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NONSTATIONARY DYNAMICS OF LONGITUDINALLY REINFORCED ELLIPTIC CYLINDRICAL SHELLS

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The forced vibrations of a longitudinally reinforced elliptic cylindrical shell under nonstationary loading are analyzed numerically. The problem is formulated and a numerical algorithm to solve it is developed. The system of differential equations is based on the Timoshenko-type theory of orthotropic shells and rods. The dynamic behavior of the shell is studied

Keywords: longitudinally reinforced elliptic cylindrical shell, forced vibrations, Timoshenko-type theory, numerical solution

Introduction. The forced vibrations of reinforced shells under various loads are addressed in many publications. The classical problem formulation for rib-reinforced shells (Kirchhoff–Love shell model, Kirchhoff–Clebsch rod model) is given in [1]. A refined form (Timoshenko-type model of shells and rods) of vibration equations is given in [3] where the dynamic behavior of reinforced shells of canonical shape (cylindrical, conical, and spherical) was studied.

Formulating and solving problems of the dynamic behavior of reinforced elliptic cylindrical shells involve substantial difficulties. The stress–strain state of a rib-reinforced elliptic cylindrical shell is determined in three stages: (i) description of forced vibrations, (ii) description of the geometry of a noncanonical structure (elliptic cylindrical shell), and (iii) description of the effect of stiffening ribs [4]. This procedure has been used very rarely. The free and forced vibrations of smooth elliptic cylindrical shells or the vibrations of reinforced circular cylindrical shells were mainly studied in [6–20]. There are very few studies on the dynamic behavior of reinforced elliptic cylindrical shells under nonstationary loads.

We will use the Timoshenko-type theory of orthotropic shells and rods to formulate the problem of the forced vibrations of orthotropic reinforced elliptic cylindrical shells, develop a numerical algorithm to solve it, and analyze the numerical results.

1. Problem Formulation. Basic Equations. Consider a reinforced elliptic cylindrical shell under a distributed internal load $P_3(s_1, s_2, t)$ (s_1, s_2 and t are the space and time coordinates). The shell is reinforced with longitudinal ribs [2, 3].

The coefficients of the first quadratic form and the curvatures of the coordinate surface of the shell are given by

$$A_1 = 1, \quad k_1 = 0, \quad A_2 = (a^2 \cos^2 \alpha_2 + b^2 \sin^2 \alpha_2)^{1/2},$$

$$k_2 = ab(a^2 \cos^2 \alpha_2 + b^2 \sin^2 \alpha_2)^{-3/2}, \quad s_1 = A_1 \alpha_1, \quad s_2 = A_2 \alpha_2, \tag{1}$$

where a and b are the semiaxes of the elliptical cross-section of the cylindrical shell.

To derive the vibration equations for a reinforced cylindrical shell, we will use the Hamilton–Ostrogradsky variational principle [2]. Performing standard transformations over the variational functional and taking into account the shell–rib interface conditions [1, 2], we obtain two groups of equations:

(i) vibration equations of a smooth elliptic cylindrical shell

$$\frac{\partial T_{11}}{\partial s_1} + \frac{\partial S}{\partial s_2} = \rho h \frac{\partial^2 u_1}{\partial t^2}, \qquad \frac{\partial S}{\partial s_1} + \frac{\partial T_{22}}{\partial s_2} - k_2 T_{23} = \rho h \frac{\partial^2 u_2}{\partial t^2},$$

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$$\frac{\partial T_{13}}{\partial s_1}+\frac{\partial T_{23}}{\partial s_2}-k_2T_{22}+P_3(s_1,s_2,t)=\rho h\frac{\partial^2 u_3}{\partial t^2},$$

$$\frac{\partial M_{11}}{\partial s_1}+\frac{\partial H}{\partial s_2}-T_{13}=\rho\frac{h^3}{12}\frac{\partial^2\varphi_1}{\partial t^2},\qquad \frac{\partial H}{\partial s_1}+\frac{\partial M_{22}}{\partial s_2}-T_{23}=\rho\frac{h^3}{12}\frac{\partial^2\varphi_2}{\partial t^2}, \tag{2}$$

(ii) vibration equations of the ith rib located along the s_1-axis:

$$\frac{\partial T_{11i}}{\partial s_1}+[S]=\rho_iF_i\left(\frac{\partial^2u_1}{\partial t^2}\pm h_{\mathrm{m}\,i}\frac{\partial^2\varphi_1}{\partial t^2}\right),$$

$$\frac{\partial T_{12i}}{\partial s_1}+[T_{22}]=\rho_iF_i\left(\frac{\partial^2u_2}{\partial t^2}\pm h_{\mathrm{m}\,i}\frac{\partial^2\varphi_2}{\partial t^2}\right),$$

$$\frac{\partial T_{13i}}{\partial s_1}+[T_{23}]=\rho_iF_i\frac{\partial^2u_3}{\partial t^2},$$

$$\frac{\partial M_{11i}}{\partial s_1}\pm h_{\mathrm{m}\,i}\frac{\partial T_{11i}}{\partial s_1}-T_{13}+[H]=\rho_iF_i\left[\pm h_{\mathrm{m}\,i}\frac{\partial^2u_1}{\partial t^2}+\left(h_{\mathrm{m}\,i}^2+\frac{I_{1i}}{F_i}\right)\frac{\partial^2\varphi_1}{\partial t^2}\right],$$

$$\frac{\partial M_{12i}}{\partial s_1}\pm h_{\mathrm{m}\,i}\frac{\partial T_{12i}}{\partial s_1}-T_{23}+[M_{22}]=\rho_iF_i\left[\pm h_{\mathrm{m}\,i}\frac{\partial^2u_2}{\partial t^2}+\left(h_{\mathrm{m}\,i}^2+\frac{I_{\mathrm{cr}\,i}}{F_i}\right)\frac{\partial^2\varphi_2}{\partial t^2}\right], \tag{3}$$

where $u_1,u_2,u_3,\varphi_1,\varphi_2$ are the components of the generalized displacement vector of the mid-surface; ρ and ρ_i are the densities of the shell and the ith rib, respectively; h is the thickness of the shell; $h_{\mathrm{m}\,i}=0.5(h+h_i)$; h_i is the cross-sectional height of the ith rib; $[f]=f^+-f^-$, where $f^{\pm}$ are the values of the functions on the right and on the left of the ith discontinuity line (projection of the center of gravity of the ith rib onto the mid-surface of the shell).

The forces/moments in the vibration equations (2) for an orthotropic shell are related to the strains by

$$T_{11}=B_{11}(\varepsilon_{11}+\nu_2\varepsilon_{22}),\quad T_{22}=B_{22}(\varepsilon_{22}+\nu_1\varepsilon_{11}),\quad T_{13}=B_{13}\varepsilon_{13},\quad T_{23}=B_{23}\varepsilon_{23},$$

$$S=B_{12}\varepsilon_{12},\quad M_{11}=D_{11}(\kappa_{11}+\nu_2\kappa_{22}),\ M_{22}=D_{22}(\kappa_{22}+\nu_1\kappa_{11}),\ H=D_{12}\kappa_{12},$$

$$\varepsilon_{11}=\frac{\partial u_1}{\partial s_1},\quad \varepsilon_{22}=\frac{\partial u_2}{\partial s_2}+k_2u_3,\quad \varepsilon_{12}=\frac{\partial u_1}{\partial s_2}+\frac{\partial u_2}{\partial s_1},\quad \varepsilon_{13}=\varphi_1+\frac{\partial u_3}{\partial s_1},$$

$$\varepsilon_{23}=\varphi_2+\frac{\partial u_3}{\partial s_2}-k_2u_2,\quad \kappa_{11}=\frac{\partial\varphi_1}{\partial s_1},\quad \kappa_{22}=\frac{\partial\varphi_2}{\partial s_2},\quad \kappa_{12}=\frac{\partial\varphi_1}{\partial s_2}+\frac{\partial\varphi_2}{\partial s_1}, \tag{4}$$

where

$$B_{11}=\frac{E_1h}{1-\nu_1\nu_2},\quad B_{22}=\frac{E_2h}{1-\nu_1\nu_2},\quad B_{12}=G_{12}h,\quad B_{13}=G_{13}h,\quad B_{23}=G_{23}h,$$

$$D_{11}=\frac{E_1h^3}{12(1-\nu_1\nu_2)},\quad D_{22}=\frac{E_2h^3}{12(1-\nu_1\nu_2)},\quad D_{12}=G_{12}\frac{h^3}{12},$$

where $E_1,E_2,G_{12},G_{13},G_{23},\nu_1,\nu_2$ are the mechanical parameters of the orthotropic material of the shell.

The forces/moments in the vibration equations (3) for the ith rib are related to the strains by

$$T_{11i}=E_iF_i\varepsilon_{11i},\quad T_{12i}=G_iF_i\varepsilon_{12i},\quad T_{13i}=G_iF_i\varepsilon_{13i},$$

$$M_{11i}=E_iI_{1i}\kappa_{11i},\quad M_{12i}=G_iI_{\mathrm{cr}\,i}\kappa_{12i},\quad \varepsilon_{11i}=\frac{\partial u_1}{\partial s_1}\pm h_{\mathrm{m}\,i}\frac{\partial\varphi_1}{\partial s_1},\quad \varepsilon_{22i}=\frac{\partial u_2}{\partial s_2}\pm h_{\mathrm{m}\,i}\frac{\partial\varphi_2}{\partial s_1},$$

$$\varepsilon_{13}=\varphi_1+\frac{\partial u_3}{\partial s_1},\quad \kappa_{11i}=\frac{\partial\varphi_1}{\partial s_1},\quad \kappa_{11i}=\frac{\partial\varphi_2}{\partial s_1},\tag{5}$$

where E_i and G_i are the material parameters of the rib; $F_i, I_{1i}, I_{\mathrm{cr}\,i}$ are geometrical parameters of the cross-section of the ith rib.

The vibration equations (2)–(5) are supplemented with appropriate boundary and initial conditions [3].

2. Numerical Problem-Solving Algorithm. The algorithm for solving the initial–boundary-value problem (2)–(5) is based on the integro-interpolation method for the construction of difference schemes with respect to the space coordinates s_1 and s_2 and an explicit approximation with respect to the time coordinate t [2, 7].

The basic difference equations over a discrete mesh (k, l, t) are

$$\frac{T^n_{11k+1/2,\,l}-T^n_{11k-1/2,\,l}}{\Delta s_1}+\frac{S^n_{k,\,l+1/2}-S^n_{k,\,l-1/2}}{\Delta s_2}=\rho h\,(u^n_{1i})_{\bar{t}t},\tag{6}$$

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for Eqs. (2) and

$$T^n_{11k\pm1/2,\,l}=B_{11}(\varepsilon^n_{11k\pm1/2,\,l}+\nu_2\varepsilon^n_{22k\pm1/2,\,l}),$$

$$T^n_{22k\pm1/2,\,l}=B_{22}(\varepsilon^n_{22k\pm1/2,\,l}+\nu_1\varepsilon^n_{11k\pm1/2,\,l}),$$

$$T^n_{13k\pm1/2,\,l}=B_{13}\varepsilon^n_{13k\pm1/2,\,l},$$

$$M^n_{11k\pm1/2,\,l}=D_{11}(\kappa^n_{11k\pm1/2,\,l}+\nu_2\kappa^n_{22k\pm1/2,\,l}),$$

$$M^n_{22k\pm1/2,\,l}=D_{22}(\kappa^n_{22k\pm1/2,\,l}+\nu_1\kappa^n_{11l\pm1/2,\,l}),$$

$$\varepsilon^n_{11k+1/2,\,l}=\frac{u^n_{1k+1,\,l}-u^n_{1k,\,l}}{\Delta s_1},\quad \varepsilon^n_{11k-1/2,\,l}=\frac{u^n_{1k,\,l}-u^n_{1k-1,\,l}}{\Delta s_1},$$

$$\varepsilon^n_{22k+1/2,\,l}=\frac{u^n_{2k+1,\,l+1/2}-u^n_{2k+1/2,\,l-1/2}}{\Delta s_2}+k_{2l}\frac{u^n_{3k+1,\,l}+u^n_{3k,\,l}}{2},$$

$$\varepsilon^n_{22k-1/2,\,l}=\frac{u^n_{2k-1,\,l+1/2}-u^n_{2k-1/2,\,l-1/2}}{\Delta s_2}+k_{2l}\frac{u^n_{3k,\,l}+u^n_{3k-1,\,l}}{2},\tag{7}$$

....

for Eqs. (4), where Δs_1 and Δs_2 are the difference steps in s_1 and s_2; indices $k, k\pm1/2, l, l\pm1/2, n$ refer to kinematic and mechanical quantities at discrete points with space coordinates s_1, s_2 and time coordinate t. Difference derivatives with respect to the space coordinates and time are denoted as in [7].

3. Numerical Results. Let us consider, as a numerical example, the dynamic behavior of a stringer-reinforced elliptic cylindrical panel under a distributed internal impulsive load. All sides of the cylindrical panel are clamped. The stringer $0\le s_1\le L$ is located in the section $s_2=0$.

The distributed impulsive load $P_3(s_1, s_2, t)$ is defined by

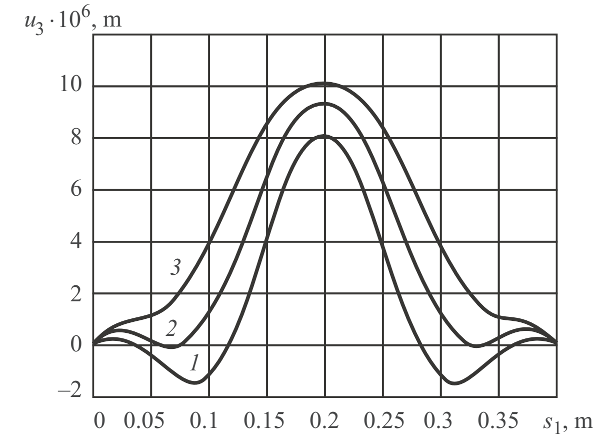

Fig. 1

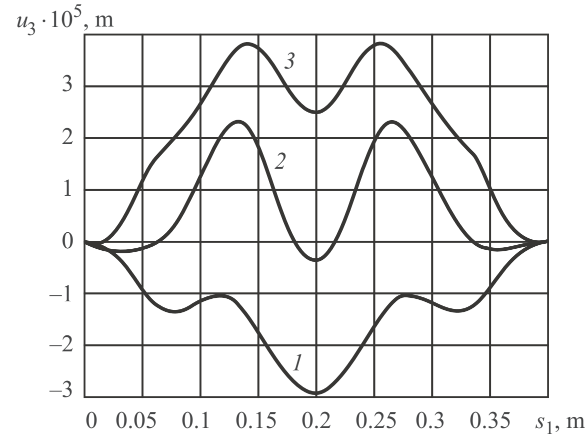

Fig. 2

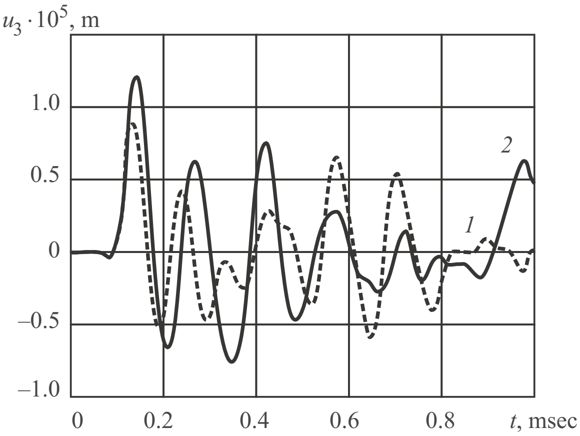

Fig. 3

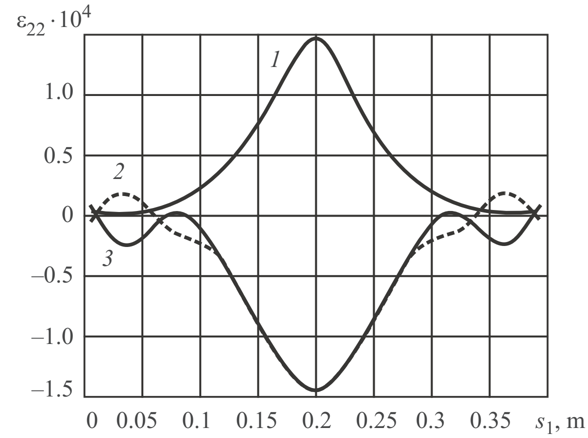

Fig. 4

$$P_3(s_1, s_2, t) = A \cdot \sin\frac{\pi t}{T}[\eta(t) - \eta(t - T)],$$

where A and T are the amplitude and duration of the load, respectively. Let $A = 10^6$ Pa, $T = 50 \cdot 10^{-6}$ sec.

The geometrical and mechanical parameters of the shell: $E_1 = E_2 = 7 \cdot 10^{10}$ Pa, $\nu_1 = \nu_2 = 0.3$, $h = 10^{-2}$ m, $L = 0.4$ m. The aspect ratio of the elliptic cross-section:

(i) $a = b = 0.1$,

(ii) $a = 1.1b$,

(iii) $a = 1.2b$.

The parameters of the rib: $E_i = E$, $F_i = a_i h_i$, $a_i = h$, $h_i = 2h$.

The computational domain is $D = \{0 \le s_1 \le L, 0 \le s_2 \le A_2 \pi / 8\}$. The calculated results are presented in the figures below.

Figures 1–8 show numerical values of $u_3, \varepsilon_{22}, \sigma_{22}$. Since the original problem includes many parameters (the kinematic and mechanical parameters have different values at different time points t and at different space points with coordinates s_1 and s_2), we will analyze the unknown quantities at instants they have the maximum magnitude. Figure 1 shows the variation in the displacement u_3 in the section $s_2 = 0$ (location of the rib) with the coordinate s_1 $(0 \le s_1 \le L)$ at the instant $t = 2.5T$.

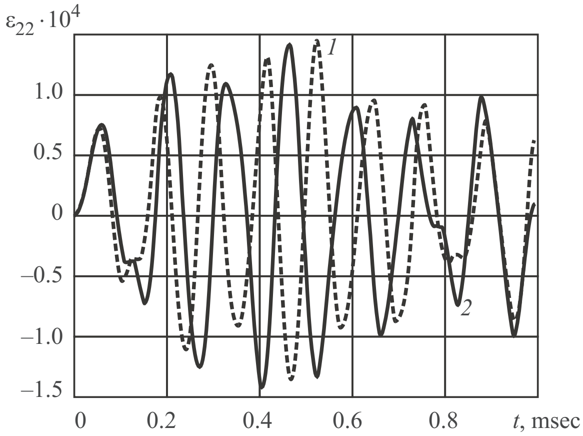

Fig. 5

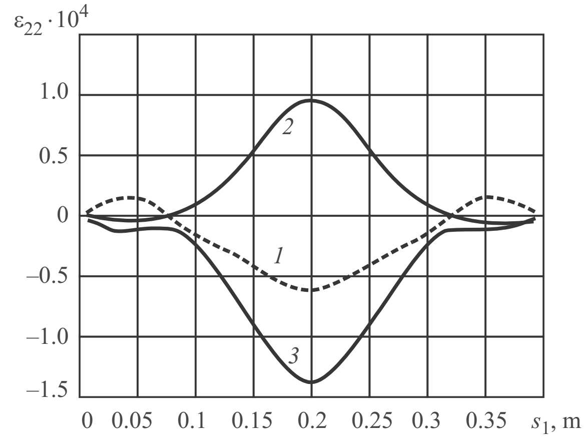

Fig. 6

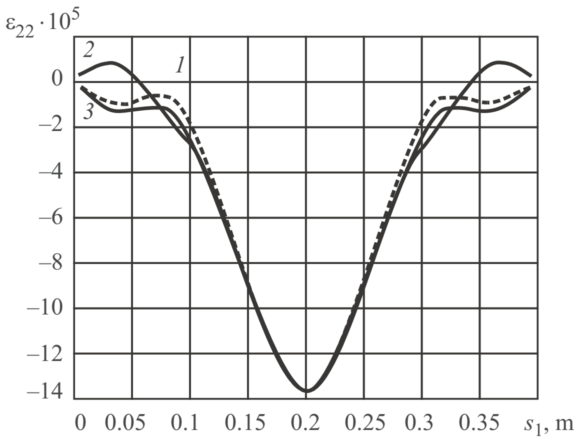

Fig. 7

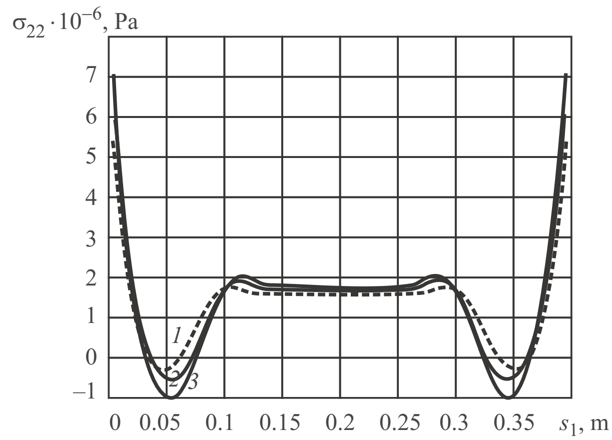

Fig. 8

Figure 2 shows the variation in u_3 in the section $s_2 = A_2\pi/16$ with the coordinate s_1 at the instant $t = 4.5T$. In Figs. 1 and 2, curve *1* corresponds to $a/b = 1$, curve *2* to $a/b = 1.1$, and curve *3* to $a/b = 1.2$.

Figure 3 shows the variation in u_3 with time t at the point $s_1 = L/2$, $s_2 = 0$ (the middle of the rib) for $a/b = 1$ (curve *1*) and $a/b = 1.1$ (curve *2*). Figure 4 shows the variation in ε_{22} with the coordinate s_1 in the section $s_2 = 0$ for $a/b = 1$ (curve *1*, $t = 10.5T$), $a/b = 1.1$ (curve *2*, $t = 8.25T$), $a/b = 1.2$ (curve *3*, $t = 9T$).

Figure 5 shows the variation in ε_{22} with time t at the point $s_1 = L/2$, $s_2 = 0$ for $a/b = 1$ (curve *1*) and $a/b = 1.1$ (curve *2*). Figure 6 shows the variation in ε_{22} with the coordinate s_1 in the section $s_2 = A_2\pi/16$ for different values of a/b at the instant $t = 9T$ (the notation is the same as in Figs. 1 and 2).

Figure 7 shows the variation in ε_{22} with the coordinate s_1 in the section $s_2 = A_2\pi/16$ for different values of a/b at the instants it reaches maximum: $t = 9.375T$ (curve *1*), $t = 8.125T$ (curve *2*), $t = 9T$ (curve *3*). Figure 8 shows the variation in σ_{22} with the coordinate s_1 in the section $s_2 = 0$ at $t = T$ for different values of a/b (the notation in Figs. 7 and 8 is the same as in Figs. 1 and 2).

Conclusions. We have found a numerical solution to the problem of the vibrations of stringer-reinforced elliptic cylindrical shells forced by a nonstationary load. A numerical algorithm for solving this class of problems has been developed. Numerical results are presented and analyzed.

REFERENCES

1. I. Ya. Amiro, V. A. Zarutskii, and V. G. Palamarchuk, *Dynamics of Ribbed Shells* [in Russian], Naukova Dumka, Kyiv (1983).
2. Ya. M. Grigorenko, E. I. Bespalova, A. B. Kitaigorodskii, and A. I. Shinkar', *Free Vibrations of Elements of Shell Structures* [in Russian], Naukova Dumka, Kyiv (1986).
3. P. Z. Lugovoi, V. F. Meish, and E. A. Shtantsel', *Nonstationary Dynamics of Inhomogeneous Shell Structures* [in Russian], Izd.-Poligr. Tsentr "Kievskii Universitet," Kyiv (2005).
4. V. F. Meish and N. V. Maiborodina, "Analysis of the nonaxisymmetric vibrations of flexible ellipsoidal shells discretely reinforced with transverse ribs under nonstationary loads," *Int. Appl. Mech.*, **44**, No. 10, 1128–1136 (2008).
5. A. Samarskii, *The Theory of Difference Schemes*, Marcel Dekker, New York (2001).
6. Ya. M. Grigorenko and A. Ya. Grigorenko, "Problems of statical and dynamical feformation of anisotropic inhomogeneous shells with variable parameters and their numerical solution (review)," *Int. Appl. Mech.*, **49**, No. 2, 123–193 (2013).
7. Ya. M. Grigorenko and S. N. Yaremchenko, "Refined design of longitudinally corrugated cylindrical shells," *Int. Appl. Mech.*, **48**, No. 2, 195–204 (2012).
8. T. Irie, G. Yamada, and Y. Kobayashi, "Free vibrations of non-circular shells with longitudinal interior partitions," *J. Sound Vibr.*, **96** (1), 133–142 (1984).
9. U. Kumar and V. Singh, "Approximate vibrations analysis of noncircular cylinders having varying thickness," *AIAA J.*, **30**, No. 7, 1929–1931 (1991).
10. M. V. Maiborodina and V. F. Mesh, "Forced vibrations of ellipsoidal shells reinforced with transverse ribs under nonstationary distributed load," *Int. Appl. Mech.*, **49**, No. 6, 693–701 (2013).
11. B. P. Patel, S. S. Gupta, M. S. Loknath, and C. P. Kadu, "Free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory," *Compos. Struct.*, **69**, 259–270 (2005).
12. P. K. Soldatos, "Mechanics of cylindrical shells with non-circular cross section: a survey," *Appl. Mech. Rev.*, **52**, No. 8, 237–274 (1999).
13. K. P. Soldatos and G. J. Tzivanidis, "Buckling and vibrations of cross-ply laminated non-circular cylindrical shells," *J. Sound Vibr.*, **82** (3), 425–434 (1982).
14. K. Suzuki and A. W. Leissa, "Free vibrations of noncircular cylindrical shells having circumferentially varying thickness," *J. Appl. Mech.*, **52**, No. 1, 149–154 (1985).
15. K. Suzuki, G. Shikanai, and A. W. Leissa, "Free vibrations of laminated composite noncircular thin cylindrical shells," *J. Appl. Mech.*, **61**, No. 4, 861–871 (1994).
16. F. Tornabene, "Free vibration analysis of functionally graded conical, cylindrical shells and annular plate structure with a four – parameter power – law distribution," *Comput. Meth. Appl. Mech. Eng.*, **198**, 2911–2935 (2009).
17. M. Firer and I. Steinman, "Nonlinear analysis of laminated noncircular cylindrical shells," *Int. J. Solids Struct.*, **32**, No. 10, 1405–1416 (1995).
18. Y. Xiang and Y. Y. Huang, "A novel semi-analytical method for solving acoustic radiation from longitudinally stiffened infinite non-circular cylindrical shells in water," *Acta Mech. Solid. Sin.*, **18**, 1–12 (2005).
19. G. Yamada, T. Irie, and Y. Tagana, "Free vibrations of non-circular cylindrical shells with variable circumferential profile," *J. Sound Vibr.*, **95**, No. 1, 117–126 (1984).
20. S. Z. Zou, Y. Y. Huang, Z. He, and Y. Xing, "New matrix method for response analysis of circumferentially stiffened non-circular shells under harmonic pressure," *Appl. Math. Mech.*, **28**, No. 10, 1397–1405 (2007).