walled cylindrical shells may result in compound multimode or multiwave dynamic deformation under certain conditions (free vibraions or external periodic loads). Many theoretical and experimental studies [2, 3, 6–9, 12–14] address the deformation of shell structures and nonlinear and resonant phenomena caused by the imposition and nonlinear interaction of several flexural vibration modes, which create preconditions for the occurrence of complex deformation modes (such as traveling circumferential waves, chaotic processes, etc.) under single-frequency excitation. It was established that even the mode shape a shell takes under purely harmonic loads can affect its dynamic instability domains (DID) [4]. For example, the principal DID of a cylindrical shell is located lower on the frequency axis and is considerably wider than the DID of a shell with alternating curvature. Such a situation is typical for composite shells [1, 8]. Therefore, experimental studies of the vibratory and wave processes in composite shell structures interacting with a fluid are of current importance. As indicated in [5], experimental methods not only give a true picture of the behavior of mechanical structures under varying loads, but also make it possible to identify the limits of validity of theoretical models. This fact was confirmed by holographic interferometry studies of the natural frequencies and modes of isotropic circular cylindrical shells.

An important task of solid mechanics is to study the nonlinear vibrations (with large deflections) of thin-walled laminated shells under combined vibratory loading. When in service, real shell structures with fluid used in aircraft and rocket technology, chemical engineering, etc. are subjected to combined vibratory loading of various types. The dynamic behavior of shells filled with a fluid is more intensive under combined two-frequency vibratory loading [10, 11] than under single-frequency loading [8, 12].

Here we will discuss test data on the nonlinear dynamic deformation of a glassfiber-reinforced plastic shell (empty or filled) subjected to longitudinal kinematic two-frequency vibrational excitation. Our primary task is to establish and analyze the relationship between the two excitation frequencies and the natural frequencies of the shell and the amplitude of the kinematic loading that cause the most intensive deformation of the shell and to analyze the effect of the filler and the way the excitation frequency varies on these processes.

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TABLE 1

g_e	f_e, →←	f_e, Hz					$2A_{max}$
		F_{st}^1 ($n = 3$)	ΔF_{st}^1	F_{tr}	F_{st}^2 ($n = 3$)	ΔF_{st}^2	
5	→	79.0–79.8	0.8	—	81.8–82.1	0.3	4
	←	77.0–79.1	2.1	—	79.1–81.2	2.4	6
10	→	78.2–79.8	1.6	—	79.8–82.7	2.9	10
	←	75.2–78.9	3.7	79.0–79.3	79.4–82.8	3.4	12
15	→	77.9–79.5	1.6	—	79.5–84.0	4.5	11
	←	74.9–78.5	3.6	78.6–79.1	79.2–84.0	4.8	13
20	→	77.2–79.0	1.8	79.1–79.3	79.4–84.0	4.6	15
	←	74.2–77.8	3.6	77.9–79.0	79.1–84.0	4.9	17

1. Test Specimen, Equipment, and Procedure. The test specimen was an elastic composite cylindrical sandwich shell with length H_{sh} = 900 mm, inside diameter D_{sh} = 320 mm, and wall thickness Δ_{sh} = 0.68 mm. A VEDS-100 electrodynamic shaker was used for longitudinal kinematic excitation. The shell was fixed on the shaker table vertically, with its upper end free and lower end inserted into a ring groove in a disk filled with epoxy resin to provide a clamped boundary condition after its curing. To produce two-frequency vibrational excitation, we used a generator built in the frame of the shaker and an external Robotron generator (one of them was used for single-frequency excitation). The vibroaccelerations of the shaker table and the shell wall were measured with IS-318 and D-14 transducers operating with the measuring unit of the shaker and an AD-1 microtransducer (with a mass of about 1 g) with a VShV-3 device. The signals (amplitudes, frequencies) from the transducers were analyzed and measured with a Bruel & Kjaer type 2031 low-frequency analyzer and V3-56 millivoltmeter. A measured wedge [1, 11] was used to measure the amplitude of the vibrating end of the shell in the antinode zone.

The shell with mass m_{sh} = 0.9 kg was filled with plastic-foam balls 15–20 mm in diameter. The mass of this filler was less than the mass of the water to maintain the integrity of the thin-walled shell.

The test shell either empty (H_f = 0) or filled to H_f = (0.3; 0.5)Nsh was subjected to vibrational excitation with frequencies Δf_e = 50–200 Hz and excitation amplitude g_e = 2–20g_0.

To assess the effect of two-frequency excitation on the deformation of the shell, either with or without filler, the shell was first subjected to single-frequency kinematic excitation. The earlier tests [1, 8, 11] showed that the single-frequency excitation of thin-walled composite shells (mainly dry) causes maximum deformation amplitudes at resonant frequencies, especially when traveling circumferential waves are generated. The dynamic instability domains in which the deformation of the shell is the most intensive at lower amplitudes of external excitation and the frequency ranges in which the deformation of the shell is a traveling wave were identified. Also the amplitudes g_e of external vibrational excitation at which such processes occur and the effect of the filler level H_f on them were assessed. What types of two-frequency excitation should be used were identified based on the information on the nonlinear processes under single-frequency axial parametric vibrational excitation.

2. Analysis of Experimental Data. ***2.1. Deformation of the Elastic Wall of a Shell under Single-Frequency Kinematic Excitation.*** Before studying the deformation of a cylindrical shell, the boundaries of the DID in which parametrical vibrations were excited were identified. Such domains for various glassfiber-reinforced plastic shells subject to external loading of relatively low amplitude were determined in [1, 8].

TABLE 2

g_e	f_e, →←	f_e, Hz		$2A_{max}$, mm
		F^1_{st} ($n = 5$)	ΔF^1_{st}	
10	→	85.0–85.6	0.6	2.5
	←	85.0–85.6	0.6	2.5
15	→	84.8–85.9	1.1	4
	←	84.2–85.9	1.7	5
20	→	84.7–86.0	1.3	6
	←	84.1–86.0	1.9	7

The experiments revealed that two circumferential modes ($n = 3$ and $n = 5$) are excited in the shell. Circumferential vibration modes of the shell wall were excited by kinematic loading of several amplitudes: $g_e = (5, 10, 15, 20)g_0$. These loads excited the circumferential mode $n = 3$ of the "dry" shell. The shell lost stability twice because of the effect of the initial imperfections at parametrical resonance: (i) in the fundamental flexural mode at the lowest natural frequency and (ii) in the conjugate mode. Experiments showed that at certain levels of external excitation g_e and filler H_f and, especially, depending on the direction of scanning the excitation frequency f_e, two DIDs overlap in this mode ($n = 3$), which causes the shell to lose stability in two conjugate modes simultaneously. The superposition and interaction of these modes result in a deformation wave traveling in the circumferential direction [1, 8, 11]. The vibrations of the "dry" shell with $n = 5$ are excited when $g_e = (10, 15, 20)g_0$. When this mode was excited, the shell lost stability twice, but only in one circumferential mode, i.e., there was no traveling wave.

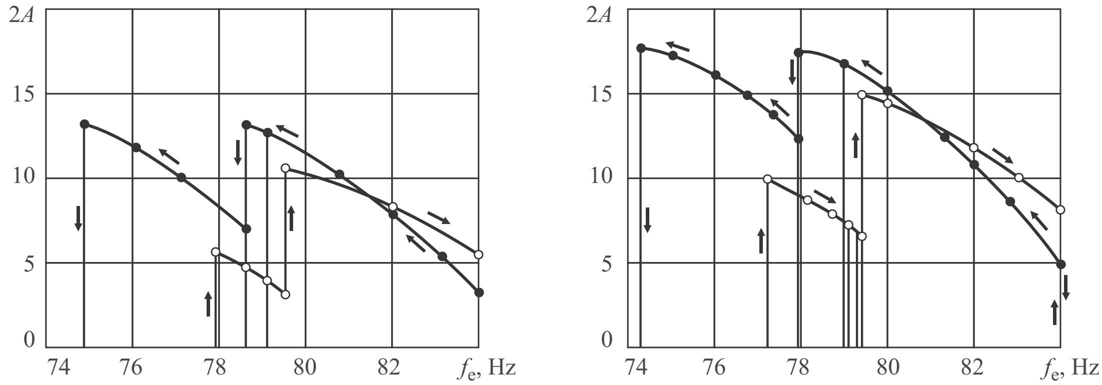
Studies showed that, due to the hysteresis effect [1, 8], the position and width of the domains in which parametrical vibrations are excited depend on the direction of scanning the excitation frequency f_e and the amplitude g_e.

Tables 1 and 2 summarize the frequency ranges for standing and traveling waves and the maximum amplitudes $2A$ of vibrations of the dry shell at the antinodes of the modes $n = 3$ and $n = 5$, respectively, for increasing (→) and decreasing (←) excitation frequency f_e. The DIDs were detected at amplitudes g_e of axial kinematic excitation indicated above.

As is seen from Tables 1 and 2, the radial vibrations of the free end of the shell with maximum amplitude $2A$ occur at $n = 3$.

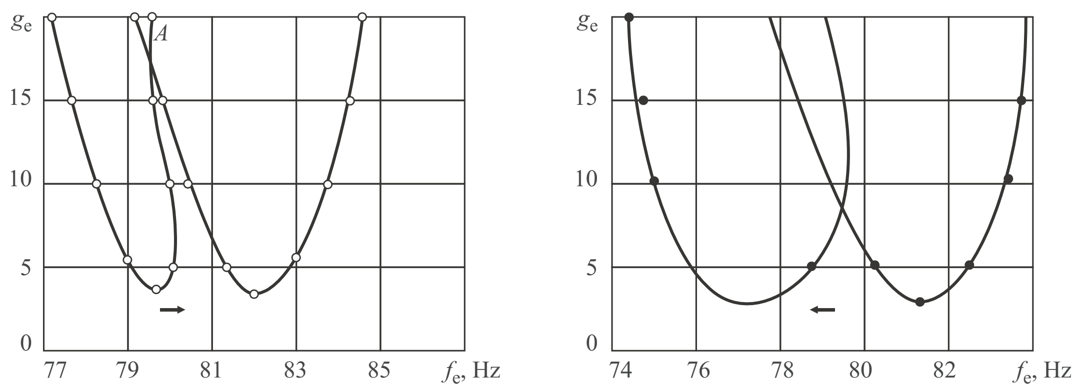
Figure 1 shows the amplitude–frequency response (AFR) of the shell undergoing parametrical vibrations in mode $n = 3$ for $g_e = 15g_0$ (Fig. 1*a*) and $g_e = 20g_0$ (Fig. 1*b*) and for increasing (open circles) and decreasing (full circles) excitation frequency f_e. As is seen, the shell loses stability twice at parametrical resonance due to the initial imperfections. For example, as the excitation frequency is increased from $f_e = 76$ Hz ($g_e = 15g_0$), the shell loses stability at $f_e = 77.9$ Hz for the first time, and the first principal circumferential mode at the lowest natural frequency ($F_{sh} = f_2/2$) is excited (Fig. 1*a*). With further increase in the excitation frequency by $f_e = 79.5$ Hz, the vibrations stop and, in 1–2 sec, the amplitude of vibrations abruptly increases and a conjugate mode is excited, i.e., the shell loses stability for the second time. Note that experiments with axial single-frequency vibrational excitation at $g_e = (5, 10, 15)g_0$ showed that the dynamic instability domains do not overlap if the excitation frequency is increased. Therefore, the conjugate modes do not interact and traveling waves are not generated (Table 1). At $g_e = 20g_0$, the domains overlap so that as the excitation frequency is increased, the shell loses stability in two modes simultaneously in some frequency range ($\Delta f_e = 79.1$–79.4 Hz), which leads to the excitation of a traveling wave (Fig. 1*b*, Table 1).

As the excitation frequency f_e is decreased, a hysteresis occurs [8], accompanied by growth of the domains of the primary and secondary parametrical resonances. This causes the two DIDs to overlap at lower amplitudes of external kinematic loading. In the dry shell, this was observed at $g_e = (10, 15, 20)g_0$, which was accompanied by the interaction of the conjugate modes of the mode $n = 3$ and the excitation of a traveling wave (Fig. 1, Table 1).



a *b*

Fig. 1



a *b*

Fig. 2

Figure 2 compares the ranges in which the parametrical vibrations of the dry shell are excited when the resonant zones corresponding to the mode $m = 1$, $n = 3$ are slowly passed from left to right (Fig. 2*a*) and from right to left (Fig. 2*b*). The vertical axis indicates the acceleration of the shaker table, and the horizontal axis indicates the frequency of kinematic excitation.

The DIDs corresponding to the conjugate modes overlap at $g_e = (19.5–20)g_0$ when f_e increases (Fig. 2*a*, domain *A*) and at $g_e = (8.5–20)g_0$ when the frequency decreases (Fig. 2*b*, domain *B*). As is seen from Figs. 1 and 2, Table 1, the domain *A* is much narrower than the domain *B* in which traveling waves are generated.

Analyzing the dependence of the DID on the amplitude g_e and filler level H_f, we discover that the DID broadens with increase in g_e and narrows down with increase in H_f. With increase in g_e, the DID broadens, regardless of whether the excitation frequency f_e is decreased or increased. The plastic-foam filler not only intensifies the damping, but also narrows the DID in which intensive circumferential vibrations of the external end of the shell are excited.

This effect is especially strong at $H_f = H_{sh}/2$. At this level of the filler, the circumferential vibrations of the free upper end of the shell in modes $n = 3$ and $n = 5$ occur when $g_e = (15; 20)g_0$ and $g_e = 20g_0$, respectively.

Thus, the most diverse nonlinear deformation of the elastic wall under single-frequency excitation is observed at the minimum resonant frequencies of conjugate modes with wave number $n = 3$. How traveling waves are excited depends on the direction of scanning the excitation frequency f_e, excitation amplitude g_e, and the filler level H_f in the shell.

2.2. Deformation of the Elastic Wall of a Shell under Two-Frequency Kinematic Vibration Excitation. Based on the test data on the deformation of a shell, dry or with plastic-foam filler, under single-frequency axial kinematic vibration excitation, we used several types of two-frequency excitation. We used the frequencies of conjugate modes with wave number $n = 3$ and nonconjugate modes with wave number $n = 5$, which correspond to two DIDs (Fig. 3) and at which the deformation processes are the most intensive (which was pointed out earlier). We mainly studied the dynamic behavior of a shell, depending on the frequency and amplitude of external two-frequency excitation.

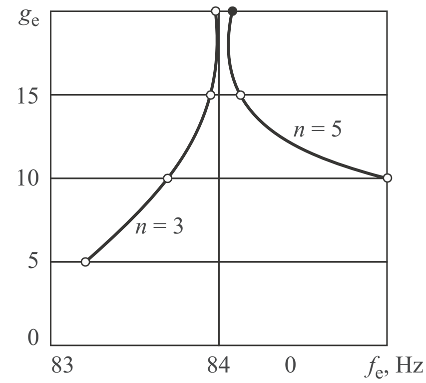


Fig. 3

First type of excitation: the primary excitation frequency f_e^1 is equal to one of the double resonant frequencies $f_e^1 = 2F_r$ of the conjugate circumferential modes and the secondary frequency f_e^2 is equal to frequencies close to the first excitation frequency, i.e., $f_e^2 \approx f_e^1 = 2F_r$.

Second type of excitation: the primary excitation frequency f_e^1 is equal to the double frequency of the traveling wave with $n = 3$ ($f_e^1 = 2F_{tr}$) and the secondary frequency (f_e^2) is to close to the double frequency of the traveling wave ($f_e^1 = 2F_{tr}$ and $f_e^2 \approx 2F_{tr}$).

Third type of excitation: the primary excitation frequency f_e^1 is equal to one of the extreme critical frequencies corresponding to the mode $n = 3$, and the secondary frequency is equal to f_e^2 corresponding to the mode $n = 5$.

The studies show that the nonlinear deformation of the composite shell is most strongly affected when the primary frequency is equal to one of the resonant frequencies F_r of the circumferential mode or the frequency F_{tr} of the traveling wave with wave number $n = 3$ and the secondary frequency is close to these frequencies: $f_e^1 = 2F_{r2}$ and $f_e^2 \approx 2F_{r2}$; $f_e^1 = 2F_{tr}$ and $f_e^2 \approx 2F_{tr}$. The deformation of the shell subject to two-frequency vibrational excitation at frequencies corresponding to two modes, f_e^1 ($n = 3$) and f_e^2 ($n = 5$), is a specific process. The deformation process is the most intensive when the frequencies f_e^1 and f_e^2 differ a little: 0.1–0.2 Hz. The elastic shell, empty ($H_f = 0$) or filled to $H_f = (0.3; 0.5)H_{sh}$, was subjected to excitation of the first type. The dry shell was subjected to two-frequency excitation with one of the frequencies (f_e^1 and f_e^2) equal to the double resonant frequency ($f_e^1 = 81.5$ Hz $= 2F_r$) of transverse vibrations of the shell in one of the conjugate modes with $n = 3$ in which vibrations occur at lower amplitude g_e and the other frequency being close to it ($f_e^2 = 81.6$ Hz) at $g_e = 4.0g_0$. This joint action produces beating [11] (at a frequency of $\omega_{tr} \approx 0.1$ Hz), which is accompanied by variation in the amplitude of kinematic excitation and the amplitude of parametrical vibrations of the shell wall.

Table 3 shows the variation in the amplitude $F = 2A$ mm of vibrations of the shell with wave number $n = 3$ and the amplitude $G = g_e$ (f_e^1 and f_e^2) of two-frequency kinematic excitation (f_e^1 and f_e^2) and the variation in the amplitude $F_1 = 2A$ mm of vibrations of the shell wall and the amplitude $G_1 = g_e$ (f_e^1) of single-frequency excitation (f_e^1) for filler levels $H_f = (0, 0.3, 0.5)H_{sh}$.

As is seen, such axial excitation ($f_e^1 = 81.5$ Hz and $f_e^2 = 81.6$ Hz) causes beating manifested as variation in the amplitude of kinematic excitation in the range $g_e = (3.5–5)g_0$ and in the amplitude of vibrations of the wall of the dry shell at antinodes in the range $2A = (4.5–7)$ mm. Under single-frequency excitation ($f_e^1 = 81.5$ Hz, $g_e = 4g_0$), the amplitude of vibrations of the shell wall $2A = 6$ mm.

Two-frequency excitation ($f_e^1 = F_r$ (85.2 Hz) and $f_e^2 = 85.3$ Hz) causes the deformation of the dry shell with wave number $n = 5$ with lower amplitudes $2A = (1.5–2)$ mm at higher excitation amplitudes $g_e = (8.5–9.2)g_0$, compared with the mode $n = 3$.

TABLE 3

Parameter	H_f/H_{sh}		
	0	0.3	0.5
f_e^1, Hz	81.5	81.7	82.2
f_e^2, Hz	81.6	81.8	82.3
F	4.5–7	3.5–5.0	1–2.5
G	3.5–5	3–4.6	3–4.2
F_1	6	4.5	3
G_1	4	4	4

Several types of two-frequency excitation were used to study the traveling-wave deformation (second type) of the shell observed in the shell either empty or filled to $H_f = (0.3; 0.5)H_{sh}$ under single-frequency axial kinematic excitation. We used the frequencies of conjugate modes with wave number $n = 3$ that occur only as traveling waves under kinematic excitation (which was pointed out earlier).

It was established that the circumferential deformation of the shell wall under single-frequency excitation depends not only on the frequency, but also on the amplitude of kinematic excitation, i.e., whether the circumferential deformation mode of the upper end of the shell is traveling or standing wave depends on the amplitude g_e. The wave is traveling when g_e is high and standing when g_e is low.

In the case of two-frequency (f_e^1 and f_e^2) excitation of the dry shell, the primary frequency f_e^1 was chosen equal to the frequency of traveling waves ($f_e^1 = F_{tr}$) when $g_e = (10, 15, 20)g_0$. At these amplitudes, the deformation of the shell wall occurs as a traveling wave with the maximum amplitude $2A$. The secondary frequency (f_e^2) is equal to close frequencies corresponding to these excitation amplitudes: $f_e^1 = 2F_{tr}$ = (79.0, 78.6, 77.9) Hz and $f_e^2 \approx 2F_{tr}$ = (Δ78.5–79.5, 78.0–80.0, 77.0–80.0) Hz.

When the dry shell is subjected to two-frequency excitation, its deformation mode alternates between traveling and standing waves in the used frequency range Δf_e because of the variation in the amplitude g_e caused by beating resulting from the interaction of the two excitation frequencies [11].

Figure 4 shows the frequency ranges in which the vibration mode of the free end of the dry shell is a traveling wave under single-frequency (curve *1*) and two-frequency (curve *2*) excitation with $g_e = (10; 15; 20)g_0$.

The deformation of the free end of the dry shell under two-frequency excitation is the most intensive when the excitation frequencies (f_e^1 and f_e^2) are equal to the frequencies f_e^1 of traveling waves and to frequencies close to them.

This type of excitation with starting amplitude $g_e = 15.0g_0$ and frequencies $f_e^1 = F_{tr}$ (78.6 Hz) and f_e^2 = 78.5 Hz generates a cyclic process in the range t = (8–9) sec when the excitation amplitude varies as $g_e = (13.0–16.0)g_0$. The wave is traveling in the ranges t = (4–5) sec and Δf_e = (78.5–79.1) Hz when $g_e = (14.5–16.0)g_0$ (Fig. 4*a*, curves *2*) and is standing in the range t = (3–4) sec when $g_e = (13.0–14.5)g_0$.

As is seen, such external excitation with varying g_e causes not only the alternation of the deformation modes of the shell wall, but also the alternation of the amplitudes of vibrations of the shell wall at antinodes in the range $2A$ = (8–15) mm. In single-frequency excitation of a traveling wave (f_e^1 = 78.6 Hz, $g_e = 15.0g_0$), the amplitude of vibrations of the dry shell $2A$ = 13 mm.

In the composite shell with filler, the cyclic processes described above occur at higher amplitudes g_e and within narrower frequency ranges.

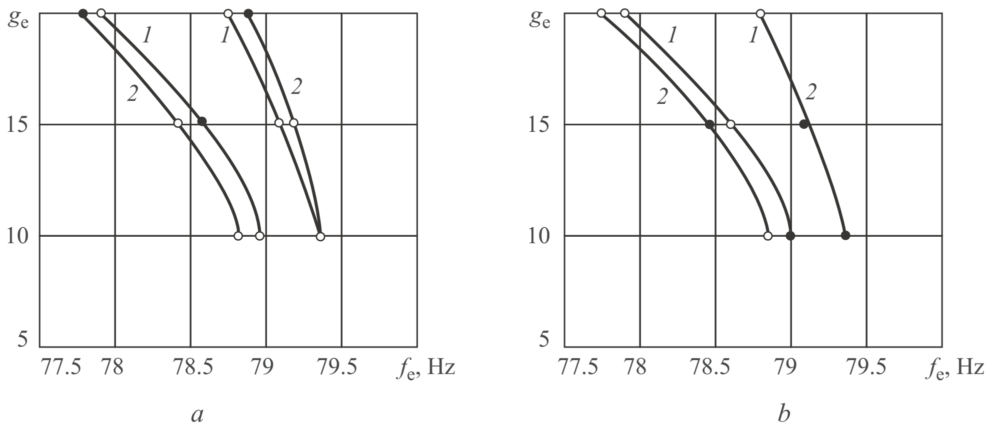


Fig. 4

A cylindrical shell filled to $H_f = H_{sh}/2$ was subject to two-frequency axial kinematic excitation with the primary frequency equal to the frequencies of traveling waves ($f_e^1 = F_{tr}$) at starting excitation amplitudes $g_e = (15; 20)g_0$ and the secondary frequency (f_e^2) equal to close frequencies at the following starting excitation amplitudes: $f_e^1 = 2F_{tr}$ (78.75, 78.2) Hz and $f_e^2 \approx 2F_{tr}$ = (Δ78.5–79.0, 77.5–78.7) Hz. As established earlier, the deformation mode of the shell with $H_f = H_{sh}/2$ under single-frequency vibrational excitation is a traveling wave with $n = 3$ when $g_e = (15–20)g_0$.

Figure 4 shows the frequency ranges in which the vibration mode of the free end of the shell with this filler level is a traveling wave under single-frequency (curve *1*) and two-frequency (curve *2*) excitation with $g_e = (15; 20)g_0$.

As in the dry shell, the cyclic deformation of the wall of the shell with filler level $H_f = H_{sh}/2$ under two-frequency excitation with $g_e = 15.0g_0$ is the most intensive when one of the frequencies is equal to the frequency of the traveling wave and the other frequency is close to it: $f_e^1 = F_{tr}$ (78.5 Hz) and f_e^2 = 77.75 Hz. With this type of excitation, the shell deforms in the range t = (8–10) sec when $g_e = (13–16)g_0$. The wave is traveling in the range t = (3–4) sec when $g_e = (15–16)g_0$ and is standing in the range t = (4–5) sec when $g_e = (13–15)g_0$. In this cyclic process, the amplitude of vibrations of the shell wall at antinodes varies: $2A$ = (9–12) mm. In single-frequency excitation of a traveling wave (f_e^1 = 78.75 Hz, $g_e = 15.0g_0$), the amplitude of vibrations of the shell with $H_f = H_{sh}/2$ reaches $2A$ = 10 mm.

The frequencies of the third type of two-frequency excitation of the dry shell ($H_f = 0$) are equal to the extreme critical frequencies [1, 8] at which radial vibrations of the free upper end of the shell in modes $n = 3$ and $n = 5$ are excited at minimum excitation amplitude g_e. Figure 3 shows the spectrum of close extreme critical frequencies of the two vibration modes depending on the excitation amplitude.

Two types of two-frequency vibrational excitation with extreme critical frequencies of modes $n = 3$ and $n = 5$ were used: type 3.1: f_e^1 = 84.0 Hz ($n = 3$) and f_e^2 = 84.2 Hz ($n = 5$) and type 3.2: f_e^1 = 84.0 Hz ($n = 3$) and f_e^2 = 84.1 Hz ($n = 5$).

How the vibrations of the shell wall with wave numbers $n = 3$ and $n = 5$ are excited depends on the frequency f_e and amplitude g_e of external single-frequency excitation. The circumferential vibration mode $n = 5$ of the free end of the shell in its DID begins to be excited at frequency f_e = 84.2 Hz in the range $g_e = (14.0–14.5)g_0$ and at frequency f_e = 84.1 Hz in the range $g_e = (18.0–18.5)g_0$, while the vibrations of the shell in mode $n = 3$ occur at lower amplitudes g_e. With these types of two-frequency excitation, the starting excitation amplitudes are: $g_{e\,st} = 14.5g_0$ for type 3.1 and $g_{e\,st} = 18.5g_0$ for type 3.2.

The two-frequency excitation causes beating [1, 8, 14] (at frequencies w_{tr} = (0.1; 0.2 Hz)), which is accompanied by a change in the excitation amplitude g_e and in the amplitude $2A$ of vibrations of the free end of the shell.

An analysis of the experimental results (Table 4) shows that these types of two-frequency axial kinematic excitation (f_e^1 ($n = 3$) and f_e^2 ($n = 5$)) with cyclic change in the excitation amplitude g_e cause the circumferential deformation mode of the shell wall to alternate between one mode ($n = 3$) and a combined mode ($n = 3 + n = 5$). For example, if f_e^1 = 84.0 Hz, f_e^2 = 84.2 Hz, and $g_{e\,st} = 14.5g_0$, the cyclic change in the excitation amplitude occurs in the range $g_e = (13.0–15.0)g_0$. When $g_e = (13.0–14.5)g_0$, only the circumferential mode $n = 3$ is excited in the range t = (5–6) sec. When $g_e = (14.5–15.0)g_0$, the combined circumferential deformation mode ($n = 3 + n = 5$) is excited in the range t = (1–2) sec. The simultaneous excitation of the two

TABLE 4

f_e, Hz, $2A$, g_e	Type of excitation	
	3.1	3.2
f_e^1 ($n = 3$)	84.0	84.0
f_e^2 ($n = 5$)	84.2	84.1
$2A$ (f_e^1 and f_e^2)	4…7	7…13
g_e (f_e^1 and f_e^2)	13…15	17…20
$2A$ (f_e^1)	4.5	6.5
$g_{e\ st}$ (f_e^1)	14.5	18.5
$2A$ (f_e^2)	3.5	5.5
$g_{e\ st}$ (f_e^2)	14.5	18.5

circumferential modes $n = 3$ and $n = 5$ considerably intensifies the deformation of the shell. This type of excitation causes the cyclic change in the amplitude of vibrations of the end of the dry shell in the range $2A = (4.0–7.0)$ mm (Table 4)). Single-frequency vibrational excitation with $g_e = 15g_0$ causes the following amplitudes of deformation of the shell wall at these frequencies: $2A = 6.0$ mm for $n = 3$ and $2A = 5.0$ mm for $n = 5$.

Under two-frequency excitation ($f_e^1 = 84.0$ Hz and $f_e^2 = 84.1$ Hz), the deformation of the dry shell with wave numbers $n = 3$ and $n = 5$ occurs at higher excitation amplitudes.

For example, at starting excitation amplitude $g_e = 18.5g_0$, the cyclic change in the excitation amplitude occurs in the range $t = (13–14)$ sec when $g_e = (17.0–20.0)g_0$. When $g_e = (17.0–18.5)g_0$, only the circumferential mode $n = 3$ is excited in the range $t = (10–11)$ sec. When $g_e = (18.5–20.0)g_0$, the combined circumferential deformation mode ($n = 3 + n = 5$) is excited in the range $t = (3–4)$ sec. This type of excitation causes the cyclic change in the amplitude of vibrations of the free end of the dry shell at antinodes in the range $2A = (7.0–13.0)$ mm (Table 4). Single-frequency vibrational excitation with $g_e = 20g_0$ causes the following amplitudes of deformation of the shell wall at these frequencies: $2A = 8.0$ mm for $n = 3$ and $2A = 7.0$ mm for $n = 5$.

If the shell is filled to $H_f = H_{sh}/2$, its deformation occurs only if the difference of the two excitation frequencies is 0.1 Hz, i.e., $f_e^1 = 84.5$ Hz ($n = 3$), $f_e^2 = 84.6$ Hz ($n = 5$), and $g_{e\ st} = 20.0g_0$. This type of excitation gives rise to a cyclic process in the range $t = (7–8)$ sec when $g_e = (8.5–20.5)g_0$. When $g_e = (18.5–20.0)g_0$, only the circumferential mode $n = 3$ is excited in the range $t = (6–7)$ sec. When $g_e = (20.0–20.5)g_0$, the combined circumferential deformation mode ($n = 3 + n = 5$) is excited in the range $t = (1–2)$ sec.

This type of excitation causes a cyclic change in the amplitude of vibrations of the end of the shell with $N_f = H_{sh}/2$ at antinodes in the range $2A = (5–9)$ mm. Single-frequency vibrational excitation with $g_e = 20.0g_0$ causes the following amplitudes of deformation of the shell wall at these frequencies: $2A = 6$ mm for $n = 3$ and $2A = 5$ mm for $n = 5$.

Conclusions. We have experimentally established that quite complex dynamic processes of deformation occur in the elastic glassfiber-reinforced plastic cylindrical shell under two-frequency vibrational excitation, unlike single-frequency excitation. These processes are the most intensive at the resonant frequencies and at lower excitation amplitudes.

Especially complex processes of deformation such as cyclic change in the amplitude and deformation mode between traveling and standing waves are due to beating and the interaction of conjugate circumferential modes in a certain frequency range and at certain excitation amplitudes and due to the interaction of various nonconjugate modes ($n = 3 + n = 5$). The plastic-foam filler intensifies the damping of vibrations.

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