International Applied Mechanics, Vol. 48, No. 5, September, 2012

FORCED RESONANT VIBRATIONS AND SELF-HEATING OF A FLEXIBLE CIRCULAR PLATE WITH PIEZOACTUATORS

I. F. Kirichok

The problem of the forced axisymmetric vibrations and self-heating of a clamped flexible circular plate with piezoelectric actuators is solved. The aspects of mechanical and electric excitation of vibrations and damping of mechanical vibrations with actuators are discussed. The effect of geometrical nonlinearity on the frequency dependence of deflections and self-heating temperature under electromechanical harmonic loading at the principal resonance of bending vibrations of the plate is studied

Keywords: clamped flexible circular plate, forced axisymmetric vibrations, self-heating, piezoactuator, temperature, active damping, geometrical nonlinearity

Introduction. Thin plates made of viscoelastic materials are widely used as elements of modern engineering structures. They are frequently subject to intensive harmonic loads at nearly resonant frequency, which cause vibrations with high amplitudes [2, 15]. Because of hysteresis losses, self-heating may occur in plates under harmonic loading. High-amplitude vibrations and self-heating may cause a structural member to lose its functions because of fatigue failure, high stresses, high temperature, etc. In this connection, there is a need to damp the forced resonant vibrations of thin plates with large deflections.

Recent trends have been toward the use of not only the passive damping of the vibrations of thin-walled structures [14, 16] but also the active damping of vibrations with piezoelectric elements incorporated into the structure [16, 21, 22] to play the role of actuators [12, 22, 23]. Determining the voltage that has to be applied to the actuator to balance the mechanical load is one of the main tasks in using active damping. Issues related to the active damping of thin-walled elements were discussed in [10, 13, 18, 20, 21, etc.] using geometrically linear formulation of thermoviscoelastic problems. Aspects of modeling the thermomechanical behavior of flexible viscoelastic plates with distributed actuators and solutions of some problems obtained with the Bubnov–Galerkin method are addressed in [6, 7, 11, 24, etc.]. The solutions of specific problems disregarded the thickness of the piezolayers of the actuator and the viscoelastic properties of the piezomaterial.

The present paper addresses the problem of the forced resonant vibrations and self-heating of a flexible circular plate with piezoelectric actuators under axisymmetric electromechanical loading. The thickness of the actuators and the viscoelastic properties of the piezoactive and passive materials will be taken into account. Geometrical nonlinearity appears as squared angles of rotation in the governing equations. The solution in time is found by expanding the unknown deflection and the radial displacement into harmonic series and retaining the first and second harmonics, respectively. The nonlinear equations of harmonic vibrations are linearized by the quasilinearization method. The linearized equations are solved by the numerical discrete-orthogonalization method.

1. Problem Formulation. Consider a sandwich plate of radius R with passive (no piezoelectric effect) isotropic core layer of thickness h_0 perfectly bonded to piezoelectric face layers (actuators) of thickness h_1 each. The materials of the layers are viscoelastic. The plate is described in a polar coordinate system r, θ, z with the origin at the center of the midsurface of the passive layer. Let the piezolayers ($h_0 / 2 \le z \le h_0 / 2 + h_1$) and ($-h_0 / 2 - h_1 \le z \le -h_0 / 2$) be polarized across the thickness in opposite directions and characterized by piezoelectric moduli d_{31} and $-d_{31}$, respectively. Both faces of the piezolayers are covered with

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine 03057, e-mail: tepm@inmech.kiev.ua. Translated from Prikladnaya Mekhanika, Vol. 48, No. 5, pp. 112–121, September–October 2012. Original article submitted May 11, 2010.

solid continuous infinitely thin electrodes. The inside electrodes are kept at zero potential. The edge $r = R$ of the plate is clamped. The mechanical vibrations of the plate are excited by axisymmetric surface pressure $\hat{q}_z = q_z(r)\cos\omega t$ harmonically varying with time t with nearly resonant circular frequency ω. The potential difference $\varphi_1(h_0/2+h_1)-\varphi_1(-h_0/2-h_1)=\mathrm{Re}(2V_A e^{i\omega t})$ applied to the actuator of radius $r = r_0$ has the same frequency as the mechanical load, which is weakened or amplified depending on the amplitude and phase of this voltage. The electrodes are short-circuited ($V_A = 0$) in the region $r > r_0$. For active damping of vibrations of the plate excited by a mechanical load, it is necessary to solve a mechanical problem to find the voltage that will balance this load.

To model the electromechanical vibrations of the plate, we assume that the Kirchhoff–Love hypotheses for mechanical variables are valid for the entire sandwich. As for the electric field variables, we assume that the following components of the electric-flux density and electric-field intensity can be neglected in the plane of each piezolayer: $D_r, D_\theta, E_r, E_\theta$. Then it follows from the electrostatic equations that the normal electric-flux density D_z is constant throughout the thickness of the piezolayer [8, 11]. The self-heating temperature is assumed to be constant throughout the thickness of the plate. Let the strains be small, and the deflections of the plate be such that the squared angles of rotation have to be kept in the kinematic equations. The equations of motion are nonlinear as well.

With the above assumptions, the statement of the problem of the forced axisymmetric electromechanical vibrations of a flexible circular plate in polar coordinates includes the equations of motion [4, 8]

$$\frac{\partial N_r}{\partial r}+\frac{1}{r}(N_r-N_\theta)=\rho_h\frac{\partial^2 u}{\partial t^2},$$

$$\frac{\partial \overline{Q}_r}{\partial r}+\frac{1}{r}\overline{Q}_r+q_z=\rho_h\frac{\partial^2 w}{\partial t^2},$$

$$\frac{\partial M_r}{\partial r}+\frac{1}{r}(M_r-M_\theta)-\overline{Q}_r-N_r\vartheta_r=0, \tag{1}$$

the constitutive equations of viscoelasticity

$$N_r = C_{11}*\varepsilon_r + C_{12}*\varepsilon_\theta,\quad N_\theta = C_{12}*\varepsilon_r + C_{11}*\varepsilon_\theta,$$

$$M_r = D_{11}*\kappa_r + D_{12}*\kappa_\theta + M_E,\quad M_r = D_{11}*\kappa_r + D_{12}*\kappa_\theta + M_E, \tag{2}$$

the kinematic equations

$$\varepsilon_r=\frac{\partial u}{\partial r}+\frac{1}{2}\vartheta_r^2,\quad \varepsilon_\theta=\frac{u}{r},\quad \kappa_r=\frac{\partial\vartheta_r}{\partial r},\quad \kappa_\theta=\frac{\vartheta_r}{r},\quad \vartheta_r=-\frac{\partial w}{\partial r}, \tag{3}$$

where the symbol "*" denotes an integral operator known from the linear theory of viscoelasticity [5]:

$$D*f=\overset{0}{D}f-\int_{-\infty}^{t}\overset{1}{D}(t-\tau)f(\tau)\tau,$$

which leads to the concept of complex moduli [8, 9] for harmonic deformation processes:

$$D*f=(D'+iD'')(f'+if''). \tag{4}$$

In (1)–(3), the following notation is used:

$$C_{11}=h_0B_{11}^E+2h_1B_{11}^s,\quad C_{12}=h_0\nu B_{11}^E+2h_1\nu_E B_{11}^s,$$

$$D_{11}=\frac{h_0^3}{12}B_{11}^E+\hat{h}^3B_{11}^s+\frac{h_1^3}{6}\gamma_{33},\quad D_{12}=\frac{h_0^3}{12}\nu B_{11}^E+\hat{h}^3\nu_E B_{11}^s+\frac{h_1^3}{6}\gamma_{33},$$

$$\hat{h}^3 = \frac{1}{6}(4h_1^3 + 6h_1^2 h_0 + 3h_1 h_0^2), \quad B_{11}^E = (1-\nu^2)^{-1} E^*,$$

$$B_{11}^s = 1/[s_{11}^E(1-\nu_E^2)], \quad \nu_E = -s_{12}^E / s_{11}^E, \quad b_{31} = d_{31}/[s_{11}^E(1-\nu_E)],$$

$$b_{33} = \varepsilon_{33}^T(1-k_p^2), \quad k_p^2 = 2d_{31}^2/[s_{11}^E \varepsilon_{33}^T(1-\nu_E)], \quad \gamma_{33} = b_{31}^2 / b_{33},$$

$$\rho_h = 2h_1\rho_1 + \rho_0 h_0, \quad M_E = (h_0 + h_1) b_{31}{}^* V_A, \quad \overline{Q}_r = Q_r - N_r \vartheta_r, \tag{5}$$

$s_{11}^E{}^*, s_{12}^E{}^*, d_{31}{}^*, \varepsilon_{33}^T{}^*$ are the temperature-independent complex compliances, piezoelectric modulus, and permittivity in the piezoelectric layers; N_r, N_θ, Q_r are forces; M_r, M_θ are moments; u, w, ϑ_r are displacements and angle of rotation; ρ_1, ρ_0 are the specific densities of the piezoactive and passive materials; E^* is the viscoelastic Young's modulus; $\nu = \text{const}$ is Poisson's ratio of the passive material.

The energy equation averaged over a cycle of vibration and over the thickness of the shell and describing the axisymmetric distribution of self-heating temperature is as follows:

$$\frac{1}{a}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{2\alpha_n}{\lambda h}(T - T_s) + \frac{1}{\lambda h}\hat{W}, \tag{6}$$

where $h = 2h_1 + h_0$, $\alpha_n = (\alpha_1 + \alpha_2)/2$; α_1, α_2 are the heat-transfer coefficients on the surfaces $z = \mp(h_0/2 + h_1)$; λ is the average thermal conductivity; a is the thermal diffusivity; T_s is the ambient temperature; $\hat{W}$ is the dissipation rate averaged over a period of vibration.

The governing equations (1)–(6) should be supplemented with mechanical and thermal boundary conditions and initial conditions. Since the central point $r = 0$ of a circular solid plate is singular, for numerical purposes we consider the plate to have a hole of rather small radius $r = \varepsilon$ on which regularity and symmetry conditions [4] for the electroelastic (1)–(3) and heat-conduction (6) equations, respectively, are prescribed:

$$N_r = 0, \quad \overline{Q}_r = 0, \quad \vartheta_r = 0 \text{ and } \partial T/\partial r = 0 \quad \text{at} \quad r = \varepsilon. \tag{7}$$

The mechanical boundary conditions on the outer edge that is free in the radial direction and clamped in the transverse direction are

$$N_r = 0, \quad w = 0, \quad \vartheta_r = 0 \quad \text{at} \quad r = R. \tag{8}$$

The thermal boundary condition and the initial condition are

$$\lambda \frac{\partial T}{\partial r} = -\alpha_R (T - T_s) \quad \text{at} \quad r = R, \quad T = T_0 \quad \text{at} \quad t = 0. \tag{9}$$

2. Problem-Solving Method. To solve this problem, we will represent the equation of motion (1) and the kinematic equations (3) for the unknowns $u, w, \vartheta_r, N_r, \overline{Q}_r, M_r$ as

$$\frac{\partial u}{\partial r} = \varepsilon_r - \frac{1}{2}\vartheta_r^2, \quad \frac{\partial w}{\partial r} = -\vartheta_r, \quad \frac{\partial \vartheta_r}{\partial r} = \kappa_r,$$

$$\frac{\partial N_r}{\partial r} = -\frac{1}{r}N_r + \frac{1}{r}N_\theta + \rho_h \frac{\partial^2 u}{\partial t^2}, \quad \frac{\partial \overline{Q}_r}{\partial r} = -\frac{1}{r}\overline{Q}_r + \rho_h \frac{\partial^2 w}{\partial t^2} - q_z,$$

$$\frac{\partial M_r}{\partial r} = -\frac{1}{r}M_r + \frac{1}{r}M_\theta + \overline{Q}_r + N_r \vartheta_r. \tag{10}$$

The constitutive equations (2) yield

$$\varepsilon_r = J_c * N_r - \nu_c * \frac{u}{r}, \quad \kappa_r = J_D * M_r + \nu_D * \frac{\vartheta_r}{r} - J_D * M_E,$$

$$N_\theta = \nu_c * N_r + B_{22} * \frac{u}{r}, \quad M_\theta = -\nu_D * M_r + D_{22} * \frac{\vartheta_r}{r} + (1+\nu_D) * M_E, \tag{11}$$

where

$$J_c = 1/C_{11}, \quad J_D = 1/D_{11}, \quad \nu_c = C_{12}/C_{11}, \quad \nu_D = -D_{12}/D_{11},$$

$$B_{22} = C_{11}(1-\nu_c^2), \quad D_{22} = D_{11}(1-\nu_D^2). \tag{12}$$

For monoharmonic loading

$$q_z = q'_z \cos \omega t - q''_z \sin \omega t \qquad (q''_z = 0) \tag{13}$$

the solution of problem (10) includes not only the principal frequency (frequency of loading), but also other harmonics because of geometrical nonlinearity, i.e., the vibratory process is polyharmonic. Solving the nonlinear problem in the single-mode approximation for the variables $A = \{w, \vartheta_r, \overline{Q}_r, M_r, M_\theta, \kappa_r\}$ describing the bending of the plate and keeping the second harmonics for the variables $B = \{u, N_r, \varepsilon_r, N_\theta\}$ describing the plane deformation of the plate, we obtain

$$A = A' \cos \omega t - A'' \sin \omega t,$$

$$B = \overset{0}{B} + \sum_{k=1}^{2} (\overset{k}{B'} \cos k\omega t - \overset{k}{B''} \sin k\omega t). \tag{14}$$

We first substitute (13) and (14) into the governing equations (10) and equate the coefficients of $\cos k\omega t$ and $\sin k\omega t$ $(k = 0, 1, 2)$. Then we eliminate the equilibrium quantities $\overset{0}{\varepsilon}_r$, $\overset{0}{N}_\theta$ and the amplitude variables $\overset{1}{\kappa}{}'_r$, $\overset{1}{k}{}''_r$, $\overset{1}{M}{}'_\theta$, $\overset{1}{M}{}''_\theta$, $\overset{k}{\varepsilon}{}'_r$, $\overset{k}{\varepsilon}{}''_r$, $\overset{k}{N}{}'_\theta$, $\overset{k}{N}{}''_\theta$ $(k = 1, 2)$ from the resulting relations. The quantities $\overset{0}{\varepsilon}_r$, $\overset{0}{N}_\theta$ are defined by the first and third formulas in (11) where the asterisk is omitted and the quantities J_c, ν_c, B_{22} are calculated from formulas (5) and (12) where the viscoelastic moduli are replaced by the equilibrium elastic moduli. The variables $\overset{1}{\kappa}{}'_r, \ldots, \overset{k}{N}{}''_\theta$ are found using procedure (4) and formulas (11) valid for each harmonic [8]. The complex stiffnesses (5), (12) are calculated at frequency ω for the index 1 and at frequency 2ω for the index 2. Finally, we obtain the following approximate system of nonlinear differential equations for amplitudes:

$$\frac{d\overset{0}{u}}{dr} = \overset{0}{J}_c \overset{0}{N}_r - \overset{0}{\nu}_c \frac{\overset{0}{u}}{r} - \frac{1}{2}(\overset{1}{\vartheta}{}'^2_r + \overset{1}{\vartheta}{}''^2_r),$$

$$\frac{du'}{dr} = J'_c N'_r + J''_c N''_r - \nu'_c \frac{u'}{r} + \nu''_c \frac{u''}{r}, \quad \frac{\partial u''}{\partial r} = -J''_c N'_r + J'_c N''_r - \nu''_c \frac{u'}{r} - \nu'_c \frac{u''}{r},$$

$$\frac{d\overset{2}{u}{}'}{dr} = \overset{2}{J}{}'_c \overset{2}{N}{}'_r + \overset{2}{J}{}''_c \overset{2}{N}{}''_r - \overset{2}{\nu}{}'_c \frac{\overset{2}{u}{}'}{r} + \overset{2}{\nu}{}''_c \frac{\overset{2}{u}{}''}{r} - \frac{1}{4}(\overset{1}{\vartheta}{}'^2_r - \overset{1}{\vartheta}{}''^2_r),$$

$$\frac{d\overset{2}{u}{}''}{dr} = -\overset{2}{J}{}''_c \overset{2}{N}{}'_r + \overset{2}{J}{}'_c \overset{2}{N}{}''_c - \overset{2}{\nu}{}''_c \frac{\overset{2}{u}{}'}{r} - \overset{2}{\nu}{}'_c \frac{\overset{2}{u}{}''}{r} - \frac{1}{2}\overset{1}{\vartheta}{}'_r \overset{1}{\vartheta}{}''_r,$$

$$\frac{dw'}{dr} = -\vartheta'_r, \quad \frac{dw''}{dr} = -\vartheta''_r,$$

$$\frac{d\vartheta'_r}{dr}=J'_D M'_r+J''_D M''_r+\nu'_D\frac{\vartheta'_r}{r}-\nu''_D\frac{\vartheta''_r}{r}-(J'_D M'_E+J''_D M''_E),$$

$$\frac{d\vartheta''_r}{dr}=-J''_D M'_r+J'_D M''_r+\nu''_D\frac{\vartheta'_r}{r}+\nu'_D\frac{\vartheta''_r}{r}+(J''_D M'_E-J'_D M''_E),$$

$$\frac{d\overset{0}{N}_r}{dr}=-\frac{1-\overset{0}{\nu}_c}{r}\overset{0}{N}_r+\frac{\overset{0}{B}_{22}}{r^2}\overset{0}{u},$$

$$\frac{dN'_r}{dr}=-\frac{1-\nu'_c}{r}N'_r-\frac{\nu''_c}{r}N''_r+\frac{B'_{22}}{r^2}u'-\frac{B''_{22}}{r^2}u''-\rho_h\omega^2 u',$$

$$\frac{dN''_r}{dr}=\frac{\nu''_c}{r}N'_r-\frac{1-\nu'_c}{r}N''_r+\frac{B''_{22}}{r^2}u'+\frac{B'_{22}}{r^2}u''-\rho_h\omega^2 u'',$$

$$\frac{d\overset{2}{N}{}'_r}{dr}=-\frac{1-\overset{2}{\nu}{}'_c}{r}\overset{2}{N}{}'_r-\frac{\overset{2}{\nu}{}''_c}{r}\overset{2}{N}{}''_r+\frac{\overset{2}{B}{}'_{22}}{r}\overset{2}{u}{}'-\frac{\overset{2}{B}{}''_{22}}{r^2}\overset{2}{u}{}''-4\rho_h\omega^2\overset{2}{u}{}',$$

$$\frac{d\overset{2}{N}{}''_r}{dr}=\frac{\overset{2}{\nu}{}''_c}{r}\overset{2}{N}{}'_r-\frac{1-\overset{2}{\nu}{}'_c}{r}\overset{2}{N}{}''_r+\frac{\overset{2}{B}{}''_{22}}{r}\overset{2}{u}{}'+\frac{\overset{2}{B}{}'_{22}}{r^2}\overset{2}{u}{}''-4\rho_h\omega^2\overset{2}{u}{}'',$$

$$\frac{d\overline{Q}{}'_r}{dr}=-\frac{1}{r}\overline{Q}{}'_r-\rho_h\omega^2 w'-q'_z,\qquad \frac{d\overline{Q}{}''_r}{dr}=-\frac{1}{r}\overline{Q}{}''_r-\rho_h\omega^2 w'',$$

$$\frac{dM'_r}{dr}=-\frac{1+\nu'_D}{r}M'_r+\frac{\nu''_D}{r}M''_r+\frac{D'_{22}}{r^2}\vartheta'_r-\frac{D''_{22}}{r^2}\vartheta''_r+\overline{Q}{}'_r+\overset{0}{N}_r\vartheta'_r$$

$$+\frac{1}{2}\overset{2}{N}{}'_r\vartheta'_r+\frac{1}{2}\overset{2}{N}{}''_r\vartheta''_r+\frac{1+\nu'_D}{r}M'_E-\frac{\nu''_D}{r}M''_E,$$

$$\frac{dM''_r}{dr}=-\frac{\nu''_D}{r}M'_r-\frac{1+\nu'_D}{r}M''_r+\frac{D''_{22}}{r^2}\vartheta'_r+\frac{D'_{22}}{r^2}\vartheta''_r+\overline{Q}{}''_r+\overset{0}{N}_r\vartheta''_r$$

$$-\frac{1}{2}\overset{2}{N}{}'_r\vartheta''_r+\frac{1}{2}\overset{2}{N}{}''_r\vartheta'_r+\frac{\nu''_D}{r}M'_E+\frac{1+\nu'_D}{r}M''_E. \tag{15}$$

The mechanical boundary conditions (7), (8) take the form

$$\overset{0}{N}_r=N'_r=N'_r=\overset{2}{N}{}'_r=\overset{2}{N}{}''_r=\vartheta'_r=\vartheta''_r=\overline{Q}{}'_r=\overline{Q}{}''_r=0\qquad(r=\varepsilon),$$

$$\overset{0}{N}_r=N'_r=N'_r=\overset{2}{N}{}'_r=\overset{2}{N}{}''_r=w'=w''=\vartheta'_r=\vartheta''_r=0\qquad(r=R). \tag{16}$$

The dissipation function $\hat{W}$ in the energy equation (6) for a flexible viscoelastic plate with a piezoelectric actuator is expressed in terms of the unknown functions of the system of equations (15) as

$$\frac{2}{\omega}\hat{W}=\sum_{k=1}2k(\overset{k}{N}{}''_r\overset{k}{\varepsilon}{}'_r-\overset{k}{N}{}'_r\overset{k}{\varepsilon}{}''_r+\overset{k}{N}{}''_\theta\overset{k}{\varepsilon}{}'_\theta-\overset{k}{N}{}'_\theta\overset{k}{\varepsilon}{}''_\theta)+M''_r\kappa'_r-M'_r\kappa''_r+M''_\theta\kappa'_\theta-M'_\theta\kappa''_\theta$$

$$+(h_0+h_1)[(b''_{31}\kappa'+b'_{31}\kappa'')V'_A-(b'_{31}\kappa'-b''_{31}\kappa'')V''_A]+2b''_{33}(V'^2_A+V''^2_A)/h_1, \tag{17}$$

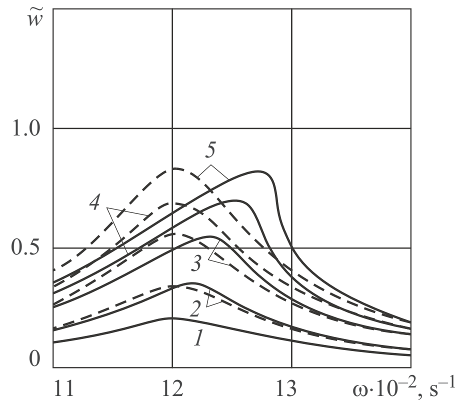

Fig. 1

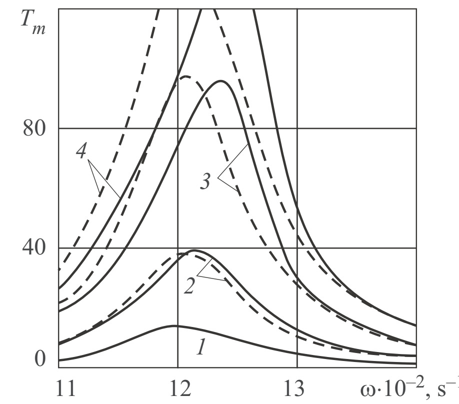

Fig. 2

where $\kappa'{}'' = \kappa_r'{}'' + \kappa_\theta'{}''$ (the index 1 in Eqs. (15)–(17) is omitted for brevity).

The nonlinear problem (15), (16) is linearized using the quasilinearization method [4, 9], and the linearized system of ordinary differential equations is integrated at each approximation using the discrete-orthogonalization method [4] and a standard software program [3]. After the dissipation function (17) is found, the nonstationary heat-conduction problem (6), (9) is solved by the explicit finite-difference method. For numerical implementation of the algorithm, we use the dimensionless space and time coordinates, $x = (r-\varepsilon)/L \; (L = R-\varepsilon)$ and $\tau = at/L^2$, and the dimensionless heat-transfer coefficients $(\gamma_n, \gamma_r) = (\alpha_n, \alpha_R)L/\lambda$.

Let us consider three ways of exciting the harmonic vibrations of the plate:

(i) radially uniform surface pressure $q_z' = q_0$,

(ii) voltage $\pm V_A$ applied to the actuator, and

(iii) combination of mechanical load q_0 and voltage $2V_A$.

To determine the voltage V_A balancing the mechanical load q_0, we will use the formula

$$V_A = k_A(x_0)q_0, \tag{18}$$

where k_A is the control ratio; $x_0 = (r_0 - \varepsilon)/L$ is the dimensionless radius of the circular actuator.

By analogy with the linear problem [13], k_A is the ratio of the maximum deflection amplitude w_p caused by a unit mechanical load ($q_0 = 1$ Pa) at linear resonance frequency to the maximum deflection w_E caused by a unit voltage ($V_A' = 1$ V, $V_A'' = 0$) applied to the actuator:

$$k_A = |w_{p\max}|/|w_{E\max}|. \tag{19}$$

If the mechanical load varies with time as in (13), then the antiphase voltage defined by (18) should vary as $V_A \cos(\omega t + \pi) = -V_A \cos \omega t$.

3. Calculated Results. For numeric purposes, we consider a circular plate made of passive polymer [17] with actuators made of viscoelastic piezoceramics TsTStB-2 [1] with the following frequency-independent mechanical characteristics (5):

$$\overset{0}{G} = \overset{k}{G}' = 794.2 \text{ MPa} = 73.1 \text{ MPa}, \quad \overset{0}{\nu} = \overset{k}{\nu} = 0.3636, \quad E = 2(1+\nu)G, \quad \overset{k}{\nu}_E' = 0.37,$$

$$\overset{k}{s}{}_{11}^{E}{}^* = (12.5 - i0.02)\cdot 10^{-12} \text{ m}^2/\text{N}, \quad \overset{k}{d}_{31}{}^* = (-1.6 + i0.0064)\cdot 10^{-10} \text{ C/m},$$

$$\overset{k}{\varepsilon}_{31}{}^* = (21 + i0.735)\cdot 10^2 \varepsilon_0, \quad \varepsilon_0 = 8.854\cdot 10^{-12} \text{ F/m}, \quad \overset{k}{\nu}_E'' = 0, \quad \gamma_n = \gamma_R = 25.5,$$

$$\rho_0 = 929 \text{ kg/m}^3, \quad \rho_1 = 7520 \text{ kg/m}^3, \quad \lambda = 0.47 \text{ W/(m}\cdot{}^\circ\text{C)}, \quad T_0 = T_s = 20\ {}^\circ\text{C}.$$

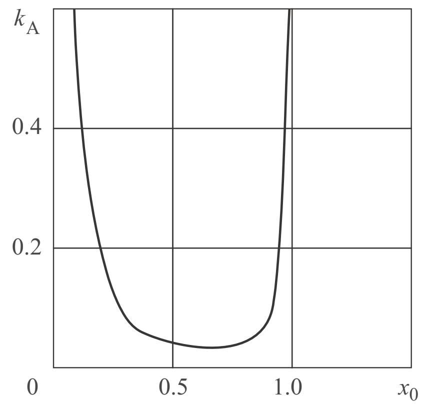

Fig. 3

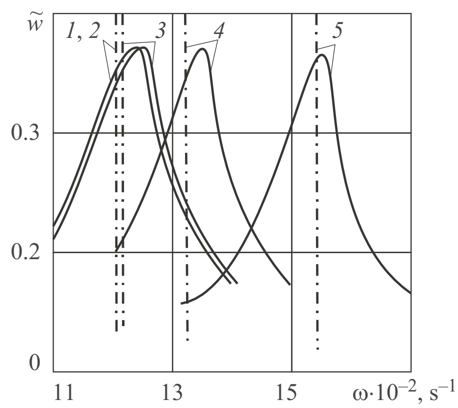

Fig. 4

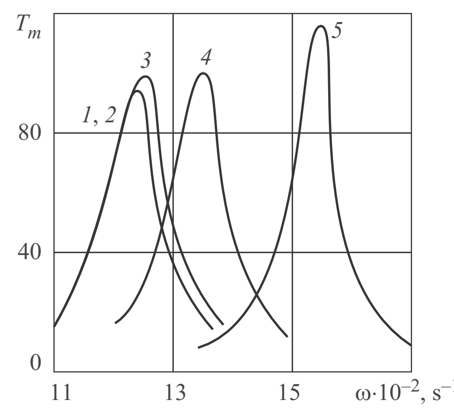

Fig. 5

The radius of the plate $R = 0.2$ m; $\varepsilon = 10^{-4}$ m; the thickness of the passive layer $h_0 = 0.01$ m.

Since the load causes mainly flexural vibrations of the plate and the deflections are maximum at the resonant frequency of the first mode, which is most intensive, we will consider the neighborhood of the first resonant frequency of the bending mode.

Figure 1 shows (curves *1–5*) the frequency dependence (AFR) of the maximum deflection $\tilde{w} = |w(x=0)|/h_0$ of the plate with piezolayers of thickness $\delta = h_1 / h_0 = 0.5 \cdot 10^{-4}$ in the linear (dashed lines) and nonlinear (solid lines) problems for the electric load $V_A = 0$ and the following amplitudes of mechanical load: $q_0 \cdot 10^{-4} = 0.15$, 0.25, 0.4, 0.5, 0.6 Pa. Figure 2 shows the frequency dependence of the steady-state ($\tau = 0.5$) maximum self-heating temperature (TFR) $T_m = (T_{\max} - T_0)$ °C for the first four values of the load (curves *1–4*). It can be seen that the linear problem formulation is sufficient when $\tilde{w} \le 0.2$. These heat-transfer conditions do not cause intensive heating. As the relative deflection ($\tilde{w} > 0.2$) is increased with increasing load, the contribution of geometrical nonlinearity is accompanied [2] by an increase in the resonant frequency and hard amplitude– and temperature–frequency response. The geometrical nonlinearity causes an insignificant decrease in the maximum amplitude of deflection and self-heating temperature at the resonant frequencies.

Figure 3 shows the control ratio k_A of a circular piezoactuator as a function of the dimensionless radius x_0 of the electrode calculated by formula (19) at linear resonance frequencies for the following values of the relative thickness of piezolayers: $\delta = 0, 10^{-4}, 10^{-3}, 10^{-2}$. It appeared that the curves of k_A versus x_0 for these values of δ agree to three decimal digits. This means that k_A for an actuator with cut electrodes does not depend on the thickness of the piezolayers and can be determined regardless of this dependence in the stiffness characteristics of the system. It can be seen that the parameter k_A is minimum within $0.54 \le x_0 \le 0.74$. The actuator with such parameters is the most effective because it balances the mechanical load at minimum voltage. Figures 4 and 5 show how the thickness of the piezolayers of an actuator of radius $x_0 = 0.7$ influences the AFR

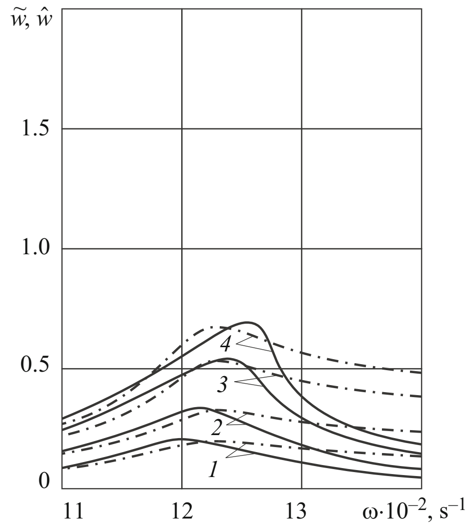

Fig. 6

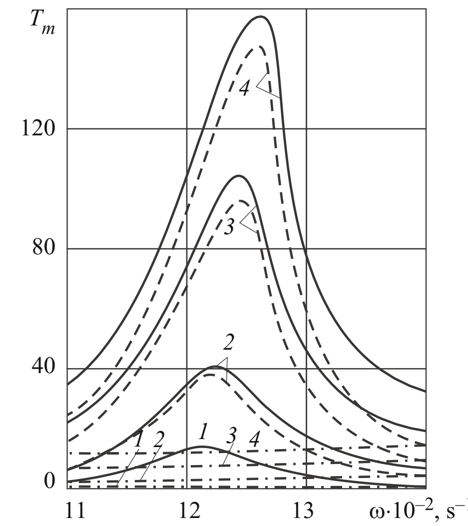

Fig. 7

and TFR, respectively, of the plate under electric loading $\pm V_A = 132$ V. Curves *1–5* correspond to the following values of the relative thickness of the piezolayer: $\delta = 0, 10^{-5}, 10^{-4}, 10^{-3}, 3\cdot10^{-2}$.

The vertical dash-and-dot lines in Fig. 4 represent the linear-resonance frequencies. An analysis of the AFR and TFR suggests that an increase in the thickness of the actuator under electric loading leads to a decrease in the deflection amplitude and an increase in the resonant frequency and self-heating temperature of the plate. The effect of the actuator thickness on the AFR and TFR can be neglected when $\delta < 10^{-4}$.

Figure 6 shows (solid lines *1–4*) the maximum deflection $\tilde{w}$ of the plate with an actuator of radius $x_0 = 0.7$ and relative thickness $\delta = 0.5\cdot10^{-4}$ subject to a mechanical load with amplitude $q_0\cdot10^{-4}$ = (0.15, 0.25, 0.4, 0.5) Pa at short-circuited electrodes ($V_A = 0$) or to an electric load V_A = 49.5, 82.5, 132, 165 V ($q_0 = 0$) applied to the actuator and balancing the mechanical load. With such types of loading, the AFRs are visually indistinguishable (on the given scale). Figure 7 shows (curves *1–4*) the frequency dependence of the maximum self-heating temperature $T_m = (T_{\max} - T_0)$ °C for electric (solid lines) and mechanical (dashed lines) loading as in Fig. 6. It can be seen that the TFRs are distinguishable, unlike the AFRs (Fig. 4). For example, the self-heating temperature under the electric loading (solid lines) is higher than that under the mechanical loading (dashed lines). This is due to the contribution of the terms containing the piezoelectric and dielectric loss moduli to the dissipation function (17).

An analysis of Figs. 6 and 7 suggests that the contribution of the geometrical nonlinearity to the AFR and TFR increases with the amplitude of loading, accompanied by an increase in the self-heating temperature of the plate. The dash-and-dot curves represent the frequency dependence of the deflection $\hat{w} = \tilde{w}\cdot10^2$ (Fig. 6) and the maximum temperature (Fig. 7) for a plate subject to a harmonic mechanical load and an electric load applied in antiphase. It is seen that active damping reduces the deflection amplitude by more than two orders of magnitude and the self-heating temperature to nearly initial level.

Conclusions. We have studied the thermomechanical processes in a clamped viscoelastic flexible circular plate with piezoactuators subjected to mechanical and electric loading. If the piezolayers of equal thickness are polarized in opposite directions, mechanical and balancing electric loads, acting separately, cause equal deflections and slightly different self-heating temperatures. This factor provides a basis for the development of a method for active damping of the mechanical vibrations of plates with piezoelectric actuators. In this case, the geometrical nonlinearity does not affect the control ratio of the actuator and can be determined by solving the linear problem.

REFERENCES

1. A. M. Bolkisev, V. L. Karlash, and N. A. Shul'ga, "Temperature dependence of the properties of piezoelectric ceramics," *Int. Appl. Mech.*, **20**, No. 7, 650–653 (1984).
2. A. S. Vol'mir, *Nonlinear Dynamics of Plates and Shells* [in Russian], Nauka, Moscow (1972).
3. Ya. M. Grigorenko, E. I. Bespalova, A. T. Vasilenko, et al., *Numerical Solution of Static Boundary-Value Problems for Orthotropic Shells of Revolution on M-220 Computer* [in Russian], Naukova Dumka, Kyiv (1971).
4. Ya. M. Grigorenko and A. P. Mukoed, *Computer-Aided Solution of Linear and Nonlinear Problems in the Theory of Shells* [in Ukrainian], Lybid', Kyiv (1992).
5. A. A. Il'yushin and B. E. Pobedrya, *Fundamentals of the Mathematical Theory of Thermoviscoelasticity* [in Russian], Nauka, Moscow (1970).
6. V. G. Karnaukhov and T. V. Karnaukhova, "Damping the resonant bending vibrations of a hinged flexible viscoelastic circular plate with sensors and actuators," *Teor. Prikl. Mekh.*, **46**, 125–131 (2009).
7. V. G. Karnaukhov, T. V. Karnaukhova, and V. F. Zrazhevskaya, "Active damping of the resonant bending vibrations of a hinged flexible circular plate with piezoactuators," *Teor. Prikl. Mekh.*, **45**, 114–123 (2009).
8. V. G. Karnaukhov and I. F. Kirichok, *Coupled Problems for Viscoelastic Plates and Shells* [in Russian], Naukova Dumka, Kyiv (1986).
9. V. G. Karnaukhov and I. F. Kirichok, *Electrothermoviscoelasticity*, Vol. 4 of the five-volume series *Mechanics of Coupled Fields in Structural Members* [in Russian], Naukova Dumka, Kyiv (1988).
10. V. G. Karnaukhov, A. V. Kozlov, and E. V. Pyatetskaya, "Damping the vibrations of viscoelastic plates with distributed piezoelectric inclusions," *Akust. Visn.*, **5**, No. 4, 15–32 (2002).
11. V. G. Karnaukhov, V. I. Kozlov, and T. V. Karnaukhova, "Modeling the forced resonant vibrations and self-heating of flexible viscoelastic plates with distributed actuators," *Fiz.-Mat. Model. Inform. Tekhnol.*, **8**, 48–68 (2008).
12. V. G. Karnaukhov and V. V. Mikhailenko, *Nonlinear Thermomechanics of Piezoelectric Inelastic Bodies under Monoharmonic Loading* [in Russian], ZhTTU, Zhitomir (2005).
13. I. F. Kirichok, O. V. P'yatets'ka, and M. V. Karnaukhov, "Flexural vibrations and self-heating of a viscoelastic circular plate with piezoelectric actuators under electromechanical monoharmonic loading," *Visn. Kyiv. Univ., Ser.: Fiz.-Mat. Nauky*, No. 2, 84–92 (2006).
14. V. V. Matveev, *Damping the Vibrations of Deformable Bodies* [in Russian], Naukova Dumka, Kyiv (1985).
15. A. N. Guz (ed.), *Mechanics of Composite Materials and Structural Members* [in Russian], in 3 vols., Naukova Dumka, Kyiv (1982–1983).
16. A. D. Nashif, D. J. Johnes, and J. P. Henderson, *Vibration Damping*, John Wiley & Sons, New York (1985).
17. N. P. Nesterenko, O. P. Chervinko, and I. K. Senchenkov, "Modeling the ultrasonic heating in structural elements of polymeric fibrous composites under normal loading," *Vest. Nats. Tekhn. Univ. "KhPI*," No. 9, 3–8 (2002).
18. I. F. Kirichok, "Resonant vibration and heating of ring plates with piezoactuators under electromechanical loading and shear deformation," *Int. Appl. Mech.*, **45**, No. 2, 215–222 (2009).
19. I. F. Kirichok, "Control of axisymmetric resonant vibrations and self-heating of shells of revolution with piezoelectric sensor and actuators," *Int. Appl. Mech.*, **46**, No. 8, 890–901 (2010).
20. I. F. Kirichok and T.V. Karnaukhova, "Forced axisymmetric vibrations and self-heating of a circular viscoelastic plate, controlled with piezoelectric sensors and actuators," *Int. Appl. Mech.*, **46**, No. 4, 429–437 (2010).
21. I. F. Kirichok and T. V. Karnaukhova, "Axisymmetric vibrations and self-heating of thermoviselastic cylindrical shells with dielectric actuators," *Int. Appl. Mech.*, **46**, No. 10, 1119–1131 (2010).
22. H. S. Tzou and G. L. Anderson (eds.), *Intelligent Structural Systems*, Kluwer, Dordrecht (1992).
23. H. S. Tzou, *Piezoelectric Shells* (*Distributed Sensing and Control of Continua*), Kluwer, Dordrecht (1993).
24. Ya. A. Zhuk and I. A. Guz, "Active damping of the forced vibration of a hinged beam with piezoelectric layers, geometrical and physical nonlinearities taken into account," *Int. Appl. Mech.*, **45**, No. 1, 94–108 (2009).