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STABILITY OF COMPOSITE CYLINDRICAL SHELLS WITH ADDED MASS INTERACTING WITH THE INTERNAL FLUID FLOW

P. S. Koval'chuk[1], L. A. Kruk[2], and V. A. Pelykh[1]

The effect of added masses on the quasistatic (divergence) and dynamic (flutter) loss of stability of cylindrical shells interacting with the internal fluid flow is studied. The dependence of the critical velocity of the fluid on the type of attachment of the added masses is analyzed

Keywords: cylindrical shell, ideal incompressible fluid, concentrated mass, critical velocity, instability, divergence and flutter

Introduction. The dynamic stability and vibrations of various fluid-conveying pipeline systems have recently attracted increasing attention [8–11, 13, 14, 18, etc.]. The prevailing majority of relevant studies are concerned with the interaction of an elastic object and a fluid flow on the assumption that this object is made of an isotropic material and has a perfect cylindrical form. However, pipelines quite often have local "inclusions" such as rigidly or elastically attached concentrated masses or solid bodies contacting with the structure over some finite area.

Dynamic problems for empty elastic shells with added masses are addressed in [1, 4–7, 12, etc.], where primary attention is given to special methods for determining the natural frequencies and vibration modes of shells and to the effect of added masses and the type and place of their attachment to the lateral surface on these frequencies and modes. Modern methods for and results of solving various problems of the nonstationary contact interaction between solids and elastic media are presented in [16]. Approaches to studying the influence of rigid inclusions and reinforced holes on the nonlinear deformation of thin isotropic and orthotropic shells were proposed in [17].

The combined effect of two factors (added masses and internal flow) on the stability and vibration of cylindrical shells was studied in [15]. Let the mass be concentrated at one point or uniformly distributed over the ring cross-section or the length of the shell. Expanding upon the studies in [15], we will analyze the stability of a fluid-conveying composite cylindrical shell with more complicated "distribution" of added mass over its surface. It is assumed, in particular, that the mass contacting with the shell is uniformly distributed over a rectangular or elliptic boundary (of the developed surface of the shell). Note that such a distribution of "added" masses is typical of pipelines after managing emergencies.

1. Starting Equations of Motion. Consider an elastic closed (orthotropic model) cylindrical shell containing a fluid moving with a constant velocity U. The geometry of the shell is shown in Fig. 1. Figure 2 shows (indicating necessary coordinates) closed rectangular (Fig. 2*a*) and elliptic (Fig. 2*b*) boundaries along which some added mass M is uniformly distributed.

Assume, as usual, that the added mass transmits only the radial force to the shell [1, 5, 6].

The rotational inertia of the mass during vibrations of the shell is also neglected. The fluid filling the shell is ideal and incompressible, and its motion is potential.

We will use a linear problem formulation to analyze the shell with uniformly distributed mass for stability. We will start with mixed equations of motion [2, 3]:

[1]S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine 03057, e-mail: volna@inmech.kiev.ua. [2]National University of Transport, 1 Suvorova St., Kyiv, Ukraine 01010, e-mail: krukles@ukr.net. Translated from Prikladnaya Mekhanika, Vol. 50, No. 5, pp. 101–110, September–October 2014. Original article submitted November 11, 2011.

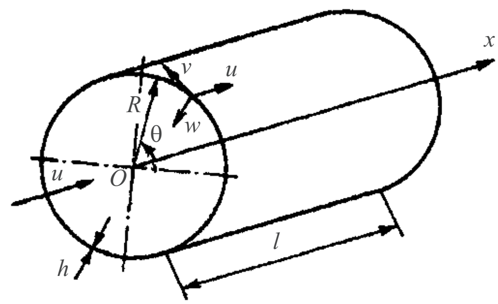

Fig. 1

$$\frac{1}{h}\nabla_D^4 w = \frac{1}{R}\frac{\partial^2 \Phi}{\partial x^2} - \rho\frac{\partial^2 w}{\partial t^2} - \varepsilon_0 \rho \frac{\partial w}{\partial t} - K_{\text{in}} - \frac{P_{\text{h}}}{h},$$

$$\frac{1}{E}\nabla_\delta^4 \Phi = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2}, \tag{1.1}$$

where ∇_D^4 and ∇_δ^4 are operators such that

$$\nabla_D^4 = D_1 \frac{\partial^4}{\partial x^4} + 2D_3 \frac{\partial^4}{\partial x^2 \partial y^2} + D_2 \frac{\partial^4}{\partial y^4},$$

$$\nabla_\delta^4 = \delta_2 \frac{\partial^4}{\partial x^4} + 2\delta_3 \frac{\partial^4}{\partial x^2 \partial y^2} + \delta_1 \frac{\partial^4}{\partial y^4},$$

where the notation is traditional for the classical theory of orthotropic shells [3]; ε_0 is the structural damping parameter; K_{in} is a function characterizing the additional "inertia" of the shell caused by the added mass (the form of this function depends on the type of attachment of the mass and its magnitude); P_{h} is the hydrodynamic pressure exerted by the fluid flow upon the shell and defined by the following well-known formula [2, 10]:

$$P_{\text{h}} = -\rho_0 \left(\frac{\partial \varphi}{\partial t} + U \frac{\partial \varphi}{\partial x} \right)_{r=R}, \tag{1.2}$$

where ρ_0 is the density of the fluid; $\varphi = \varphi(x, r, \Theta, t)$ is the perturbed velocity potential (x, r, Θ are cylindrical coordinates).

Let the shell be simply supported at the ends $x = 0$ and $x = l$ [3, 10]. Then the dynamic deflection of the shell can be expanded into a two-parameter series:

$$w = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} (f_{nm}^{(1)} \cos s_n y + f_{nm}^{(2)} \sin s_n y) \sin \lambda_m x, \tag{1.3}$$

where, as usually, $f_{nm}^{(1,2)}$ are unknown functions of time, which are generalized coordinates; $s_n = n/R$ and $\lambda_m = m\pi/l$ are the circumferential and longitudinal wave numbers, respectively.

The expression of the potential φ corresponding to (1.3) and derived from the solution of the following boundary-value problem can be found in [14, 15]:

$$\Delta\varphi = 0 \qquad (0 \le x \le l,\ 0 \le r \le R,\ \ 0 \le \Theta \le 2\pi),$$

$$\frac{\partial \varphi}{\partial r} = -\left(\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} \right) \quad \text{at} \quad r = R,$$

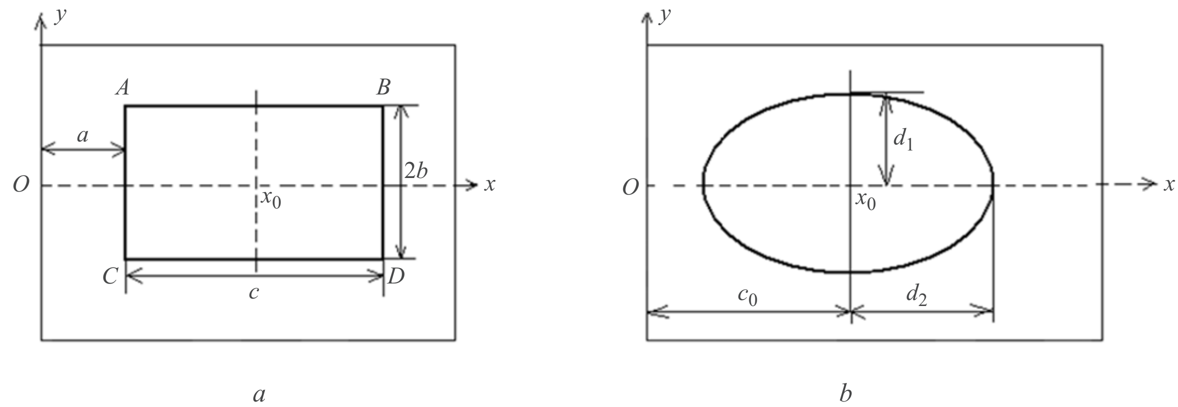

Fig. 2

$$\frac{\partial\varphi}{\partial r} < \infty \quad \text{at} \quad r = 0.$$

Substituting it into (1.2) and applying the Bubnov–Galerkin (BG) method to (1.1), we obtain a system of differential equations for the generalized displacements $f_{nm}^{(1,2)}$. Solving it, it is possible to find the critical velocities of the fluid at which the shell undergoes quasistatic (divergent) or dynamic (flutter) buckling and to analyze the effect of added masses on these velocities.

2. Stability of a Shell with Mass Distributed along a Rectangular Boundary. Let the added mass be distributed along a rectangular boundary with two sides *AB* and *CD* parallel to the shell axis (Fig. 2*a*). As follows from [15], the function K_{in} can be expressed as $K_{\text{in}} = K_1 + K_2$, where K_1 is the component characterizing the "inertial" effect of the mass distributed along the parallel sides *AC* and *BD* on the shell,

$$K_1 = \left\{ \frac{M}{2(2b+c)h} \left[\delta(y-b) + \delta(y+b) \right] \int_a^{a+c} \delta(x-\xi) d\xi \right\} \frac{\partial^2 w}{\partial t^2},$$

K_2 represents the effect of the mass distributed along the sides *AB* and *CD*:

$$K_2 = \left\{ \frac{M}{2(2b+c)h} \left[\delta(x-a) + \delta(x-(a+c)) \right] \int_{-b}^{b} \delta(y-\eta) d\eta \right\} \frac{\partial^2 w}{\partial t^2},$$

where c and $2b$ are the side lengths of the rectangle; $\delta = \delta(z)$ is the delta function.

Let $\cos s_n y \sin \lambda_m x$ and $\sin s_n y \sin \lambda_m x$ be weight functions.

Applying the BG method yields

$$\ddot{f}_{nm}^{(1)} + (\omega_{nm}^2 - \alpha_{nm} U^2) f_{nm}^{(1)} + \varepsilon_{nm} \dot{f}_{nm}^{(1)} + \sum_{q=1}^{\infty} \beta_{nq}^{(m)} U \dot{f}_{nq}^{(1)} + \frac{2k_0 G_1^{nm}}{M_{nm}} = 0,$$

$$\ddot{f}_{nm}^{(2)} + (\omega_{nm}^2 - \alpha_{nm} U^2) f_{nm}^{(2)} + \varepsilon_{nm} \dot{f}_{nm}^{(2)} + \sum_{q=1}^{\infty} \beta_{nq}^{(m)} U \dot{f}_{nq}^{(2)} + \frac{2k_0 G_2^{nm}}{M_{nm}} = 0$$

$$(n = 0, 1, 2, \dots, \quad m = 1, 2, \dots), \tag{2.1}$$

where the following notation is used [13]:

$$\omega_{nm}^2 = \frac{\omega_{nm\,0}^2}{M_{nm}}, \quad \omega_{nm\,0}^2 = \frac{1}{\rho}\left[\frac{1}{h}\Delta_D(\lambda_m, s_n) + \frac{\lambda_m^4}{R^2 \Delta_\delta(\lambda_m, s_n)}\right],$$

$$M_{nm} = 1 + \frac{\rho_0}{\rho}\frac{K_{nm}}{h\lambda_m}, \quad \alpha_{nm} = \frac{\rho_0}{\rho h}\frac{K_{nm}\lambda_m}{M_{nm}}, \quad \varepsilon_{nm} = \frac{\varepsilon_0}{M_{nm}},$$

$$\beta_{nq}^{(m)} = \frac{4\rho_0}{\rho h l}\frac{\lambda_m[1-(-1)^{m-q}]}{(\lambda_m^2 - \lambda_q^2)M_{nm}} K_{nq},$$

$$K_{nm} = \frac{I_n(\lambda_m R)}{I_n'(\lambda_m R)}, \quad I_n'(\lambda_m R) = \frac{I_{n-1}(\lambda_m R) + I_{n+1}(\lambda_m R)}{2} \quad (q = 1, 2, \ldots), \tag{2.2}$$

Δ_D and Δ_δ are operators such that

$$\Delta_D(\lambda_m, s_n) = D_1\lambda_m^4 + 2D_3\lambda_m^2 s_n^2 + D_2 s_n^4,$$

$$\Delta_\delta(\lambda_m, s_n) = \delta_2\lambda_m^4 + 2\delta_3\lambda_m^2 s_n^2 + \delta_1 s_n^4,$$

$k_0 = 2M / M_s, M_s$ is the mass of the shell ($M_s = 2\pi R l \rho h$); G_1^{nm} and G_2^{nm} are functions such that

$$G_1^{nm} = \frac{h}{M}\int_0^l \int_0^{2\pi R} (K_1 + K_2)\cos s_n y \sin \lambda_m x \, dxdy,$$

$$G_2^{nm} = \frac{h}{M}\int_0^l \int_0^{2\pi R} (K_1 + K_2)\sin s_n y \sin \lambda_m x \, dxdy. \tag{2.3}$$

Considering the properties of the δ-function

$$\int_\alpha^\beta f(x)\delta(x-\xi)\,dx = f(\xi) \quad (\alpha \le x \le \beta),$$

$$\int_\alpha^\beta \delta(x-\eta)\,d\eta = e(x-\alpha) - e(x-\beta),$$

where $e = e(z)$ is a unit function ($e(z) = 0$ for $z < 0$ and $e(z) = 1$ for $z > 0$), we can express G_i^{nm} $(i = 1, 2)$ in the general case as

$$G_i^{nm} = \frac{1}{d}\left\{\int_a^{c_1}\left[F_i^{nm}(x, b) + F_i^{nm}(x, -b)\right]dx + \int_{-b}^{b}\left[F_i^{nm}(a, y) + F_i^{nm}(c_1, y)\right]dy\right\}, \tag{2.4}$$

where d is the perimeter of the rectangle ($d = 2(2b + c)$, $c_1 = a + c$);

$$F_1^{nm} = \sum_{p=0}^\infty \sum_{q=1}^\infty (\ddot{f}_{pq}^{(1)} \cos s_p y + \ddot{f}_{pq}^{(2)} \sin s_p y)\sin \lambda_q x \cos s_n y \sin \lambda_m x,$$

$$F_2^{nm} = \sum_{p=0}^\infty \sum_{q=1}^\infty (\ddot{f}_{pq}^{(1)} \cos s_p y + \ddot{f}_{pq}^{(2)} \sin s_p y)\sin \lambda_q x \sin s_n y \sin \lambda_m x. \tag{2.5}$$

Setting up the characteristic equation for system (2.1) with (2.3), (2.4), (2.5) and analyzing its roots, we can find out when and in what mode the shell–mass system will lose stability interacting with the fluid flow. Note that the shell is stable while all characteristic numbers $\lambda = i\Omega$ are on the left half-plane of the complex plane [2]. Otherwise (if at least one characteristic number λ appears on the right half-plane), either static (divergence) or dynamic (flutter) loss of stability may occur. The characteristic number passes to the right half-plane through the origin of coordinates ($\mathrm{Im}\,\lambda = 0$) in the former case and through the point at which $\mathrm{Im}\,\lambda \neq 0$ in the latter case. Proceeding from these conditions, we can determine the critical velocities of the fluid at which either monotonic bulging or vibrations with time-dependent amplitude occur. Let us consider, as an illustration, the four-mode approximation of the deflection w (1.3):

$$w = f_1 \cos sy \sin \lambda_1 x + f_2 \sin sy \sin \lambda_1 x + f_3 \cos sy \sin \lambda_2 x + f_4 \sin sy \sin \lambda_2 x, \tag{2.6}$$

where $s = n/R$; the parameters λ_1 and λ_2 correspond to the low axial modes ($\lambda_1 = \pi/l, \lambda_2 = 2\pi/l$), which are usually used in studying the interaction of these shells with a fluid flow [2, 4, 14]. The system of equations for the unknown functions $f_1, \ldots, f_4$ becomes

$$\ddot{f}_1 + \omega_{11} f_1 + \varepsilon_1 \dot{f}_1 + \beta_1 U \dot{f}_3 + \sum_{k=1}^{4} \chi_{k1} \ddot{f}_k = 0,$$

$$\ddot{f}_2 + \omega_{11} f_2 + \varepsilon_1 \dot{f}_2 + \beta_1 U \dot{f}_4 + \sum_{k=1}^{4} \chi_{k2} \ddot{f}_k = 0,$$

$$\ddot{f}_3 + \omega_{22} f_3 + \varepsilon_2 \dot{f}_3 + \beta_2 U \dot{f}_1 + \sum_{k=1}^{4} \chi_{k3} \ddot{f}_k = 0,$$

$$\ddot{f}_4 + \omega_{22} f_4 + \varepsilon_2 \dot{f}_4 + \beta_2 U \dot{f}_2 + \sum_{k=1}^{4} \chi_{k4} \ddot{f}_k = 0, \tag{2.7}$$

where

$$\omega_{11} = \omega_{n1}^2 - \alpha_1 U^2, \quad \omega_{22} = \omega_{n2}^2 - \alpha_2 U^2, \quad \varepsilon_1 = \varepsilon_0 / M_{01}, \quad \varepsilon_2 = \varepsilon_0 / M_{02},$$

$$\beta_1 = \beta_{n3}^{(1)}, \quad \beta_2 = \beta_{n4}^{(2)}, \quad \alpha_1 = \alpha_{n1}, \quad \alpha_2 = \alpha_{n2}.$$

The constant coefficients χ_{kj} $(k, j = 1, \ldots, 4)$ are determined from (2.5),

$$\chi_{11} = \frac{1}{d}\left[(c_1 - a) - \frac{1}{2\lambda_1}(\sin 2\lambda_1 c_1 - \sin 2\lambda_1 a)\cos^2 sb + \left(b + \frac{1}{2s}\sin 2sb\right)(\sin^2 \lambda_1 a + \sin_2 \lambda_1 c)\right],$$

$$\chi_{12} = \frac{1}{d}\left\{\frac{1}{4}\left[(c_1 - a) - \frac{1}{2\lambda_1}(\sin 2\lambda_1 c_1 - \sin 2\lambda_1 a)\right](\sin^2 \lambda_1 a + \sin^2 \lambda_1 c_1)\right.$$

$$\left. - \frac{1}{4s}(\cos 2sb_1 - \cos^2 sy_0)(\sin^2 \lambda_1 a + \sin^2 \lambda_1 c_1)\right\} \quad (y_0 = 0), \quad \text{etc.} \tag{2.8}$$

The characteristic equation has the form

$$\Delta_0 = \begin{vmatrix} \omega_{11} + (1+\chi_{11})\lambda^2 + \varepsilon_1 \lambda & \chi_{21}\lambda^2 & \chi_{31}\lambda^2 + \beta_1 U\lambda & \chi_{41}\lambda^2 \\ \chi_{12}\lambda^2 & \omega_{11} + (1+\chi_{22})\lambda^2 + \varepsilon_1 \lambda & \chi_{32}\lambda^2 & \chi_{42}\lambda^2 + \beta_1 U\lambda \\ \chi_{13}\lambda^2 + \beta_2 U\lambda & \chi_{23}\lambda^2 & \omega_{22} + (1+\chi_{33})\lambda^2 + \varepsilon_2 \lambda & \chi_{43}\lambda^2 \\ \chi_{14}\lambda^2 & \chi_{24}\lambda^2 + \beta_2 U\lambda & \chi_{34}\lambda^2 & \omega_{22} + (1+\chi_{44})\lambda^2 + \varepsilon_2 \lambda \end{vmatrix} = 0,$$

TABLE 1

$\overline{U}_{\mathrm{d}}$	n							
	2	3	4	5	6	7	8	9
$\overline{U}_{\mathrm{d}}^{(1)}$	9.033	5.763	4.119	**3.439**	3.586	4.441	5.834	7.661
$\overline{U}_{\mathrm{d}}^{(2)}$	11.717	8.445	6.428	5.115	4.303	**3.937**	4.017	4.518

i.e., is generally of the eighth order:

$$\sum_{k=1}^{9} e_k \lambda^{k-1} = 0, \tag{2.9}$$

where e_k ($k = 1, \ldots, 9$) are constant coefficients depending on the geometrical and physical parameters of the shell, the velocity U of the fluid, and the magnitude of the added mass.

Analyzing (2.9), we see that divergent buckling of the shell occurs at fluid velocities satisfying the following equation [2, 14, 15]:

$$e_1 = (\omega_1^2 - \alpha_1 U^2)(\omega_2^2 - \alpha_2 U^2) = 0. \tag{2.10}$$

The minimum of U obtained from (2.10) is the divergence velocity U_{d}, which appears independent of the added mass.

Consider a shell with the following parameters:

$$E_1 = 2.15{\cdot}10^9 \text{ Pa}, \quad E_2 = 1.23{\cdot}10^9 \text{ Pa}, \quad G = 2.15{\cdot}10^8 \text{ Pa},$$

$$\mu_1 = 0.32, \quad \rho = 1.65{\cdot}10^3 \text{ kg/m}^3, \quad R = 0.16 \text{ m}, \quad l = 5R, \quad h = 2.5{\cdot}10^{-3} \text{ m}.$$

The fluid has density $\rho_0 = 10^3$ kg/m^3. Here E_1 and E_2 are the elastic moduli in the longitudinal and circumferential directions, respectively; G is the shear modulus; μ_1 and μ_2 are Poisson's ratios ($E_1\mu_2 = E_2\mu_1$).

Table 1 summarizes the values of divergence velocities $U_{\mathrm{d}}^{(1)} = \omega_1 / \sqrt{\alpha_1}$ and $U_{\mathrm{d}}^{(2)} = \omega_2 / \sqrt{\alpha_2}$ obtained from (2.10).

It can be seen that divergent loss of stability occurs at $\overline{U}_{\mathrm{d}} = \overline{U}_{\mathrm{d}}^{(1)}$, the circumferential mode in which monotonic bulging occurs corresponding to $n = 6$ (hereafter $\overline{U} = U / k_{00}$, where $k_{00} = (\pi^2 / l)\sqrt{D_1 / \rho h}$ [10]). Let us illustrate the determination of the flutter velocity $U = U_{\mathrm{f}}$ by analyzing some typical cases.

Let $c = 0$, $b = 0$, $x_0 = a$, i.e., the mass is concentrated at the point $(x_0, 0)$.

In [15], it was shown that in this case ($\varepsilon_0 = 0$), Eq. (2.9) can be represented by a product of two equations of the fourth order (in λ). The first of these equations can be used to determine the flutter velocity of the shell with a mass attached at a nodal point. The dimensionless values of this velocity are summarized in Table 2. The flutter velocity at which flutter instability occurs the earliest (at the minimum flow velocity) is shown in boldface. In this case, the circumferential wave number $n = 6$.

Table 3 illustrates the influence of the mass M attached at the point $x = l / 2$ on the flutter velocity $\overline{U}_{\mathrm{f}}$ (as before, $\varepsilon_0 = 0$). Comparing Tables 2 and 3 reveals that the added mass decreases the flatter velocity $\overline{U}_{\mathrm{f}}$ (boldfaced). The larger the added mass, the lower the flutter velocity.

Let now $c = 0$, $2b = 2\pi R$, $x_0 = a$, i.e., the mass is distributed over a closed transverse ring with longitudinal coordinate x_0. Table 4 summarizes the values of $\overline{U}_{\mathrm{f}}$ for $x_0 = l / 2$.

The effect of the mass (of the same magnitude) on the flutter velocity is stronger than in the previous case.

Similar results are obtained for the mass distributed over an open transverse ring (for $b < \pi R$).

Table 5 collects values of $\overline{U}_{\mathrm{f}}$ for an added mass uniformly distributed along the length of the shell ($c = l$, $b = 0$).

TABLE 2

n	2	3	4	5	6	7	8
$\overline{U}_{\mathrm{f}}$	13.343	9.452	7.142	5.709	**4.958**	4.987	5.958

TABLE 3

M / M_{s}	n						
	2	3	4	5	6	7	8
0.05	13.341	9.451	7.141	5.707	**4.956**	4.984	5.955
0.15	13.338	9.447	7.137	5.704	**4.952**	4.978	5.950
0.25	13.334	9.444	7.134	5.701	**4.948**	4.972	5.944

The effect of the mass so attached is similar to the effect of the concentrated mass attached at a point. The larger the mass, the stronger the effect of the type of its attachment. This conclusion is confirmed numerically for segments $c < l$ ($c \neq 0$) of the shell length.

The above results allow evaluating the effect of added masses distributed along the whole rectangular boundary on U_{f}. It is obvious that this effect is the strongest when the masses are distributed along the arcs *AC* and *BD* and is weaker when the masses are distributed along the sides *AB* and *CD* parallel to the shell axis. Note that this comparison is correct if the mass per unit length is the same on the intervals *AC*, *BD* and on *AB*, *CD*.

For $\varepsilon_0 \neq 0$, the numerical analysis shows that dynamic (flutter) loss of stability occurs directly after divergence, i.e., the flutter velocities U_{f} are equal ($U_{\mathrm{f}} = U_{\mathrm{d}}^{(2)}$).

For the mass *M* within the limits $0 \le M \le 0.25 M_{\mathrm{s}}$, the value of $\overline{U}_{\mathrm{f}}$ is hardly dependent on the type of attachment.

3. Stability of a Shell with Mass Distributed along an Elliptic Boundary. A similar approach can be used for stability analysis of the shell with added mass shown in Fig. 2. It should be noted that the coordinates *x* and *y* of the elliptic boundary with semiaxes d_1 and d_2 are related by a well-known equation, which yields the following expressions for the function $y = y(x)$ for $y < 0$ and $y > 0$, respectively:

$$y = y_1 = -\frac{d_1}{d_2}\sqrt{d_2^2 - (x - c_0)^2},$$

$$y = y_2 = +\frac{d_1}{d_2}\sqrt{d_2^2 - (x - c_0)^2}.$$

Since the arc length element $d\overline{s}$ of an ellipse is defined by

$$d\overline{s} = \sqrt{(dx)^2 + (dy)^2} = F(x)\,dx \qquad \left(F(x) = \sqrt{\frac{d_2^4 + (d_1^2 - d_2^2)(x - c_0)^2}{d_2^2 [d_2^2 - (x - c_0)^2]^2}},\ c_0 - d_2 \le x \le c_0 + d_2 \right),$$

we can finely derive expressions for the functions G_1^{nm} and G_2^{nm} in Eqs. (2.3):

TABLE 4

M / M_s	n						
	2	3	4	5	6	7	8
0.05	13.338	9.447	7.137	5.704	**4.953**	4.978	5.946
0.15	13.328	9.438	7.128	5.694	**4.941**	4.962	5.923
0.25	13.318	9.428	7.119	5.685	**4.930**	4.945	5.899

TABLE 5

M / M_s	n						
	2	3	4	5	6	7	8
0.05	13.342	9.451	7.141	5.707	4.957	4.985	5.956
0.15	13.338	9.447	7.138	5.704	4.953	4.981	5.953
0.25	13.335	9.444	7.135	5.701	4.950	4.977	5.950

$$G_1^{nm} = \frac{1}{l_0 h} \int\limits_{c_0-d_2}^{c_0+d_2} \left\{ \frac{\partial^2 w(x, y, t)}{\partial t^2} \cos s_n y \sin \lambda_m x \left[\delta(y-y_1) + \delta(y-y_2) \right] \right\} F(x)\, dx,$$

$$G_2^{nm} = \frac{1}{l_0 h} \int\limits_{c_0-d_2}^{c_0+d_2} \left\{ \frac{\partial^2 w(x, y, t)}{\partial t^2} \sin s_n y \sin \lambda_m x \left[\delta(y-y_1) + \delta(y-y_2) \right] \right\} F(x)\, dx, \tag{3.1}$$

where l_0 is the length of an ellipse (expressed in terms of an elliptic integral of the second kind).

It is obvious that the governing equations, which can be used to analyze the effect of added mass, as before, have the form (2.11) in view of (2.10).

Unlike a rectangular boundary, however, the coefficients χ_{kj} obtained with the BG method cannot be represented in closed form in terms of elementary functions (similarly to (2.8)). To determine them, it is necessary (according to formulas (3.1)) to use direct numerical integration.

Conclusions. We have proposed and numerically tested a method for stability analysis of cylindrical composite (orthotropic model) shells with added mass interacting with the internal fluid flow.

Divergent (monotonic) and flutter (vibratory) loss of stability of the shell occurring at certain fluid velocities have been considered. The effect of various types of attachment of added masses to the shell (along a ring, length, rectangular and elliptic boundaries) on the critical velocities of the fluid flow has been studied.

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