International Applied Mechanics, Vol. 49, No. 6, November, 2013

COUPLED PROCESSES OF DEFORMATION AND LONG-TERM DAMAGE OF PHYSICALLY NONLINEAR LAMINATED MATERIALS

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A theory of long-term damage of laminated materials with physically nonlinear components is proposed. The damage of components is modeled by random micropores. The damage criterion for a microvolume is characterized by its stress-rupture strength. It is determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its critical value, which is the tensile strength, according to the Huber–Mises criterion, and assumed to be a random function of coordinates. An equation of damage (porosity) balance in the components at an arbitrary time is formulated. Algorithms of calculating the time dependence of microdamage and macrostresses are developed and respective curves are plotted. The influence of the nonlinearity of the matrix on the macrostress–macrostrain and damage curves is studied

Keywords: laminated material, physical nonlinearity, stochastic structure, long-term damage, effective characteristics, porosity balance equation

Introduction. Long-term loads lower than the ultimate ones may cause sudden failure of structural elements. This is due to the occurrence and development of dispersed microdamages, which commonly lead to the formation of main cracks. Physically, the damage of a material may be considered as dispersed defects such as microcracks, microvoids, or destroyed microvolumes. They reduce the effective or bearing portion of the material that resists loads.

Experimental data on and observation of the real behavior of structural members and structures suggest that damage can be either short-term (occurring instantaneously after the application of stresses or strains) or long-term (building up with time after the application of load). A structural theory of short-term microdamage of homogeneous and composite materials was proposed in [8]. It employs the mechanics of microinhomogeneous bodies of stochastic structure and models dispersed microdamages by quasispherical micropores [5]. Long-term damage is the accumulation of dispersed microdamages such as micropores and microcracks. At the microscopic level, the strength of a material is inhomogeneous, i.e., the ultimate strength and stress-rupture curves for a microvolume are random functions of coordinates with certain distribution density or cumulative distribution. When a macrospecimen is subject to constant stresses, some microvolumes whose ultimate strength is less than the equivalent stress are damaged, i.e., microcracks or micropores form in their place. Microvolumes where the stress is less than, yet close to the ultimate strength are damaged after a lapse of time, which depends on the difference between the applied stress and the ultimate microstrength. A theory of the long-term damage of homogeneous, particulate, and laminated materials was developed in [9] based on models and methods of the mechanics of stochastically inhomogeneous materials.

The stress–strain behavior of many materials becomes nonlinear under quite high loads. This type of nonlinearity is typical of metals and polymers at high temperatures. Therefore, it is important to generalize the theory of long-term damage of laminated materials [10, 11] based on models and methods of the mechanics of stochastically inhomogeneous materials to physically nonlinear laminated materials. The damage of the components (plies) of a laminated material is modeled by dispersed microvolumes destroyed to become random micropores. The failure criterion for a single microvolume is determined by its

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 1063-7095/13/4906-0650

stress-rupture strength described by a fractional or exponential power function, which is, in turn, determined by the dependence of the time to brittle failure on the difference between the equivalent stress and its critical value, which characterizes the ultimate strength according to the Huber–Mises criterion. The ultimate strength is assumed to be a random function of coordinates whose one-point distribution is described by a power function on some interval or by the Weibull function. The effective properties and the stress–strain state of a laminated material with random microdamages are determined from the stochastic equations of elasticity of laminated materials with porous components. We will derive a damage (porosity) balance equation from the properties of the distribution functions and ergodicity of the random field of ultimate microstrength, and the dependence of the time to brittle failure for a microvolume on its stress state and ultimate microstrength for given macrostrains and an arbitrary time. The macrostress–macrostrain relationship and the porosity balance equations for a laminated material with porous components describe the coupled and interacting processes of deformation and long-term damage. We will use an iteration method to develop algorithms for calculating the microdamage and macrostresses as functions of time and to plot the respective curves. The influence of nonlinearity on the deformation and microdamage of laminated materials will be analyzed.

1. The physically nonlinear deformation of a laminated material with N isotropic components is described as the dependence of the bulk (K_ν) and shear (μ_ν, $\nu = 1, 2, \dots, N$) moduli on strains. The microdamage of the composite components caused by loading is modeled by random quasispherical micropores occurring in those microvolumes where the stresses exceed the ultimate microstrength. The macrostresses $\langle \sigma_{ij} \rangle$ and macrostrains $\langle \varepsilon_{ij} \rangle$ in an elementary macrovolume are related by

$$\langle \sigma_{ij} \rangle = (\lambda_{11}^* - \lambda_{12}^*)\langle \varepsilon_{ij} \rangle + (\lambda_{12}^* \langle \varepsilon_{rr} \rangle + \lambda_{13}^* \langle \varepsilon_{33} \rangle)\delta_{ij},$$

$$\langle \sigma_{33} \rangle = \lambda_{13}^* \langle \varepsilon_{rr} \rangle + \lambda_{33}^* \langle \varepsilon_{33} \rangle, \quad \langle \sigma_{i3} \rangle = 2\lambda_{44}^* \langle \varepsilon_{i3} \rangle \quad (i, j, r = 1, 2), \tag{1.1}$$

where $\lambda_{11}^*, \lambda_{12}^*, \lambda_{13}^*, \lambda_{33}^*, \lambda_{44}^*$ are the effective elastic moduli dependent on the macrostrains $\langle \varepsilon_{ij} \rangle$ due to physical nonlinearity and microdamage.

Denote the bulk and shear moduli of the skeleton of the νth component by K_ν and μ_ν, its porosity by p_ν, and the volume fraction of the porous νth component by c_ν ($\nu = 1, \dots, N$). The effective moduli of a physically nonlinear laminated composite with porous components can be determined using the following algorithm. The effective moduli λ_{11}^*, λ_{12}^*, λ_{13}^*, λ_{33}^*, λ_{44}^* of the composite are expressed [2, 7] in terms of the moduli $\lambda_{\nu p}$, $\mu_{\nu p}$ ($\nu = 1, 2, \dots, N$) of its components as

$$\lambda_{11}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle^2 + 4\left\langle \frac{\mu_p(\lambda_p + \mu_p)}{\lambda_p + 2\mu_p} \right\rangle,$$

$$\lambda_{12}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle^2 + 2\left\langle \frac{\lambda_p \mu_p}{\lambda_p + 2\mu_p} \right\rangle, \quad \lambda_{44}^* = \left\langle \frac{1}{\mu_p} \right\rangle^{-1},$$

$$\lambda_{13}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle, \quad \lambda_{33}^* = \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1}, \tag{1.2}$$

where

$$\langle f_p \rangle = \sum_{\nu=1}^{N} c_\nu f_{\nu p}. \tag{1.3}$$

The effective moduli $K_{\nu p}, \lambda_{\nu p}, \mu_{\nu p}$ of the porous νth component are defined by the following formulas, according to [2, 6]:

$$K_{\nu p} = K_{\nu p}(\langle \varepsilon_{lm}^{1\nu} \rangle) = \frac{4K_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)(1 - p_\nu)^2}{3K_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)p_\nu + 4\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)(1 - p_\nu)}, \quad \lambda_{\nu p} = K_{\nu p} - \frac{2}{3}\mu_{\nu p},$$

$$\mu_{\nu p} = \mu_{\nu p}(\langle \varepsilon_{lm}^{1\nu} \rangle) = \frac{[9K_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle) + 8\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)]\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)(1 - p_\nu)^2}{3K_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)(3 - p_\nu) + 4\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle)(2 + p_\nu)} \quad (\nu = 1, \ldots, N), \tag{1.4}$$

where $\langle \varepsilon_{ij}^{1\nu} \rangle$ are the average strains in the undamaged portion of the νth component. Since they are expressed in terms of the elastic moduli K_ν, μ_ν of the components, which, in turn, are functions of the average strains in the undamaged portion of the νth component, they can be determined using the following iterative algorithm. Their $(n+1)$th approximation is related to the nth approximation by

$$\langle \varepsilon_{kg}^{1\nu} \rangle^{(n+1)} = \frac{1}{(1 - p_\nu)} \left\{ \frac{\mu_{\nu p}(\langle \varepsilon_{lm}^{1\nu} \rangle^{(n)})}{\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle^{(n)})} \langle \varepsilon_{kg}^{\nu} \rangle + \frac{1}{3} \left[\frac{K_{\nu p}(\langle \varepsilon_{lm}^{1\nu} \rangle^{(n)})}{K_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle^{(n)})} - \frac{\mu_{\nu p}(\langle \varepsilon_{lm}^{1\nu} \rangle^{(n)})}{\mu_\nu(\langle \varepsilon_{lm}^{1\nu} \rangle^{(n)})} \right] \langle \varepsilon_{rr}^{\nu} \rangle \delta_{kg} \right\}$$

$$(\nu = 1, \ldots, N). \tag{1.5}$$

The average strains $\langle \varepsilon_{ij}^{\nu} \rangle$ are determined in terms of the macrostrains $\langle \varepsilon_{ij} \rangle$ by the formulas

$$\langle \varepsilon_{ij}^{1} \rangle = \ldots = \langle \varepsilon_{ij}^{N} \rangle = \langle \varepsilon_{ij} \rangle, \quad \langle \varepsilon_{i3}^{\nu} \rangle = \frac{1}{\mu_{\nu p}} \left\langle \frac{1}{\mu_p} \right\rangle^{-1} \langle \varepsilon_{i3} \rangle,$$

$$\langle \varepsilon_{33}^{\nu} \rangle = \frac{1}{\lambda_{\nu p} + 2\mu_{\nu p}} \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle^{-1} \left[\left\langle \frac{\lambda_p}{\lambda_p + 2\mu_p} \right\rangle - \lambda_{\nu p} \left\langle \frac{1}{\lambda_p + 2\mu_p} \right\rangle \right] \langle \varepsilon_{rr} \rangle + \langle \varepsilon_{33} \rangle$$

$$(i, j = 1, 2, \ \nu = 1, \ldots, N). \tag{1.6}$$

The zero-order approximation represents physically linear components.

We will use the Huber–Mises criterion [3] as a condition for the formation of a microdamage in a microvolume of the undamaged portion of the components:

$$I_{\langle \sigma \rangle}^{1\nu} = k_\nu \quad (\nu = 1, 2, \ldots, N), \tag{1.7}$$

where $I_{\langle \sigma \rangle}^{1\nu} = (\langle \sigma_{ij}^{1\nu} \rangle' \langle \sigma_{ij}^{1\nu} \rangle')^{1/2}$ is the second invariant of the deviatoric average-stress tensor $\langle \sigma_{ij}^{1\nu} \rangle'$ in the undamaged portion of the νth component; k_ν is the ultimate microstrength, which is a random function of coordinates. The average stresses $\langle \sigma_{ij}^{1\nu} \rangle$ in the undamaged portion of the νth component are related to the average stresses $\langle \sigma_{ij}^{\nu} \rangle$ as follows [6]:

$$\langle \sigma_{ij}^{1\nu} \rangle = \frac{1}{1 - p_\nu} \langle \sigma_{ij}^{\nu} \rangle. \tag{1.8}$$

If the invariant $I_{\langle \sigma \rangle}^{1\nu}$ does not reach the critical value k_ν in some microvolume of the νth component, then, according to the stress-rupture criterion, failure will occur in some time τ_k^ν, which depends on the difference between $I_{\langle \sigma \rangle}^{1\nu}$ and k_ν. In the general case, this dependence can be represented as some function:

$$\tau_k^\nu = \varphi_\nu(I_{\langle \sigma \rangle}^{1\nu}, k_\nu), \tag{1.9}$$

where $\varphi_\nu(k_\nu, k_\nu) = 0$ and $\varphi_\nu(0, k_\nu) = \infty$ according to (1.9).

The one-point distribution function $F_\nu(k_\nu)$ in the undamaged portion of the νth component can be approximated by a power function on some interval:

$$F_\nu(k_\nu)=\begin{cases}0, & k_\nu<k_{\nu 0},\\ \left(\dfrac{k_\nu-k_{\nu 0}}{k_{\nu 1}-k_{\nu 0}}\right)^{n_\nu}, & k_{\nu 0}\le k_\nu\le k_{\nu 1},\\ 1, & k_\nu>k_{\nu 1},\end{cases} \tag{1.10}$$

or by the Weibull function

$$F_\nu(k_\nu)=\begin{cases}0, & k_\nu<k_{\nu 0},\\ 1-\exp[-m_\nu(k_\nu-k_{\nu 0})^{n_\nu}], & k_\nu\ge k_{\nu 0},\end{cases} \tag{1.11}$$

where $k_{\nu 0}$ is the minimum value of ultimate microstrength in the νth component; $k_{\nu 1}, m_\nu, n_\nu$ are deterministic constants describing the behavior of the distribution function and determined by fitting experimental microstrength scatter or stress–strain curves.

The random field of ultimate microstrength k_ν is statistically homogeneous in real materials, and its correlation scale and the size of single microdamages and the distances between them are negligible compared with the macrovolume. Then the random field k_ν and the distribution of microstresses in the component under uniform loading are ergodic, and the distribution function $F_\nu(k_\nu)$ defines the fraction of the undamaged portion of the component in which the ultimate strength is less than k_ν. Therefore, if the stresses $\langle\sigma_{ij}^{1\nu}\rangle$ are nonzero, the function $F_\nu(I_{\langle\sigma\rangle}^{1\nu})$ defines, according to (1.7), (1.10), and (1.11), the fraction of damaged microvolumes of the skeleton of the component. Since the damaged microvolumes are modeled by pores, we can write a porosity balance equation [8]:

$$p_\nu=p_{\nu 0}+(1-p_{\nu 0})F_\nu(I_{\langle\sigma\rangle}^{1\nu}), \tag{1.12}$$

where $p_{\nu 0}$ is the initial porosity of the component and, according to (1.8),

$$I_{\langle\sigma\rangle}^{1\nu}=\frac{1}{1-p_\nu}I_{\langle\sigma\rangle}^{\nu}\qquad (I_{\langle\sigma\rangle}^{\nu}=(\langle\sigma_{ij}^{\nu}\rangle'\langle\sigma_{ij}^{\nu}\rangle')^{1/2}). \tag{1.13}$$

Given macrostrains $\langle\varepsilon_{ij}\rangle$, the average stresses $\langle\sigma_{ij}^{\nu}\rangle$ in the νth component are related to the macrostrains $\langle\varepsilon_{ij}\rangle$ as follows [2]:

$$\langle\sigma_{ij}^{\nu}\rangle=2\mu_{\nu p}\langle\varepsilon_{ij}\rangle+\frac{\lambda_{\nu p}}{\lambda_{\nu p}+2\mu_{\nu p}}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}$$

$$\times\left[\left(\left\langle\frac{\lambda_p}{\lambda_p+2\mu_p}\right\rangle+2\mu_{\nu p}\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle\right)\langle\varepsilon_{rr}\rangle+\langle\varepsilon_{33}\rangle\right]\delta_{ij},$$

$$\langle\sigma_{33}^{1}\rangle=\ldots=\langle\sigma_{33}^{N}\rangle=\left\langle\frac{1}{\lambda_p+2\mu_p}\right\rangle^{-1}\left(\left\langle\frac{\lambda_p}{\lambda_p+2\mu_p}\right\rangle\langle\varepsilon_{rr}\rangle+\langle\varepsilon_{33}\rangle\right),$$

$$\langle\sigma_{i3}^{1}\rangle=\ldots=\langle\sigma_{i3}^{N}\rangle=2\left\langle\frac{1}{\mu_p}\right\rangle^{-1}\langle\varepsilon_{i3}\rangle\qquad (i,j,r=1,2,\ \nu=1,\ldots,N). \tag{1.14}$$

If the stresses $\langle\sigma_{ij}^{\nu}\rangle$ act for some time t, then, according to the stress-rupture criterion (1.9), those microvolumes of the νth component are damaged that have k_ν such that

$$t\ge\tau_k^{\nu}=\varphi_\nu(I_{\langle\sigma\rangle}^{1\nu},k_\nu), \tag{1.15}$$

where $I^{1\nu}_{\langle\sigma\rangle}$ is defined by (1.13).

The time to brittle failure τ^{ν}_{k} for the νth component of real materials at low temperatures is finite beginning only from some value of $I^{1\nu}_{\langle\sigma\rangle} > 0$. In this case, the durability function $\varphi_{\nu}(I^{1\nu}_{\langle\sigma\rangle}, k_{\nu})$ can be represented as

$$\varphi_{\nu}(I^{1\nu}_{\langle\sigma\rangle}, k_{\nu}) = \tau_{0\nu}\left(\frac{k_{\nu} - I^{1\nu}_{\langle\sigma\rangle}}{I^{1\nu}_{\langle\sigma\rangle} - \gamma_{\nu}k_{\nu}}\right)^{n_{1\nu}} \qquad (\gamma_{\nu}k_{\nu} \le I^{1\nu}_{\langle\sigma\rangle} \le k_{\nu},\ \gamma_{\nu} < 1), \tag{1.16}$$

where some typical time $\tau_{0\nu}$, exponent $n_{1\nu}$, and coefficient γ_{ν} are determined from the fit of experimental durability curves for the νth component.

Substituting (1.16) into (1.15), we arrive at the inequality

$$k_{\nu} \le I^{\nu}_{\langle\sigma\rangle}\frac{1 + \bar{t}_{\nu}^{-1/n_{1\nu}}}{1 + \gamma_{\nu}\bar{t}_{\nu}^{-1/n_{1\nu}}} \qquad \left(\bar{t}_{\nu} = \frac{t}{\tau_{0\nu}}\right). \tag{1.17}$$

Considering the definition of the distribution function $F_{\nu}(k_{\nu})$, we conclude that the function $F_{\nu}[I^{1\nu}_{\langle\sigma\rangle}\psi_{\nu}(\bar{t}_{\nu})]$, where

$$\psi_{\nu}(\bar{t}_{\nu}) = \frac{1 + \bar{t}_{\nu}^{-1/n_{1\nu}}}{1 + \gamma_{\nu}\bar{t}_{\nu}^{-1/n_{1\nu}}} \tag{1.18}$$

defines the fraction of the destroyed microvolumes in the undamaged (before loading) portion of the νth component at the time $\bar{t}_{\nu}$. Then, in view of (1.9), the porosity balance equation for the νth component subject to long-term damage can be represented as

$$p_{\nu} = p_{0\nu} + (1 - p_{0\nu})F_{\nu}\left[\frac{I^{\nu}_{\langle\sigma\rangle}}{1 - p_{\nu}}\psi_{\nu}(\bar{t}_{\nu})\right], \tag{1.19}$$

where p_{ν} is a function of dimensionless time $\bar{t}_{\nu}$, and $I^{\nu}_{\langle\sigma\rangle}$ is defined by (1.14).

If the time τ^{ν}_{k} is finite for arbitrary values of $I^{1\nu}_{\langle\sigma\rangle}$, which may be observed at high temperatures, then the durability function can be approximated by an exponential power function:

$$\varphi_{\nu}(I^{\nu}_{\langle\sigma\rangle}, k_{\nu}) = \tau_{0\nu}\left\{\exp m_{1\nu}\left[(k_{\nu}/I^{\nu}_{\langle\sigma\rangle})^{n_{1\nu}} - 1\right] - 1\right\}^{n_{2\nu}}, \tag{1.20}$$

which has enough constants $\tau_{0\nu}$, $m_{1\nu}, n_{1\nu}, n_{2\nu}$ to fit experimental curves. Substituting (1.20) into (1.15), we arrive at the inequality

$$k_{\nu} \le I^{\nu}_{\langle\sigma\rangle}\left[1 + \frac{1}{m_{1\nu}}\ln(1 + \bar{t}_{\nu}^{-1/n_{2\nu}})\right]^{1/n_{1\nu}} \qquad \left(\bar{t}_{\nu} = \frac{t}{\tau_{0\nu}}\right). \tag{1.21}$$

Considering the definition of the distribution function $F_{\nu}(k_{\nu})$, we conclude that the function $F_{\nu}[I^{1\nu}_{\langle\sigma\rangle}\psi_{\nu}(\bar{t}_{\nu})]$, where

$$\psi_{\nu}(\bar{t}_{\nu}) = \left[1 + \frac{1}{m_{1\nu}}\ln(1 + \bar{t}_{\nu}^{-1/n_{2\nu}})\right]^{1/n_{1\nu}}, \tag{1.22}$$

defines the fraction of the destroyed microvolumes in the undamaged (before loading) portion of the νth component at the time $\bar{t}_{\nu}$. Then, in view of (1.1), the porosity balance equation for the νth component subject to long-term damage can be represented in the form (1.19), where p_{ν} is a function of dimensionless time $\bar{t}_{\nu}$, and $I^{\nu}_{\langle\sigma\rangle}$ is defined by (1.14).

At $\bar{t}_\nu = 0$, the porosity balance equation (1.19) with (1.14), (1.18) (or (1.22)) defines the short-term (instantaneous) damage of the νth component. As time elapses, Eqs. (1.19) with (1.14), (1.18) (or (1.22)) defines its long-term damage, which consists of short-term damage and additional time-dependent damage.

Equations (1.1), (1.2)–(1.6), (1.19), (1.14), (1.10) (or (1.11)), (1.18) (or (1.22)) form a closed-form system describing the coupled processes of statistically homogeneous physically nonlinear deformation and long-term damage of a laminated material. The physical nonlinearity of its components affects the way pores form during deformation, and the porosity of the material has an effect on its stress–strain curve. This is why the nonlinearity of the stress–strain curve of the laminated composite is determined by the physical nonlinearity of its components and the increase in the porosity during physically nonlinear deformation.

To describe the coupled processes of physically nonlinear deformation and long-term damage, it is necessary to find the macrostrain-dependent effective elastic moduli by the iterative algorithm (1.2)–(1.6) and to determine the porosity from Eqs. (1.14), (1.10) (or 1.11)), (1.18) (or (1.22)) also by an iterative method. At the *n*th step of the iterative process (1.2)–(1.6), Eq. (1.19) is represented as

$$f_\nu^{(n)} = p_\nu - p_{\nu 0} - (1 - p_{\nu 0}) F_\nu \left[\frac{I_{\langle\sigma\rangle}^{\nu(n)}}{1 - p_\nu} \psi_\nu(\bar{t}_\nu) \right]. \tag{1.23}$$

Then the root p_ν of Eq. (1.23) at the *m*th step of some iterative process can be expressed as

$$p_\nu^{(m,n)} = A_\nu f_\nu^{(n)}(p_\nu^{(m-1)}), \tag{1.24}$$

where A_ν is an operator on the function $f_\nu^{(n)}(p_\nu)$. The root is found as follows:

$$p_\nu = \lim_{\substack{m\to\infty \\ n\to\infty}} p_\nu^{(m,n)}. \tag{1.25}$$

2. Let us analyze, as an example, the coupled processes of nonlinear deformation and long-term microdamage of a two-component laminated composite with linear elastic reinforcement and microdamaged nonlinear elastic matrix with bulk strains being linear and shear strains described by a linear-hardening diagram, i.e.,

$$\langle\sigma_{rr}^2\rangle = K_2 \langle\varepsilon_{rr}^2\rangle, \quad \langle\sigma_{ij}^2\rangle' = 2\mu_2(S_2)\langle\varepsilon_{ij}^2\rangle', \tag{2.1}$$

where the bulk modulus K_2 does not depend on the strains, and the shear modulus $\mu_2(S_2)$ is described by

$$\mu_2(S_2) = \begin{cases} \mu_{20}, & T_2 \le T_{20}, \\ \mu_2' + \left(1 - \dfrac{\mu_2'}{\mu_{20}}\right)\dfrac{T_{20}}{2S_2}, & T_2 \ge T_{20}, \end{cases} \tag{2.2}$$

$$S_2 = (\langle\varepsilon_{ij}^2\rangle'\langle\varepsilon_{ij}^2\rangle')^{1/2}, \quad T_2 = (\langle\sigma_{ij}^2\rangle'\langle\sigma_{ij}^2\rangle')^{1/2}, \quad T_{20} = \sqrt{2/3}\,\sigma_{20}, \tag{2.3}$$

where $\langle\varepsilon_{ij}^2\rangle'$ and $\langle\sigma_{ij}^2\rangle'$ are the strain and stress deviators in the matrix; σ_{20} is the tensile proportional limit assumed to be independent of the coordinates; μ_{20} and μ_2' are the material constants of the matrix.

The root p_2 of Eq. (1.23) can be found by the secant method [1]. Since the root p_2 falls within $[p_{20}, 1]$, which follows from

$$f_2^{(n)}(p_{20}) \le 0, \quad f_2^{(n)}(1) \ge 0, \tag{2.4}$$

the zero approximation $p_2^{(0,n)}$ is

$$p_2^{(0,n)} = \frac{a_2^{(0)} f_2^{(n)}(b_2^{(0)}) - b_2^{(0)} f_2^{(n)}(a_2^{(0)})}{f_2^{(n)}(b_2^{(0)}) - f_2^{(n)}(a_2^{(0)})}, \tag{2.5}$$

where $a_2^{(0)} = p_{20}, b_2^{(0)} = 1$. The subsequent approximations of the secant method are found in the iterative process

$$p_2^{(m,n)} = A_2 f_2^{(n)}(p_2^{(m-1,n)}) \equiv \frac{a_2^{(m)} f_2^{(n)}(b_2^{(m)}) - b_2^{(m)} f_2^{(n)}(a_2^{(m)})}{f_2^{(n)}(b_2^{(m)}) - f_2^{(n)}(a_2^{(m)})}, \tag{2.6}$$

$$a_2^{(m)} = a_2^{(m-1)}, \quad b_2^{(m)} = p_2^{(m-1,n)} \quad \text{for} \quad f^{(n)}(a^{(m-1)}) f^{(n)}(p^{(m-1,n)}) \le 0,$$

$$a_2^{(m)} = p_2^{(m-1,n)}, \quad b_2^{(m)} = b_2^{(m-1)} \quad \text{for} \quad f_2^{(n)}(a_2^{(m-1)}) f_2^{(n)}(p_2^{(m-1,n)}) \ge 0$$

$$(m = 1, 2, \dots),$$

which proceeds until

$$\left| f_2^{(n)}(p_2^{(m,n)}) \right| < \delta, \tag{2.7}$$

where δ is the error of the root.

We analyzed the coupled processed of nonlinear deformation and long-term microdamage of a laminated material for the Weibull distribution (1.11) and for the fractional power durability function $\psi_2(\bar{t}_2)$ defined by (1.22). The reinforcement is linear elastic plies with the following characteristics [4] and volume fraction:

$$K_1 = 38.89 \text{ GPa}, \quad \mu_1 = 29.17 \text{ GPa}, \quad c_1 = 0, \ 0.25, \ 0.5, \ 0.75, \ 1.0 \tag{2.8}$$

and the matrix is described by the linear-hardening diagram (2.1), (2.2) with the following constants [2, 4]:

$$K_2 = 3.33 \text{ GPa}, \quad \mu_{20} = 1.11 \text{ GPa}, \quad \mu'_2 = 0.331 \text{ GPa} \tag{2.9}$$

and the following proportional limits and minimum tensile microstrength ($\sigma_{2p} = \sqrt{3/2}\, k_{20}$):

$$\sigma_{20} = 0.003 \text{ GPa}, \quad \sigma_{2p} = 0.011 \text{ GPa}, \tag{2.10}$$

and

$$p_{02} = 0, \quad k_{02}/\mu_2 = 0.01, \quad m_2 = 1000, \quad \alpha_2 = 2, \quad \gamma_2 = 0.05, \quad n_{12} = 1. \tag{2.11}$$

If

$$\langle \varepsilon_{33} \rangle \ne 0, \quad \langle \sigma_{11} \rangle = \langle \sigma_{22} \rangle = 0, \tag{2.12}$$

then, according to (1.1), the macrostress $\langle \sigma_{33} \rangle$ is related to the macrostrain $\langle \varepsilon_{33} \rangle$ by

$$\langle \sigma_{33} \rangle = \frac{1}{\lambda_{11}^* + \lambda_{12}^*} [(\lambda_{11}^* + \lambda_{12}^*)\lambda_{33}^* - 2(\lambda_{13}^*)^2] \langle \varepsilon_{33} \rangle. \tag{2.13}$$

In the porosity balance equation (1.19), (1.14), (1.11), (1.18) we use

$$\langle \varepsilon_{11} \rangle = \langle \varepsilon_{22} \rangle = -\frac{\lambda_{13}^*}{\lambda_{11}^* + \lambda_{12}^*} \langle \varepsilon_{33} \rangle, \tag{2.14}$$

which is equivalent to (2.12).

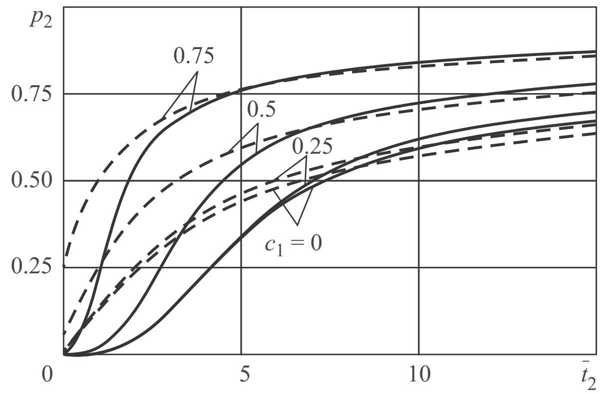

Fig. 1

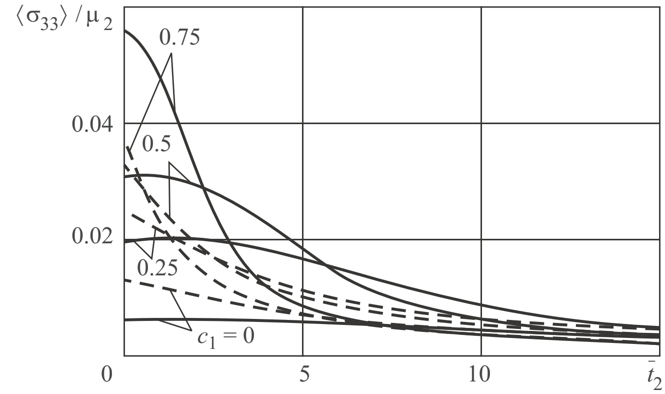

Fig. 2

Figure 1 shows (solid lines) the porosity p_2 of the linear-hardening matrix as a function of time $\bar{t}_2$ for macrostrain $\langle\varepsilon_{11}\rangle = 0.005$ and for different values of c_1. For comparison, the same figure shows (dashed lines) p_2 versus $\bar{t}_2$ for a linear matrix (the notation is the same in Fig. 2). As is seen, the physical nonlinearity of the matrix has a significant effect on the microdamage of the laminated material. The microdamage of the material with linear-hardening matrix sets in at greater values of $\bar{t}_2$ and occurs more intensively than in the material with linear elastic matrix, i.e., at great values of $\bar{t}_2$, the porosity of the composite with linear-hardening matrix is higher than in the composite with linear elastic matrix.

Figure 2 shows (solid lines) the macrostress $\langle\sigma_{33}\rangle / \mu_2$ for laminated materials with linear-hardening matrix and linear elastic matrix as a function of time $\bar{t}_2$ for macrostrain $\langle\varepsilon_{11}\rangle = 0.005$ and for different values of c_1. At small values of $\bar{t}_2$, the physical nonlinearity of the matrix has a significant effect on the stress state of the laminated material. At great values of $\bar{t}_2$, the effect of nonlinearity on the stress state is weak.

REFERENCES

1. Ya. S. Berezikovich, *Approximate Calculations* [in Russian], GITTL, Moscow–Leningrad (1949).
2. A. N. Guz, L. P. Khoroshun, G. A. Vanin, et al., *Materials Mechanics*, Vol. 1 of the three-volume series *Mechanics of Composite Materials and Structural Members* [in Russian], Naukova Dumka, Kyiv (1982).
3. L. M. Kachanov, *Fundamentals of Fracture Mechanics* [in Russian], Nauka, Moscow (1974).
4. A. F. Kregers, "Mathematical modeling of the thermal expansion of spatially reinforced composites," *Mech. Comp. Mater.*, **24**, No. 3, 316–325 (1988).
5. V. P. Tamuzs and V. S. Kuksenko, *Microfracture Mechanics of Polymeric Materials* [in Russian], Zinatne, Riga (1978).
6. L. P. Khoroshun, "Saturated porous media," *Int. Appl. Mech.*, **12**, No. 12, 1231–1237 (1976).
7. L. P. Khoroshun, "Conditional-moment method in problems of the mechanics of composite materials," *Int. Appl. Mech.*, **23**, No. 10, 989–996 (1987).
8. L. P. Khoroshun, "Principles of the micromechanics of material damage. 1. Short-term damage," *Int. Appl. Mech.*, **34**, No. 10, 1035–1041 (1998).
9. L. P. Khoroshun, "Principles of the micromechanics of material damage. 2. Long-term damage," *Int. Appl. Mech.*, **43**, No. 2, 127–135 (2007).
10. L. P. Khoroshun and E. N. Shikula, "Deformation and long-term damage of layered materials with stress-rupture microstrength described by an exponential power function," *Int. Appl. Mech.*, **45**, No. 8, 873–881 (2009).
11. L. P. Khoroshun and E. N. Shikula, "Coupled deformation and long-term damage of layered materials with stress-rupture microstrength described by a fractional-power function," *Int. Appl. Mech.*, **45**, No. 9, 991–999 (2009).