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DETERMINING THE AXISYMMETRIC THERMOELASTOPLASTIC STATE OF THIN SHELLS WITH ALLOWANCE FOR THE THIRD INVARIANT OF THE DEVIATORIC STRESS TENSOR

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A technique for determining the axisymmetric thermoelastoplastic state of thin shells with allowance for the third invariant of the deviatoric stress tensor is developed. The technique is based on the theory of thin shells that incorporates transverse shear and torsional strains. The equations of thermoplasticity relating the stress components in Euler coordinates with the components of the linear part of the finite-strain tensor are used as constitutive equations. The nonlinear scalar functions in the constitutive equations are determined from reference tests on tubular specimens under proportional loading at different temperatures and stress mode angles. The boundary-value problem is solved by numerically integrating a system of ordinary differential equations using Godunov's discrete orthogonalization. The thermoelastoplastic stress–strain state of a corrugated shell is analyzed as an example

Keywords: thermoelastoplastic state, thin shell, stress mode, shell of revolution

Introduction. The elastoplastic stress–strain state (SSS) of shells of revolution under isothermal loading was analyzed in [2, 3] taking the third invariant of the deviatoric stress tensor into account. The papers [2, 3] outline techniques for solving axisymmetric problems of plasticity for thin shells based on the Kirchhoff–Love [2] and straight-line [3] hypotheses and the constitutive equations from [5, 6]. These equations relate the stress components in Euler coordinates and the components of the linear part of the finite-strain tensor. The chosen stress and strain measures are energy-consistent. These equations employ the nonlinear relationships between the first invariants of the stress and strain tensors and between the second invariants of the deviatoric stress and strain tensors. These relationships are established in tests on tubular specimens under proportional loading at different values of the stress mode angle, which depends on the third invariant of the deviatoric stress tensor. The constitutive equations from [5, 6] were modified in [7] to include the case of nonisothermal loading and used in [1] to solve, based on the Kirchhoff–Love hypotheses, an axisymmetric problem of thermoplasticity for thin shells having temperature-dependent material properties and undergoing no torsion. In contrast to [1], we will outline a technique developed to solve a similar problem based, however, on the kinematic straight-line element model that allows for transverse shear and torsional strains.

1. Problem Statement. Consider a thin shell of revolution made by joining elements with differently shaped meridian. The position of an arbitrary point of the shell is defined by curvilinear orthogonal coordinates s, φ, ς, where $s\,(s_0 \le s \le s_n)$ is the arc length of the middle meridian, φ is the circumferential coordinate, and ς is the distance from the point to the mid-surface. Let the shell be such that ςk_s and ςk_φ (k_s and k_φ are the principal curvatures of the mid-surface) can be neglected compared with unity. We will also assume (as in shell theory) that the normal stress $\sigma_{\varsigma\varsigma}$ may be neglected compared with the other normal stresses:

$$\sigma_{\varsigma\varsigma} = 0. \tag{1.1}$$

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Let the shell, being undeformed and at temperature $T = T_0$, be subject to nonuniform heating and loading that cause an axisymmetric SSS, torsion, and large plastic strains. The unloading is not accompanied by secondary plastic strains, and creep strains are negligibly small compared with instantaneous elastoplastic strains. We will use a geometrically linear quasistatic problem formulation. The loading process is divided into short steps such that the loading history is described in the best possible way. At each step of loading, the problem of thermoplasticity will be solved with the method of successive approximations.

2. Kinematic and Static Equations. To solve the problem, we will use the kinematic equations of the straight-element model [3]:

$$\varepsilon_s = u' + k_s w, \qquad \varepsilon_\varphi = \rho u + k_\varphi w,$$

$$\kappa_s = \psi_s', \qquad \kappa_\varphi = \rho \psi_s,$$

$$\Omega = v' - \rho v, \qquad \tau = \psi_\varphi' - \rho \psi_\varphi, \qquad \rho = \frac{\cos\theta}{r}, \tag{2.1}$$

where

$$\psi_s = \vartheta_s + \gamma_s, \quad (s, \varphi),$$

$$\vartheta_s = k_s u - w', \qquad \vartheta_\varphi = k_\varphi v,$$

$$k_s = \theta', \qquad k_\varphi = \frac{\sin\theta}{r},$$

$$(\ldots)' = \frac{d(\ldots)}{ds}, \tag{2.2}$$

where ε_s and ε_φ are the strain components of the mid-surface in the s- and φ-directions; κ_s and κ_φ are parameters characterizing the variation in the curvature of the midsurface in the same directions; Ω and τ are parameters characterizing the variation in the angle between the s- and φ-axes; u and v are the displacements of particles of the midsurface in the s- and φ-directions; w is deflection; ψ_s and ψ_φ are the complete angles of rotation of the straight element; ϑ_s and ϑ_φ are the angles of rotation of the normal to the midsurface; γ_s and γ_φ are the angles of rotation caused by transverse shear; r is the parallel radius of the midsurface; $(\pi - \theta)$ is the angle between the normal to this surface and the z-axis of revolution; (s, φ) denotes the circular permutation of the indices s and φ.

The strain components at an arbitrary point of the shell are related to the strain components of its midsurface as follows:

$$\varepsilon_{ss} = \varepsilon_s + \varsigma \kappa_s, \qquad 2\varepsilon_{s\varphi} = \Omega + \varsigma \tau,$$

$$2\varepsilon_{s\varsigma} = \gamma_s \qquad (s, \varphi). \tag{2.3}$$

The static equilibrium equations for a shell element under axisymmetric loading and torsion are as follows [3]:

$$(rN_s)' - r\rho N_\varphi + rk_s Q_s + rq_s = 0,$$

$$(rN_{\varphi s})' + r\rho N_{\varphi s} + rk_\varphi Q_\varphi + rq_\varphi = 0,$$

$$(rM_s)' - r\rho M_\varphi - rQ_s + rm_s = 0,$$

$$(rM_{\varphi s})' + r\rho M_{\varphi s} - rQ_\varphi + rm_\varphi = 0,$$

$$(rQ_s)' - rk_s N_s - rk_\varphi N_\varphi + rq_\varsigma = 0, \tag{2.4}$$

where N_s, Q_s, and M_s are the normal and transverse forces and bending moment acting in the section $s =$const; N_φ, Q_φ, and M_φ are the normal and transverse forces and bending moment acting in the section $\varphi =$const; $N_{\varphi s}$ and $M_{\varphi s}$ are the shearing force and twisting moment acting in the same sections; q_s, q_φ, and q_ς are distributed loads referred to the midsurface; m_s and m_φ are the distributed moments induced by these loads.

3. Constitutive Equations. We will use the equations from [7] describing nonisothermal loading when the material properties depend on temperature. These equations relate the components of the stress tensor σ_{ij} in Euler coordinates and the components of the linear part of the finite-strain tensor ε_{ij}. According to assumption (1.1), we represent the equations [7] as Hooke's law with additional stresses:

$$\sigma_{ss} = \frac{2G}{1-\nu}(\varepsilon_{ss} + \nu\varepsilon_{\varphi\varphi}) - \sigma_{ss}^{\mathrm{ad}},$$

$$\sigma_{\varphi\varphi} = \frac{2G}{1-\nu}(\nu\varepsilon_{ss} + \varepsilon_{\varphi\varphi}) - \sigma_{\varphi\varphi}^{\mathrm{ad}},$$

$$\sigma_{s\varphi} = 2G\varepsilon_{s\varphi} - \sigma_{s\varphi}^{\mathrm{ad}},$$

$$\sigma_{s\varsigma} = 2G\varepsilon_{s\varsigma} - \sigma_{s\varsigma}^{\mathrm{ad}},$$

$$\sigma_{\varphi\varsigma} = 2G\varepsilon_{\varphi\varsigma} - \sigma_{\varphi\varsigma}^{\mathrm{ad}}, \tag{3.1}$$

where $\sigma_{ij}^{\mathrm{ad}}$ are the additional stresses determined from the SSS found at the previous iteration:

$$\sigma_{ss}^{\mathrm{ad}} = \frac{2G}{1-\nu}\left[e_{ss}^{\mathrm{p}} + \nu e_{\varphi\varphi}^{\mathrm{p}} + (1+\nu)(\varepsilon_T + \varepsilon_0^P)\right],$$

$$\sigma_{\varphi\varphi}^{\mathrm{ad}} = \frac{2G}{1-\nu}\left[\nu e_{ss}^{\mathrm{p}} + e_{\varphi\varphi}^{\mathrm{p}} + (1+\nu)(\varepsilon_T + \varepsilon_0^P)\right],$$

$$\sigma_{s\varphi}^{\mathrm{ad}} = 2Ge_{s\varphi}^{\mathrm{p}},$$

$$\sigma_{s\varsigma}^{\mathrm{ad}} = 2Ge_{s\varsigma}^{\mathrm{p}},$$

$$\sigma_{\varphi\varsigma}^{\mathrm{ad}} = 2Ge_{\varphi\varsigma}^{\mathrm{p}}. \tag{3.2}$$

The strain $\varepsilon_{\varsigma\varsigma}$ is determined from assumption (1.1) and the equation [7] for the stress $\sigma_{\varsigma\varsigma}$:

$$\varepsilon_{\varsigma\varsigma} = -\frac{1}{1-\nu}\left[\nu(\varepsilon_{ss} + \varepsilon_{\varphi\varphi}) + (1-2\nu)(e_{ss}^{\mathrm{p}} + e_{\varphi\varphi}^{\mathrm{p}}) - (1+\nu)(\varepsilon_T + \varepsilon_0^{\mathrm{p}})\right], \tag{3.3}$$

where $\varepsilon_T = \alpha_T(T - T_0)$ is the thermal strain; G, ν, and α_T are the temperature-dependent shear modulus, Poisson's ratio, and thermal linear expansion coefficient, respectively; $\varepsilon_0^{\mathrm{p}}$ is the plastic component of the mean strain $\varepsilon_0 = \frac{1}{3}(\varepsilon_{ss} + \varepsilon_{\varphi\varphi} + \varepsilon_{\varsigma\varsigma})$; e_{ij}^{p} are the components of the deviatoric plastic-strain tensor. These equations describe the deformation of the body's element along small-curvature paths and differ from the well-known equations [4] by the term $\varepsilon_0^{\mathrm{p}}$,

$$\sigma_0 = F_1(\varepsilon_0 - \varepsilon_T, T, \omega_\sigma), \tag{3.4}$$

where $\sigma_0 = \frac{1}{3}(\sigma_{ss} + \sigma_{\varphi\varphi})$ is the mean stress; ω_σ is the stress mode angle,

$$\omega_\sigma = \frac{1}{3}\arccos\left[-\frac{3\sqrt{3}}{2}\frac{J_3(D_\sigma)}{S^3}\right], \tag{3.5}$$

where $J_3(D_\sigma) = \det(s_{ij})$ is the third invariant of the deviatoric stress tensor D_σ; s_{ij} are the components of the deviatoric stress tensor; S is the shear-stress intensity,

$$S=\left\{\frac{1}{3}(\sigma_{ss}^2-\sigma_{ss}\sigma_{\varphi\varphi}+\sigma_{\varphi\varphi}^2)+\sigma_{s\varphi}^2+\sigma_{s\varsigma}^2+\sigma_{\varphi\varsigma}^2\right\}^{1/2}. \tag{3.6}$$

It is assumed that the components of the deviatoric strain tensor in Eqs. (3.1) have elastic ($e_{ij}^{\mathrm{e}}=\frac{s_{ij}}{2G}$) and plastic ($e_{ij}^{\mathrm{p}}$) components. The plastic components are determined from the coaxialty of the director stress tensor and the director plastic strain rate tensor, i.e., $\frac{s_{ij}}{S}=\frac{de_{ij}^{\mathrm{p}}}{d\Gamma^{\mathrm{p}}}$, where $d\Gamma^{\mathrm{p}}$ is the plastic shear strain intensity differential. The total components of the deviatoric plastic strain tensor and the total shear plastic strain intensity are determined as the sums of their increments over all steps of loading:

$$e_{ij}^{\mathrm{p}}=\sum_k \Delta_k e_{ij}^{\mathrm{p}},\qquad \Delta_k e_{ij}^{\mathrm{p}}=\left\langle\frac{s_{ij}}{S}\right\rangle\Delta_k\Gamma^{\mathrm{p}}, \tag{3.7}$$

$$\Gamma^{\mathrm{p}}=\sum_k \Delta_k\Gamma^{\mathrm{p}}, \tag{3.8}$$

where the angular brackets denote averaging over the kth step.

The increment of the shear plastic strain intensity $\Delta_k\Gamma^{\mathrm{p}}$ is determined from the nonlinear formula

$$S=F_2(\Gamma,T,\omega_\sigma). \tag{3.9}$$

We assume that, when the body's element deforms along small-curvature paths, the shear strain intensity Γ includes elastic (Γ^{e}) and plastic (Γ^{p}) components:

$$\Gamma=\Gamma^{\mathrm{e}}+\Gamma^{\mathrm{p}}=\frac{S}{2G}+\Gamma^{P}. \tag{3.10}$$

The scalar functions F_1 and F_2 are determined from tests on tubular specimens under proportional loading at different temperatures T and stress mode angles ω_σ. At a fixed temperature, these functions are stress–strain curves at different values of the angle ω_σ. At a fixed angle, these functions describe instantaneous thermomechanical surfaces [4]. In [5–7], the functions F_1 and F_2 were determined in tests on tubular specimens under a combination of tensile force and internal pressure at different temperatures and $\omega_\sigma=0°, 30°, 60°$. It was established by calculation that when stress–strain curves are plotted for non-reference values of T and ω_σ, these functions permit linear interpolation. Then $\varepsilon_0^{\mathrm{p}}$ and $\Delta\Gamma_p$ can be determined from the corresponding stress–strain curves by the formulas

$$\varepsilon_0^{\mathrm{p}}=|\varepsilon_0-\varepsilon_T|-\frac{F_1}{K},\qquad \Delta\Gamma^{\mathrm{p}}=\frac{S-F_2}{2G}, \tag{3.11}$$

where $K=\frac{2G(1+\nu)}{1-2\nu}$ is the dilatation modulus.

Transforming Eqs. (3.1) to forces and moments, we arrive at the following constitutive equations:

$$\begin{pmatrix}\vec{X}_s\\ \vec{X}_\varphi\end{pmatrix}=\begin{bmatrix}[C] & [K]\\ [K]^T & [D]\end{bmatrix}\begin{pmatrix}\vec{\varepsilon}_s\\ \vec{\varepsilon}_\varphi\end{pmatrix}-\begin{pmatrix}\vec{X}_s^{\mathrm{ad}}\\ \vec{X}_\varphi^{\mathrm{ad}}\end{pmatrix},$$

$$\vec{Q}=[L]\vec{\gamma}-\vec{Q}^{\mathrm{ad}}, \tag{3.12}$$

where

$$\vec{X}_s=\{N_s,N_{s\varphi},M_s,M_{s\varphi}\}^T,$$

$$\vec{\varepsilon}_s = \{\varepsilon_s, \omega_s, \kappa_s, \tau_s\}^T \quad (s, \varphi),$$

$$\vec{Q} = \{Q_s, Q_\varphi\}^T, \qquad \vec{\gamma} = \{\gamma_s, \gamma_\varphi\}^T,$$

$$\vec{X}_s^{\text{ad}} = \{N_s^{\text{ad}}, N_{s\varphi}^{\text{ad}}, M_s^{\text{ad}}, M_{s\varphi}^{\text{ad}}\}^T \qquad (s, \varphi),$$

$$\vec{Q}^{\text{ad}} = \{Q_s^{\text{ad}}, \sigma_\varphi^{\text{ad}}\}^T, \tag{3.13}$$

where $[C]$, $[K]$, $[D]$, $[L]$ are stiffness matrices obtained by integrating Eqs. (3.1) over the thickness of the shell; $N_s^{\text{ad}}, \ldots, Q_\varphi^{\text{ad}}$ are the additional forces and moments.

4. Governing System of Equations. Equations (2.1)–(2.4), (3.12), (3.13) constitute a closed system of equations for determining the SSS at each iteration of an arbitrary step of loading. We use these equations to reduce the thermoplactic problem to the system of differential equations

$$\vec{Y}' = [P]\vec{Y} + \vec{f} \tag{4.1}$$

for the vector $\vec{Y}$ of unknown functions in terms of which the boundary conditions at the shell ends are formulated:

$$\vec{Y} = \{\vec{N}, \vec{u}\}^T,$$

$$\vec{N} = r\{N_s, N_{\varphi s}, M_s, M_{\varphi s}, Q_s\}^T,$$

$$\vec{u} = \{u, v, \psi_s, \psi_\varphi, w\}^T. \tag{4.2}$$

The boundary conditions at $s = s_0$ and $s = s_n$ are

$$[G]\vec{Y} = \vec{g} \qquad (i = 0, n), \tag{4.3}$$

where $[P]$ and $\vec{f}$ are the matrix of the system of equations and the vector of free terms; $[G]$ and $\vec{g}_i$ are the given matrix and vector of boundary conditions.

At each step of loading, the thermoplastic problem is solved by the method of successive approximations in each of which the values of the additional stresses σ_{ij}^{ad} are corrected. In each approximation, the boundary-value problem (4.1)–(4.3) is reduced to Cauchy problems, which are integrated by the Runga–Kutta method in combination with Godunov's discrete orthogonalization.

5. Problem-Solving Algorithm. Let the values of $\sigma_{ij}^{(k-1)}$, $\varepsilon_{ij}^{(k-1)}$, $\sigma_{ij}^{\text{ad}(k-1)}$, $\Gamma^{\text{p}(k-1)}$, $\varepsilon_0^{\text{p}(k-1)}$ $(i, j = s, \varphi, \varsigma)$ have been found at the $(k-1)$th step of loading. As the first approximation of the kth step, we solve the boundary-value problem (4.1)–(4.3) with known $\sigma_{ij}^{\text{ad}(k-1)}$, which yields the vector of unknown functions $\vec{Y}^{k,1}$. These functions are then used to determine, at each point of the shell, the strains $\varepsilon_{ss}^{k,1}, \varepsilon_{s\varphi}^{k,1}, \varepsilon_{s\varsigma}^{k,1}, (s, \varphi)$ (2.3) and then the strain $\varepsilon_{\varsigma\varsigma}^{k,1}$ (3.3), the strains ε_{ss}, $\varepsilon_{s\varphi}$, and ε_T corresponding to the first approximation of the current step, while e_{ss}^{p}, $e_{\varphi\varphi}^{\text{p}}$, ε_0^{p} corresponding to the last approximation of the previous step. Next, we determine the mean strain $\varepsilon_0^{k,1} = \frac{1}{3}(\varepsilon_{ss}^{k,1} + \varepsilon_{\varphi\varphi}^{k,1} + \varepsilon_{\varsigma\varsigma}^{k,1})$, the components of the stress tensor $\sigma_{ij}^{k,1}$ (3.1), the mean stress $\sigma_0^{k,1} = \frac{1}{3}(\sigma_{ss}^{k,1} + \sigma_{\varphi\varphi}^{k,1})$, the components of the deviatoric stress tensor $s_{ij}^{k,1}$, and the shear-stress intensity $S^{k,1}$ (3.6).

After that, we determine the stress mode angle $\omega_\sigma^{k,1}$ (3.5) and the shear-strain intensity $\Gamma^{k,1} = \Gamma_{\text{p}}^{(k-1)} + \dfrac{S^{k,1}}{2G(T_k)}$ at each point of the shell. By linear interpolation of surface (3.9) with respect to temperature and angle $\omega_\sigma^{k,1}$, we find

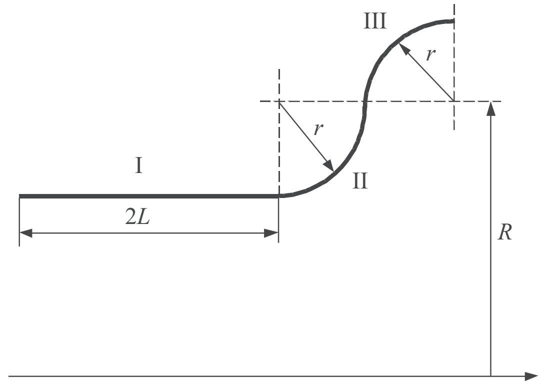

Fig. 1

$S_{\text{ad}}^{k,1} = F_2(\Gamma^{k,1}, T_k, \omega_{\sigma}^{k,1})$ and the increment of the plastic shear strain intensity $\Delta_{k,1}\Gamma^{\text{p}} = \dfrac{S^{k,1} - S_{\text{ad}}^{k,1}}{2G(T_k)}$. If $\Delta_{k,1}\Gamma^{\text{p}} \ge 0$, the process is active loading; otherwise, it is unloading, and it is necessary to set $\Delta_{k,1}\Gamma^{\text{p}} = 0$. The loading direction found in the first approximation is used in all the subsequent approximations of the current step. The increment $\Delta_k\Gamma^{\text{p}}$ is determined as the sum of increments found in all approximations, while the total plastic-strain intensity is found from (3.8). Next, we determine the increments of the deviatoric plastic-strain tensor $\Delta_{k,1}e_{ij}^{\text{p}} = \left\langle \dfrac{s_{ij}^{k,1}}{S^{k,1}} \right\rangle \Delta_{k,1}\Gamma^{p}$ and the total deviatoric plastic-strain tensor $e_{ij}^{\text{p}(k,1)}$ (3.7).

In the case of active loading, we find $F_1^{k,1}$ and $\varepsilon_0^{\text{p}(k,1)} = |\varepsilon_0^{k,1} - \varepsilon_T| - \dfrac{F_1^{k,1}}{K(T_k)}$ by linear interpolation with respect to temperature and angle $\omega_{\sigma}^{k,1}$ using (3.4). In the case of unloading, we set $\varepsilon_0^{\text{p}(k,1)} = \varepsilon_0^{\text{p}(k-1)}$. Then the additional stresses $\sigma_{ij}^{\text{ad}(k,1)}$ (2.7) are corrected and the boundary-value problem is solved in the second approximation to determine the vector of unknown functions $\vec{Y}^{k,2}$, the strains $\varepsilon_{ss}^{k,2}$, $\varepsilon_{s\varphi}^{k,2}$, $\varepsilon_{s\varsigma}^{k,2}$ (s, φ), and the strain $\varepsilon_{\varsigma\varsigma}^{k,2}$, and so on. Unlike the first approximation, the strains ε_{ss} and $\varepsilon_{\varphi\varphi}$ in (3.3) correspond to the second approximation, while e_{ss}^{p}, $e_{\varphi\varphi}^{\text{p}}$, ε_0^{p} to the first approximation. The process of successive approximations at the *k*th step is terminated once the relative changes the increments of the plastic shear strain intensity $\Delta_k\Gamma^{\text{p}}$ over the step found in two successive approximations has differed by a small amount δ, which is the error of solution of the plastic problem.

In [1], the convergence of the process of successive approximations was tested by comparing the values of σ_0 found by the formula $\sigma_0 = \dfrac{\sigma_{ss} + \sigma_{\varphi\varphi}}{3}$ and formula (3.4) and it was shown that it is sufficient to trace the increment $\Delta_k\Gamma^{\text{p}}$ alone.

To validate the results obtained with the above algorithm, it is necessary to refine the spatial mesh and steps of loading.

6. Example. Let us analyze the thermoelastoplastic SSS of a corrugated shell whose middle meridian is shown in Fig. 1. The geometrical parameters: $R = 0.14$ m, $r = 0.04$ m, $2L = r\pi$ m, thickness $h = 0.004$ m. The shell is composed of a cylindrical segment of length $2L$ and two toroidal segments of arc length L each. The segments are numbered as I, II, III and are smoothly joined without discontinuity of the derivative θ'.

The shell is in a stationary temperature field varying across the thickness as

$$T = T_2 + \frac{T_1 - T_2}{\ln\left(\dfrac{r - h/2}{r + h/2}\right)} \ln\left(\frac{r + \varsigma}{r + h/2}\right),$$

TABLE 1

$\varepsilon_0 - \varepsilon_T$	$T = 293$ K			$T = 773$ K		
	$\omega_\sigma = 0°$	$\omega_\sigma = 30°$	$\omega_\sigma = 60°$	$\omega_\sigma = 0°$	$\omega_\sigma = 30°$	$\omega_\sigma = 60°$
0.0000	0.0	0.0	0.0	0.0	0.0	0.0
0.0002	88.9	88.9	88.9	71.8	71.8	71.8
0.0006	243.8	207.1	121.4	165.9	131.2	88.1
0.0010	261.9	214.9	133.1	178.5	138.8	99.9
0.0024	295.7	233.4	154.2	205.3	151.4	115.5
0.0030	306.0	240.4	160.7	207.4	154.0	119.4
0.0040	316.6	252.0	169.0	211.4	159.0	123.9
0.0060	336.6	267.0	179.5	215.4	164.0	129.3
0.0080	356.6	282.0	186.2	219.0	169.0	133.0
0.0200	373.0	365.0	188.0	239.8	197.0	154.0
0.0400	411.6	431.6	199.5	274.5	243.7	189.0

TABLE 2

Γ	$T = 293$ K			$T = 773$ K		
	$\omega_\sigma = 0°$	$\omega_\sigma = 30°$	$\omega_\sigma = 60°$	$\omega_\sigma = 0°$	$\omega_\sigma = 30°$	$\omega_\sigma = 60°$
0.0000	0.0	0.0	0.0	0.0	0.0	0.0
0.0002	32.2	32.2	32.2	26.0	26.0	26.0
0.0100	166.6	166.6	166.6	104.3	104.3	104.3
0.0200	188.9	180.0	191.6	128.6	128.6	128.6
0.0400	220.5	204.0	223.0	149.0	137.0	164.6
0.0600	243.0	222.8	247.8	166.0	145.2	187.6
0.0800	261.9	235.3	268.0	178.6	152.6	200.4
0.1000	276.1	246.3	287.9	183.8	156.7	211.9
0.1600	310.0	279.3	325.3	199.4	169.0	231.8
0.2000	332.6	301.3	350.2	209.8	177.2	245.1
0.2500	360.9	328.8	381.4	222.8	187.5	261.7

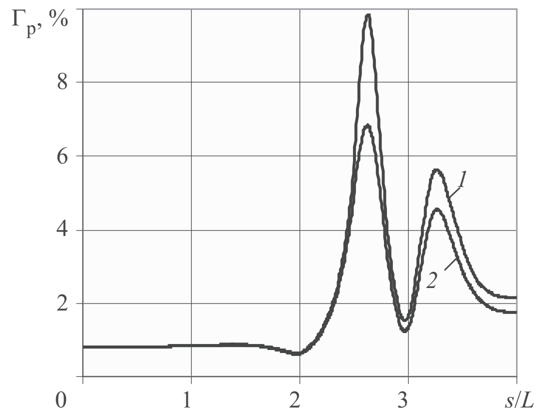

Fig. 2

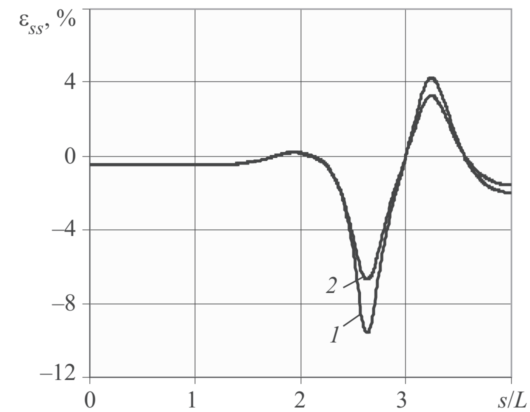

Fig. 3

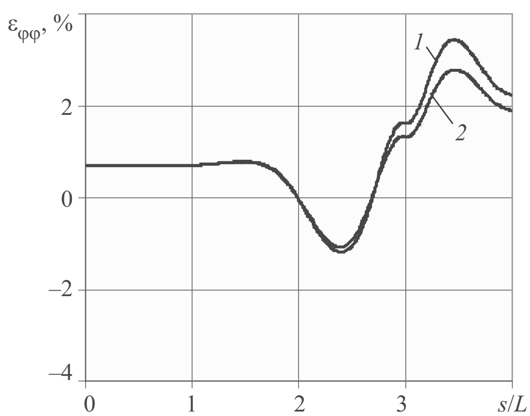

Fig. 4

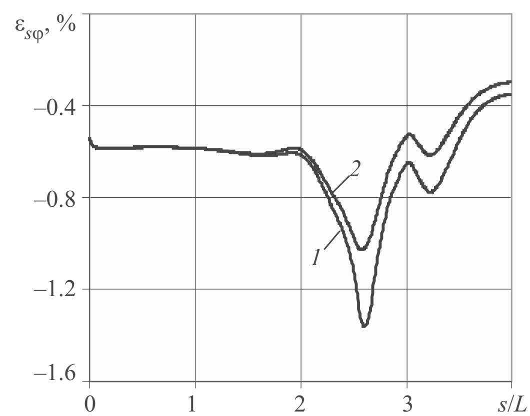

Fig. 5

where $T_1 = 473$ K is the temperature of the inside surface; $T_2 = 323$ K is the temperature of the outside surface; $T_0 = 293$ K is the initial temperature.

The shell is subject to internal pressure q_ς, axial compressive force N_s^*, and shearing force $N_{\varphi s}^*$ that are proportional to the parameter p: $q_\varsigma = p$ MPa, $N_s^* = -0.1p$ MPa·m, $N_{\varphi s}^* = -0.1p$ MPa·m.

Let $p = 0.5, 1.0, 1.2, 1.4, \ldots, 3.6, 3.8$. The boundary conditions: $N_s = N_s^*$, $N_{s\varphi} = N_{\varphi s}^*$, $M_{s\varphi} = Q_s = \psi_s = 0$ at $s = s_0$ and $Q_s = u = v = \psi_s = \psi_\varphi = 0$ at $s = s_n$. The shell is made of Kh18N10T steel with Poisson's ratio $\nu = 0.27$ and linear thermal expansion coefficient $\alpha_T = 12\cdot10^{-6}$ K^{-1}. The values of the functions F_1 and F_2 are summarized in Tables 1 and 2, respectively.

The number of points of integration along the length of each segment $K_s^i = 401, 201, 201$, and the number of points of integration across the thickness $K_\varsigma = 17$. The error of solution $\delta = 0.001$. To check the accuracy of the results, we compared them with those obtained with a double mesh spacing ($K_s^i = 201, 101, 101$, $K_\varsigma = 9$). The former partition provides convergence (the difference between maximum strains is no greater than 5%). We also compared the results obtained with and without regard to the stress mode. In the latter case, it was assumed that $\varepsilon_0^p = 0$ ($\sigma_0 = K\varepsilon_0$), and function (3.9) was determined in tension tests ($\omega_\sigma = 60°$).

By solving the problem, we establish that all elements of the shell are subject to active loading. The plastic strains on the surfaces of the shell are large. The angle ω_σ at different points varies from 0 to 59°. At the beginning of the process, the results obtained in both cases are similar. At $\Gamma_p \approx 2\%$, the difference between strains does not exceed 7%. Then this difference increases, reaching 44% at the last step.

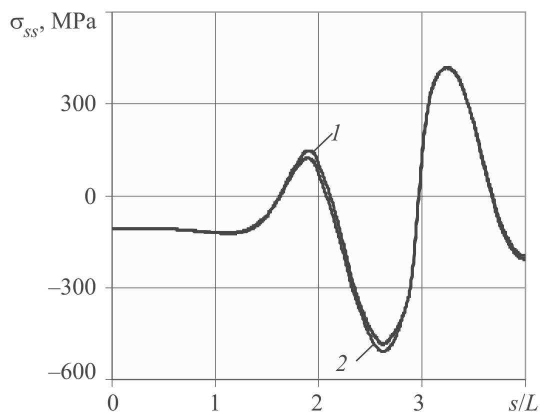

Fig. 6

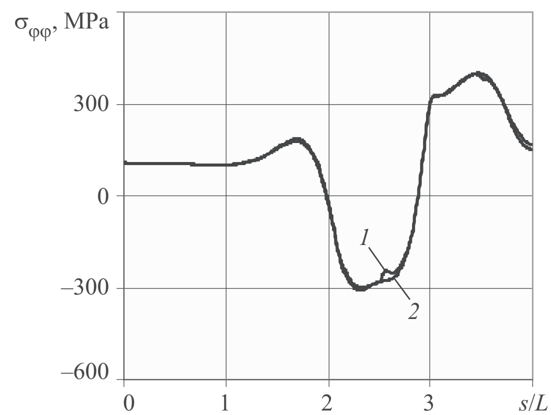

Fig. 7

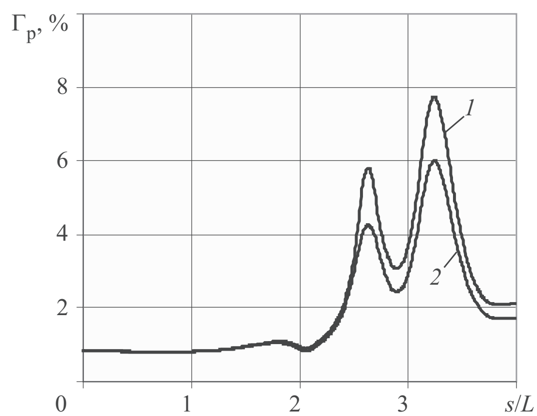

Fig. 8

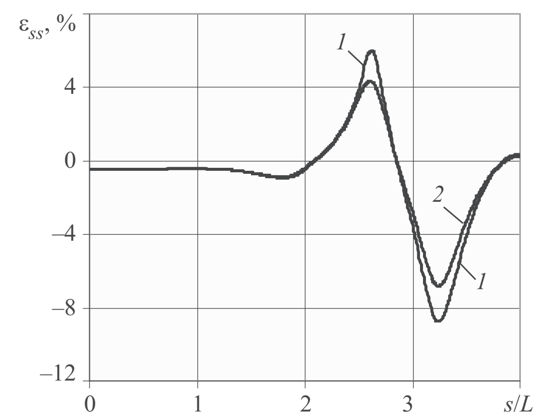

Fig. 9

Figures 2–9 show calculated strains on the outside surface at the last step of the process with (curves *1*) and without (curves *2*) regard to the stress mode.

Figure 2 demonstrates the meridional distribution of the plastic shear strain intensity Γ_p. Figures 3–5 show the variation in the strains ε_{ss}, $\varepsilon_{\varphi\varphi}$, $\varepsilon_{s\varphi}$ in the same direction.

It can be seen that the maximum values of the strains ε_{ss} obtained in the two cases differ by 45%, the strains $\varepsilon_{\varphi\varphi}$ differ by 25%, and the strains $\varepsilon_{s\varphi}$ by 33%. Contrastingly, the values of the stresses σ_{ss} and $\sigma_{\varphi\varphi}$ (Figs. 6 and 7) differ insignificantly between the cases.

For comparison, Figs. 8 and 9 show the meridional distribution of the plastic shear strain intensity Γ_p and the strains ε_{ss} on the inside surface. As is seen, the maximum strains on the inside surface are somewhat lower than on the outside surface.

Conclusions. We have developed a technique for solving the axisymmetric problem of thermoplasticity for thin isotropic shells based on constitutive equations that incorporate the third invariant of the deviatoric stress tensor. The nonlinear scalar functions in the constitutive equations have been determined in reference tests on tubular specimens under proportional loading at different temperatures and stress mode angles. The boundary-value problem has been reduced to the numerical integration of a system of ordinary differential equations. The example considered has demonstrated that the results obtained with and without regard to the stress mode are in good agreement only for plastic strain intensities lower than 2%. As the strains increase, the agreement becomes worse and the difference between the results reaches 45%.

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