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STRESS–STRAIN ANALYSIS OF NONTHIN CONICAL SHELLS WITH THICKNESS VARYING IN TWO COORDINATE DIRECTIONS

O. A. Avramenko

The stress–strain state of nonthin conical shells with thickness varying in two coordinate directions is examined using the approach developed to solve boundary-value problems. Displacement and stress fields in such shells are determined and analyzed

Keywords: nonthin orthotropic conical shells, refined theory, variable thickness, displacement and stress fields

Introduction. Thin and nonthin conical shells with variable thickness are used as structural elements in many fields of engineering and construction [4, 8, 10, 15, 18]. Varying the pattern of variation in the thickness of the shell, while keeping the volume of the structure constant, it is possible to find the rational parameters of shells [1, 2, 5, 6].

Here we analyze the stress–strain state of nonthin conical shells with thickness varying in two coordinate directions by using the approach [3] based on the spline-approximation and discrete-orthogonalization methods. To solve boundary-value problems, we will use the refined theory of conical shells based on the straight-line hypothesis [4, 11–13, 16, 17].

1. Problem Formulation. We will use an orthogonal coordinate system s, θ, γ, where s is the longitudinal coordinate on the datum surface, θ is the azimuth angle, and γ is the normal (to the surface) coordinate. The radius of the circular cross-section is given by

$$r(s) = r_0 + s\cos\varphi, \tag{1}$$

where r_0 is the radius of the datum plane; φ is the angle between the normal and the axis of revolution.

The radius of curvature R_θ in the θ-direction is given by

$$R_\theta = r / \sin\varphi. \tag{2}$$

Let the thickness of the shell vary in the circumferential direction as follows:

$$h(s,\theta) = h_0\left[1-\alpha\left(\frac{s}{l}-1\right)^2\right](1+\beta\cos\theta). \tag{3}$$

For the shell of constant thickness, we set $h = H = \text{const}$. For the volume of the shell to remain constant while the parameters α and β are varied, it is necessary that

$$\int_0^L\int_0^{2\pi} h(s,\theta)\,ds\,d\theta = \int_0^L\int_0^{2\pi} H\,ds\,d\theta, \tag{4}$$

whence

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, 3 Nesterova St., Kyiv, Ukraine 03057, e-mail: avrolya@front.ru. Translated from Prikladnaya Mekhanika, Vol. 48, No. 3, pp. 117–126, May–June 2012. Original article submitted June 1, 2010.

$$h_0 = H / (1 - \alpha / 3). \quad (5)$$

2. Governing Equations. The system of governing equations for nonthin conical shells with thickness varying in both directions can be written as follows using (1)–(4) [3]:

$$\frac{\partial^2 u}{\partial \theta^2} = b_{11} u + b_{12} \frac{\partial u}{\partial s} + b_{13} \frac{\partial u}{\partial \theta} + b_{14} \frac{\partial^2 u}{\partial s^2} + b_{15} v + b_{16} \frac{\partial v}{\partial s} + b_{17} \frac{\partial v}{\partial \theta} + b_{18} \frac{\partial^2 v}{\partial s \partial \theta} + b_{19} w + b_{1,10} \frac{\partial w}{\partial s},$$

$$\frac{\partial^2 v}{\partial \theta^2} = b_{21} u + b_{22} \frac{\partial u}{\partial s} + b_{23} \frac{\partial u}{\partial \theta} + b_{24} \frac{\partial^2 u}{\partial s \partial \theta} + b_{25} v + b_{26} \frac{\partial v}{\partial s} + b_{27} \frac{\partial v}{\partial \theta} + b_{28} \frac{\partial^2 v}{\partial s^2}$$

$$+ b_{29} w + b_{2,10} \frac{\partial w}{\partial \theta} + b_{2,11} \frac{\partial \psi_s}{\partial \theta} + b_{2,12} \frac{\partial^2 \psi_s}{\partial s \partial \theta} + b_{2,13} \psi_\theta + b_{2,14} \frac{\partial \psi_\theta}{\partial s} + b_{2,15} \frac{\partial^2 \psi_\theta}{\partial s^2},$$

$$\frac{\partial^2 w}{\partial \theta^2} = b_{31} u + b_{32} \frac{\partial u}{\partial s} + b_{33} v + b_{34} \frac{\partial v}{\partial \theta} + b_{35} w + b_{36} \frac{\partial w}{\partial s} + b_{37} \frac{\partial w}{\partial \theta}$$

$$+ b_{38} \frac{\partial^2 w}{\partial s^2} + b_{39} \psi_s + b_{3,10} \frac{\partial \psi_s}{\partial s} + b_{3,11} \psi_\theta + b_{3,12} \frac{\partial \psi_\theta}{\partial \theta} + b_{3,13} q_\gamma,$$

$$\frac{\partial^2 \psi_s}{\partial \theta^2} = b_{41} u + b_{42} \frac{\partial u}{\partial s} + b_{43} \frac{\partial u}{\partial \theta} + b_{44} \frac{\partial^2 u}{\partial s^2} + b_{45} v + b_{46} \frac{\partial v}{\partial s} + b_{47} \frac{\partial v}{\partial \theta} + b_{48} \frac{\partial^2 v}{\partial s \partial \theta}$$

$$+ b_{49} w + b_{4,10} \frac{\partial w}{\partial s} + b_{4,11} \psi_s + b_{4,12} \frac{\partial \psi_s}{\partial s} + b_{4,13} \frac{\partial \psi_s}{\partial \theta} + b_{4,14} \frac{\partial^2 \psi_s}{\partial s^2} + b_{4,15} \psi_\theta$$

$$+ b_{4,16} \frac{\partial \psi_\theta}{\partial s} + b_{4,17} \frac{\partial \psi_\theta}{\partial \theta} + b_{4,18} \frac{\partial^2 \psi_\theta}{\partial s \partial \theta},$$

$$\frac{\partial^2 \psi_\theta}{\partial \theta} = b_{51} u + b_{52} \frac{\partial u}{\partial s} + b_{53} \frac{\partial u}{\partial \theta} + b_{54} \frac{\partial^2 u}{\partial s \partial \theta} + b_{55} v + b_{56} \frac{\partial v}{\partial s} + b_{57} \frac{\partial v}{\partial \theta} + b_{58} \frac{\partial^2 v}{\partial s^2}$$

$$+ b_{59} w + b_{5,10} \frac{\partial w}{\partial \theta} + b_{5,11} \psi_s + b_{5,12} \frac{\partial \psi_s}{\partial s} + b_{5,13} \frac{\partial \psi_s}{\partial \theta} + b_{5,14} \frac{\partial^2 \psi_s}{\partial s \partial \theta} + b_{5,15} \psi_\theta$$

$$+ b_{5,16} \frac{\partial \psi_\theta}{\partial s} + b_{5,17} \frac{\partial \psi_\theta}{\partial \theta} + b_{5,18} \frac{\partial^2 \psi_\theta}{\partial s^2} \quad (0 \le s \le L,\ 0 \le \theta \le 2\pi), \quad (6)$$

where u, v, and w are the displacements of particles of the coordinate surface; ψ_s, ψ_θ are the complete angles of rotation of the straight-line element; q_γ is the surface load. The coefficients b_{ij} depend on s and θ (see [3] for their expressions). The elasticity relations are

$$N_s = C_{11} \varepsilon_s + C_{12} \varepsilon_\theta, \quad N_\theta = C_{12} \varepsilon_s + C_{22} \varepsilon_\theta, \quad N_{s\theta} = C_{66} \varepsilon_{s\theta} + \frac{2 \sin \varphi}{r} D_{66} \kappa_{s\theta},$$

$$N_{\theta s} = C_{66} \varepsilon_{s\theta}, \quad M_s = D_{11} \kappa_s + D_{12} \kappa_\theta, \quad M_\theta = D_{12} \kappa_s + D_{22} \kappa_\theta,$$

$$M_{\theta s} = M_{s\theta} = 2 D_{66} \kappa_{s\theta}, \quad Q_s = K_1 \gamma_s, \quad Q_\theta = K_2 \gamma_\theta, \quad (7)$$

where

$$C_{11} = \frac{E_s h}{1 - \nu_s \nu_\theta}, \quad C_{12} = \nu_\theta C_{11}, \quad C_{22} = \frac{E_\theta h}{1 - \nu_s \nu_\theta}, \quad C_{66} = G_{s\theta} h,$$

TABLE 1

θ	α	s / L	wE / q_0				
			β = –0.3	β = –0.2	β = 0	β = 0.2	β = 0.3
0	0	0.2	439.79	382.58	303.39	251.56	231.9
		0.4	625.48	550.607	444.90	373.39	345.55
		0.6	849.87	46.07	597.42	496.15	456.82
		0.8	960.30	818.09	626.58	505.38	460.42
	0.2	0.2	448.94	400.14	327.39	276.62	256.73
		0.4	599.23	539.11	448.22	382.80	356.53
		0.6	798.73	708.86	575.86	482.70	446.04
		0.8	882.20	755.38	583.27	473.42	432.46
	0.3	0.2	452.42	410.17	342.56	292.83	272.87
		0.4	582.29	531.55	450.03	388.47	363.18
		0.6	770.29	688.48	564.44	475.87	440.68
		0.8	842.28	723.59	561.58	457.56	418.64
$\frac{\pi}{2}$	0	0.2	304.50	304.00	303.39	303.27	303.4
		0.4	446.27	445.67	444.90	444.67	444.77
		0.6	597.93	597.87	597.41	596.47	595.82
		0.8	626.58	626.87	626.58	625.14	624.00
	0.2	0.2	332.33	329.71	327.39	328.89	331.09
		0.4	454.14	451.01	448.22	449.98	452.57
		0.6	579.73	577.80	575.86	576.42	577.64
		0.8	584.89	584.26	583.27	582.65	582.48
	0.3	0.2	350.02	346.00	342.56	345.13	348.67
		0.4	458.68	454.04	450.03	452.99	457.05
		0.6	570.21	567.22	564.44	565.86	568.13
		0.8	564.10	562.96	561.58	561.41	561.77

TABLE 1 (continued)

θ	α	s / L	wE / q_0				
			β = –0.3	β = –0.2	β = 0	β = 0.2	β = 0.3
π	0	0.2	231.92	251.57	303.39	382.55	439.73
		0.4	345.58	373.40	444.90	550.56	625.41
		0.6	456.85	496.17	597.41	746.03	849.79
		0.8	460.44	505.39	626.58	818.06	960.24
	0.2	0.2	256.77	276.65	327.39	400.09	448.82
		0.4	356.58	382.84	448.21	539.04	599.10
		0.6	446.08	482.73	575.86	708.80	798.61
		0.8	432.49	473.44	583.27	755.34	882.12
	0.3	0.2	272.92	292.87	342.56	410.09	452.28
		0.4	363.24	388.52	450.03	531.46	582.12
		0.6	440.73	475.91	564.44	688.41	770.15
		0.8	418.67	457.59	561.58	723.55	842.20

$$D_{11} = \frac{E_s h^3}{12(1-\nu_s \nu_\theta)}, \quad D_{12} = \nu_\theta D_{11}, \quad D_{22} = \frac{E_J h^3}{12(1-\nu_s \nu_\theta)}, \quad D_{66} = \frac{G_{s\theta} h^3}{12},$$

$$K_1 = \frac{5}{6} h G_{s\gamma}, \quad K_2 = \frac{5}{6} h G_{\theta\gamma}, \tag{8}$$

$h = h(s,\theta)$ is the thickness of the shell wall; $E_s, E_\theta, \nu_s, \nu_\theta$ are Young's moduli and Poisson's ratios along the coordinate axes s and θ; $G_{s\theta}, G_{s\gamma}, G_{\theta\gamma}$ are the shear moduli.

Supplementing the governing equations (6) with boundary conditions at the ends, we arrive at a two-dimensional boundary-value problem.

3. Problem-Solving Method. To solve problems of this class, we will use the spline-collocation method [7, 9, 14] to separate variables and use the stable numerical discrete-orthogonalization method [4] to solve the resulting boundary-value problem for a system of ordinary differential equations.

Note that system (6) contains no higher than second-order derivatives of the unknown functions with respect to the coordinate s. Then we can use cubic spline-functions [9]. If the following boundary conditions are set at the ends $s = 0$ and $s = L$:

$$u = v = w = \psi_s = \psi_\theta = 0, \tag{9}$$

then the solution of the boundary-value problem (6) can be represented as

$$u(s,\theta) = \sum_{i=0}^{N} u_i(\theta)\varphi_{1i}(s), \quad v(s,\theta) = \sum_{i=0}^{N} v_i(\theta)\varphi_{2i}(s), \quad w(s,\theta) = \sum_{i=0}^{N} w_i(\theta)\varphi_{3i}(s),$$

TABLE 2

θ	α	s/L	σ_s^+				
			β = –0.3	β = –0.2	β = 0	β = 0.2	β = 0.3
0	0	0	–27.663	–23.526	–17.908	–14.32	–12.982
		0.2	4.633	3.834	2.614	1.780	1.469
		0.4	6.094	5.291	4.297	3.717	3.502
		0.6	11.117	10.258	8.981	7.988	7.555
		0.8	24.805	21.627	16.864	13.553	12.272
		1.0	–35.003	–28.747	–20.334	–15.043	–13.098
	0.2	0	–30.752	–26.658	–20.809	–16.886	–15.385
		0.2	5.084	4.539	3.549	2.770	2.456
		0.4	6.080	5.427	4.609	4.113	3.922
		0.6	11.152	10.294	8.973	7.932	7.479
		0.8	23.469	20.289	15.611	12.419	11.196
		1.0	–31.001	–25.566	–18.208	–13.550	–11.834
	0.3	0	–32.938	–28.907	–22.914	–18.755	–17.134
		0.2	5.183	4.848	4.056	3.359	3.057
		0.4	6.050	5.476	4.748	4.296	4.118
		0.6	11.182	10.306	8.944	7.868	7.403
		0.8	22.808	19.617	14.972	11.834	10.639
		1.0	–28.953	–23.957	–17.158	–12.833	–11.235
$\frac{\pi}{2}$	0	0	–18.003	–17.958	–17.908	–17.907	–17.926
		0.2	–18.003	2.601	2.615	2.591	2.564
		0.4	4.290	4.296	4.298	4.287	4.276
		0.6	8.953	8.970	8.981	8.957	8.933
		0.8	16.840	16.861	16.864	16.816	16.772
		1.0	–20.321	–20.342	–20.334	–20.266	–20.208
	0.2	0	–21.103	–20.948	–20.809	–20.886	–21.009
		0.2	3.564	3.558	3.549	3.546	3.547
		0.4	4.635	4.622	4.609	4.614	4.622

TABLE 2 (continued)

θ	α	s / L	σ_s^+				
			β = –0.3	β = –0.2	β = 0	β = 0.2	β = 0.3
$\frac{\pi}{2}$	0.2	0.6	8.945	8.964	8.973	8.950	8.925
		0.8	15.567	15.599	15.611	15.556	15.503
		1.0	–18.279	–18.251	–18.208	–18.181	–18.174
	0.3	0	–23.336	–23.112	–22.914	–23.043	–23.229
		0.2	4.119	4.092	4.056	4.079	4.099
		0.4	4.791	4.770	4.749	4.761	4.777
		0.6	8.913	8.933	8.944	8.919	8.892
		0.8	14.913	14.952	14.972	14.911	14.851
		1.0	–17.278	–17.223	–17.158	–17.157	–17.180
π	0	0	–12.984	–14.322	–17.908	–23.524	–27.659
		0.2	–12.984	1.780	2.615	3.835	4.633
		0.4	3.502	3.716	4.298	5.291	6.095
		0.6	7.556	7.988	8.981	10.258	11.116
		0.8	12.273	13.554	16.864	21.626	24.805
		1.0	–13.099	–15.043	–20.334	–28.746	–35.001
	0.2	0	–15.388	–16.889	–20.808	–26.654	–30.743
		0.2	2.457	2.771	3.549	4.539	5.084
		0.4	3.923	4.113	4.609	5.426	6.079
		0.6	7.479	7.932	8.973	10.293	11.151
		0.8	11.196	12.419	15.611	20.289	23.469
		1.0	–11.835	–13.551	–18.208	–25.564	–30.999
	0.3	0	–17.138	–18.758	–22.914	–28.901	–32.927
		0.2	3.057	3.360	4.056	4.847	5.182
		0.4	4.119	4.297	4.749	5.475	6.049
		0.6	7.403	7.868	8.944	10.307	11.182
		0.8	10.638	11.834	14.972	19.617	22.808
		1.0	–11.236	–12.834	–17.158	–23.955	–28.949

TABLE 3

θ	α	s/L	σ_s^-				
			$\beta = -0.3$	$\beta = -0.2$	$\beta = 0$	$\beta = 0.2$	$\beta = 0.3$
0	0	0	27.096	22.821	17.083	13.469	12.133
		0.2	0.753	0.698	0.801	0.948	1.010
		0.4	7.353	6.384	4.932	3.909	3.512
		0.6	9.702	7.947	5.561	4.109	3.600
		0.8	2.792	2.451	2.292	2.332	2.361
		1.0	64.565	54.523	40.837	32.052	28.769
	0.2	0	31.542	26.832	20.363	16.183	14.617
		0.2	0.902	0.652	0.545	0.611	0.656
		0.4	7.668	6.631	5.056	3.945	3.515
		0.6	9.426	7.689	5.371	3.986	3.505
		0.8	3.197	2.851	2.653	2.643	2.647
		1.0	59.388	50.122	37.518	29.441	26.428
	0.3	0	34.686	29.702	22.742	18.173	16.442
		0.2	1.126	0.732	0.458	0.447	0.473
		0.4	7.886	6.810	5.167	4.006	3.557
		0.6	9.326	7.589	5.300	3.946	3.478
		0.8	3.457	3.086	2.846	2.800	2.788
		1.0	56.828	47.947	35.877	28.153	25.274
$\frac{\pi}{2}$	0	0	17.131	17.113	17.083	17.061	17.054
		0.2	0.814	0.806	0.801	0.809	0.817
		0.4	4.968	4.950	4.932	4.936	4.946
		0.6	5.633	5.597	5.561	5.575	5.601
		0.8	2.360	2.322	2.292	2.322	2.359
		1.0	40.909	40.890	40.837	40.765	40.722
	0.2	0	20.581	20.469	20.363	20.406	20.486
		0.2	0.545	0.544	0.5447	0.545	0.546
		0.4	5.079	5.068	5.0563	5.054	5.058

TABLE 3 (continued)

θ	α	s / L	σ_s^-				
			β = –0.3	β = –0.2	β = 0	β = 0.2	β = 0.3
$\frac{\pi}{2}$	0.2	0.6	5.431	5.401	5.371	5.381	5.401
		0.8	2.704	2.676	2.653	2.676	2.704
		1.0	37.615	37.58	37.518	37.465	37.443
	0.3	0	23.095	22.909	22.742	22.839	22.987
		0.2	0.445	0.451	0.458	0.451	0.445
		0.4	5.184	5.176	5.167	5.161	5.161
		0.6	5.354	5.327	5.300	5.307	5.325
		0.8	2.888	2.864	2.845	2.864	2.888
		1.0	35.989	35.946	35.877	35.836	35.825
π	0	0	12.134	13.470	17.083	22.820	27.093
		0.2	1.010	0.948	0.801	0.699	0.754
		0.4	3.512	3.910	4.932	6.383	7.353
		0.6	3.601	4.109	5.560	7.946	9.701
		0.8	2.361	2.332	2.293	2.450	2.792
		1.0	28.771	32.053	40.837	54.521	64.561
	0.2	0	14.618	16.185	20.363	26.831	31.539
		0.2	0.656	0.610	0.545	0.653	0.903
		0.4	3.515	3.945	5.056	6.630	7.668
		0.6	3.505	3.986	5.371	7.687	9.425
		0.8	2.647	2.643	2.653	2.851	3.198
		1.0	26.430	29.443	37.518	50.120	59.384
	0.3	0	16.444	18.174	22.742	29.699	34.681
		0.2	0.472	0.447	0.458	0.733	1.128
		0.4	3.557	4.006	5.167	6.810	7.888
		0.6	3.477	3.945	5.301	7.590	9.327
		0.8	2.788	2.799	2.845	3.086	3.457
		1.0	25.275	28.154	35.877	47.945	56.823

$$\psi_s(s,\theta)=\sum_{i=0}^{N}\psi_{si}(\theta)\varphi_{4i}(s), \qquad \psi_\theta(s,t)=\sum_{i=o}^{N}\psi_{\theta i}(\theta)\varphi_{5i}(s), \tag{10}$$

where $u_i(\theta), v_i(\theta), w_i(\theta), \psi_{si}(\theta), \psi_{\theta i}(\theta)$ are the unknown functions of θ; $\psi_{ji}(s)$, $j=\overline{1,5}$ are linear combinations of cubic B-splines on a uniform mesh Δ: $0=s_0<s_1<\ldots<s_N=L$ that satisfy the boundary conditions.

Since the unknown functions are equal to zero at the ends, we have

$$\varphi_{j0}(s)=-4B_3^{-1}(s)+B_3^0(s),$$

$$\varphi_{j1}(s)=B_3^{-1}(s)-\frac{1}{2}B_3^0(s)+B_3^1(s),$$

$$\varphi_{ji}(s)=B_3^i(s) \quad (i=2,3,\ldots,N-2),$$

$$\varphi_{j,N-1}(s)=B_3^{N+1}(s)-\frac{1}{2}B_3^N(s)+B_3^{N-1}(s),$$

$$\varphi_{jN}(s)=-4B_3^{N+1}(s)+B_3^N(s). \tag{11}$$

Substituting expressions (10) with (11) into (6), using the spline-collocation method, and requiring them to be satisfied on $N+1$ lines $s=\xi_i$ $(i=\overline{1,N+1})$, we arrive at a system of ordinary differential equations of order $10(N+1)$ that has the following Cauchy form:

$$\frac{d\overline{R}}{d\theta}=A(\theta)\overline{R}+\bar{f}(\theta), \tag{12}$$

$\overline{R}=\{u_0,u_1,\ldots,u_N,v_0,v_1,\ldots,v_N,w_0,w_1,\ldots,w_N,\psi_{s0},\psi_{s1},\ldots,\psi_{sN},\psi_{\theta 0},\psi_{\theta 1},\ldots,\psi_{\theta N}\}^{\mathrm{T}}$ is a vector function of θ; $\bar{f}(\theta)$ is the vector of right-hand sides; A is a square matrix whose elements depend on θ.

Let us consider circumferentially closed shells using the symmetry conditions for $\theta=0$ and $\theta=\pi/2$. Then the boundary conditions can be represented as

$$A_1\overline{R}=\bar{a}_1 \qquad (\theta=0),$$

$$A_2\overline{R}=\bar{a}_2 \qquad (\theta=\pi/2), \tag{13}$$

where A_1 and A_2 are rectangular matrices; $\bar{a}_1$ and $\bar{a}_2$ are the associated vectors.

To solve the boundary-value problem for the system of equations (12) with the boundary conditions (13), we use the stable numerical discrete-orthogonalization method [1, 6]. Substituting $u_i(\theta), v_i(\theta), w_i(\theta), \psi_{si}(\theta), \psi_{\theta i}(\theta)$ into (10), we find the displacements and complete angles of rotation of the normal in the original problem and use them to determine the stress–strain state of the shell.

4. Analysis of the Numerical Results. Consider a closed nonthin conical shell with thickness varying in two coordinate directions, its volume being constant.

The shell is under a surface load q_γ = const and is made of a transversally isotropic material with the following parameters $G_{s\gamma}=G_{\theta\gamma}=G'=E/40$, where $E_s=E_\theta=E$ is Young's modulus in the isotropy plane, and Poisson's ratio $\nu_s=\nu_\theta=\nu=0.3$. The shell ends $s=0$ and $s=L$ are clamped. The input data: $L=30$, $r_0=12.5$; half-cone angle $\psi=\pi/6$.

If $H=1$, then for $\alpha=0, 0.2, 0.3$, we have $h_o=1, 1.07, 1.11$, respectively.

Table 1 collects the values of deflection for $\theta=0, \pi/2, \pi$, different values of α and β, and different values of s. As is seen, the deflection increases with the parameter α for all values of thickness and angle θ.

As the parameter β varies within $-0.3\le\beta\le0.3$, the deflection almost doubles for $\theta=0$ and decreases for $\theta=\pi$. When $\theta=\pi/2$, the influence of β on the deflection is weak (the thickness of the shell remains constant with variation in this parameter).

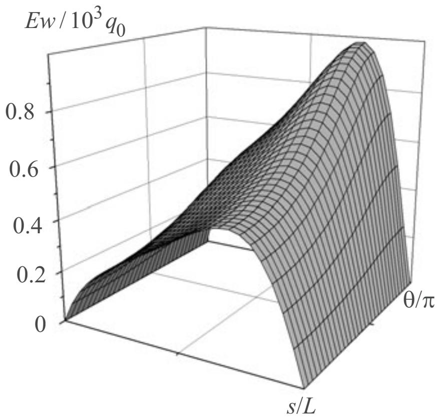

Fig. 1

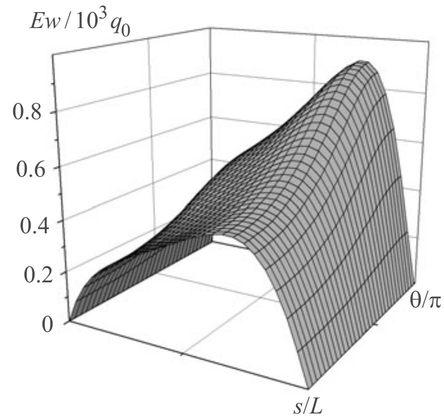

Fig. 2

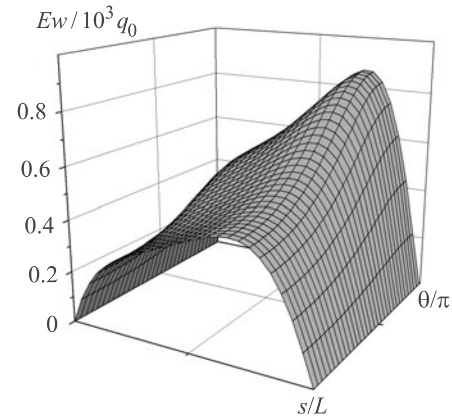

Fig. 3

Table 2 summarizes the values of stress σ_s^+ on the outside surface of the shell for $\theta = 0, \pi/2, \pi$, different values of α and β, and different values of s. It can be seen that the stress σ_s^+ is negative near the ends and positive in the middle of the shell. The stress σ_s^+ first increases to $s/L = 0.8$ and then abruptly decreases. The parameter β has a stronger effect on the stress than the parameter α at the ends. As the parameter β varies within $-0.3 \le \beta \le 0.3$, the stress on the outside surface decreases by a factor of 3.

Table 3 shows how the parameters α and β influence the distribution of the stress σ_s^- over the outside surface for $\theta = 0$, $\pi/2$, π and different values of s.

The stress σ_s^- on the inside surface is tensile (unlike the stress on the outside surface) for all values of s. The stresses at the ends are maximum.

The parameter β has a stronger effect on the stress on the inside surface than α does. The stress σ_s^- at $s/L = 0$ is lower than at $s/L = 1$ by a factor of 2.4 for $\alpha = 0$, a factor of 1.85 for $\alpha = 0.2$, and a factor of 1.6 for $\alpha = 0.3$.

Figures 1, 2, 3 show three-dimensional distributions of the deflection for $\alpha = 0$, 0.2, 0.3, respectively, and $\beta = 0.3$. They qualitatively demonstrate the mutual influence of the parameters α and β. For example, as the thickness parameter increases from 0 to 0.2 in the s-direction, the maximum deflection decreases in the θ-direction by 10%, and as the thickness parameter increases from 0 to 0.3, the maximum deflection decreases by 15%.

Thus, varying the values of the parameters α and β in (3) and keeping the volume of the shell constant, we can find the rational distributions of deflections and stresses.

Conclusions. Using an approach developed, we have analyzed the stress–strain state of nonthin conical shells with thickness varying in two coordinate directions. The distributions of displacements and stresses presented as tables and plots have been analyzed.

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